Module 7
Version 03
Sections 1–3

Math 7

Module 7

Statistics and Probability

BRITISH COLUMBIA

OPEN SCHOOL BC

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Module 7 Table of Contents

Course Overview

Welcome to Mathematics 7!

In this course you will continue your exploration of mathematics. You'll have a chance to practise and review the math skills you already have as you learn new concepts and skills. This course will focus on math in the world around you and help you to increase your ability to think mathematically.

Organization of the Course

The Mathematics 7 course is made up of seven modules. These modules are:

Module 1: Numbers and Operations

Module 2: Fractions, Decimals, and Percents

Module 3: Lines and Shapes

Module 4: Cartesian Plane

Module 5: Patterns

Module 6: Equations

Module 7: Statistics and Probability

Organization of the Modules

Each module has either two or three sections. The sections have the following features:

Pretest	This is for students who feel they already know the concepts in the section. It is divided by lesson, so you can get an idea of where you need to focus your attention within the section.
Section Challenge	This is a real-world application of the concepts and skills to be learned in the section. You may want to try the problem at the beginning of the section if you're feeling confident. If you're not sure how to solve the problem right away, don't worry—you'll learn all the skills you need as you complete the lessons. We'll return to the problem at the end of the section.

Each section is divided into lessons. Each lesson is made up of the following parts:

Student Inquiry	Inquiry questions are based on the concepts in each lesson. This activity will help you organize information and reflect on your learning.
Warm-up	This is a brief drill or review to get ready for the lesson.
Explore	This is the main teaching part of the lesson. Here you will explore new concepts and learn new skills.
Practice	These are activities for you to complete to solidify your new skills. Mark these activities using the answer key at the end of the module.

At the end of each module you will find:

Resources	Templates to pull out, cut, colour, or fold in order to complete specific activities. You will be directed to these as needed.
Glossary	This is a list of key terms and their definitions for the module.
Answer Key	This contains all of the solutions to the Pretests, Warm-ups and Practice activities.

Thinking Space

The column on the right hand side of the lesson pages is called the Thinking Space. Use this space to interact with the text using the strategies that are outlined in Module 1. Special icons in the Thinking Space will cue you to use specific strategies (see the table below). Remember, you don't have to wait for the cues—you can use this space whenever you want!

Just Think It: Questions	Write down questions you have or things you want to come back to.
Just Think It: Comments	Write down general comments about patterns or things you notice.
Just Think It: Responses	Record your thoughts and ideas or respond to a question in the text.
Sketch It Out	Draw a picture to help you understand the concept or problem.
Word Attack	Identify important words or words that you don't understand.
Making Connections	Connect what you are learning to things you already know.

More About the Pretest

There is a pretest at the beginning of each section. This pretest has questions for each lesson in the sections. Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in the section. Mark your answers using the key found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Materials and Resources

There is no textbook required for this course. All of the necessary materials and exercises are found in the modules.

In some cases you will be referred to templates to pull out, cut, colour, or fold. These templates will always be found near the end of the module, just in front of the answer key.

You will need a calculator for some of the activities and a geometry set for Module 3 and Module 7.

If you have Internet access, you might want to do some exploring online. The Math 7 Course Website will be a good starting point. Go to:

http://www.openschool.bc.ca/courses/math/math7/mod4.html

and find the lesson that you're working on. You'll find relevant links to websites with games, activities, and extra practice. Note: access to the course website is not required to complete the course.

Icons

In addition to the thinking space icons, you will see a few icons used on the left-hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.

Explore Online

Warm-up

Explore

Practice

Answer Key

Use a Calculator

Module 7 Overview

Everyday in the newspaper and on television, we are bombarded with statistics.

> "Today's weather forecast states an 85% chance of rain. Don't forget your umbrella today!"
>
> "**Forever White**'s lab-developed ingredients are 52% more powerful and 35% more effective than other teeth-whitening products."
>
> "With **Super-Gro** plant fertilizer, the average houseplant grows three times faster than normal!"

It's important for us to understand where these numbers come from and to be able to discriminate between reliable and unreliable data. After all, you need to make decisions every day about what products you buy, where you get your news from and whether you should bring your umbrella of put on suncreen.

This module will help you make sense of some of the statistics you see every day. We'll start off by learning all about reading, interpreting and creating circle graphs. In the second section we'll look at some sets of data and explore the ways that we can describe the sets of numbers. We'll finish off the module with an exploration of probability.

Section Overviews

Section 7.1: Circle Graphs

Data—we collect it, interpret it, and summarize the results. You've probably seen data presented in several ways. For example, you may have made bar graphs, or t-tables in other math lessons, in other courses, or even just to help you organize information in your day.

In this section you will learn another way to organize data. This section is about circle graphs: how to read, understand, analyze, and create them.

Section 7.2: Measures of Central Tendency

If someone gave you a list of numbers and asked you to describe the set, how would you answer? What kinds of things would you need to look at in the list in order to describe it accurately? Do the numbers fit together in any ways?

These are some questions that statisticians ask when they analyze data. In order to understand numerical data, statisticians look for ways to measure the trends in data sets. In this section we will look at statistics—how to read them, understand them, and choose the best measure to represent them.

Section 7.3: Probability

What are the chances that you'll run into your best friend at the park? What about running into your favourite celebrity? Are you more likely, or less likely to run into your favourite celebrity than your best friend? How do you know?

In this section we'll explore probability. You'll figure out the likelihood of certain events happening and express the probabilities using ratios, fractions and percents. Along the way you'll get to conduct experiments by rolling dice, flipping coins, picking coloured tokens and even digging through your dresser drawer!

Section 7.1: Circle Graphs

Section
1

Contents at a Glance

Learning Outcomes

By the end of this section, you will be better able to:

- identify characteristics of circle graphs.
- read and interpret circle graphs.
- create your own circle graphs.
- solve problems that involve interpreting and creating circle graphs.

Pretest 7.1

Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in this section. Mark your answers using the key found at the end of the module.

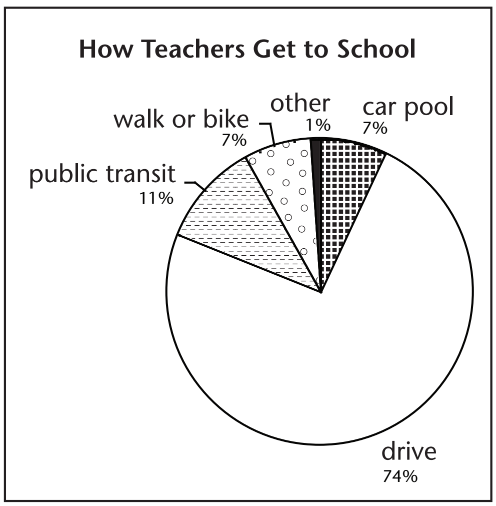
If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson 7.1A

Use the following graph to answer the pre-test questions for Lesson 7.1A and B.

A company surveyed 5000 teachers about how they get to school. The data collected are shown in the following circle graph.



1. a. What percentage of teachers walk or ride a bicycle to work?

   b. What percentage of teachers use a vehicle to get to work?

   c. What is the title of this graph?

Lesson 7.1B

1. a. How many of teachers use public transit to get to school?

   b. How many teachers car pool?

Lesson 7.1C

1. A container of yogurt contains 228 g water, 54 g of carbohydrates, 12 g of protein, and 6 g of fat.

   a. Display the data in a circle graph.

   b. What percentage of the yogurt is fat?

Turn to the Answer Key at the end of the Module and mark your answers.

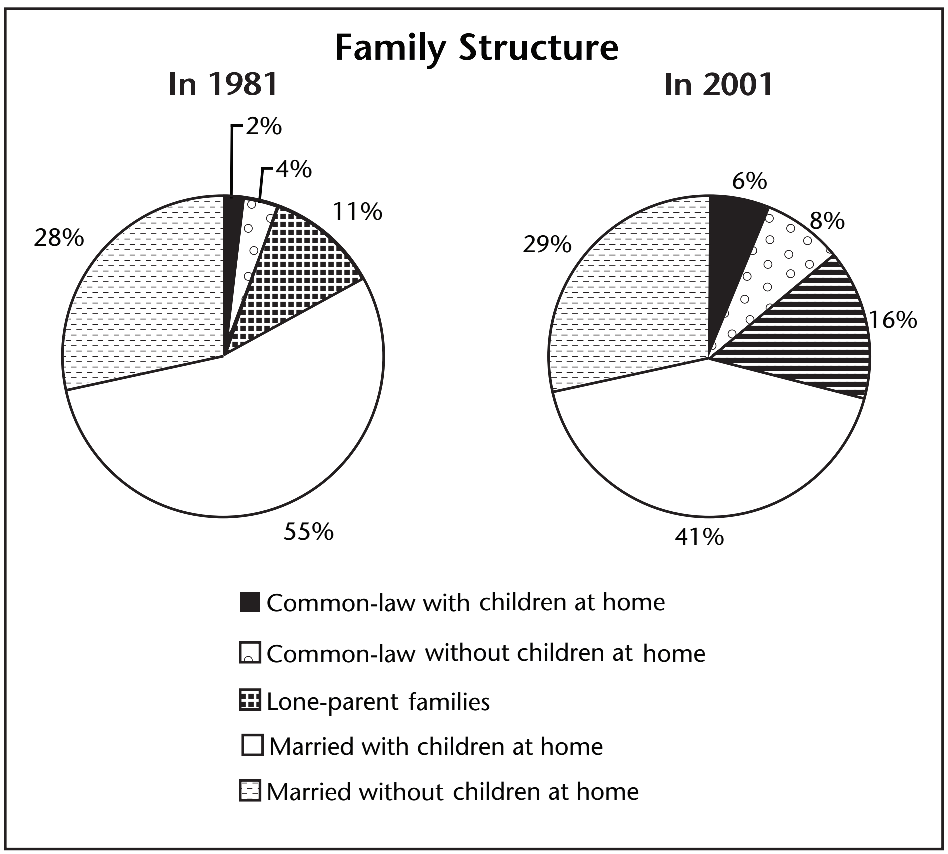
Section Challenge

Changing Families

Being able to read a variety of graph types can help you in many areas. You may have seen circle graphs in some of your other classes in school or in the newspaper or on TV. Reading and creating circle graphs are useful skills to have!

You may have seen statistics from the Canadian Census of Population. The census is a survey that asks people all over Canada a variety of questions about their age, sex, marital status, education, employment, income, language, and many other things. The census is conducted every five years. Not only does the census provide a large collection of facts about people in Canada, but it also provides a way to track trends over time.

Below are two circle graphs that describe some of the different family structures in Canada. The data come from the 1981 census and the 2001 census.



Source: Statistics Canada, 1981 and 2001 Census of Population

1. Use the two graphs above to answer the following questions.

    a. For each year, list the five types of families in order from most common to least common. Are your two lists the same or different?

    b. There were move lone-parent families in 2001 than in 1981. What other differences do you notice between the two graphs?

    c. Another census was done in 2006. Based on the differences and similarities you noted between the 1981 and 2001 data, what predictions can you make about the 2006 data on family structure in Canada?

2. Below are data from the 2006 census that describe some of the different family structures in Canada. Complete the table and then create a circle graph based on the data in the table.

Family Structure	Number of Families in 2006	Percentage of Families in 2006
Common-law with children at home	618 150	
Common-law without children at home	758 715	
Lone-parent families	1 414 060	
Married with children at home	3 443 775	
Married without children at home	2 662 135	
Total	**8 896 840**	

3. Looking at the circle graph you created, how accurate were your predictions (in question 1c)? Explain your answer.

If you're not sure how to solve the questions now, don't worry. You'll learn all the skills you need to solve the problem as your work through this section. Give them a try now or wait until the end of the section—it's up to you!

Lesson 7.1A: Circle Graphs

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
What are some characteristics of a circle graph?		answer
		example
Why would someone choose to use a circle graph to display data?		answer
		example

Lesson 7.1A: Circle Graphs

Thinking Space

Introduction

By now your brain is filled with a wealth of circle knowledge. You may remember exploring circles in Module 3. In this module you will get an opportunity to apply this information to a very real life concept.

Circle graphs or pie charts are a useful way to represent data. But as you will see through the lesson, there are certain times when circle graphs are more effective in communicating data than others.

Let’s start by reviewing some of the concepts you have already covered.

Explore Online

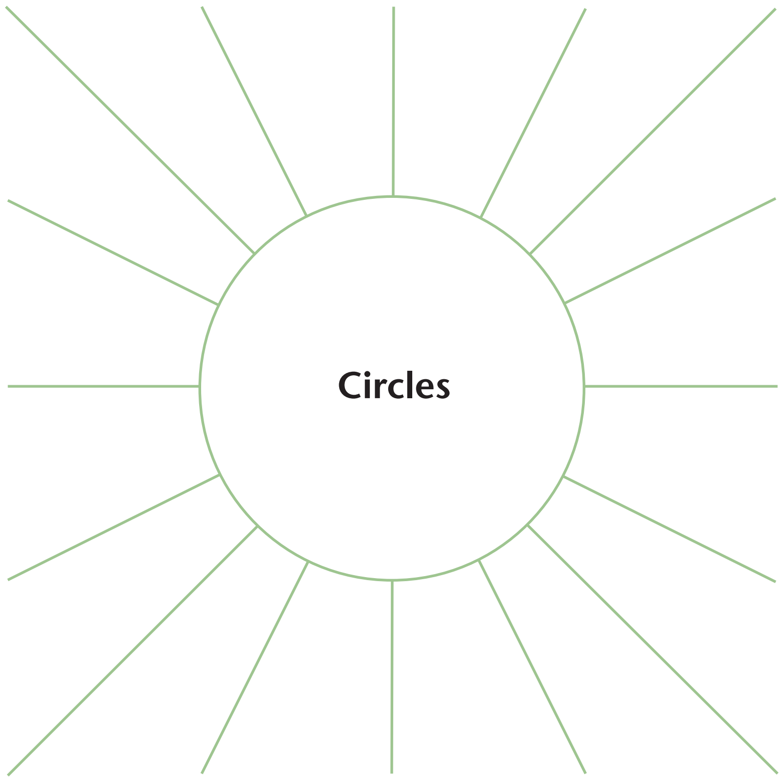
Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:

http://www.openschool.bc.ca/courses/math/math7/mod5.html

Look for *Lesson 7.1A: Circle Graphs* and check out some of the links!

Warm-up

What do you already know about circles? Warm up with the knowledge you gathered as you worked through previous modules, grades, or experiences you have had in the world.



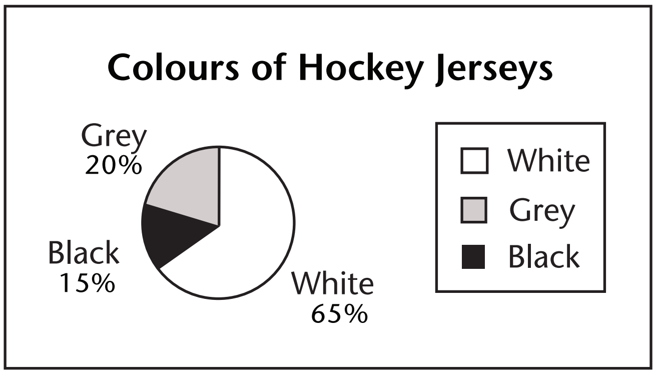
If you need some help, or want to add more to really get your brain ready, you could go back to some earlier lessons and review.

Explore
Why Do We Use Circle Graphs?

Thinking Space

1. Circle Graphs Are Easy to Read and Understand

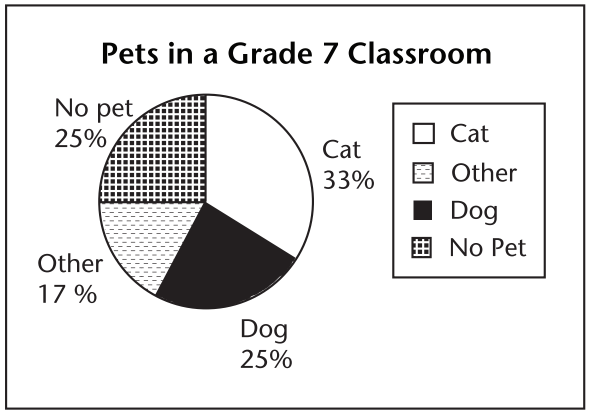
Circle graphs are a useful and a convenient form of displaying data. Depending on the type of data you are communicating, a circle graph could be just the method you need. Look at the following circle graph. Why would someone choose a circle graph over other types, such as bar or line graphs?



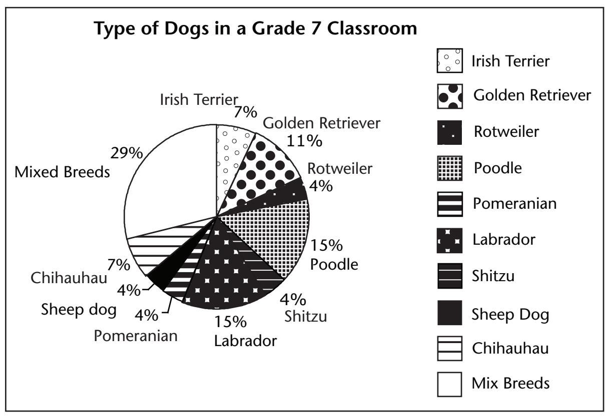
You probably noticed the difference in sizes of the sectors very quickly. **Sectors** are the pieces of the circle graph that look like slices of pie. When you can compare the sectors easily with each other, you can quickly determine features of the data that the graph represents. For instance, you probably noticed that a larger sector means more players wear that colour jersey. The smallest sector represents the least popular colour.

Let's look at two more circle graphs.

The graph to the right shows the types of pets that grade 7 students have. You can quickly see which pet is the most common, and which pet is the least common.



This graph shows the types of dogs that Grade 7 students have. It is more difficult to read than the graph of pet types (above) because there are so many categories.



Circle graphs are easier to understand when there aren't too many sectors.

Thinking Space

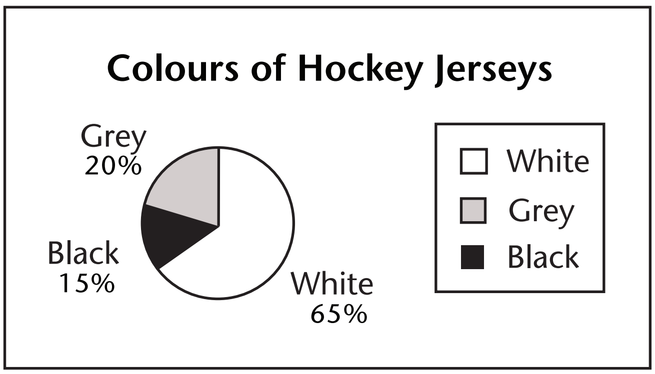
Thinking Space

A **pie chart**, another word for a circle graph, is easier to read than a list of data.

Look at the example below of a data list compared with the circle graph of the hockey jersey colour.

Can you see the difference in how it's displayed? Which one helps you understand the information better?

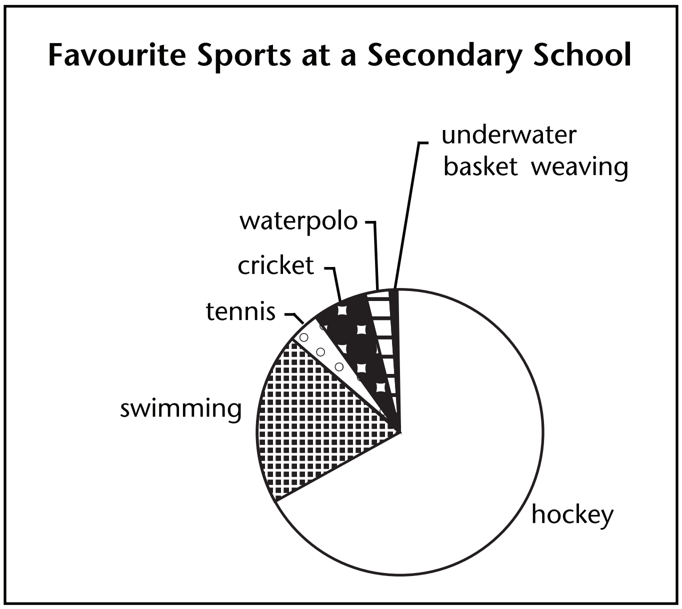
What should the new hockey jersey colour be?
White (W) Grey (G) Black (B)
W B G G W B W W W G B W W W W G B B B W G W W W W G G G G G G W W W G G G B W W W G B B B B B B W G R

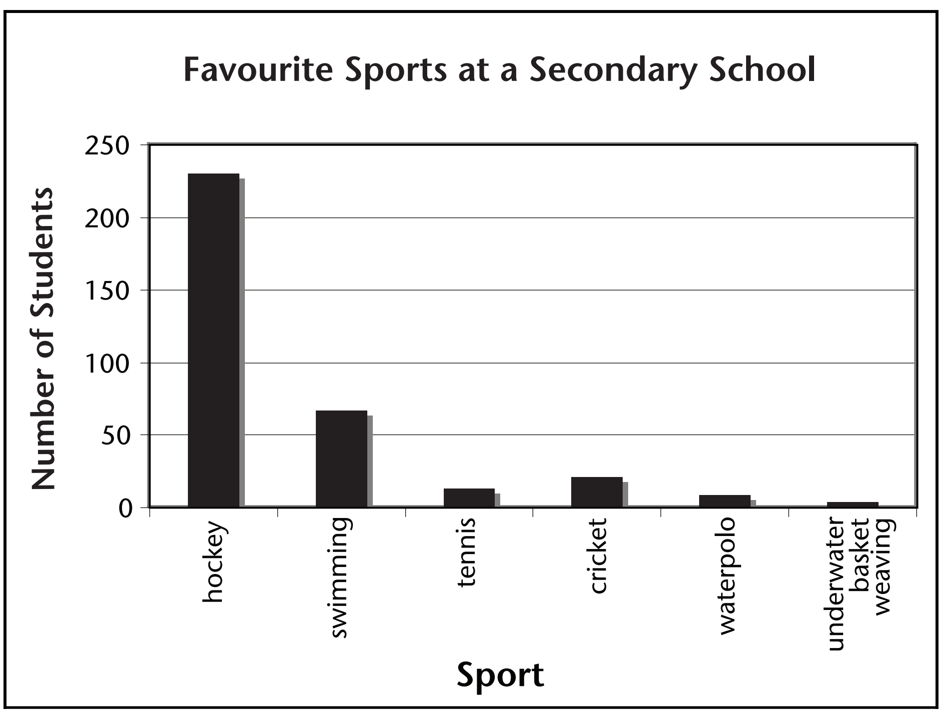


2. Circle Graphs are Useful When There Are Large Differences in Data Points

Thinking Space

The second reason why people would choose a circle graph over other kinds of graphs is to display data that has big differences. When sectors are close to the same size, it is difficult to see the differences. But circle graphs are quite effective when the sectors are very different in size—we are able to see the differences very easily. Compare the pie chart and the bar graph below. Can you see why the pie chart would be useful in this situation?





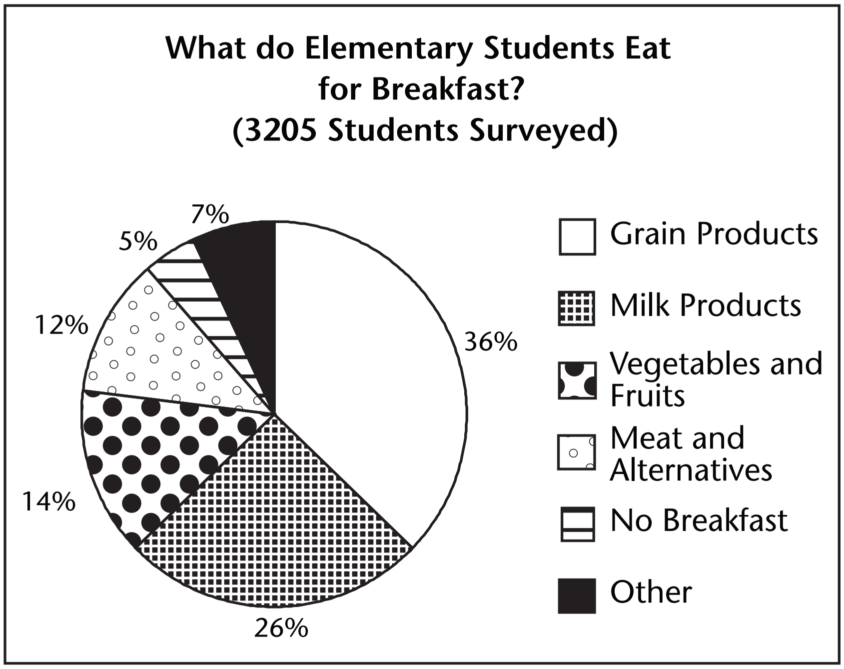
Thinking Space

Before we go any further, let's review a few important details. Reminding ourselves about concepts we have already learned will help us continue working with circle graphs.

What Characteristics Do Circle Graphs Have?

Circle graphs have sectors.

Look at this circle graph. Can you see which sector is the biggest? How do you know? What are some clues that tell you?



Circle graphs express data by the size of the sectors, but also by showing percentages of each sector.

You may have noticed that as the sector increases in size, the percentage and the central angle increases. This is an important feature to remember when working with circle graphs.

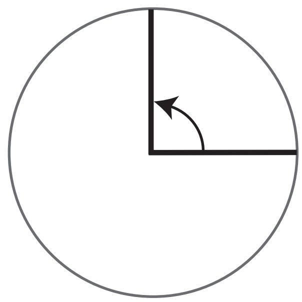
Circle graphs have percentages.

Percentages compare a portion to a whole, or 100%. Add up all the percentages in the graphs above. What do you notice?

Did you know that in all circle graphs the percentages add up to 100%? You have learned a lot about percentages and fractions in other lessons. Can you see how what you know about percentages and fractions can help you out now?

Circle graphs have central angles.

Remember when you worked with central angles of circles in previous modules? A central angle is an angle formed by two radii of a circle.



We can also connect this concept to the skate park.

Have you ever seen a skateboarder perform a 360 pop-shuvit? In this trick, the skateboarder spins the board around in a full circle under his or her feet. The "360" in the name of this trick comes from the fact that the board spins 360°—a full circle.

Thinking Space

Do you know any other skateboarding tricks with angles in their names? How about tricks in other sports?"

Thinking Space

Think about this when you are working with circles—the central angle of a circle is always 360°. However the sectors are broken up, all the angles will always add up to 360°.

In a circle graph:

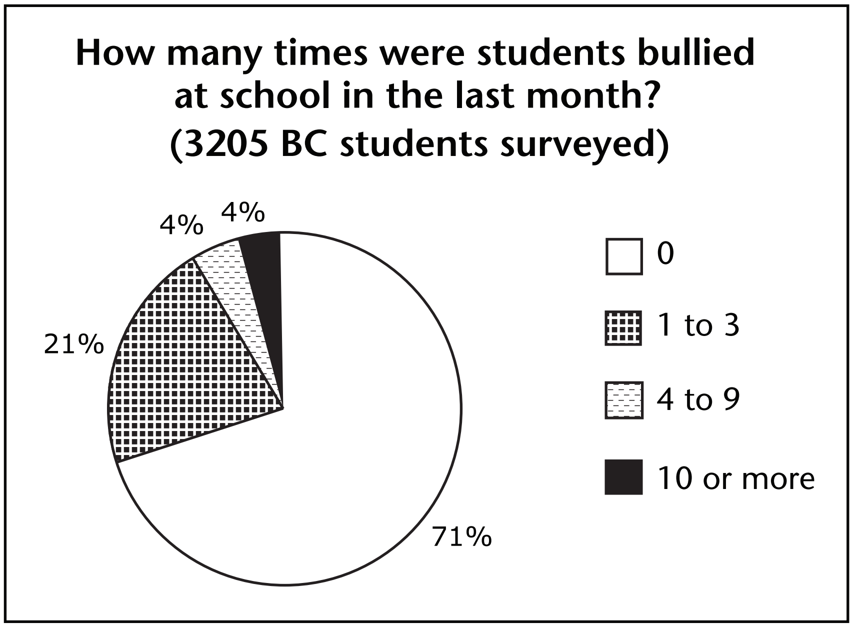
- all of the angles add up to 360°
- all of the percentages add up to 100%

Circle graphs have features similar to other graphs.

Although there are characteristics of circle graphs that are unique, there are also parts of circle graphs that are similar to other graphs: basic features that we can look at to give us information to help us read and understand the data. Circle graphs have:

- a title that tells us what the graph is about
- a legend that helps us connect the labels and sectors

Practice 1



Source: Statistics Canada, Census at School, 2006/2007

1. What is the title of this graph?

2. What are the different categories or sectors represented on the graph?

3. Which sector is the largest category and what does it represent?

4. Add up all the percentages on the circle graph. What do they add up to?

5. What is 100% of the total that we are measuring?

6. Write the labels for each sector of this graph.

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 7.1B: Understanding Circle Graphs

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
How do I read a circle graph?		answer
		example
How do I change percentages on a circle graph into quantities?		answer
		example

Lesson 7.1B: Understanding Circle Graphs

Thinking Space

Introduction

This lesson will focus on how to read, understand, and interpret circle graphs. By the end of this lesson, you will be able to answer questions based on the information presented in this kind of graph.

As you come across circle graphs in newspapers, websites, television shows and other media, this lesson will help you to the understand the information being presented to you in a more meaningful way.

Explore Online

Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:

http://www.openschool.bc.ca/courses/math/math7/mod5.html

Look for *Lesson 7.1B: Understanding Circle Graphs* and check out some of the links!

Warm-up

1. Convert each decimal to a fraction

   a. $0.2 = \frac{\quad}{10}$ b. $0.34 = \frac{34}{\quad}$

   c. $0.09 = \frac{\quad}{\quad}$

2. Write each fraction as a percent.

   a. $\frac{3}{4}$ b. $\frac{9}{12}$

   c. $\frac{5}{8}$ d. $\frac{12}{22}$

   e. $\frac{34}{51}$

3. Write each percent as a decimal.

   a. 21% b. 56%

   c. 4% d. 71%

   e. 1%

4. Complete each statement with: <, >, or =

   a. 0.4 ___ 40% b. 20% ___ $\frac{1}{4}$

   c. $\frac{2}{10}$ ___ 5.3 d. 41% ___ 4.1

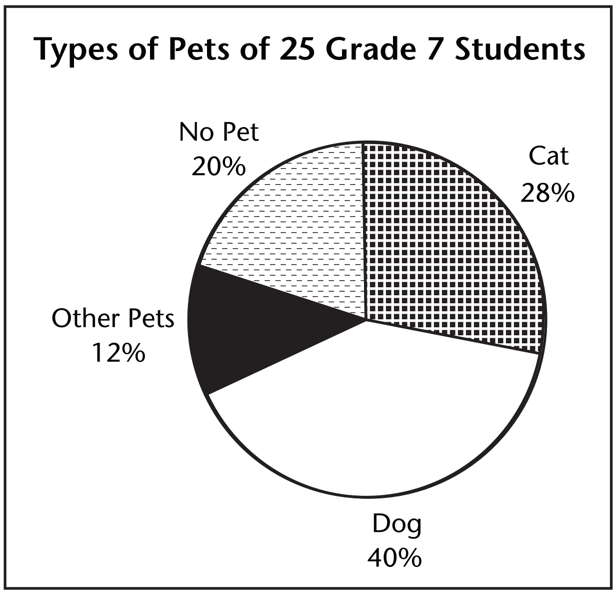
Turn to the Answer Key at the end of the Module and mark your answers.

Explore
Reading Circle Graphs

Thinking Space

Now that you know all the features of circle graphs, put them together to figure out how to read a circle graph. As we discussed before, circle graphs have sectors, and each sector is labeled with a percentage.

Here is an example. What conclusions can we come to about this circle graph by just looking at it?



The graph above shows the percentages of students in a classroom that have pets.

Hmmn. I know percentages are parts of a total. Since the circle is made up of percentages, the circle must represent a total of something.

Let's look at some steps that will help us read circle graphs.

Step 1: What is the total?

In the example above, the total of students on the graph is 25.

Thinking Space

Step 2: Restate the information about the graph using the total and the title.

Example:

The title tells us the graph is about pets and Grade 7 students. We can use this information, as well as the total, to make this statement:

"This graph is displaying the pets of students in a total class of 25."

Step 3: Writing data sentences.

The last step is to write data sentences—short statements that describe the data represented by each sector of the graph.

Here are some data sentences using the example above:

Students with cats as pets are 28% of 25 students.

Students with dogs are 40% of 25 students.

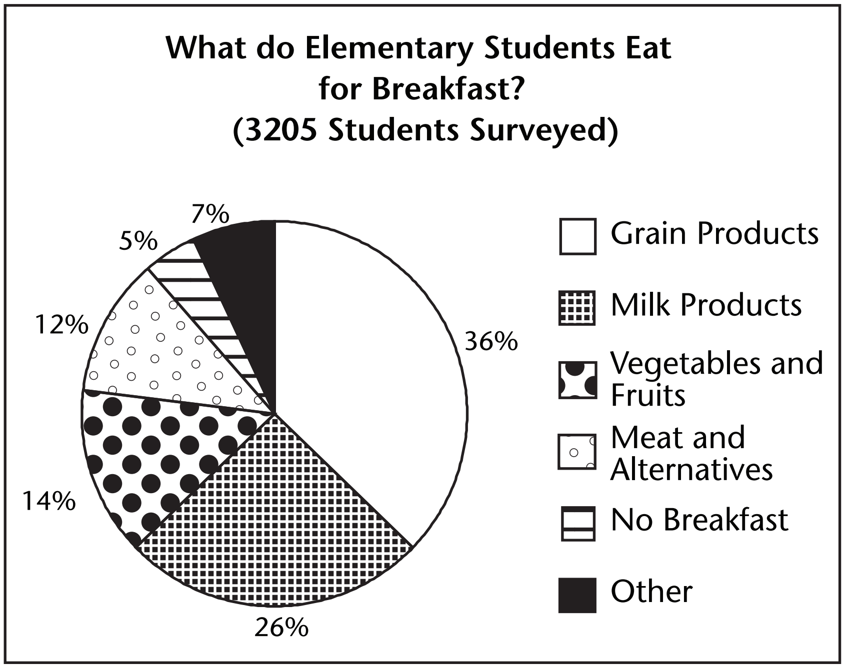
Students with no pets are 20% of 25 students.

Students with other kinds of pets are 12% of 25 students.

Being able to write these data sentences will help you later when we interpret circle graphs. But before we go further, let's practise reading circle graphs.

Practice 1

1. Look at the circle graph below and then follow the steps to help you read and understand the data presented.



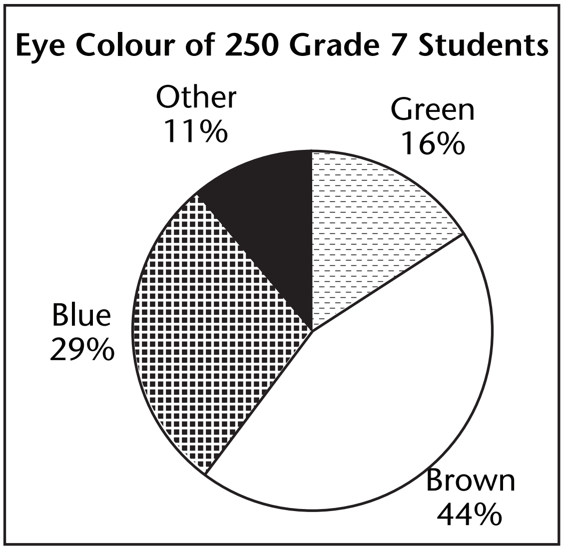
Source: Statistics Canada, Census at School, 2006/2007.

Step 1: What is the total?

Step 2: Restate the information using information from the title.

Step 3: Write the data sentences for each sector.

2. Look at the circle graph below and then follow the steps to help you read and understand the data presented.



Step 1: What is the total?

Step 2: Restate the information using information from the title.

Step 3: Write the data sentences for each sector.

Turn to the Answer Key at the end of the Module and mark your answers.

Explore
Interpreting Circle Graphs

Thinking Space

Even though we can now state graph information in our own words and create data sentences, there's still some important information that we can't see on the graph. Look at this data sentence made from a graph earlier in the lesson.

28% of 25 grade 7 students have cats as pets.

Although this sentence tells us the percentage of Grade 7 students in this class that have cats as pets, and the number of students in the class, it doesn't tell us how many students actually have cats as pets. To find this out, we need to turn the data sentence into an equation or number phrase like this:

28% of 25

As you can see we just took out most of the words.

This might look more familiar to you from other lessons. This is a percentage question now:

28% of 25 is?

(If you want to review solving problems about percent, review Section 3 of Module 2.)

$$0.28 \times 25 = 7$$

Now we know the answer to our original question, How many students in this class have a cat for a pet? Seven students in this class have a cat as a pet.

Solve the rest of the data sentences for this circle graph by following the steps above. The first one has been done for you.

Data Sentence	Question	Equation	Solution	Statement
28% of 25 Grade 7 students have cats as pets.	28% of 25 is?	$0.28 \times 25 =$	7	7 students in the class have a cat as a pet.
40% of 25 Grade 7 students have dogs as pets.				
20% of 25 Grade 7 students have no pets.				
12% of 25 Grade 7 students have other kinds of pets.				

Thinking Space

!

Compare your answers with the solutions below:

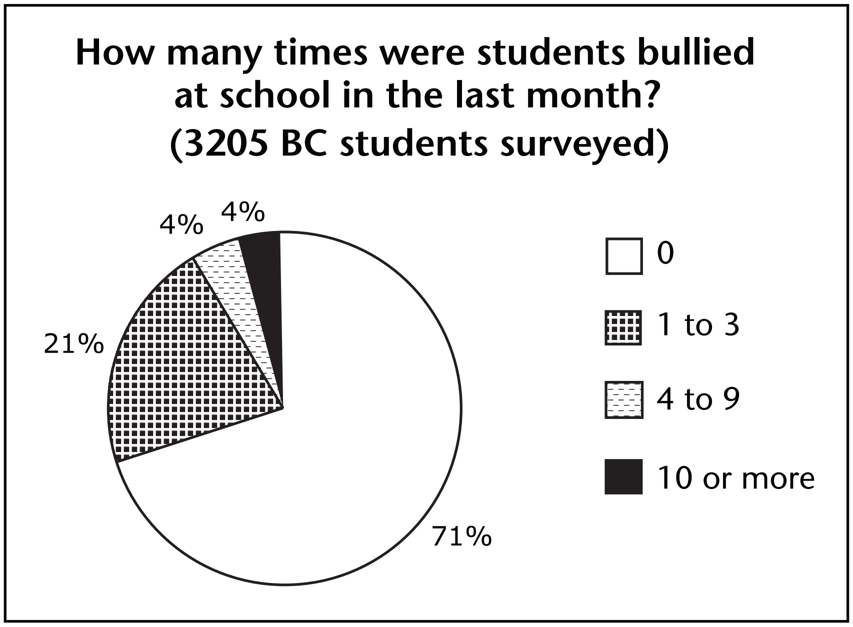
Data Sentence	Question	Equation	Solution	Statement
28% of 25 Grade 7 students have cats as pets.	28% of 25 is?	$0.28 \times 25 =$	7	7 students in the class have a cat as a pet.
40% of 25 Grade 7 students have dogs as pets.	40% of 25 is?	$0.40 \times 25 =$	10	10 out of 25 students in the class have dogs as pets.
20% of 25 Grade 7 students have no pets.	20% of 25 is?	$0.20 \times 25 =$	5	5 out of 25 students in the class have no pets.
12% of 25 Grade 7 students have other kinds of pets.	12% of 25 is?	$0.12 \times 25 =$	3	3 out of 25 students in the class have other kinds of pets.

Let's do one more together before you practise.

Thinking Space

Sometimes statistics tell us how our favourite hockey players are performing, or show us specific preferences among people. Statistics can also help us learn about serious issues such as environmental patterns, crime rates, and economic trends.

The graph below is one that we looked at in Lesson A. Now that we know how to read it, let's interpret it together.



Source: Statistics Canada, Census at School, 2006/2007.

Try answering these questions, using the skills you just learned about interpreting circle graphs.

1. a. What percentage of students have been bullied 1–3 times in the last month?

   b. Create a data sentence for the number of students who have been bullied 1–3 times in the last month.

2. a. What percentage of students have been bullied 4–9 times in the last month?

   b. Create a data sentence for the number of students who have been bullied 4–9 times in the last month.

3. a. What percentage of students have been bullied 10 or more times in the last month?

   b. Create a data sentence for the number of students who have been bullied 10 or more times in the last month.

4. a. What percentage of students have not been bullied in the last month?

   b. Create a data sentence for the number of students who have not been bullied in the last month.

Compare your answers with the solutions below:

Data Sentence	Question	Equation	Solution	Quantity Statement
21% of 150 students have been bullied 1–3 times in the last month	21% of 3205 is? $\frac{21}{100}=0.21$	0.21 x 3205 =	673.05	Approximately 673 out of 3205 students have been bullied 1–3 times at school in the last month.
4% of 150 students have been bullied 4–9 times in the last month	4% of 3205 is? $\frac{4}{100}=0.04$	0.04 x 3205 =	128.2	Approximately 128 out of 3205 students have been bullied 4–9 times at school in the last month.
4% of 150 students have been bullied 10 or more times in the last month	4% of 3205 is ? $\frac{4}{100}=0.04$	0.04 x 3205 =	128.2	Approximately 128 out of 3205 students have been bullied 10 or more times school in the last month.
71% of 150 students have not been bullied in the last month	71% of 3205 is? $\frac{71}{100}=0.71$	0.71 x 3205 =	2275.55	Approximately 2276 out of 3205 students have not been bullied at school in the last month.

Thinking Space

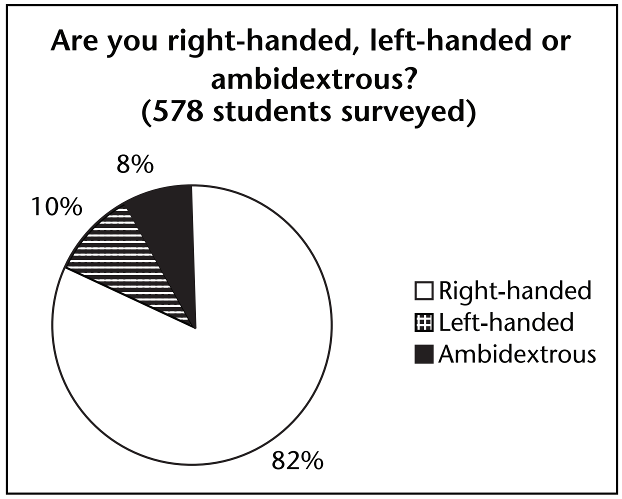
Interpreting the data is more than just reading numbers. Serious issues deserve careful examination. After looking closely at this graph, think about the following questions to help you reflect on the data.

- What do you think the graph tells us about bullying in BC schools?
- Are you surprised by the number of students who are bullied in school?
- Do these statistics reflect your experiences in school?
- Could you see how using this data might help to prevent or reduce bullying in schools?

Well done. When you are confident with your circle-graph-reading skills, move on to the practice section in this lesson.

Practice 2:

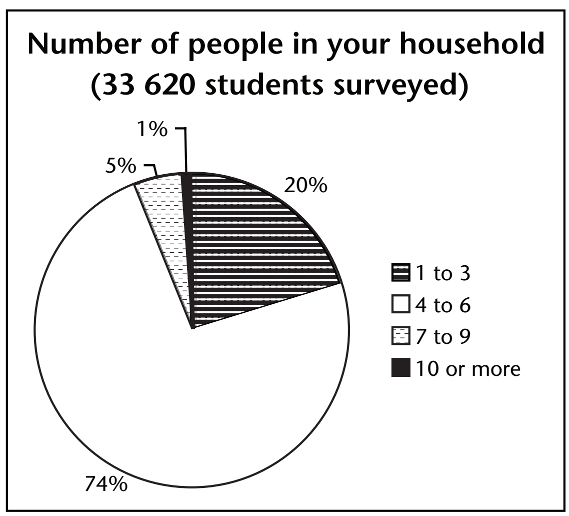
1. Ambidextrous means someone who is equally capable with both their left and right hands. 578 students across Canada were asked about which hand they use. The graph summarizes the results of the survey.



   a. What is the most common handedness in grade seven students?

   b. How many students are ambidextrous?

2. This circle graph shows the family size of students in schools across Canada.



Source: Statistics Canada, Census at School, 2006/2007.

   a. What is the most common family size?

   b. How many students have 1–3 people in their household?

   c. How many students have less than 10 people in their household?

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 7.1C: Making Circle Graphs

Thinking Space

Introduction

You can read them, and you can interpret them—the next step is learning how to make them. After this lesson, you will be able to create and label your own circle graph.

Warm-up

Review proportions, percent, and use of a protractor.

1. Find the following percents:

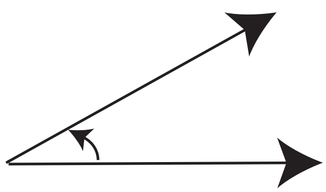
   a. $\frac{45}{100}$ = ______%

   b. $\frac{25}{50}$ = ______%

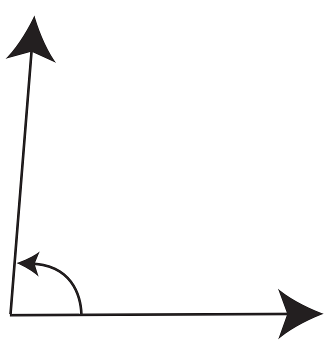
   c. $\frac{30}{75}$ = ______%

2. Use a protractor to measure the following angles:

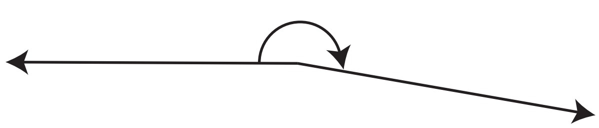
   a.



   b.



   c.



3. Use a protractor to draw the following angles:

   a. 30°

   b. 98°

   c. 205°

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Making Circle Graphs

The following data table lists organizations that 3205 BC youth would choose to donate to if they had $1,000.

If you had $1,000 to give to charity, what type of organization would you choose?						
	Health	International aid	Arts, culture and sports	Wildlife/ animals	Environmental	Other
BC	1006	542	314	651	272	420
Source: Statistics Canada, Census at School, 2006/2007.						

Let's look at the different ways we can create a circle graph using this data.

Making a Circle Graph Using a 100% Circle Template

Step 1: Write each number as a percent of a total.

$$\frac{1006}{3205} = 0.3138\ldots \approx 31\%$$

Organization	Number of People	Percent of total
Health	1006	31%
International Aid	542	
Arts, Culture, Sports	314	
Wild life/ animals	651	
Environmental	272	
Other	420	

Fill in the rest if the chart. Compare your solutions to the ones below. Add up all of your percentages. What do you notice?

Thinking Space

What organization would you choose if you had $1,000 to give?

It looks like we're rounding to the nearest whole number percent.

The percentages should add up to 100%. This is a great way to check your work to see if you have calculated correctly.

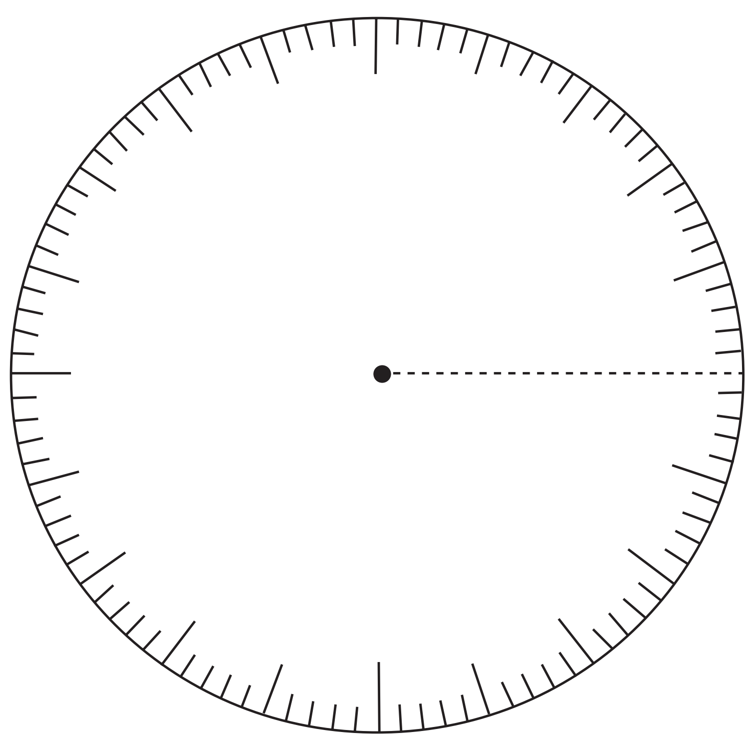
Thinking Space

Organization	Number of People	Percent of total
Health	1006	31%
International Aid	542	**17%**
Arts, Culture, Sports	314	**10 %**
Wild life/ animals	651	**20%**
Environmental	272	**9%**
Other	420	**13%**

Step 2: Draw the radius.

This is a 100% circle template. As you can see, it is divided evenly all the way around to help you create your sectors, and each small mark is 1%. The second step when making a circle graph is to draw the radius to any point on the rim you choose for your starting point.

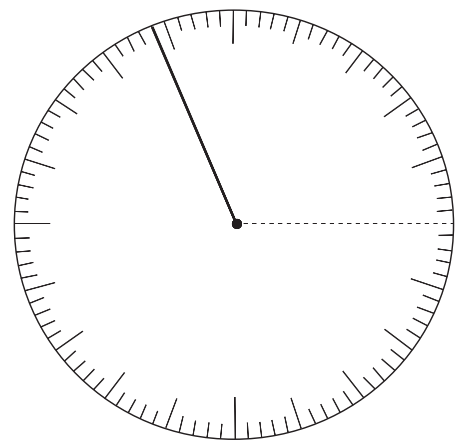
Draw the radius like this.



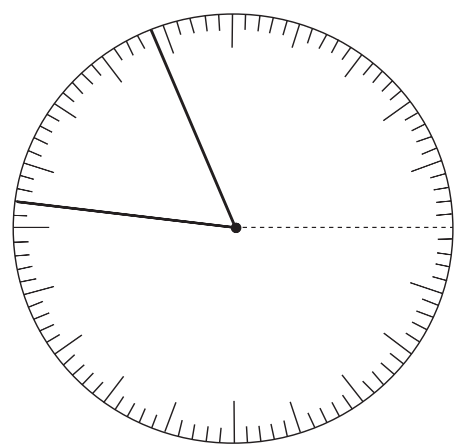
Step 3: Draw the sectors by counting the ticks around. The longer dashes represent 5%. This will help us count out percents easily. Let's put the first piece of data on our circle graph.

Thinking Space

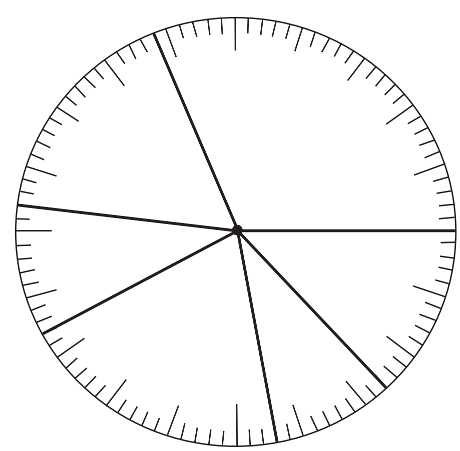
Our first piece of data states that 31% of student voted for health. Starting at 0% count out 31% around the edge of the template. Use the 5% ticks to help you. The sector should now look like this:



To add the next piece of data, International Aid with 17%, we start counting from where the last sector finished. Our graph now looks like this:



We continue in this manner until we have placed all of our data on our graph.

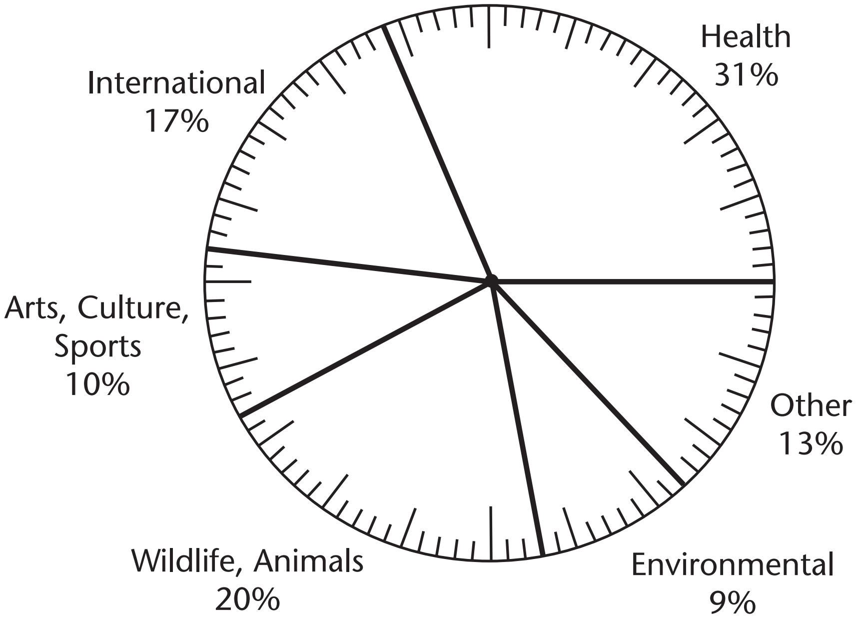


Thinking Space

Step 4: Label the circle graph.

Labelling is important, because it shows the reader what your graph is representing. Name each sector and provide the percentages like this:

Charities Students Would Choose to Donate To if Given $1000. (3205 BC Students Surveyed)



Finally, we write the title on the top.

Legends

Sometimes we include legends to communicate the colour or pattern of the sectors. To do this, all you need to do is create a box with a sample of each colour in it, and what it represents. An example of a legend that could be used for this circle graph is:

Note: If you have a legend, you don't need to label the sectors directly. The coloured labels in the legend tell you what each of the sectors represents. This can be very useful if there are several sectors, since putting labels directly on the graph can make the graph look cluttered.

Good work so far. When you are ready, move on to the practice.

Thinking Space

Practice 1

1. For this activity you'll need

- the 100% circle template (located at the back of this module)
- a pencil
- a ruler

1. Use the following data to create a circle graph on the 100% circle template.

   25 656 Canadian elementary students were asked "How many days last week did you do an intense physical activity?"

Number of Days	Number of Students
0 to 1	5131
2 to 3	8210
4 to 5	6927
6 to 7	5388

Source: Statistics Canada, Census at School, 2006/2007.

Step 1: Write each number as a percent of a total.

Total: ___________

Sector Label	Number of Students	Percent of Total
0 to 1		
2 to 3		
4 to 5		
6 to 7		

Step 2: Draw the radius.

Step 3: Draw the sectors.

Step 4: Label the circle graph and add a title and a legend.

2. Use the following data to create a circle graph on the 100% circle template.

   7964 Canadian secondary students were asked "How many days last week did you do an intense physical activity?"

Number of Days	Number of Students
0 to 1	2389
2 to 3	2549
4 to 5	1752
6 to 7	1274

Source: Statistics Canada, Census at School, 2006/2007.

Step 1: Write each number as a percent of a total.

Total: ___________

Sector Label	Number of Students	Percent of Total
0 to 1		
2 to 3		
4 to 5		
6 to 7		

Step 2: Draw the radius.

Step 3: Draw the sectors.

Step 4: Label the circle graph and add a title and a legend.

3. Use the two circle graphs you created (in questions 1 and 2) to compare the number of days per week that elementary students and secondary students exercise.

   a. Which group of students exercises more frequently?

b. What are some possible reasons for your answer in a?

4. Read the survey question carefully. What factors might have contributed to how students responded to the survey?

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Making a Circle Graph Using Central Angles

Thinking Space

You may not always have access to a 100% circle template. Let's look at how we can make a circle graph using our knowledge of central angles. You will need a protractor and a compass to complete this part of the lesson.

Method of Communication	Number of Students
Internet chat or MSN	2843
In person	2604
Telephone (land-line)	1163
Cell phone	661
Text messaging	414
E-mail	159
Other	119

Source: Statistics Canada, Census at School, 2006/2007.

Step 1: Write each number as a percent.

Let's use a table to organize the information. We'll look at the first row of the table as an example.

Total Number of students surveyed: 7964

Method of Communication	Number of Students	Percent of Total
Internet chat or MSN	2843	
In person	2604	
Telephone (land-line)	1163	
Cell phone	661	
Text messaging	414	
E-mail	159	
Other	119	

Continue to fill in the "Percent of Total" column in the above table. Check your answers with the table below.

Total Number of students surveyed: 7964

Method of Communication	Number of Students	Percent of Total
Internet chat or MSN	2843	35.7%
In person	2604	**32.7%**
Telephone (land-line)	1163	**14.6%**
Cell phone	661	**8.3%**
Text messaging	414	**5.2%**
E-mail	159	**2.0%**
Other	119	**1.5%**

Step 2: Find the central angles for each of the sectors.

Each row in the table will be a sector in our circle graph. We need to know the central angle of each sector in the graph. Let's add a column to our table for "Central Angle." We'll use the first row as an example.

Total Number of students surveyed: 7964

Method of Communication	Number of Students	Percent of Total	Central Angle
Internet chat or MSN	2843	35.7%	
In person	2604	32.7%	
Telephone (land-line)	1163	14.6%	
Cell phone	661	8.3%	
Text messaging	414	5.2%	
E-mail	159	2.0%	
Other	119	1.5%	

When finding the central angle, it is helpful to use words to express what we need.

35.7% of the circle graph is the “Internet chat or MSN” sector

Since we know that all circles have a 360° central angle, we can find the angle of the sector.

35.7% of 360° is:

We know from earlier that what we have just created is a data sentence. All we need to do now is solve for this value, and we will have the central angle for this sector:

35.7% of 360° is

35.7% x 360° = ?

0.357 x 360° = 128.5°

We’ll round our answer to the nearest degree, so our final answer is 129°.

Total Number of students surveyed: 7964

Method of Communication	Number of Students	Percent of Total	Central Angle
Internet chat or MSN	2843	35.7%	129°
In person	2604	32.7%	
Telephone (land-line)	1163	14.6%	
Cell phone	661	8.3%	
Text messaging	414	5.2%	
E-mail	159	2.0%	
Other	119	1.5%	

Use this method to fill in the “Central Angle” column in the above table. Check your answers with the table on the next page.

Thinking Space

360° in a circle
- just like a 360
pop-shuvit

?

Why do we round
to the nearest
degree?

Total Number of students surveyed: 7964

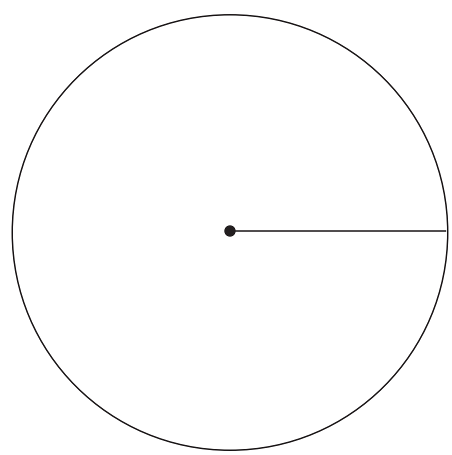
Method of Communication	Number of Students	Percent of Total	Central Angle
Internet chat or MSN	2843	35.7%	129°
In person	2604	32.7%	118°
Telephone (land-line)	1163	14.6%	53°
Cell phone	661	8.3%	30°
Text messaging	414	5.2%	19°
E-mail	159	2.0%	7°
Other	119	1.5%	5°

Add up all your central angles. What do you notice?

You may notice that sometimes your angles don't quite add up to 360°. This is usually due to rounding errors. Because it's easier to work with whole numbers when drawing angles, we have to round. This means the values for the central angles aren't quite as accurate as if we hadn't rounded. Keeping one or two decimal places in the percents can help.

Step 3: Draw the circle for the graph.

To draw the circle graph, use your compass to draw a circle. Remember, draw a point on your page and then use that point as the centre of the circle.



Draw your baseline radius by using a ruler to join a starting at the centre and connecting it to 0 degrees like we did earlier in the lesson. This makes a great starting point for the next step.

Thinking Space

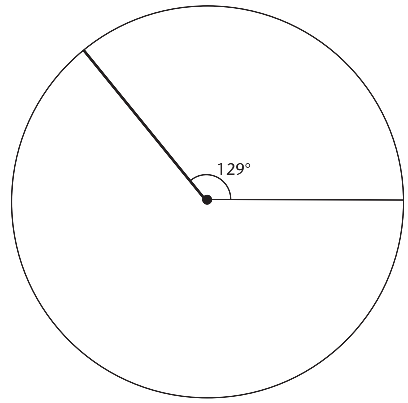
I noticed that we rounded our percents to the nearest tenth. This makes our calculations more accurate than if we rounded to the nearest whole number percent.

I remember drawing circles in module 3!

Step 4: Draw in your sectors.

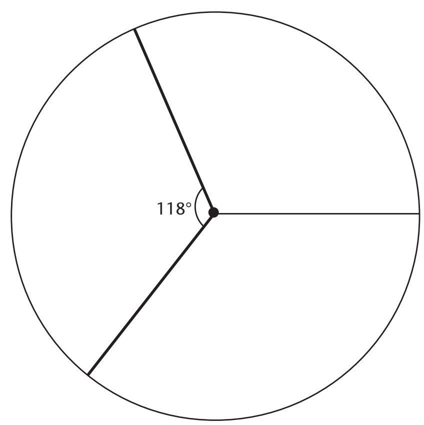
Thinking Space

Using your protractor and radius, draw your first sector by measuring off the correct number of degrees, starting from the baseline radius. The central angle for our first sector is 129°. Your circle graph should now look like this:



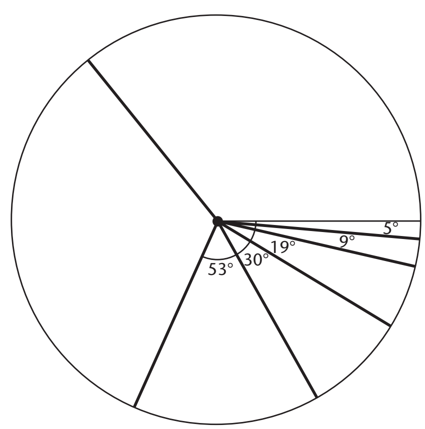
Note: We wrote the angles in here so you can see the steps clearly. When you're creating a circle graph, you don't need to label the angles.

Create the next sector. It may be more comfortable when adding your new sectors to turn your paper around. Our circle graph now looks like this:



Continue like this until you have filled in all 360° with the sectors and their angles. Keep turning your paper until the whole circle is divided into sectors with the correct central angles.

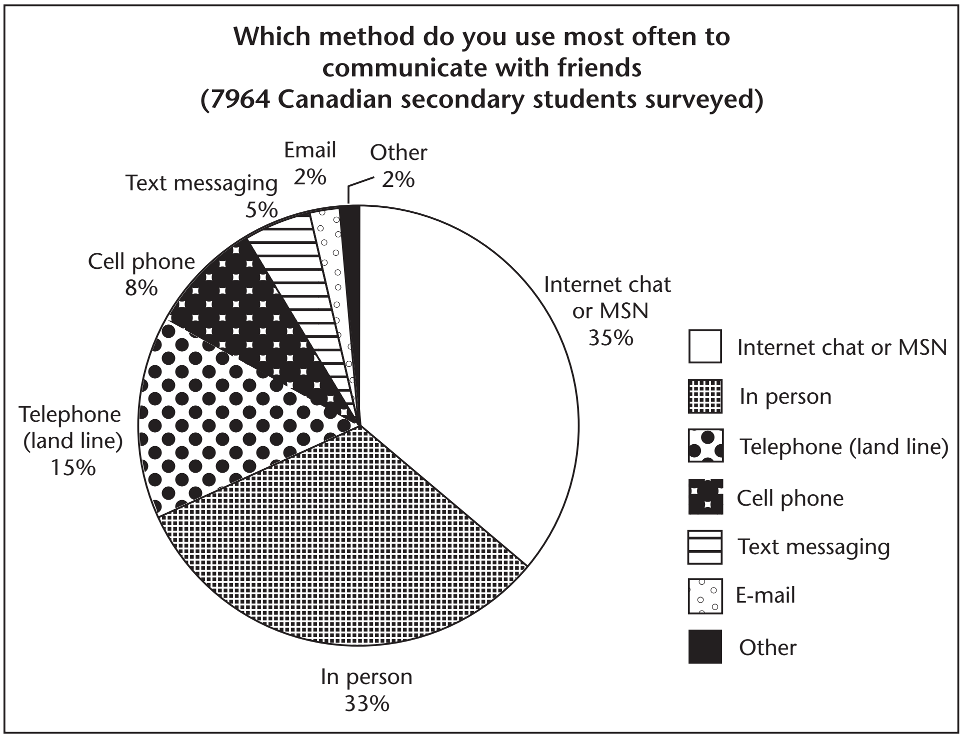
Your graph now looks like this:



Thinking Space

Step 5: Label the circle graph.

Just like before, providing labels and titles for your graph is an important part of communicating data to the reader. Finish off your graph by labelling all sectors with their category name and percentage value, as well as providing a title at the top. You may use a legend rather than placing labels directly on the sectors.



Now it's your turn to make your own circle graph using central angles.

Practice 2:

1. Rahul sorted and counted his hockey cards and decided to make a circle graph of the types of cards that he had.

Player's Position	Number of Hockey Cards	Percent of Total	Decimal Value	Central Angle
Forward	25			
Defence	16			
Goal	3			
Totals				

a. Complete Rahul's table.

b. Draw a circle graph to display the data. (Use the central angle method for creating your graph.)

2. At a local music store, 250 people were surveyed about their favourite Canadian artist. The table below shows the results. Complete the table, and then construct a circle graph by hand to display the data.

Canadian Music Artist	Number of People	Percentage	Measurement of Central Angle
Anne Murray	31		
Bryan Adams	52		
Alanis Morissette	45		
Céline Dion	90		
Gordon Lightfoot	32		
Total			

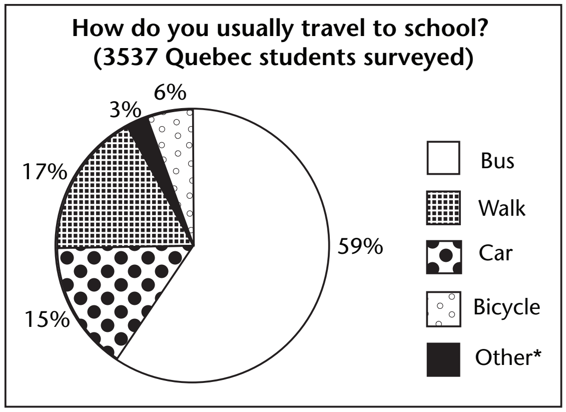
3. BC students were asked how they usually travel to school. The following table shows the results of the survey. Complete the table, and then construct a circle graph by hand to display the data.

Mode of Transportation	Number of Students	Percent of Total	Central Angle
Bus	452		
Walk	1247		
Car	1298		
Bicycle	125		
Other*	83		
Totals	3205		

*indicates the following categories: skateboard, subway/metro, moped, inline skate, train and motorcycle

Source: Statistics Canada, Census at School, 2006/2007.

4. The circle graph below shows the data from a similar survey conducted in Quebec.



*indicates the following categories: skateboard, subway/metro, moped, inline skate, train and motorcycle

Source: Statistics Canada, Census at School, 2006/2007.

Compare this graph with the one you made for the BC survey.

a. What percentage of students walk to school in BC?

b. What percentage of students walk to school in Quebec?

c. What are some possible reasons for this difference?

Turn to the Answer Key at the end of the Module and mark your answers.

Section Summary

In this section you learned all about circle graphs. Circle graphs (also known as pie charts) are a convenient way of displaying data visually. This type of graph is, very simply, a circle divided into wedge-shaped pieces. These pieces are called sectors. Each sector represents a particular category of the data you want to display. The size of each sector is proportional to the amount it represents.

To draw a circle graph, you can either use a 100% circle template or you can start from scratch.

The 100% circle template has markings on it that represent 1%. If you have a template, you only need to find the percentages for each category. Then, simply count out the number of markings on the template to match the percentage of each category and draw lines in to make your sectors. Give your circle graph a title, and label each sector. You may choose to colour code the sectors and include a legend.

If you don't have a template, you can draw a circle graph using a compass and a protractor. A summary of the steps is given below.

1. Using the data you are given, write each number as a percent of the total.
2. Find the central angle for each sector.
   - The central angle of a circle is 360°. Use the percent from each category to calculate the central angle of each sector.
3. Draw the circle.
   - Mark the centre point and use your compass to draw the circle. Use a ruler (or the straightedge of your protractor) to draw the baseline radius from the centre point to the edge of the circle.
4. Draw in the sectors.
   - Use the central angles you calculated in Step 2, and your protractor, draw in the sectors.
5. Label your graph.
   - Provide a title for the graph, labels for each sector, and a legend if necessary.

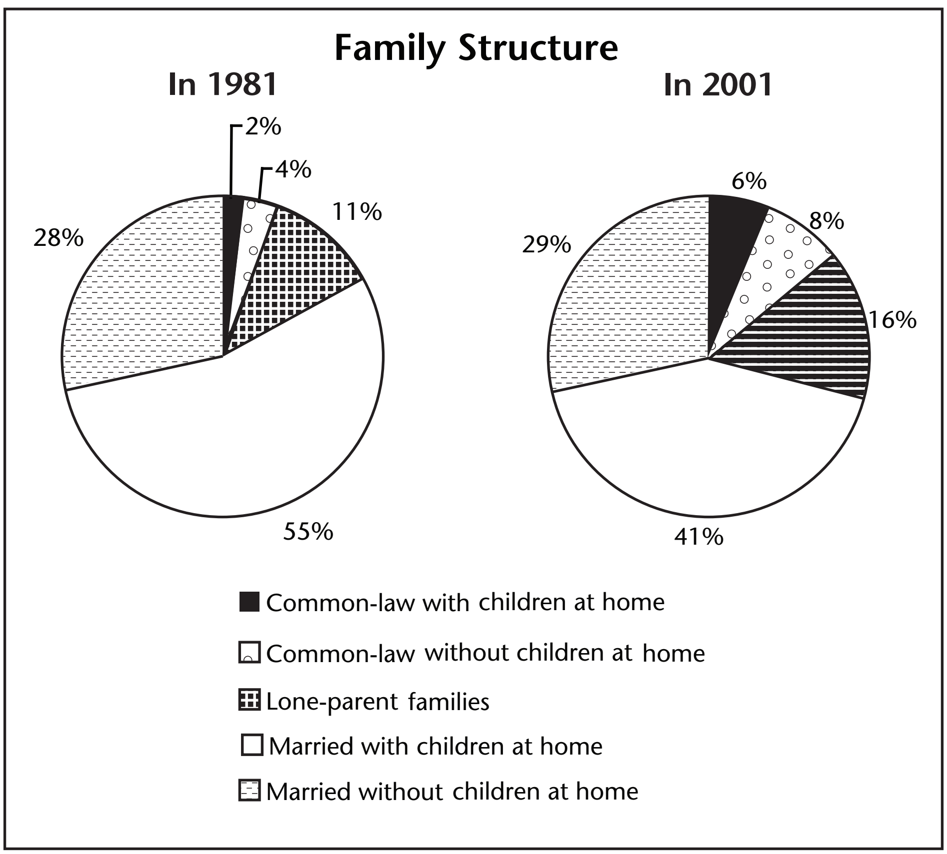
Section Challenge

Changing Families

Being able to read a variety of graph types can help you in many areas. You may have seen circle graphs in some of your other classes in school or in the newspaper or on TV. Reading and creating circle graphs are useful skills to have!

You may have seen statistics from the Canadian Census of Population. The census is a survey that asks people all over Canada a variety of questions about their age, sex, marital status, education, employment, income, language, and many other things. The census is conducted every five years. Not only does the census provide a large collection of facts about people in Canada, but it also provides a way to track trends over time.

Below are two circle graphs that describe some of the different family structures in Canada. The data come from the 1981 census and the 2001 census.



Source: Statistics Canada, 1981 and 2001 Census of Population

1. Use the two graphs to answer the following questions.

   a. For each year, list the five types of families in order from most common to least common. Are your two lists the same or different?

   b. Lone-parent families were more numerous in 2001 than in 1981. What other differences do you notice between the two graphs?

   c. Another census was done in 2006. Based on the differences and similarities you noticed between the 1981 and 2001 data, what predictions can you make about the 2006 data on family structure in Canada?

2. Below are data from the 2006 census that describe some of the different family structures in Canada. Complete the table and then create a circle graph based on the data in the table.

Family Structure	Number of Families in 2006	Percentage of Families in 2006
Common-law with children at home	618 150	
Common-law without children at home	758 715	
Lone-parent families	1 414 060	
Married with children at home	3 443 775	
Married without children at home	2 662 135	
Total	**8 896 840**	

3. Looking at the circle graph you created, how accurate were your predictions (in question 1c)? Explain your answer.

Section
2

Section 7.2: Measures of Central Tendency

Contents at a Glance

Learning Outcomes

By the end of this section, you will be better able to:

- define range, mean, median and mode.
- determine the range, mean, median, and mode for a set of data.
- decide which measure of central tendency is the most appropriate to describe a data set.
- identify outliers in a data set.
- describe how outliers affect the mean, median, and mode for a data set.

Pretest 7.2

Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in this section. Mark your answers using the key found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson 7.2A

Determine the range for each set of data:

1. 23, 56, 75, 34, 55, 66, 75, 39, 69

2. 556, 754, 643, 757, 987, 335, 236, 225, 675

Lesson 7.2B

Determine the median and mode of each data set.

1. 4, 8, 8, 9, 3, 4, 4

2. 125, 83, 115, 94, 109, 115, 89, 104

Lesson 7.2C

These are CoCo's golf scores this year:

118, 112, 116, 120, 112, 117, 96, 90, 92, 81, 83, 92, 92

1. Determine the mode, median, and the mean of CoCo's golf scores.

2. Which value is the best measure of central tendency of CoCo's golf scores? Explain your choice.

Lesson 7.2D

Duncan asked 15 people their age when they entered an amusement park. Here are the ages:

17, 25, 33, 38, 24, 8, 4, 27, 15, 26, 12, 4, 67, 23, 15

1. Determine the mode, mean, and median of the ages with and without the outlier.

2. Would you choose to include or not to include the outlier when determining an age that best represents the age of visitors at the amusement park.

Turn to the Answer Key at the end of the Module and mark your answers.

Section Challenge

Gas Relief!!!

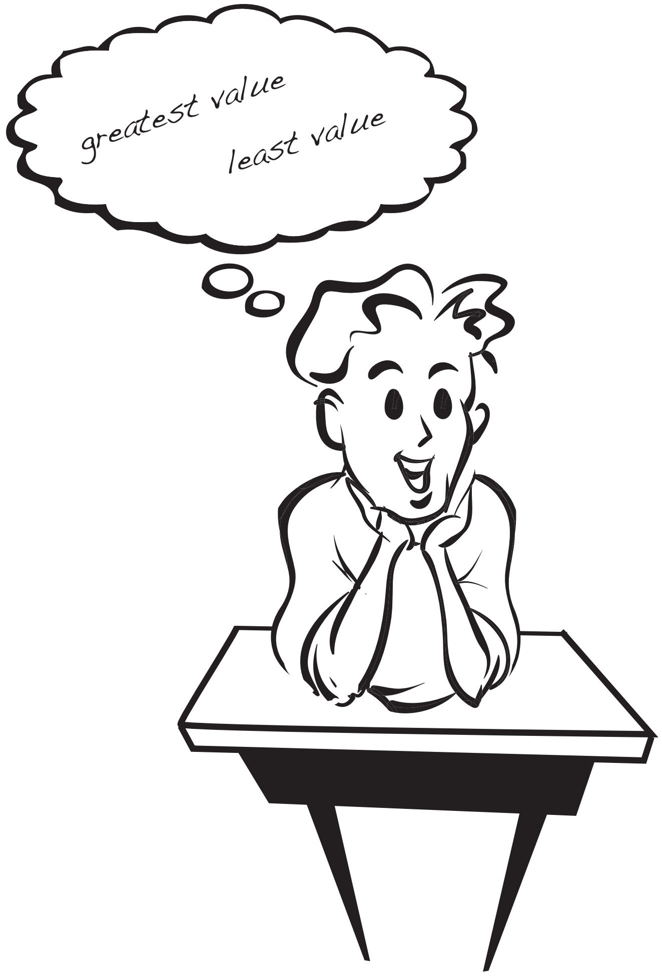
Elan is looking to buy a new car. Because fuel prices have increased so drastically within the past year, she is considering buying a hybrid vehicle. Hybrid vehicles combine the energy from fuel with electricity. There are many cars on the market, so Elan researched all kinds of cars before deciding to take three on a test drive. Use the following data to decide which three cars Elan should consider when purchasing a car.

Car	Fuel Efficiency in city (L for each 100 km)
1 Honda Civic	7.4
2 Honda Civic hybrid	4.7
3 Mazda 3	8.4
4 Toyota Prius hybrid	4.0
5 Toyota Yaris	7.0
6 Camry Hybrid	5.7
7 Saturn Hybrid	8.5
8 Mini Cooper	7.1
9 Chevrolet Cobalt	9.2

1. Find the range of fuel efficiency for the listed cars.
2. Find the mode, median, and mean for the fuel efficiency values.
3. How can you use these values to decide which three cars Elan should consider to test-drive?
4. Given the research and analysis of the data above, would you agree or disagree with the statement that Hybrid vehicles, on average, are better on gas than regular cars?

Lesson 7.2A: Data Sets and Range

Student Inquiry



This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
What does the range of a set of data tell me?		answer
		example
How do I find the range of a set of data?		answer
		example

Lesson 7.2A: Data Sets and Range

Thinking Space

Introduction

Statistics provide us with information about pretty much everything. Regardless of your interest, you can find statistics for the topic. You can find statistics on hockey teams, populations, food content, exam marks, heart rate...everything.

Many companies and websites bring in an income purely on providing statistics to the public to help inform them about certain topics, products, geographic regions, or whatever they happen to be researching. Statistics are a great way to summarize a lot of data.

One statistic is the range of the data set we're looking at. This lesson will show you how to find it!

Warm-up

Find the following words in the puzzle below:

- statistics
- median
- average
- mean
- graph
- range
- mode
- outlier
- data

s	a	a	r	g	s	i	m	u
c	i	r	t	t	e	s	m	a
i	o	e	n	a	i	o	d	a
t	h	s	m	a	d	u	g	t
s	p	i	h	e	a	t	a	n
i	r	e	p	m	d	l	i	s
t	i	g	a	e	g	i	l	n
a	v	e	r	a	g	e	a	a
t	d	e	g	n	a	r	i	n
s	a	a	u	r	e	s	g	c

Turn to the Answer Key at the end of the Module and mark your answers.

Explore
Range

Thinking Space

A basketball coach has put her players into two teams. She wants to make sure the teams are well-matched physically. She asks the players to measure their heart rate, height, and mass.

Team 1	Heart rate (beats/minute)	Height (m)	Mass (kg)
Edison	78	1.6	45
Brian	72	1.7	59
Tatiana	74	1.62	48
Tysheim	69	1.78	66
Celina	62	1.81	63

Team 2	Heart rate (beats/minute)	Height (m)	Mass (kg)
Provie	70	1.72	60
Andon	70	1.54	46
Brandon	72	1.64	50
Onazette	76	1.62	45
Shakeema	64	1.8	61

The chart displays the data collected about the players' heart rate, height, and mass.

Let's look at the data for heart rate for Team 1 and Team 2.

Team 1 Heart rate (beats/minute)
78
72
74
69
62

Team 2 Heart rate (beats/minute)
70
70
72
76
64

Let's find the **range**. The range is the difference between the greatest and least value in a set of data.

Range of heart rate for Team 1	Range of heart rate for Team 2
Greatest value: 78 Least value: 62 Difference: 78 – 62 = 16 Therefore the range of heart rates for Team 1 is 16 beats per minute.	Greatest value: 76 Least value: 64 Difference: 76 – 64 = 12 Therefore the range of heart rates for Team 2 is 12 beats per minute.

Do you see how this information would help the coach create equally balanced teams? Now it's your turn. Find the range of height and mass for each team.

Range of heights for Team 1	Range of heights for Team 2
Greatest value: Least value: Difference:	Greatest value: Least value: Difference:

Range of mass for Team 1	Range of mass for Team 2
Greatest value: Least value: Difference:	Greatest value: Least value: Difference:

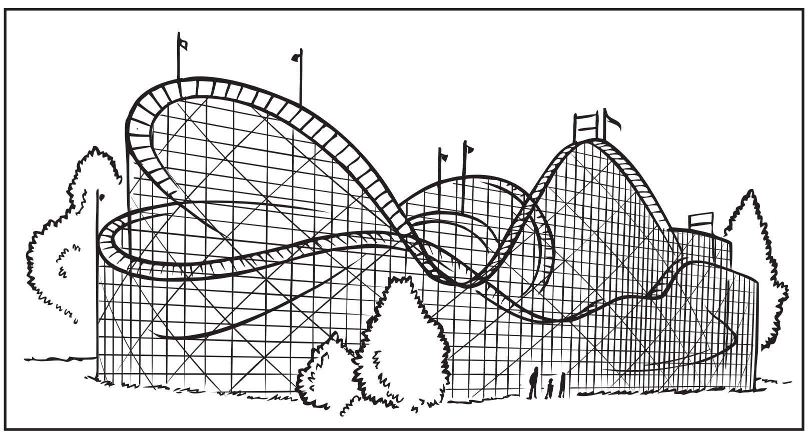
Now let's compare the teams to see if the coach made an equal division of her players.

Thinking Space

Range	Team 1	Team 2
Heart Rate	16 b/m	12 b/m
Height	0.21 m	0.26 m
Mass	21 kg	16 kg

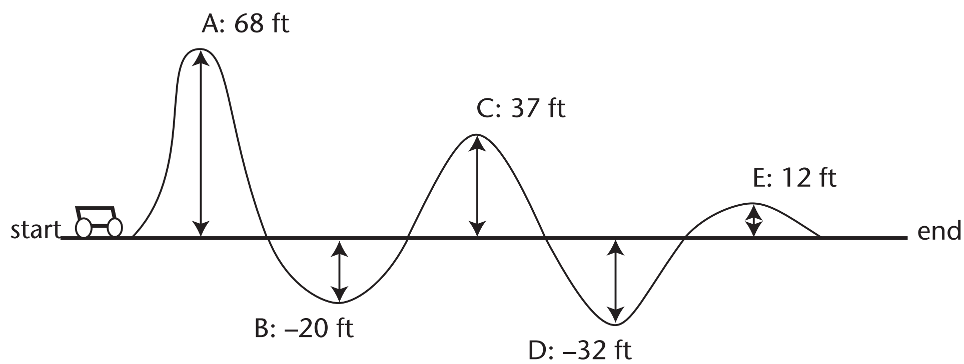
When we look at all the ranges together, we can see that the coach did a great job dividing her players according to their heart rate and height, because the ranges are similar. When we look at the mass, however, we see that the differences in ranges are greater. After finding the ranges, this coach may have gone back and made some changes to ensure that these ranges were closer in comparison of the two teams.

Did this give you a better idea of how we can use range to help us compare and analyze data? Let's do one more example of finding range in a data set together.



The rollercoaster at Play Land in Vancouver was built in 1958 and is the one of the oldest wooden roller coasters still in use in the world. Most roller coasters are made of steel. Another interesting fact about this rollercoaster is that after the initial ascent, the entire rollercoaster is completely powered by gravity only, which is intense. If you have ever ridden on it, you will know how fast it rips up and down those tracks. It speeds along at up to 75 km/h.

At its highest elevation the rollercoaster climbs up to 68 ft., or 20.73 metres. Once it drops from this initial climb, the graph below shows measurements of its height at different positions along the track.



Thinking Space

Fill in this table of the elevations of this rollercoaster at each point.

Location along the ride	Start	A	B	C	D	E	Ending
Elevation relative to starting point	0						

Find the range of heights for this rollercoaster ride. Compare your results to the solution below.

Solution:

Location along the ride	Start	A	B	C	D	E	Ending
Elevation relative to starting point	0	68	–20	37	–32	12	0

The greatest value is 68 ft.

The lowest value is –32 ft.

The difference is:

$$68 - (-32) = 68 + 32 = 100$$

You've done a great job so far! As you can see, ranges help us understand data sets by beginning to describe them. While working through the practice questions, think about how you could use range to help you describe a set of data.

Practice 1

Thinking Space

1. Determine the range in gas prices in Kelowna from March 2002 to March 2005.

Date	March 2002	March 2003	March 2004	March 2005
Price($/L)	0.732	0.877	0.89	0.974

2. List a set of 10 different values that have a range of 15.

3. Calculate the range for the given data sets:

   a. 6, 10, 5, 7, 3, 2

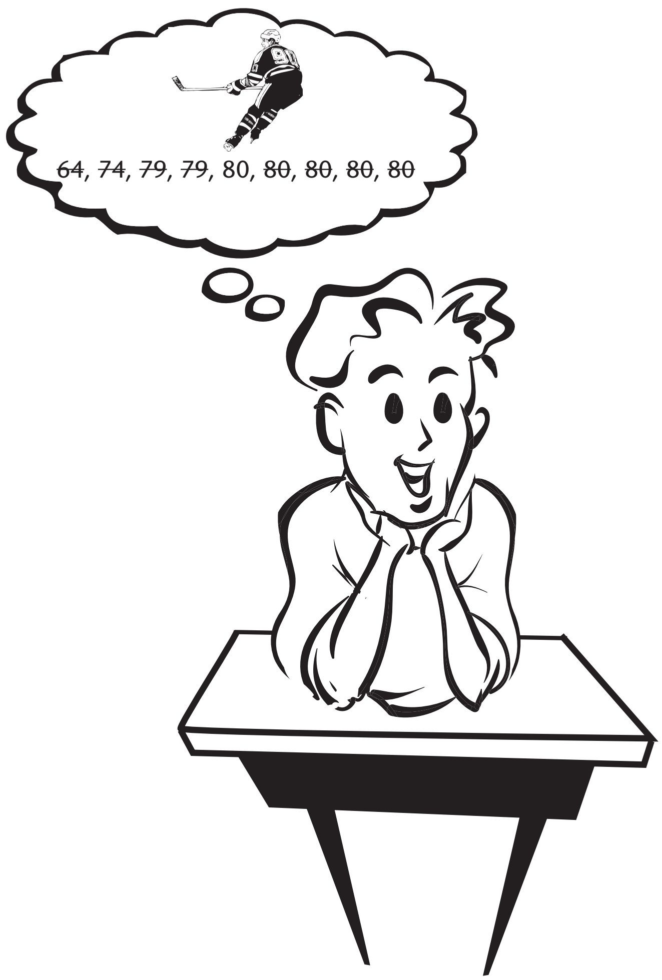
   b. 23, 56, 33, 68, 53, 24, 78, 43

   c. 12.5, 5.3, 2.9, 5.6, 4.2

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 7.2B: Measures of Central Tendency— Median and Mode

Student Inquiry



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When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
How do I find the median and mode of a set of data?		answer
		example
Why can the median and mode of a data set be different values?		answer
		example

Lesson 7.2B: Measures of Central Tendency—Median and Mode

Thinking Space

Introduction

Although the range helps us start to describe sets of data, we are limited if we rely on only this method. There is much more information we can pull out of data sets, and much more information we need to understand in order to describe sets of data accurately. In this lesson we will look at different measures of central tendency.

Warm-up

Place the following sets of numbers in order from least to greatest:

1. 21, 61, 52, 68, 53, 34

2. 41, 3, 18, 1, 22, 27, 24,38,

3. 6, 11, 74, 1, 11, 35, 63, 3, 10, 62, 6, 48, 37, 63

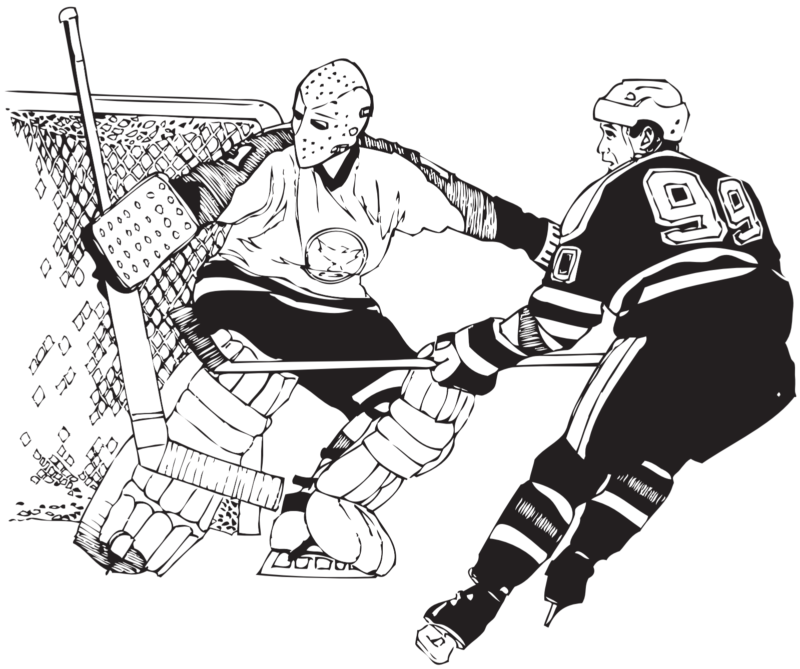
Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Median and Mode

Thinking Space

Wayne Gretzky is a celebrated hockey player and model of dedication to a sport. He was awarded many trophies in his hockey career. Awards committees use data sets to help them determine which player to award each season.



The awards to players each season are as follows:

- Hart Memorial Trophy (HM) is the most valuable player award.
- Lester B Pearson Trophy (LBP) is awarded to the most valuable player as voted by peers.
- Art Ross Trophy (AR) is awarded to the highest scorer each season.
- Conn Smythe Trophy (CS) is given to the player judged most valuable to his team during the NHL's Stanley Cup playoffs.

Here are some of Wayne Gretzky's statistics as an Edmonton Oiler.

Year	Games played	Points	Trophies
1979–80	79	137	LBM, HM
1980–81	80	164	AR,HM
1981–82	80	212	AR,HM,LBP
1982–83	80	196	AR,HM,LBP
1983–84	74	205	AR,HM,LBP
1984–85	80	208	AR,HM,CS
1985–86	80	215	AR,HM,LBP
1986–87	79	183	AR,HM
1987–88	64	149	CS

Let's use Wayne's hockey stats to learn how to find the mode and median. The mode and median are both methods to find a central data value. As you will see, although they both measure a central value, they differ slightly in their methods. See if you can spot how they are different.

Mode: A value of a data set. It represents the value or item that occurs most often in a set of data.

Median: A value of a data set. It represents the middle value of an ordered set of data.

The first step in finding median and mode is to order a set of data from least to greatest. Look at this data set listing Wayne Gretzky's games played each year.

Games played
79
80
80
80
74
80
80
79
64

List the data set in order from least to greatest.

64, 74, 79, 79, 80, 80, 80, 80, 80

Now that the set is in order, we can easily determine the mode and median.

Thinking Space

It's no wonder the city has a major road named after him!

Thinking Space

Mode: 80

The mode is the most frequently occurring value. In our list, we can see that 80 games occur most often.

Median: 80

When finding the median, we need to find the value directly in the middle. Cross off a value one at a time at each end like this:

~~64~~, ~~74~~, ~~79~~, ~~79~~, 80, ~~80~~, ~~80~~, ~~80~~, ~~80~~

We can see that the value in the middle is 80. This is our median.

In this example, we see that median and mode are the same value. It is easy then to come to the conclusion that 80 would be a good value to report as a central value. But this is not always the case.

Find the median and mode of Wayne Gretzky's points scored each year for his time as a player on the Edmonton Oilers. Compare you results with the solutions on the next page.

List the data set in order from least to greatest.	
Mode: most occurring	Median: The middle value

Solution:

	Games played	Points
Mode	80	No mode
Median	80	196

Did you have difficulty finding the mode? When there is no value that is listed more than once, we state that there is no mode. In this case, we rely on the median to describe our central value.

Do you think this method gives us an accurate way to describe Wayne's performance over these years?

Do you think there is one method that better represents how Wayne performed during these seasons?

Let's do one more example together before moving on.

Annabel and Polly decided to plant seedlings as part of a science project. After 2 weeks they measured and recorded the heights of the seedlings in centimetres.

39, 42, 44, 56, 78, 67, 76, 45, 33, 67, 44, 40, 45, 87, 97,
54, 55, 65, 33, 24, 67, 99, 105, 35, 77, 43, 67, 88, 55, 33

Use the chart to find the mode and median of the heights of the seedlings. Which method do you think best describes the typical height of a seedling at this time?

List the data set in order from least to greatest.	
Mode: most occurring	Median: The middle value
I think the ____________best describes the most typical height of a seedling at this time.	

Thinking Space

Compare your results to the solution that follows.

Solution:

List the data set in order from least to greatest.	
24,33,33,33,35,39,40,42,43,44,44,44,45,45,54,55,55, 56,65,67,67,67,76,77,78,87,87,87,87,88,97,99,105	
Mode: most occurring	Median: The middle value
87 cm occurs the most often	The 2 middle values are 54 and 55. if we find the exact middle between these two values its is 54.5.
I think the *median* best describes the most typical height of a seedling at this time.	

Median and Mode Using a Frequency Table

Sometimes frequency tables are used to collect data. When this is the case, median and mode need an extra step. Take a look at this example.

A survey was taken about how much high school students are making at their part-time jobs.

Wage	Number of Students
\$7.00	2
\$8.00	4
\$9.00	1
\$10.00	5
\$11.00	2

Thinking Space

?

Do you have any questions about finding the median or mode?

Thinking Space

Since there are 2 students who make $7.00/ hour, and 4 students who make $8.00 / hour, we list the wages like this:

7, 7, 8, 8, 8, 8

We continue like this until all of our values are listed in order. Now our data set looks like this:

7, 7, 8, 8, 8, 8, 9, 10, 10, 10, 10, 10, 11, 11

Now that we have our data set in order from least to greatest, we can continue finding the mode and median of this set.

Mode: find the most frequently occurring value. When working with frequency table, this is much easier. Because the table already tells you how many of each value is reported, all we have to look for is the value that is reported the most times. In this example, 5 students are making $10.00/hour, and that is more than for any other wage.

Our mode is: $10.00/hour.

Median: find the middle value. Remember it does not have to be a value listed—if you have two numbers left, find the middle value of those 2 values.

7, 7, 8, 8, 8, 8, 9, 10, 10, 10, 10, 10, 11, 11

Our median is: $9.50/hour.

Let's practise together finding median and mode using a frequency table.

Thinking Space

The following frequency chart shows the heights of boys on a basketball team.

Height (cm)	Number of players
150	2
155	1
160	2
165	3
170	5

1. What is the mode for the heights?
2. What is the median height?

Compare your results to the solution that follows.

List the data set in order from least to greatest.	
Mode: most occurring	Median: The middle value

Solution:

List the data set in order from least to greatest.	
150, 150, 155, 160, 160, 165, 165, 165, 170, 170, 170, 170, 170	
Mode: most occurring	Median: The middle value
170 cm	165 cm

You have done really well working through this lesson. Continue now to the practice questions before moving onto the next lesson, learning about our final measure of central tendency: mean.

Practice 1

1. What are the mode and median of each set of data?

   a. 4, 2, 9, 6, 4

      Mode:

      Median:

   b. 21, 15, 18, 21, 20, 19

      Mode:

      Median:

   c. 3, 8, 5, 12, 10, 8, 2

      Mode:

      Median:

2. In one week, a store in the mall sold the following numbers of CDs from their Rock section.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
32	45	37	54	46	58	32

   a. What were the mode and median for the CD sales (from the Rock section) that week?

      Mode:

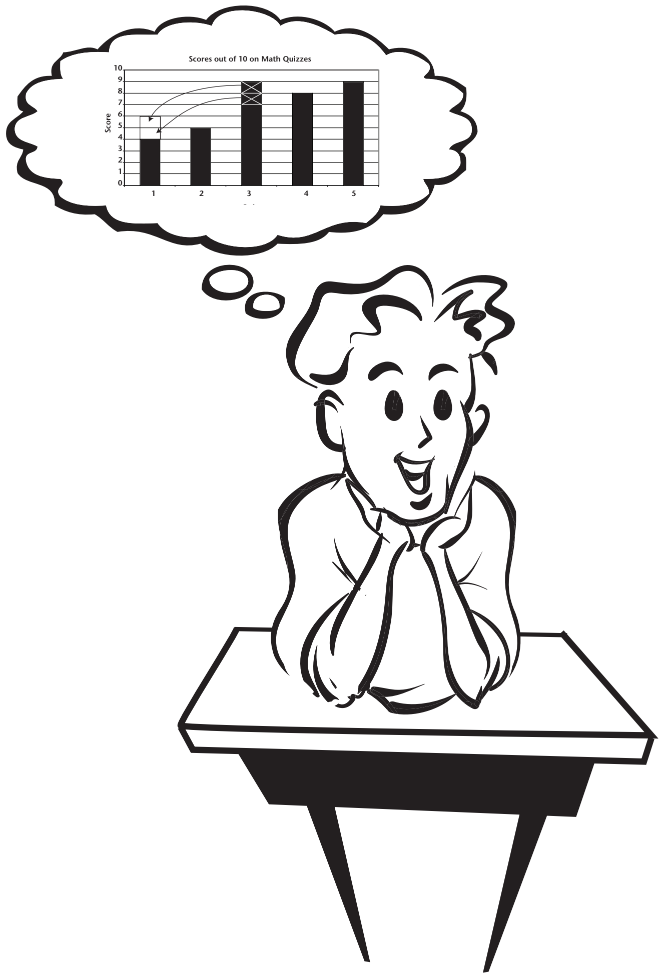
      Median:

b. Does mode or median better represent sales of CDs (from the Rock section) during this week?

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 7.2C: Measures of Central Tendency—Mean

Student Inquiry



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When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
How do I find the mean of a set of data?		answer
		example
What does the mean tell me about a set of data ?		answer
		example
How is the mean different from the median and mode?		answer
		example

Lesson 7.2C: Measures of Central Tendency—Mean

Thinking Space

Introduction

We learn about the different measures of central tendency because the more ways we can understand a set of data, the better we can describe, compare, and contrast that data. The last method of central tendency we will look at is the **mean**. By the end of this lesson, you will be able to determine which methods to use (and which methods not to use) in order to best describe a data set.

Explore Online

Looking for more practice or just want to play some fun games? If you have internet access, go to the Math 7 website at:

http://www.openschool.bc.ca/courses/math/math7/mod7.html

Look for *Lesson 7.2C: Measures of Central Tendency—Mean* and check out some of the links!

Warm-up

Complete the addition practice to get ready for this lesson:

91 + 64 =	89 + 65 =	92 + 47 =
57 + 43 =	87 + 19 =	49 + 37 =
43 + 76 =	78 + 40 =	22 + 39 =
91 + 66 =	97 + 20 =	55 + 43 =
53 + 24 =	24 + 29 =	94 + 61 =
61 + 50 =	87 + 10 =	35 + 37 =
32 + 99 =	79 + 61 =	83 + 64 =

Turn to the Answer Key at the end of the Module and mark your answers.

Explore
Mean

Thinking Space

Shannon and Brandon both reported the same scores on their math quizzes in a term. Their scores out of 10 were 8,9,4,5,9. Their teacher asked them both to determine a score which best described their work in the term.

Each student chose a different method to determine their central value to represent how they performed that term. Calculate the mode for Shannon and the median for Brandon.

Shannon chose to use the mode	Brandon chose to use the median

Which method would you have chosen to put on your report card?

Although both Shannon and Brandon have justified their decision and found a measure of central tendency, can you see how these methods might limit or misrepresent a data set?

We have to be careful when relying on these methods to describe data. It is important to consider which methods are most appropriate.

Their teacher Ms. Preece challenged the students to find a better way to calculate their term mark. What did they do? Do you know? They used **mean**.

Let's use Shannon and Brandon's math scores as the data set to figure out what mean is. Mean is a value that represents a set of data. It is determined by sharing the sum of the data evenly among the values in the set. Another word used to represent this concept is "average." You may have heard it used before.

Mean: Using Counters

Thinking Space

We will use counters to show how we can find the mean using the scores from Brandon and Shannon's quizzes.

For this activity you'll need to do some cutting. Go to the templates section (at the back of the module) and locate the counters template. Once you have the template, follow the instructions below.

Cut out and colour the counters:

- Colour 4 red
- Colour 5 yellow
- Colour 9 blue
- Colour 8 green
- Colour 9 purple

After you have colored the dots, sort them into the piles based on their colour. These colored dots now represent the math scores of Shannon and Brandon. Can you see how the red pile represents their score of 4?

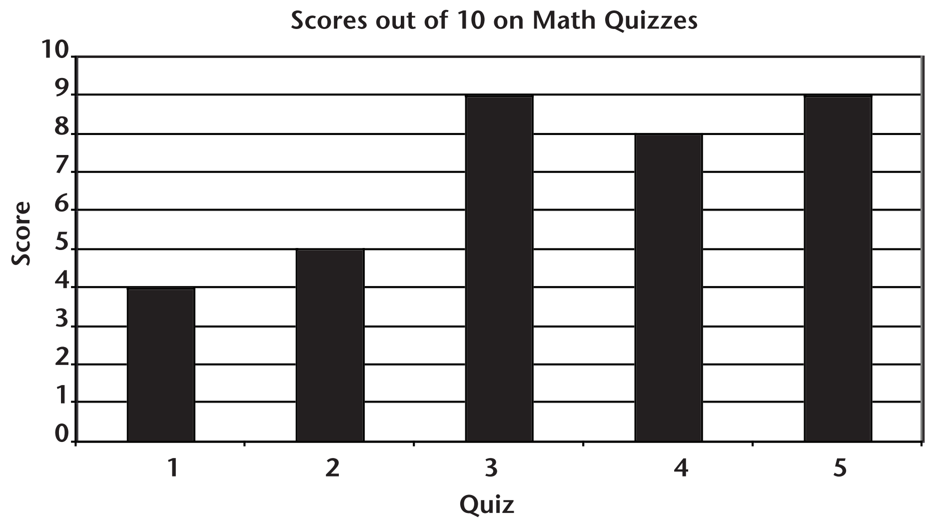
Now using these counters, move them around so that every pile has the same amount. The mean shows us what a typical or average score would be. If the students had received the same score on every quiz, it would be easy to determine what the mean is. Because the scores are different on every quiz, rearranging the counters so that the piles are even will tell us what their typical score on the quizzes are. This is the mean.

After rearranging the piles, you should have 7 counters in each pile. Which piles did you take from? Which piles did you give to? This concept of sharing is exactly what the mean is. Sharing to make values even tells us what a typical measure would be.

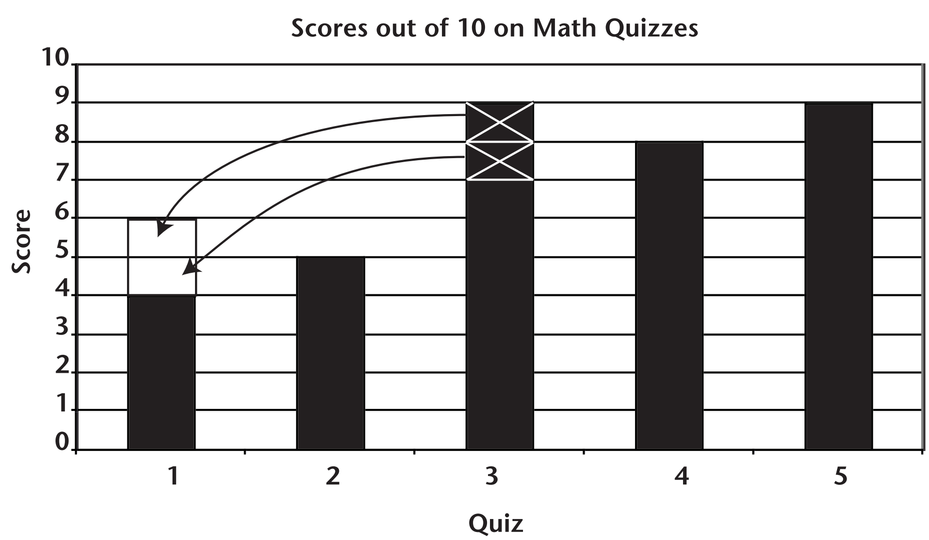
Mean: Using a Bar Graph

Thinking Space

We can also show what mean is by using this bar graph of Shannon and Brandon's math scores.

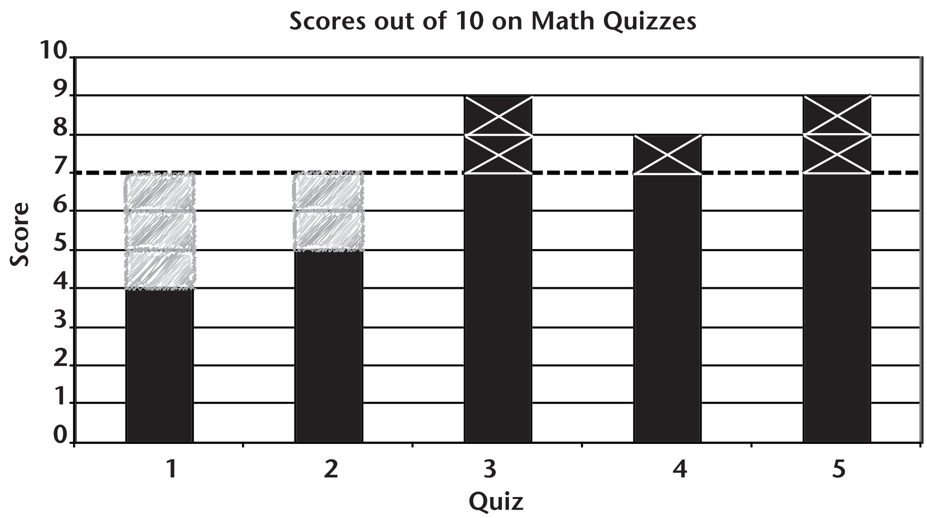


Looking at this bar graph we can see that some bars are really tall and some bars are really short. We can share the bars to make them even. Then we will see what the mean is. For example, Quiz 3 has a score of 9, but Quiz 1 has a score of 4. We can share by giving some of the Quiz 3 bar to the Quiz 1 bar like this:



If we keep sharing the tall bars with the shorter bars, the graph eventually looks like this:

Thinking Space



The mean value for this set of data using a bar graph is a score of 7.

Sometimes it's easier to understand a process if we look at in a variety of ways. Let's try to find the mean using another method.

This matches the mean we got when we used the counters!

Mean: Using Calculations

Now that you have found the mean using a bar graph and counters, showing you how to solve the mean with calculations will make more sense. The same idea of "sharing" still applies.

First we add up all the values:

$$4 + 5 + 9 + 8 + 9 = 35$$

The second step is to divide the total of the set by the amount of values within it. In this example, we have 5 values. Therefore, to find the mean we divide our total, 35, by the amount of values, 5.

$$35 \div 5 = 7$$

By calculating, we can see that our mean is the same as the other methods: 7.

Do you think that 7 is a more representative score of Shannon and Brandon's performance this term in math? How does it compare to the median 8 and the mode 9?

Which method helped you understand mean the best? Let's do some questions together involving mean. Choose the method that works the best for you.

Here are the ages of a group of children in a choir:

9, 10, 11, 7, 14, 11, 10, 8, 7, 7, 11, 11

1. What is the mean age?

2. If two new members joined and they are both 7 years old, what is the new mean age?

Use the chart to answer the questions. Then compare your answer to the solution that follows:

	1. Age of children in the choir	2. Age of children in choir with two new members
Data Set		
Sum of Values		
Number of Values		
Mean	Mean A = $\frac{\text{total sum}}{\text{number of values}}$	Mean B = $\frac{\text{total sum}}{\text{number of values}}$

Thinking Space

The method that works best for finding the mean is...

Solution:

	1. Age of children in the choir	2. Age of children in choir with two new members
Data Set	7,7,7,8,9,10,10,11,11,11,11,14	7,7,7,7,7,8,9,10,10,11,11,11,11,14
Sum of Values	116	130
Number of Values	12	14
Mean	$\text{Mean } A = \frac{\text{total sum}}{\text{number of values}}$ $= \frac{116}{12}$ $= 9.7$ The mean age is 9.6 years old.	$\text{Mean } B = \frac{\text{total sum}}{\text{number of values}}$ $= \frac{130}{14}$ $= 9.3$ The mean age after the two new members join is 9.3 years old.

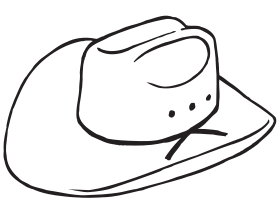
Thinking Space

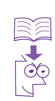
Putting It All Together

Now that you know how to find the mode, median, and the mean, you can use all of them to describe a set of data. It is good to describe a set of data by using several measures. As you have seen, sometimes one method does not give a fully accurate representation of a set. Just like Shannon and Brandon's teacher wanted a fair value to express their performance, we want a fair value to express our data. Knowing that you are looking for what is "typical" is a good way to determine which method is the most appropriate method to choose.

You may have noticed that mean is similar to a concept you may already be aware of. "Mean" is often referred to as "the average." However, statisticians use the word "average" to refer to any measure of middle or "central tendency" of a data set.

Let's look at an example together that incorporates all that we have learned so far.





The Calgary Stampede is a famous exhibition that includes a summer fair and a rodeo. People dress up like cowboys and spend their days riding horses, listening to country singers, and eating Alberta beef!

The stampede had record attendance in 2006.

Thinking Space

Date	Attendance
July 4	31 308
July 5	86 070
July 6	134 693
July 7	146 490
July 8	78 963
July 9	127 471
July 10	123 035
July 11	106 711
July 12	144 813
July 13	160 253
July 14	122 711

Using mode, median, and mean, determine the most appropriate value to represent the average amount of visitors per day at the Calgary stampede.

Use this chart if you need help with the steps, and compare your answer to the solution below.

<table>
<tr><td colspan="2">List the data set in order from least to greatest.</td></tr>
<tr><td>Mode: most occurring</td><td>Median: The middle value</td></tr>
<tr><td></td><td></td></tr>
</table>

Mean or average: sharing the sum evenly among a set

Data Set	
Sum of Values	
Number of Values	
Mean	

Mode:

Median:

Mean:

Thinking Space

I think the (mode/median/mean) is the most appropriate method to represent the average because:

Solution:

List the data set in order from least to greatest. 31 308, 78 963, 86 070, 106 711, 122 711, 123 035, 127 471, 134 693, 144 813, 146 490, 160 253	
Mode: most occurring	Median: The middle value
There is no value that occurs more than once, therefore there is no mode.	~~31 308~~, ~~78 963~~, ~~86 070~~, ~~106 711~~, ~~122 711~~, 123 035, ~~127 471~~, ~~134 693~~, ~~144 813~~, ~~146 490~~, ~~160 253~~ Median: 123 035

Mean or average: sharing the sum evenly among a set

Data Set	31 308, 78 963, 86 070, 106 711, 122 711, 123 035, 127 471, 134 693, 144 813, 146 490, 160 253
Sum of Values	1 262 518
Number of Values	11
Mean	Mean A $= \frac{\text{total sum}}{\text{number of values}}$ $= \frac{1\ 262\ 518}{11}$ $= 114\ 774.364$

Thinking Space

The mean amount of visitors per day at the Calgary stampede is 114 774. Round to the nearest whole number because you cannot have a fraction of a person.

Mode: *no mode*

Median: *123 035*

Mean: *114 774*

I think the median and the mean are the most appropriate methods to represent the average because: *they are similar to each other. The mode is not a good representation, because there were never two days where the same amount of visitors came to the stampede.*

Practice 1

1. The following table shows the populations of Canada's provinces and territories as of July 1, 2007.

Province/Territory	Population
Newfoundland and Labrador	506 275
Prince Edward Island	138 627
Nova Scotia	934 147
New Brunswick	749 782
Quebec	7 700 807
Ontario	12 803 861
Manitoba	1 186 679
Saskatchewan	996 869
Alberta	3 473 984
British Columbia	4 380 256
Yukon Territory	30 989
Northwest Territories	42 637
Nunavut	31 113

   a. Determine the mean population of all provinces or all territories.

   b. Is your answer an accurate value representing the typical population of a province or territory in Canada? Explain.

2. This chart describes average heights of grade 7 students in each province. What is the mean height of Grade 7 students across Canada? Round your answer to the nearest tenth.

Province/Territory	Height (cm)
Newfoundland and Labrador	156.5
Prince Edward Island	158.9
Nova Scotia	156.7
New Brunswick	151.5
Quebec	155.2
Ontario	155.2
Manitoba	157.8
Saskatchewan	154.9
Alberta	150.6
British Columbia	153.6

Source: Census at School, 2003–2004

3. A set of values has a sum of 84 and a mean of 12. How many values are in the set?

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 7.2D: Outliers—Skewing the Data!

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
What is an outlier?		answer
		example
How do I find an outlier?		answer
		example
What effect does an outlier have on a data set?		answer
		example

Lesson 7.2D: Outliers—Skewing the Data!

Thinking Space

Introduction

Statistics are a great way to communicate information about data. Understanding statistics involves critical examination.

Media is gaining more and more of an influence in our lives. It's important to make sure we look at data presented in magazines and on TV from multiple perspectives. We wouldn't want to be misled!

Explore Online

Looking for more practice or just want to play some fun games? If you have internet access, go to the Math 7 website at:

http://www.openschool.bc.ca/courses/math/math7/mod7.html

Look for *Lesson 7.2D: Outliers—Skewing the Data!* and check out some of the links!

Warm-up

Complete the subtraction practice to get ready for this lesson:

56 – 23 =	43 – 12 =	26 – 11 =	98 – 23 =
12 – 4 =	56 – 25 =	99 – 24 =	65 – 7 =
32 – 18 =	90 – 64 =	101 – 23 =	87 – 19 =
29 – 19 =	63 – 59 =	82 – 59 =	134 – 76 =

Turn to the Answer Key at the end of the Module and mark your answers.

Explore
What is an outlier?

Thinking Space

An **outlier** is a value that is much larger or smaller than other values.

What is The Affect of an Outlier? The True Story of Dora

Let's look at an example to understand the effects of outliers.

Dora's cafeteria makes the best food ever! Dora is a favorite in the school, known for her great tasting lunches. She wants to make sure that she feeds all the students, while at the same time not wasting any food. (Although the teacher next door really likes it when there are leftovers!) Normally Dora makes 400 lunches a day, and every day she has too much food.

Dora recorded how many lunches were bought every day for 15 days.

Day	Lunches bought
1	345
2	323
3	357
4	313
5	120
6	298
7	359
8	355
9	327
10	301
11	311
12	345
13	333
14	354
15	362

Then she calculated the mean to determine a central measure, or typical amount of lunches sold in a day.

Calculate the mean and compare your results to Dora's solution below.

Thinking Space

Data set:	
Sum of values	
Number of values	
Mean $= \dfrac{\text{sum}}{\text{number of values}}$	

Dora's Solution:

Data set:	120, 298, 301, 311, 313, 323, 327, 333, 345, 345, 354, 355, 357, 359, 362
Sum of values	4803
Number of values	15
Mean $= \dfrac{\text{sum}}{\text{number of values}}$	Mean $= \dfrac{\text{sum}}{\text{number of values}}$ $= \dfrac{4803}{15}$ $= 320$

When Dora looked at her value, she was confused. If she chose 320 lunches as her average day, she would not have enough food for most of the days. She looked back at her data and noticed that out of the 15 days, 10 of them showed MORE than 320 lunches bought.

"This is not typical!" she said to herself.

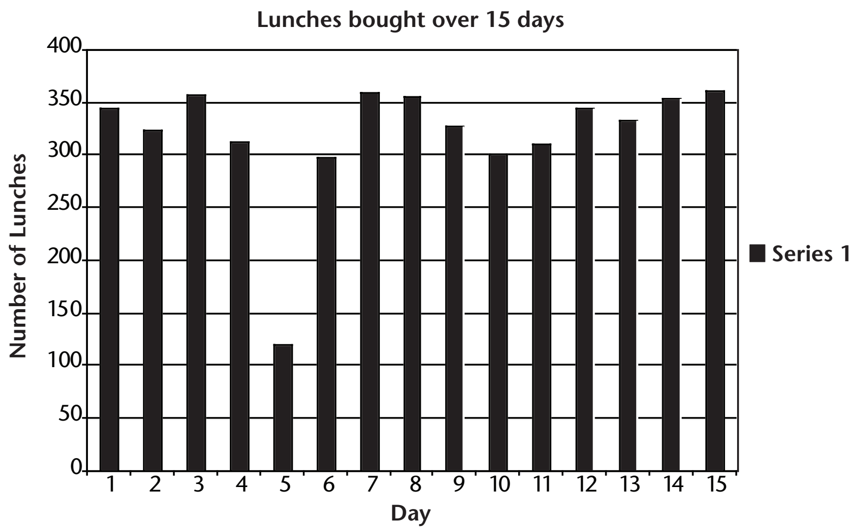
What was Dora to do? Not having enough food just wasn't a solution… she would be dealing with rioting hungry students!

Dora decided to find out what was going on. She looked back over the 15 days and realized that on one of the days there was a huge field trip. All of the Grade 8 and 9 classes were out at Stanley Park doing science experiments. She knew exactly what to do!

Dora knew she has been the victim of an outlier. Because so many students were away on the day of the field trip, her data was skewed.

How do I find outliers?

To find the outlier, let's look at a graph of Dora's data:



On most days, the number of students buying lunches was about the same… except one. On Day 5, only 120 students bought lunch. How would including this day in our calculation affect our average value?

Dora decided that because Day 5 wasn't a typical day, she wouldn't include it in finding her central value.

Thinking Space

?

When was the field trip?

Help Dora find the mean of her data WITHOUT including the outlier, Day 5, of her data collection. Compare your data to Dora's solution below:

Thinking Space

Data set:	
Sum of values	
Number of values	
Mean $= \dfrac{\text{sum}}{\text{number of values}}$	
Mean without outlier	

Dora's Solution:

Data set:	298, 301, 311, 313, 323, 327, 333, 345, 345, 354, 355, 357, 359, 362
Sum of values	4683
Number of values	14
Mean $= \dfrac{\text{sum}}{\text{number of values}}$	$\text{Mean} = \dfrac{\text{sum}}{\text{number of values}} = \dfrac{4683}{14}$
Mean without outlier	335

Thinking Space

Dora looked at her new mean value. When she compared it to her data she found that her new value was a much better representation of her typical day.

Look at the graph again. Lines have been drawn to represent the 320 mean value and the 335 mean value. Can you see how excluding the outlier gave Dora a better representation of a typical day?

Count the bars that are over our new mean value, then count the bars that are below. The amount above and below are close to being balanced.

The new mean is a good measure of central value.

Outliers affect measures of central tendency. The smaller the set of data, the more an outlier has an affect.

As readers of data, we need to take outliers into consideration. Knowing when to include or exclude values that are not typical will prevent data from being skewed, or misleading.

When you're deciding if a value is an outlier, ask yourself:

Does this value represent a typical scenario?
Does this value represent what would normally happen?

When you are ready, move on to your practice questions identifying outliers and determining their affects.

Practice 1

The last two teams in the playoffs in the 2008 hockey season were the Detroit Red Wings and the Pittsburgh Penguins. The table below summarizes the goals scored by each team in the four rounds of the finals.

Pittsburgh	Team Played	Number of Games	Goals Scored	Average Goals Scored per Game
Round 1	Ottawa	4	16	
Round 2	NY Rangers	5	15	
Round 3	Philadelphia	5	24	
Round 4	Detroit	6	10	

Detroit	Team Played	Number of Games	Goals Scored	Average Goals Scored per Game
Round 1	Nashville	6	17	
Round 2	Colorado	4	21	
Round 3	Dallas	6	17	
Round 4	Pittsburg	6	17	

1. Complete the two tables above.
2. Find the mean, mode, and median of the "average goals scored per game" for each team.

3. Based on these values, who do you think is the stronger team? Explain your answer.

4. Can you identify any outliers that might skew your statistics?

Turn to the Answer Key at the end of the Module and mark your answers.

Section Summary

In this section we explored several ways to describe data sets. We started by looking at range, the difference between the greatest and least value in a set of data. This method of describing data is useful sometimes, but not always.

Sometimes it's better to describe a data set using a measure of central tendency. We learned how to find the median, mode, and mean of a data set, and explored how each method is useful in different situations.

In the last lesson we saw how outliers can affect (or skew) your data.

Section Challenge

Gas Relief!!!

Elan is looking to buy a new car. Because fuel prices have increased so drastically within the past year, she is considering buying a hybrid vehicle. Hybrid vehicles combine the energy from fuel with electricity. There are many cars on the market, so Elan researched all kinds of cars before deciding to take three on a test drive. Use the following data to decide which three cars Elan should consider when purchasing a car.

Car	Mileage in city (L for each 100km)
1 Honda Civic	7.4
2 Honda Civic hybrid	4.7
3 Mazda 3	8.4
4 Toyota Prius hybrid	4.0
5 Toyota Yaris	7.0
6 Camry Hybrid	5.7
7 Saturn Hybrid	8.5
8 Mini Cooper	7.1
9 Chevrolet Cobalt	9.2

1. Find the range of fuel efficiency for the listed cars.
2. Find the mode, median, and mean for the fuel efficiency values.
3. How can you use these values to decide which three cars Elan should consider to test-drive?
4. Given the research and analysis of the data above, would you agree or disagree with the statement that Hybrid vehicles, on average, are better on gas than regular cars

Section 7.3: Probability

Section 3

Contents at a Glance

Learning Outcomes

By the end of this section, you will be better able to:

- describe probability.
- express probabilities as ratios, fractions, and percents.
- identify independent events.
- find the sample space for two independent events using organized lists or tree diagrams.
- compare experimental and theoretical probability.

Pretest 7.3

Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in this section. Mark your answers using the key found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson 7.3A

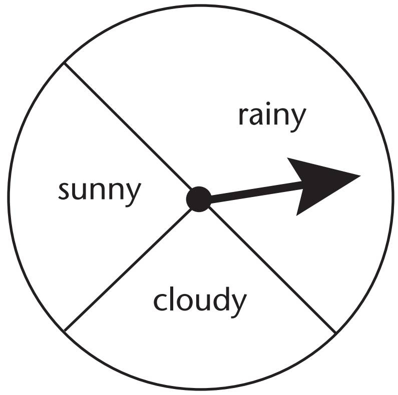
1. Rebecca read that the probability of rain tomorrow was 30%.

    a. Write this probability as a fraction.

    b. Is it more likely or less likely that it will rain tomorrow?

Lesson 7.3B

1. A class predicts the weather using the following spinner:

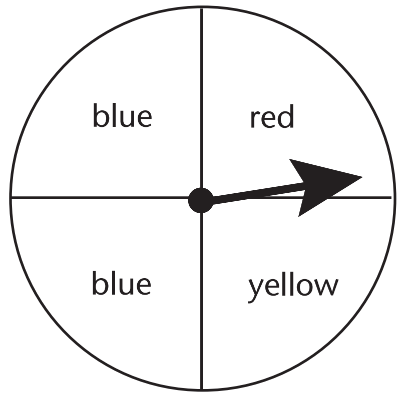


   a. Explain why each spin is an independent event.

   b. Explain why rainy, sunny, and cloudy are not equally likely events.

Lesson 7.3C

Consider an experiment in which you spin the spinner below and flip a coin. What is the sample space for this experiment?



Lesson 7.3D

Roll one die twice.

1. What is the sample space for this experiment?

2. Determine the theoretical probability that the numbers rolled will be doubles.

3. Determine the experimental probability of the outcomes by carrying out an experiment using 36 trials.

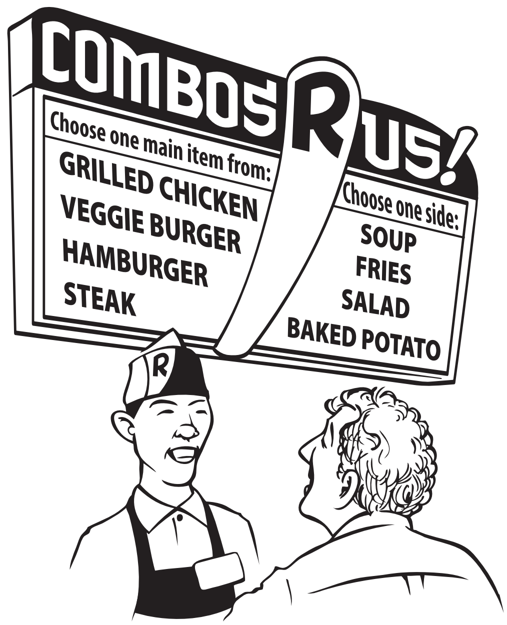
Turn to the Answer Key at the end of the Module and mark your answers.

Section Challenge

You are standing in line waiting to make your order at Combos R Us, a popular restaurant in your neighborhood. As you are waiting, thinking of what to order, you wonder what others might prefer to eat tonight.

You wonder how many meals do these cooks need to know how to make? What are the chances 2 people will order the same meal? Questions flood your head…. you must be hungry!

The specialty combo meal allows you to choose from the following selections:

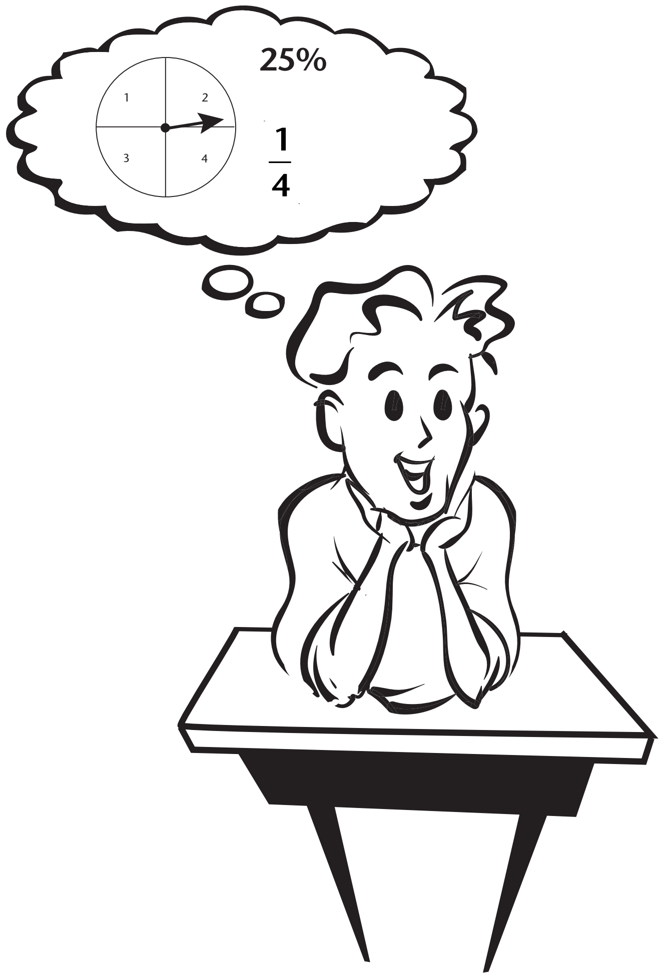


Use the menu above to answer the probability questions below.

1. How many different combos can you choose from at Combos R Us? In other words, what is the sample space for this restaurant's combo menu?
2. What is the probability that the next customer in line will not order French fries as their side?
3. What is the probability that the next customer will order steak and a baked potato?
4. What combination would you choose? What is the probability of your dinner getting chosen by someone else?

Lesson 7.3A: Expressing Probabilities

Student Inquiry



This activity will help you get ready for, learn, and review the information in the upcoming lesson.

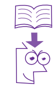
When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
What is probability and how do I find it?		answer
		example
What are the different ways that I can express probability?		answer
		example
What is a probability statement, and how do I determine which one to use?		answer
		example

Lesson 7.3A: Expressing Probabilities

Thinking Space



Introduction

In this lesson you will explore what probability is, and begin to learn the different ways probability can be represented. As well, we will look at probability statements, and how they can help communicate how probable an event actually is.

Explore Online

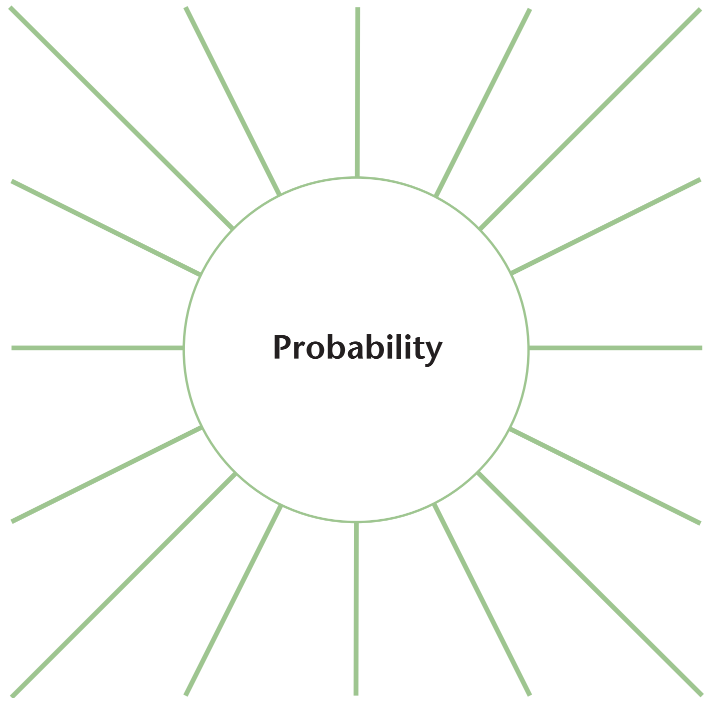
Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:

http://www.openschool.bc.ca/courses/math/math7/mod7.html

Look for *Lesson 7.3A: Expressing Probabilities* and check out some of the links!

Warm-up

1. What do you know about probability? Warm up with knowledge you gathered as you worked through previous modules, grades, or experiences you have had in the world.



2. Draw a picture that represents each fraction.

   a. $\frac{2}{6}$

   b. $\frac{3}{4}$

   c. $\frac{1}{2}$

   d. $\frac{1}{10}$

   e. $\frac{2}{5}$

3. Write a percent for the following fractions.

   a. $\frac{1}{4}$

   b. $\frac{2}{10}$

   c. $\frac{3}{4}$

   d. $\frac{1}{5}$

   e. $\frac{5}{10}$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

What is probability?

Thinking Space

Understanding probability is like understanding chances. Let's use the analogy of a surprise trip to Disneyland as an example to give us a better idea of how probability works.

Imagine you are going on a trip. Your parents have just told you that after all your hard work this year, you get to go to Disneyland the week before Spring Break. This is great! Not only do you get to go to Disneyland, but you get to have an extra week of vacation!

You go to school and tell all your friends. They are a little jealous, because most of them are doing what they usually do for the break: camping, staying home, or heading to the island to visit Grandma, and none of them get to have an extra week of spring break.

You think to yourself, "YES! I am SO lucky!"

Why do you think this? This is probability.

What are the chances that other people around you are going to Disneyland at the same time as you? You're thinking that:

> *It is nearly impossible that other people from my school will get to go to Disneyland the same time as me.*

Well, maybe it's not quite impossible, but it's sure not very likely!

Now imagine it is the day you leave. Your bags are packed, and your little sister is wearing all her favorite Minnie Mouse gear. Once you are at the airport, does the probability of our original question change?

What are the chances now that other people around you are going to Disneyland at the same time as you?

There are probably more people at the airport who are going to Disneyland than at school, so you're wondering:

> *Is it more likely, or less likely that people at the airport are going to Disneyland with me?*

It's more likely.

probably looks similar to probability

Thinking Space

You are sitting in your seat, and you hear the announcer say, "Welcome aboard, our flight today to Los Angeles is on time."

Hmmm, you think. Around you, some people are wearing Buzz Lightyear hats, and others are wearing business suits. You peek up to First Class and think that there might even be a celebrity or two.

Now let's go back to our question: What are the chances that other people around you are going to Disneyland at the same time as you?

What is the probability (or chance) that the people on this plane are going to Disneyland? It's definitely higher than it was at school. The chances were increased at the airport, but now that you are on the plane, chances have gone up even more that people are going to Disneyland. Although not everyone is going:

it is <u>more likely</u> that they are.

The big Disneyland shuttle bus is there to greet you at the door. Everyone in line is wearing a Mickey Mouse hat given to them by the shuttle bus driver. The chances have changed again. <u>Everyone</u> on the bus is going to Disneyland! This is more than school, more than the airport, and more than the airplane.

On the shuttle bus:

it is <u>certain</u> that everyone is going to Disneyland!

It is safe to say that 100% of the people on the shuttle are heading to Disneyland whether they want to or not!

Probability Statements

There were some important words or phrases that were used in the story that are an important part of probability:

- impossible
- less likely
- more likely
- certain

Using these phrases to describe the chances of an event is called a probability statement. We can determine these statements by looking at this continuum.

Thinking Space

0% impossible ← less likely — 50% — more likely → 100% certain

Think back to the story. We know that on the shuttle bus to Disneyland, 100% of the people were heading to Disneyland. But what about the other places: school, the airport, or the airplane?

Probability

Probability is the likelihood or chance of an event occurring. It can be represented as a fraction or a percent. Let's use the Disneyland example to practise finding probability and to create some probability statements.

Using the data given in the chart on the next page, find the probability of people going to Disneyland in each setting. We will use the school setting as an example. Say you went to a school of 250 students.

$$\text{Probability} = \frac{\text{favourable outcome}}{\text{possible outcomes}} = \frac{\text{going to Disneyland}}{\text{total number of kids in the school}} = \frac{1(\text{you!})}{250}$$

The probability of a student from school going to Disneyland would be $\frac{1}{250}$ or 1:250. Expressing this ratio as a percent would look like this: 0.4 %. This is a very small amount.

0% impossible ← less likely — 50% — more likely → 100% certain

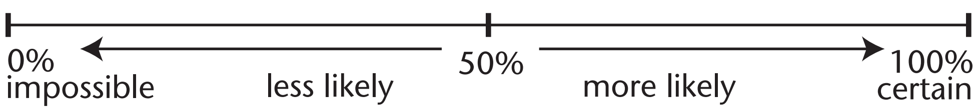
It would be **nearly impossible** for a student to take a week off before spring break and go to Disneyland!

Thinking Space

See if you can find the probability of people going to Disneyland for the rest of the trip. The data for each place is given in the table below. For example, the total number of passengers on the shuttle bus to Disneyland is 35 people.

Place the locations on the continuum after you have found their probability percentage.

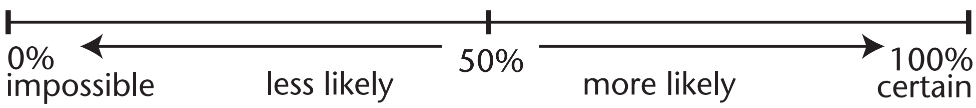
	Favourable Outcome	**Possible Outcome**	**Probability**			
Location	**People Going to Disneyland**	**Total Number of People**	**Ratio**	**Fraction**	**Percent-age**	**Probability Statement**
school	1	250	1:250	$\frac{1}{250}$	0.4%	near impossible
airport at 5:45 am	421	2300				
your airplane to Los Angeles	199	348				
shuttle bus to Disney-land	35	35				



Compare your results to the solutions on the next page.

Location	People Going to Disneyland	Total Number of People	Ratio	Percentage	Probability Statement
school	1	250	1:250	0.4%	near impossible
airport at 5:45 am	421	2300	421:2300	18%	Less likely
your airplane to Los Angeles	199	348	199:348 0.57	57%	More likely
shuttle bus to Disneyland	35	35	35:35	100%	Certain

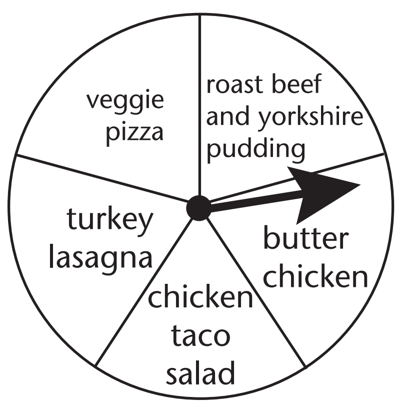
Thinking Space



Representing Probabilities

Let's look at one last example together.

For Mother's Day Jeannette made a spinner that helps her mom decide what's for dinner. The sections are all labeled with her family's favourite food.



For each question express probability in three ways: as a ratio, as a fraction, and as a percentage. Provide a probability statement for each answer.

1. What is the probability of spinning lasagna for dinner?
2. What is the probability of spinning a dinner involving meat?
3. What is the probability of spinning macaroni and cheese for dinner?
4. What is the probability of the family spinning a favourite dinner?

Let's complete these together.

For each question express probability in three ways: as a ratio, as a fraction, and as a percentage. Provide a probability statement for each answer.

1. What is the probability of spinning lasagna for dinner?

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

Favourable outcome: One of the spaces on the spinner says lasagna. There is 1 favourable outcome.

Possible outcomes: There are 5 possible dinners to choose from. There are 5 possible outcomes.

The probability of choosing lasagna for dinner is $\frac{1}{5}$.

Ratio: 1:5

Fraction: $\frac{1}{5}$

Percent: 20%

The P (probability) of choosing lasagna is "less likely" than spinning a dinner that is not lasagna.

Thinking Space

What? Macaroni & cheese wasn't on our list!

!

2. What is the probability of spinning a dinner involving meat?

   Now you try filling in the blanks on this one. If you're not sure, look back at the last one to see how we completed it.

   Favourable:____________________________.

   There is/are ________________.

   Possible: __________________________

   $P = \dfrac{\text{favourable}}{\text{possible}}$

   P =

   Ratio:

   Fraction:

   Percent:

   Did you get 4:5? $\frac{4}{5}$? 80%? If not, go back and check through your solution. There are four dinners with meat out of a total of five possible dinners on the spinner.

   The probability of choosing a dinner with meat is "more likely" than choosing a dinner without meat.

3. What is the probability of spinning macaroni and cheese for dinner?

   Favourable: macaroni and cheese. There is no macaroni and cheese dinner on the spinner. There are 0 favourable outcomes.

   Possible: There are 5 possible dinners.

   $P = \dfrac{\text{favourable}}{\text{possible}}$

   $P = \dfrac{0}{5}$

   Ratio: 0:5

   Fraction: $\dfrac{0}{5}$

   Percent: 0%

   The probability of choosing a dinner with macaroni and cheese is impossible.

Thinking Space

It's impossible, because it's not even on the spinner!

4. What is the probability of the family spinning a favourite dinner?

Favourable: favorite dinner, they are all favorites.
There are 5 favourable outcomes.

Possible: There are 5 possible dinners.

$$P = \frac{\text{favourable}}{\text{possible}}$$

$$P = \frac{5}{5}$$

Fraction: $\frac{5}{5}$

Percent: 100%

The probability of choosing a dinner that is a family favourite is certain.

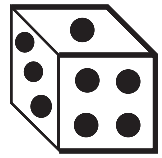
When you are feeling confident, move onto the practice.

Thinking Space

That's why they have a spinner – so they always have a favourite meal.

Practice 1

1. Determine the probability of a die to roll:

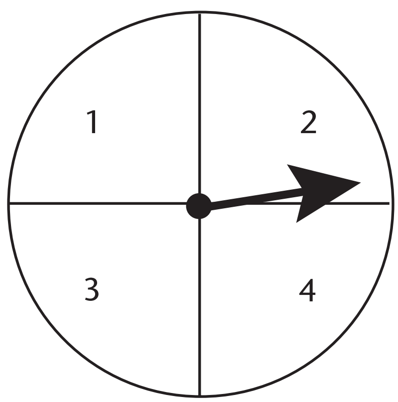


   a. an even number
   b. a 2
   c. a 3 or a 6

   Record each probability as a fraction, ratio, and a percent.

   Determine the probability statement of each event.

2. The numbered spinner below has equal probability that the spinner will land on any section.



   Write the probability of the following events as a fraction and a percent:

   a. spinning 2

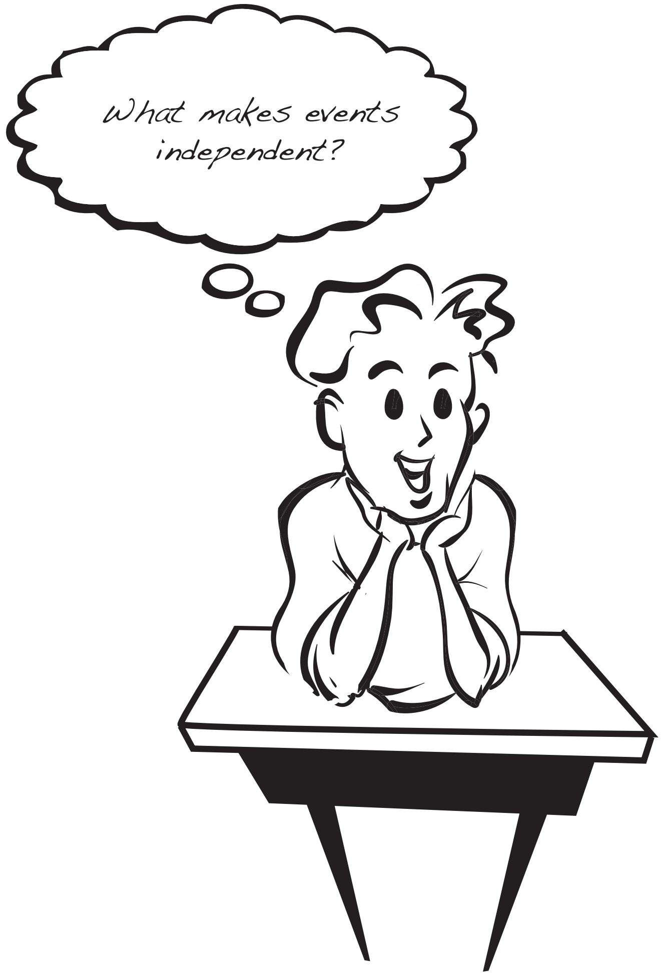
b. spinning 4

c. spinning a 1 or a 3

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 7.3B: Independent Events

Student Inquiry



This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
What are independent events?		answer
		example

Lesson 7.3B: Independent Events

Thinking Space

Introduction

You have worked closely with probability involving single, independent events. Throughout this module, you will be working with two independent events. Before we can move on to probability though, let's review independent events and how to identify them.

Explore Online

Looking for more practice or just want to play some fun games? If you have internet access, go to the Math 7 website at:

http://www.openschool.bc.ca/courses/math/math7/mod7.html

Look for *Lesson 7.3B: Independent Events* and check out some of the links!

Warm-up

Describe the probability of the following events:

1. getting a head when flipping a coin

2. rolling a 1 on a die

3. drawing a heart from a deck of cards

4. turning a year older on your next birthday

5. a turtle growing wings

Turn to the Answer Key at the end of the Module and mark your answers.

Explore
Probability of Independent Events

Thinking Space

Two events are **independent** of each other if the probability of one is not affected by the probability of the other.

There are many examples of probability involving two independent events. One example of this could be in your very own dresser.

Suppose in the top drawer of your dresser you have four shirts. One is red, two are green, and one is purple.

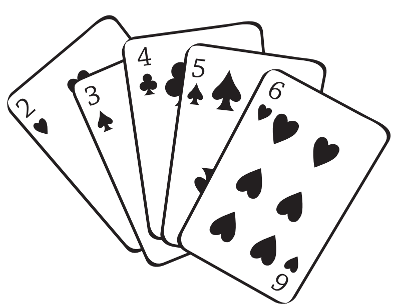
In the second drawer of your dresser you have five pairs of shorts: two pairs of khaki shorts, one pair of jean shorts, and two pairs of board shorts.

In deciding your outfit, will the shirt you pull out affect the shorts you pull out from a different drawer? Other than making sure your outfit matches when you get dressed in the morning, the probability of choosing a shirt and shorts are independent of each other. Or, in other words, choosing one won't change the probability of choosing another. The shirt that you pull out is independent of the shorts, because it won't affect what type of shorts you pull out.

Let's look at some more examples of this concept.

Examine these two experiments and determine if they are independent events or not.

You have cards numbered 2 to 6.



Experiment #1	Experiment #2
Draw a card, then return it to the pile before drawing another.	Draw a card, then draw another without putting the first one back.

Thinking Space

Which experiment involves independent events? We know that if an event is independent, it does not affect the probability of the next. In this example, the event is drawing a card; so, if the event is independent, each draw should have the same probability.

Experiment #1

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

The probability of drawing any of the cards is $\frac{1}{5}$ or 20% because there is only one of each card. If we put the card back into the deck, this probability does not change. There is still a 20% chance than any of the cards will be chosen randomly.

Compare that to Experiment #2.

Experiment #2

The first card we draw will have the same probability as above. 20%, or $\frac{1}{5}$. In the second draw, however, the probability changes. Do you know why?

Let's look at our ratio again:

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

Since we don't put the card back, the possible amount of cards left in the pile is now four. Our new ratio is $\frac{1}{4}$ or 25%.

In this experiment, the events are *not* independent. Why? The first draw changes the probability of the second draw.

So are events that are not independent of each other called dependent events?

The question to ask yourself is:

Does the probability change from one trial to the next?

Thinking Space

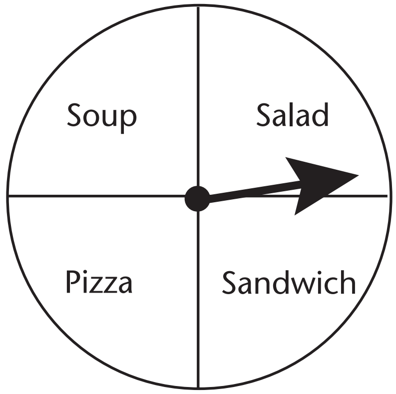
Being confident with the probability ratio will help you answer this question. Remember:

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

I think you are ready to try finding probability involving two independent events. Before you go, try moving on to the practice section and practise identifying independent events.

Practice 1

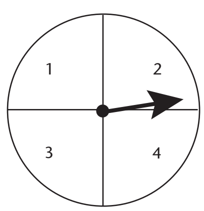
1. Take a look at this spinner below. If you used this spinner to decide your lunch for the next two days, would it be independent if:



   a. You eliminated the choices once you spun and had that lunch?

   b. You had the lunch that was spun, even if you had it the day before?

2. A spinner and a coin are tossed. Conducting an experiment with a spinner and a coin involves independent events. Why?



3. Design an experiment involving:

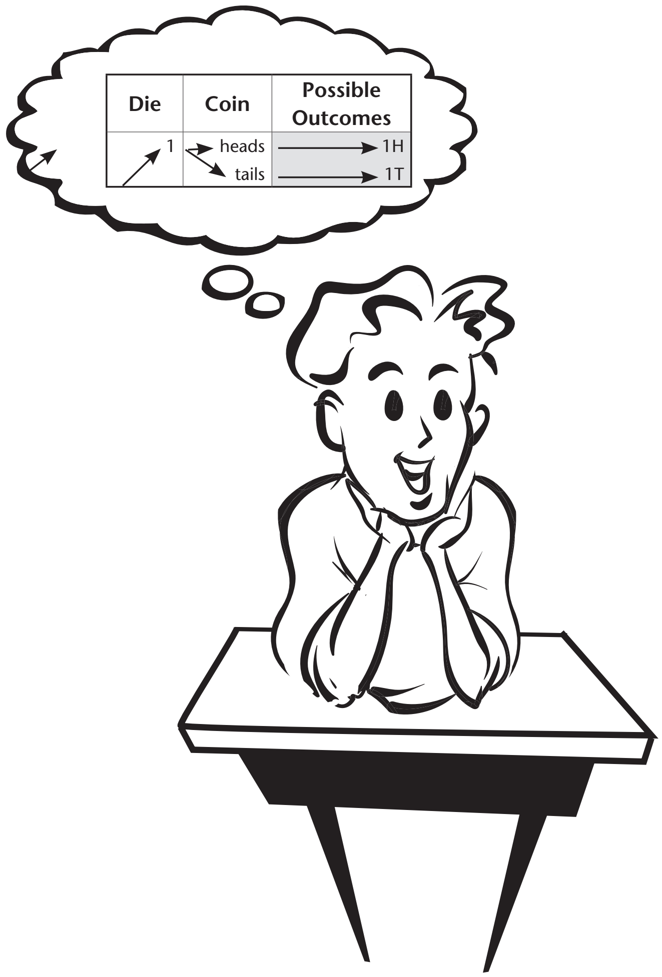
   a. a single event

   b. 2 independent events

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 7.3C: Sample Space

Student Inquiry



This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
What is sample space, and how do I find it?		answer
		example
What are some organizational strategies I can use to help me find sample space?		answer
		example

Lesson 7.3C: Sample Space

Thinking Space

Introduction

We already know that two events are independent of each other if the probability of one is not affected by the probability of the other. Before we can determine the probability of these events, however, we need to figure out how many possible outcomes there are.

When we work through problems with two independent events, our data sets can become very large! Having strategies to organize all of our information is crucial to solving probability problems. In this lesson we'll look at ways to organize our information, making it easy to find probabilities.

Explore Online

Looking for more practice or just want to play some fun games? If you have internet access, go to the Math 7 website at:

http://www.openschool.bc.ca/courses/math/math7/mod7.html

Look for *Lesson 7.3C: Sample Space* and check out some of the links!

Warm-up

Unscramble the words below based on the vocabulary you've learned so far in this section. Once the terms are unscrambled, make sure you know what each means. If there are any terms that you are unsure of, please review the glossary.

1. RPOLIBYAIBT

2. AIXMLPEETNRE

3. EERTAHTIOLC

4. ELIBMPOSSI

5. PETTNSDAEEEDVNNNI

6. ASEPEPLSACM

7. IRTDAAEMEGR

8. NZOIIREDSTGLA

Turn to the Answer Key at the end of the Module and mark your answers.

Explore
Sample Space

Thinking Space

If we remember that

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

we know that in order to solve these new types of problems, we need to find each of these values.

Once we have organized the outcomes, we can find the favourable and possible outcomes, and continue solving our probability problems.

Our organizational charts help us list all of the possible outcomes. Another term used to describe the set of possible outcomes is the sample space. **Sample space** is a list of all the possible outcomes of an experiment.

There are different ways to find sample space. Two examples that we'll explore are:

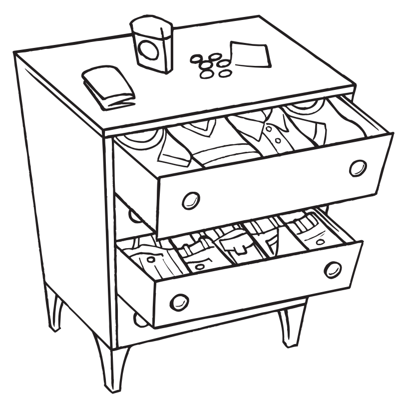
- organized lists
- tree diagrams

Probability and Sample Space Using Organized Lists

Let's use the example of the clothes in the drawers from the earlier lesson to find the sample space using an organized list.

The top drawer of your dresser has four shirts: one is red, one is green, one is blue, and one is purple.

In the second drawer of your dresser, you have three pairs of shorts: one pair of khaki shorts, one pair of jean shorts, and one pair of board shorts.



To create an organized list, we place all the possible outcomes for one independent event on the top (the short drawer), and the outcomes for the other independent event on the left of the chart like this (the shirt drawer):

	Khaki Short	**Jean Short**	**Board Short**
Red Shirt			
Green Shirt			
Blue Shirt			
Purple Shirt			

Next we find all the combinations by combining each square, similar to a times-tables chart, but instead of numbers, we are building outfits.

	Khaki Short	**Jean Short**	**Board Short**
Red Shirt	Red, Khaki	Red, Jean	Red, Board
Green Shirt	Green, Khaki	Green, Jean	Green, Board
Blue Shirt	Blue, Khaki	Blue, Jean	Blue, Board
Purple Shirt	Purple, Khaki	Purple, Jean	Purple, Board

These 12 combinations are our sample space. These are the **total possible outcomes** or combinations for the outfits that these shirts and these shorts could make.

Sample space is important in finding probability. The size of the sample space is the number of possible outcomes.

Let's practise finding the sample space using an organized list for another example involving two independent events.

Thinking Space

the probability ratio is:

$$\text{Probability} = \frac{\text{favourable outcome}}{\text{possible outcomes}}$$

Rolling two dice is an example of an experiment with two independent events. Find the sample space if you roll two dice and add the results.

Step 1: Build the table.

		Die 1					
		1	2	3	4	5	6
Die 2	1						
	2						
	3						
	4						
	5						
	6						

Along the top row, put in all the possible outcomes from rolling Die 1.

In this column, put in all the possible outcomes from rolling Die 2.

Step 2: Fill in the sample space.

In this example, since we're rolling dice, we'll use numbers. For each box, find the sum of the two dice you rolled.

		Die 1					
		1	2	3	4	5	6
Die 2	1						
	2						
	3						
	4						
	5						
	6						

To fill this box, take your outcome from Die 1 and add it to your outcome from Die 2.

1 + 1 = 2

Put a "2" in this box.

Your completed chart should look like this:

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

The highlighted area is the sample space.

Great, now let's try finding sample spaces using tree diagrams.

Thinking Space

die = 1

dice= more than 1

Have you ever played a game where you had to roll 2 dice? How did you know how many spaces to move on the board?

Probability and Sample Space Using Tree Diagrams

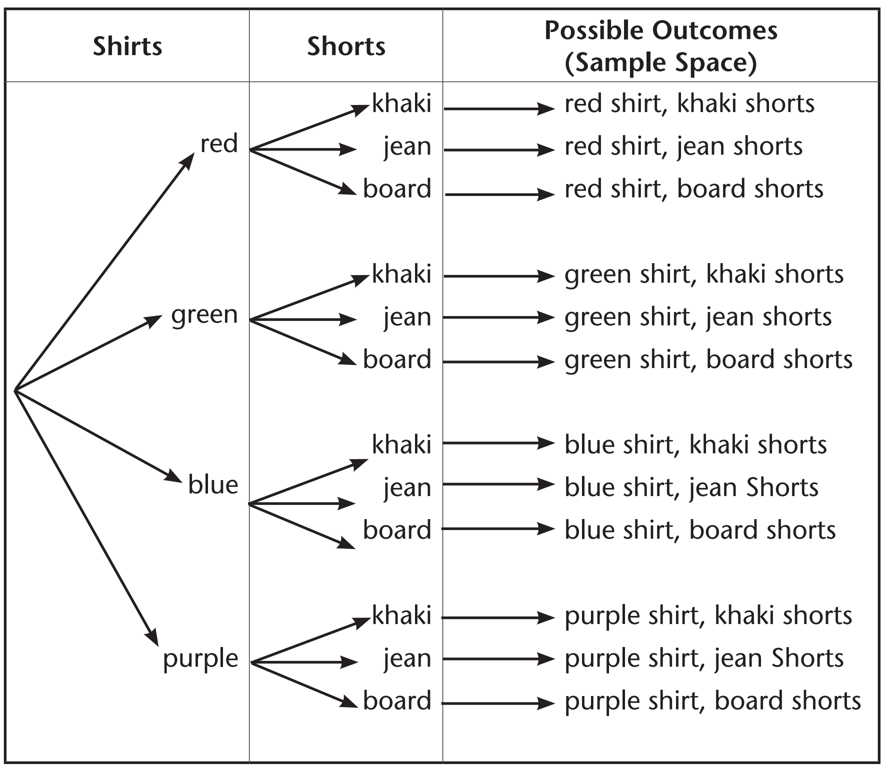
To illustrate how to use a tree diagram, let's go back to the dresser with the clothes!

The top drawer of your dresser has four shirts: one is red, one is green, one is blue, and one is purple.

In the second drawer of your dresser, you have three pairs of shorts: one pair of khaki shorts, one pair of jean shorts, and one pair of board shorts.

Arrange the sample space to illustrate the total possible outcomes or outfits.

Here's an example of a tree diagram:



It might look complicated, but if you follow these steps, you'll be making your own tree diagrams in no time!

When drawing a tree diagram, we need a little more space than when working with an organized list.

Thinking Space

!

Why do you think it's called a 'tree diagram?

How to Draw a Tree Diagram

Step 1: Draw a box with 3 columns like this:

Step 2: Fill in the columns.

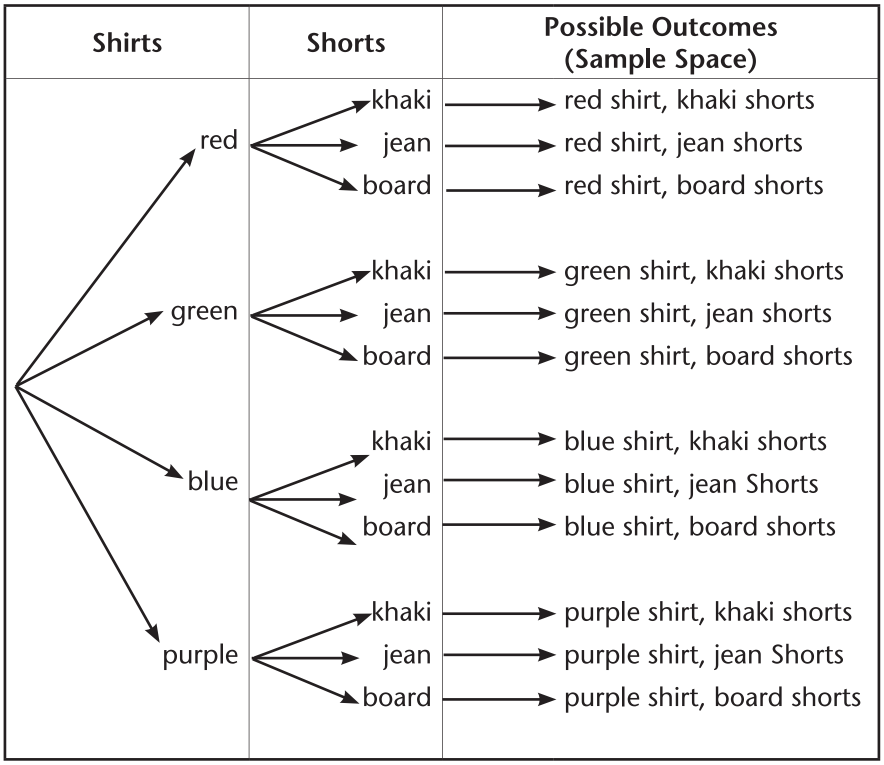
Column 1: independent event #1 outcomes

Column 2: independent event #2 outcomes for each outcome in Column 1

Column 3: possible outcomes: this is the **sample space**

Shirts	Shorts	Possible Outcomes (Sample Space)
	khaki	
red	jean	
	board	
	khaki	
green	jean	
	board	
	khaki	
blue	jean	
	board	
	khaki	
purple	jean	
	board	

Step 3: Draw the tree and the outcomes.



Thinking Space

Now you try one. Find the sample space by creating a tree diagram. Compare your results to the solutions below.

A die is rolled and a coin is flipped. They are independent of each other because the rolling of the die does not affect the flipping of the coin, nor does the coin affect the die. Make a tree chart to find the sample space.

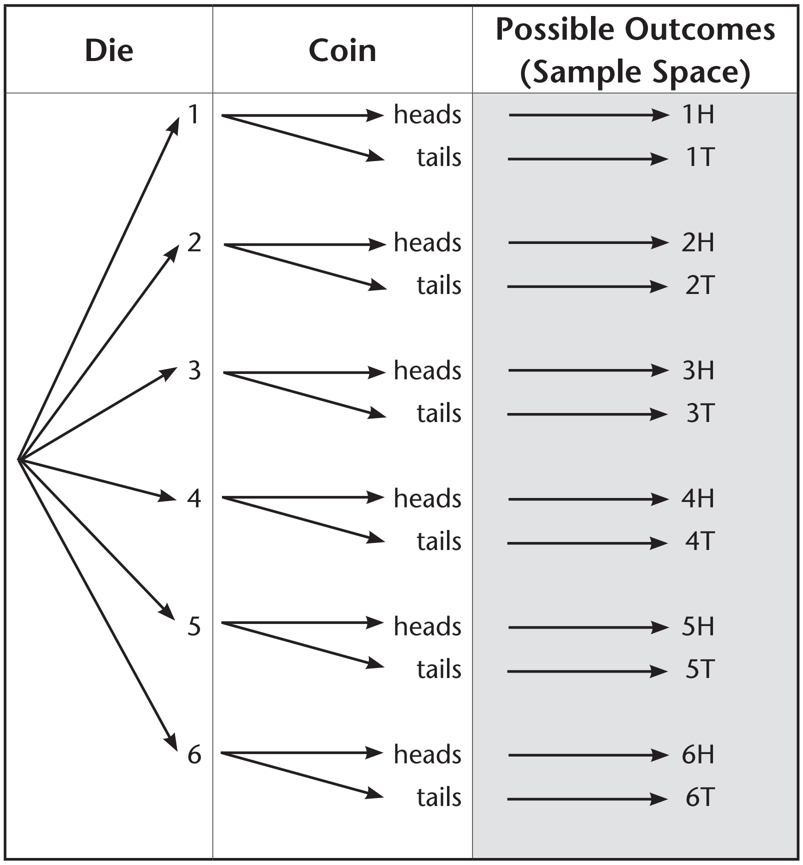
Step 1: Draw a box with 3 columns.

Step 2: Fill in the columns.

Step 3: Draw the tree and the outcomes.

Step 4: Highlight the total possible outcomes or sample space.

Solution:



You can now easily see that the sample space is: 1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T.

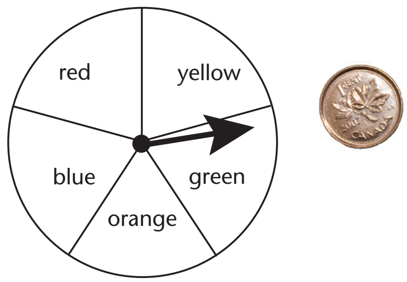
Now that you have tried both methods, which one do you like the best?

Hint: Finding the sample space in this example will help you complete the section challenge at the end of the section.

Are you ready to practise on your own? Try the practice questions to make sure you're on the right track.

Practice 1

1. Find the sample space for an experiment involving flipping a coin and spinning the spinner below.



a. using an organizational list

b. using a tree diagram

2. Find the sample space of this experiment. Choose your method of solving either by using an organized list or a tree diagram.

   It's Saturday night and Will's family is ordering food and renting a DVD. They like pizza, sushi, and hamburgers. They will rent a comedy, a drama, an action movie, or a TV series.

   What are the different dinner/DVD combinations? In other words, what's the sample space.

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 7.3D: Experimental Probability

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
What is theoretical probability?		answer
		example
What is experimental probability?		answer
		example
What is the relationship between the two?		answer
		example

Lesson 7.3D: Experimental Probability

Introduction

Have you ever performed a science experiment? In a science experiment, before you perform the actual experiment, you usually make a prediction about what is going to happen. In science, we call this prediction a hypothesis. In a probability experiment a prediction is called theoretical probability.

What? Did you think that experiments were just for scientists? No way! We also perform experiments in math. In fact, this lesson is all about math experiments. Let's take a look…

Explore Online

Looking for more practice or just want to play some fun games? If you have internet access, go to the Math 7 website at:

http://www.openschool.bc.ca/courses/math/math7/mod7.html

Look for *Lesson 7.3D: Experimental Probability* and check out some of the links!

Warm-up

1. There are 5 pairs of socks in your drawer. Two pairs have holes in them.

   What's the probability of choosing a pair of socks <u>without</u> holes.

2. Roll a die.

   a. What's the probability of rolling a 3?

   b. What's the probability of rolling an odd number?

   c. What's the probability of rolling a 7?

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Theoretical Probabilities versus Experimental Probabilities

Thinking Space

Before we use experiments to test probabilities, there are some terms we should know and understand.

When experimenting with probability, a prediction is called **theoretical probability**. Theoretical probability tells us about the likelihood that an event will occur. We can figure out the theoretical probability using the following ratio that we are already familiar with:

$$\text{Theoretical Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

!

Oh! So the 'probability' we've been talking about all along is actually called 'theoretical probability'.

Theoretical probability is different from **experimental probability**. They may sound similar, but it's important to recognize and understand their differences.

!

Until this point, we have been able to figure out probabilities without doing any experiments. This is because we have been making predictions.

> **Theoretical probability** is determined BEFORE an experiment, to help us predict an outcome.
>
> **Experimental probability** is determined AFTER an experiment to test and compare the results to the predictions made earlier.

The predictions, *theoretical probabilities*, help us to compare the actual results, the *experimental probabilities*.

Let's practise finding theoretical probabilities before moving on to performing experiments.

Theoretical Probabilities

Thinking Space

You have already mastered the first two steps in determining theoretical probabilities:

- organize the data (by using organized list or tree diagram)
- find the sample space

Let's use some of our earlier examples to explore further.

Remember the clothes-in-the-drawer example? This was the sample space we created.

	Khaki Short	**Jean Short**	**Board Short**
Red Shirt	Red, Khaki	Red, Jean	Red, Board
Green Shirt	Green, Khaki	Green, Jean	Green, Board
Purple Shirt	Purple, Khaki	Purple, Jean	Purple, Board

We'll use the sample space to answer the question below.

> What is the probability of randomly pulling a red shirt and khaki shorts out of the two drawers?

Remember: the theoretical probability ratio states that:

$$\text{Theoretical Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

We already know the sample space is the set of total possible outcomes. By looking at our organized list, we see that there are 9 possible outfits.

The number of possible outcomes is 9.

$$\text{Theoretical Probability} = \frac{\text{favourable outcomes}}{9}$$

Now we just need to figure out the favourable outcomes.

From the table we see that there is only one box that contains the combination red shirt and khaki shorts. There is one favourable outcome in this example.

$$\text{Theoretical Probability} = \frac{1}{9} = 11\%$$

Let's try another example together using a sample space we have already created.

Here is the sample space of the example we used in the previous lesson.

A coin is flipped and a die is rolled. They are independent of each other because the rolling of the die and the flipping of the coin do not affect each other.

Die	Coin	Possible Outcomes (Sample Space)
1	heads	1H
	tails	1T
2	heads	2H
	tails	2T
3	heads	3H
	tails	3T
4	heads	4H
	tails	4T
5	heads	5H
	tails	5T
6	heads	6H
	tails	6T

Let's use the sample space to answer this question.

> What is the theoretical probability that you will roll an even number and flip heads?

Thinking Space

Do you remember how to write $\frac{1}{9}$ as a percent?

Remember:

$$\text{Theoretical Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

Thinking Space

The sample space shows that there are 12 possible outcomes.

$$\text{Theoretical Probability} = \frac{\text{favourable outcomes}}{12}$$

There are 3 possible outcomes in the sample space that are favourable outcomes.

roll 2, flip head (2H)
roll 4, flip head (4H)
roll 5, flip head (6H)

$$\text{Theoretical Probability} = \frac{3}{12} = \frac{1}{4} = 25\%$$

Now that we've explored theoretical probability, let's talk about experimental probability.

Experimental Probabilities

Experimental probabilities are determined AFTER an experiment has occurred. Let's do an experiment!

For this experiment we are going to find theoretical and experimental probability when pulling coloured chips out of a bag and rolling a die. These two events are independent of each other. Once you determine your predictions (theoretical probability), you will perform the actual experiment, or see how close your results were to the theoretical probability.

For this experiment you will need:

- a container or bag that you can't see through
- 1 die
- 4 different colours of crayons or markers (green, red, blue, yellow)
- 12 chips from the 'Chips' template
- the "Trial Chart" template found at the back of this module

Go now and get the "Chips" and "Trial Chart" pages from the Template section at back of this module and continue with the instructions that follow.

Colour 12 chips on your template as follows:

- 6 green
- 3 blue
- 2 red
- 1 yellow

Cut out the chips you coloured and place them in the container or bag.

In this experiment, we pull chips out of a bag and roll a die. We determine the theoretical probability BEFORE doing our experiment. After we do our experiment, we can compare it to our actual results.

These are the questions we want to be able to answer at the end of our experiment:

1. What is the probability of rolling an odd number and pulling out a green chip?
2. What is the probability of rolling a 6 and pulling out a yellow chip?
3. What is the probability of rolling a 1 and pulling out a blue or a red chip?

Conclusion:

4. How does experimental probability compare to theoretical probability after 20 trials? After 50 trials?

Thinking Space

Finding the theoretical probability in this experiment is like writing a hypothesis in a science experiment.

Our first step in finding theoretical probability is finding our sample space:

> You can find the sample space of this experiment by using either a tree diagram or an organized list.

Thinking Space

Sample Space:

	1	2	3	4	5	6
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Blue	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**
Blue	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**
Blue	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**
Red	**R1**	**R2**	**R3**	**R4**	**R5**	**R6**
Red	**R1**	**R2**	**R3**	**R4**	**R5**	**R6**
Yellow	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**

Now that we have our sample space, we can make predictions about the theoretical probability of each question above. How many possible outcomes are there?

Our probability ratio is:

$$\text{Theoretical Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

Whatever the question asks us to find is what's "favourable." Let's look at number one:

Thinking Space

1. What is the probability of rolling an odd number and pulling out a green chip?

The favourable outcomes in this question are outcomes that are green and odd. Look at our sample space. Let's highlight all the combinations that are green and odd.

	1	2	3	4	5	6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Blue	B1	B2	B3	B4	B5	B6
Blue	B1	B2	B3	B4	B5	B6
Blue	B1	B2	B3	B4	B5	B6
Red	R1	R2	R3	R4	R5	R6
Red	R1	R2	R3	R4	R5	R6
Yellow	Y1	Y1	Y3	Y4	Y5	Y6

There are 18 favourable outcomes.

There are 48 possible outcomes in our sample space.

The probability for question 1 is:

$$\frac{\text{favourable}}{\text{possible}}$$
$$= \frac{18}{72} = \frac{1}{4}$$
$$= 0.25$$
$$= 25\%$$

There is a 25% probability that a green chip will be pulled and an odd number will be rolled.

Are you ready to try? Find the theoretical probability for question #2.

2. What is the probability of rolling a 6 and pulling out a yellow chip?

   Using the same sample space, highlight all the combinations involving a 6 and a yellow chip.

	1	2	3	4	5	6
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Green	**G1**	**G2**	**G3**	**G4**	**G5**	**G6**
Blue	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**
Blue	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**
Blue	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**
Red	**R1**	**R2**	**R3**	**R4**	**R5**	**R6**
Red	**R1**	**R2**	**R3**	**R4**	**R5**	**R6**
Yellow	**Y1**	**Y1**	**Y3**	**Y4**	**Y5**	**Y6**

   Great, now complete your probability ratio:

$$\frac{\text{favourable}}{\text{possible}}$$

$$=$$

   The probability of rolling a 6 and pulling out a yellow chip is ________%.

   Make sure your solution matches:

$$\frac{\text{favourable}}{\text{possible}}$$
$$= \frac{1}{72}$$
$$= 0.013\overline{8}$$
$$\cong 1.4\%$$

Great! Now find the theoretical probability of question 3.

Thinking Space

3. What is the probability of rolling a 1 and pulling out a blue or a red chip?

Using the same sample space, highlight all the combinations involving 1 and a blue chip.

Then, highlight all the combinations involving 1 and a red chip.

	1	2	3	4	5	6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Green	G1	G2	G3	G4	G5	G6
Blue	B1	B2	B3	B4	B5	B6
Blue	B1	B2	B3	B4	B5	B6
Blue	B1	B2	B3	B4	B5	B6
Red	R1	R2	R3	R4	R5	R6
Red	R1	R2	R3	R4	R5	R6
Yellow	Y1	Y1	Y3	Y4	Y5	Y6

Great, now complete your probability ratio:

$$\frac{\text{favourable}}{\text{possible}}$$
$$=$$

The probability of rolling a 1 and pulling out a blue or a red chip is _______%.

Make sure your solution matches:

$$\frac{\text{favourable}}{\text{possible}}$$
$$= \frac{5}{72}$$
$$= 0.069\overline{4}$$
$$= 6.9\%$$

Thinking Space

?

What do I do when it says blue OR red chip?

Thinking Space

To sum it up, our theoretical probability predictions are as follows:

There is a 38% probability that a green chip will be pulled and an odd number will be rolled.

There is a 2% probability of rolling a 6 and pulling a yellow chip.

There is a 10% probability of rolling a 1 and pulling out a blue or a red chip.

Great job! Now do the actual experiment.

Experiment #1

Pull out 20 chips randomly from the bag. Each time you pull out a chip, roll the die. Record your results on the trial chart you got from the templates at the back of the module. So, for example, if you pull a green chip and roll a 4, your data collection would look like this:

Trial	1	2	3	4
Chip	Green			
Die	4			

Make sure you put the chip that you pulled back in the bag. Our events have to be independent.

Experiment #2

Do the same experiment again, but this time do 50 trials. Use the trial chart in the templates at the back of this module.

Are you tired? I hope not—we still have work to do! Now that we have our actual data, let's answer the questions using experimental probability.

The ratio for experimental probability is:

$$\text{Experimental Probability} = \frac{\text{favourable outcomes}}{\text{number of trials}}$$

Thinking Space

The part of the ratio that is different from when we calculated theoretical probability is the denominator. When we calculated *Theoretical Probability*, the denominator was the number of possible outcomes.

When we calculate *Experimental Probability*, the denominator is the the number of trials.

You will find the experimental probability based on the data YOU collected.

Our collected data is our new sample space. We use it to find and highlight the favourable outcomes just like we did when determining theoretical probability.

Experiment #1

1. What is the probability of rolling an odd number and pulling out a green chip?

   Go through your trials and circle with green all the favourable outcomes for question 1. You are looking for all the trials in which you rolled an odd number and pulled a green chip.

$$\text{experimental probability} = \frac{\text{favourable outcomes}}{\text{number of trials}}$$

$$EP = \frac{\underline{\qquad}}{20 \text{ trials}}$$

$$EP = \frac{\underline{\quad}}{20}$$

$$EP = \underline{\qquad}\%$$

2. What is the probability of rolling a 6 and pulling out a yellow chip?

   Go through your trials and circle with yellow all the favourable outcomes for question 2. You are looking for all the trials in which you rolled a 6 and pulled a yellow chip.

$$\text{experimental probability} = \frac{\text{favourable outcomes}}{\text{number of trials}}$$

$$EP = \frac{\underline{\qquad}}{20 \text{ trials}}$$

$$EP = \frac{\underline{\quad}}{20}$$

$$EP = \underline{\qquad}\%$$

3. What is the probability of rolling a 1 and pulling out a blue or a red chip?

Thinking Space

$$\text{experimental probability} = \frac{\text{favourable outcomes}}{\text{number of trials}}$$

$$EP = \frac{}{20 \text{ trials}}$$

$$EP = \frac{}{20}$$

$$EP = \underline{}\%$$

Experiment #2

Now do the same for Experiment #2. This time the “total trials” is 50.

1. What is the probability of rolling an odd number and pulling out a green chip?

$$\text{experimental probability} = \frac{\text{favourable outcomes}}{\text{number of trials}}$$

$$EP = \frac{}{50 \text{ trials}}$$

$$EP = \frac{}{50}$$

$$EP = \underline{}\%$$

2. What is the probability of rolling a 6 and pulling out a yellow chip?

$$\text{experimental probability} = \frac{\text{favourable outcomes}}{\text{number of trials}}$$

$$EP = \frac{}{50 \text{ trials}}$$

$$EP = \frac{}{50}$$

$$EP = \underline{}\%$$

Thinking Space

3. What is the probability of rolling a 1 and pulling out a blue or a red chip?

$$\text{experimental probability} = \frac{\text{favourable outcomes}}{\text{number of trials}}$$

$$EP = \frac{\_\_\_\_\_\_}{50 \text{ trials}}$$

$$EP = \frac{\_\_}{50}$$

$$EP = \_\_\_\%$$

Conclusion

Now we'll compare our results to the theoretical probabilities (predictions) that we calculated earlier in the lesson, to answer question 4.

4. How does experimental probability compare to theoretical probability after 20 trials? (**Hint**: use the percentages.)

Experiment #1		
	Theoretical	Experimental
1. odd, green	25%	
2. 6, yellow	1.4%	
3. 1, blue or red	6.9%	

After 50 trials?

Experiment #2		
	Theoretical	Experimental
1. odd, green	25%	
2. 6, yellow	1.4%	
3. 1, blue or red	6.9%	

Was experimental probability closer to theoretical probability in Experiment #1 (20 trials) or Experiment #2 (50 trials)?

Could you make a prediction about how it might change again after 100 trials, or 1000 trials?

Did you notice that the gap between the theoretical and the experimental probabilities got smaller as the number of trials increased?

In conclusion, our answer to number 4 is:

As the experimental trials increase, the theoretical and the experimental probabilities get closer together.

You did great working through this experiment! Try practising now, using all the skills you learned about probability throughout this section.

Thinking Space

Practice 1

1. Use a chart to figure out this sample space.

   Rolling 2 dice is an example of independent events. If you roll 2 dice:

   a. What is the theoretical probability of rolling a sum of 8?

		Roll 1					
		1	**2**	**3**	**4**	**5**	**6**
Roll 2	1						
	2						
	3						
	4						
	5						
	6						

   b. What is the theoretical probability of rolling a difference of 3? You'll need to make a new chart of your sample space to answer this question.

		Roll 1					
		1	**2**	**3**	**4**	**5**	**6**
Roll 2	1						
	2						
	3						
	4						
	5						
	6						

   c. What is the theoretical probability of rolling a sum of 7?

		Roll 1					
		1	**2**	**3**	**4**	**5**	**6**
Roll 2	1						
	2						
	3						
	4						
	5						
	6						

2. Each morning, Duncan chooses a job from the job jar to help his mom out with the chores. After each morning, he puts the chore back for the next day.

   After a week Duncan recorded the chores he pulled from the jar. These were his results:

   Monday—walk dog

   Tuesday—make bed

   Wednesday—make bed

   Thursday—take out garbage

   Friday—walk dog

   Saturday—load dishwasher

   Sunday—take out garbage

   a. There is one event in this experiment. "Duncan chooses a job from the job jar.

      Determine the sample space of this experiment.

b. What is the theoretical probability of Duncan walking the dog?

c. Last week, Duncan did 7 trials of this experiment. What was the experimental probability of Duncan walking the dog? Round your answer to the nearest whole number.

Turn to the Answer Key at the end of the Module and mark your answers.

Section Summary

In this section we took a look at the likelihood of certain events occurring—in other words, probability. You learned how to express probabilities as ratios, fractions and percents.

We know that events don't always happen one at a time, so we started looking at the probability of two events happening. The events we looked at in this section were independent. Two events are independent when they have no effects on each other. In order to find the theoretical probability for two independent events, you need to figure out what all the possible outcomes are. We call this the sample space. In this section we looked at two possible ways to find the sample space: organized lists and tree diagrams.

You also learned that there are different kinds of probability.

Theoretical probability is the prediction you make about the likelihood of an event occurring, based on the possible outcomes.

$$\text{Theoretical Probability} = \frac{\text{\# of favourable outcomes}}{\text{\# of possible outcomes}}$$

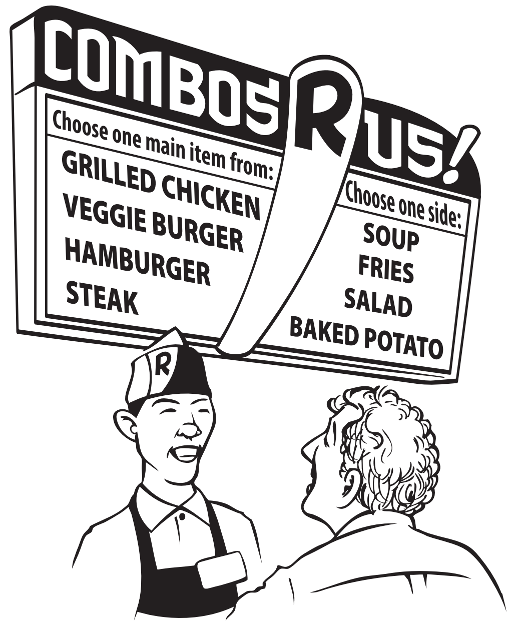
Experimental probability is based on the actual results of a probability experiment. We calculate experimental probability using the same formula as above. In this case, we use the actual number of times an event occurred as the favourable outcome, and the number of trials in the experiment is our total possible outcomes.

$$\text{Experimental Probability} = \frac{\text{\# of favourable outcomes}}{\text{\# of possible outcomes}} = \frac{\text{\# of favourable outcomes}}{\text{\# of trials}}$$

Theoretical and experimental probabilities can be calculated for one event, or for multiple events.

Section Challenge

Now that we have completed this section, we have the skills needed to solve our introductory problem. Try answering the questions. Compare your results to the solutions below.



You are standing in line waiting to make your order at a popular restaurant in your neighborhood. As you are waiting thinking what to order, you wonder what others might prefer to eat tonight.

How many meals do these cooks have to know how to make? What are the chances of two people ordering the same meal? Questions flood your head….you must be hungry!

The specialty combo meal allows you to choose from the following selections:

Use the menu above to answer these probability questions.

1. How many different combos can you choose from at Combos R Us? In other words, what is the sample space for this restaurant's combo menu?
2. What is the probability that the next customer in line will not order French fries as their side?
3. What is the probability that the next customer will order steak and a baked potato?
4. What combination would you choose? What is the probability of your dinner getting chosen by someone else?

Answer Key Table of Contents

Answer to Pretest 7.1

1. a. 7%

   b. 92%

   c. How teachers get to school

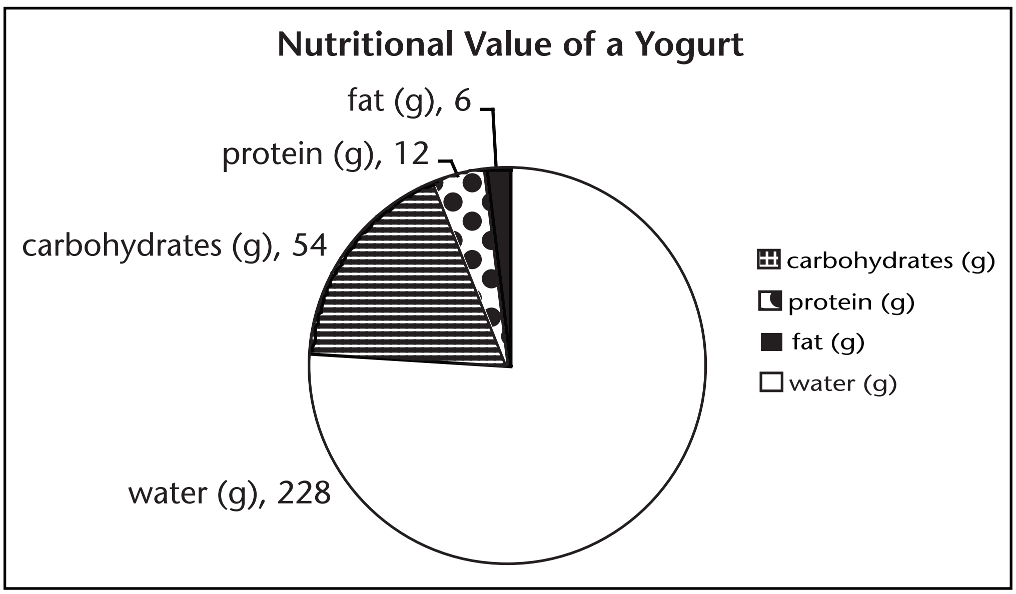
Lesson 7.1B

1. a. 11% of 5000 = 550

   b. % 0f 5000 = 350

Lesson 7.1C

1. a. Display the data in a circle graph.



   b. 2%

Answer to Lesson 7.1A Practice 1

1. How many times have were students bullied at school in the last month? (3205 BC students surveyed)

2. number of times 0

   number of times 1 to 3

   number of times 4 to 9

   number of times 10 more

3. number of times 0
4. 100%
5. 3205
6. 4%, 4%, 21%, 71%

Answer to Lesson 7.1B Warm-up

1.

a. $0.2 = \frac{2}{10}$

b. $0.34 = \frac{34}{100}$

c. $0.09 = \frac{9}{100}$

2.

a. $\frac{3}{4} = 75\%$

b. $\frac{9}{12} = 75\%$

c. $\frac{5}{8} = 63\%$

d. $\frac{12}{22} = 55\%$

e. $\frac{34}{51} = 67\%$

3.

a. 0.21

b. 0.56

c. 0.04

d. 0.71

e. 0.01

4.

a. $0.4 = 40\%$

b. $20\% < \frac{1}{4}$

c. $\frac{2}{10} < 5.3$

d. $41\% < 4.1$

Answer to Lesson 7.1B Practice 1

1. Step 1: 3205 students

   Step 2: The graph displays what 3205 students usually eat for breakfast.

   Step 3: Students who ate grain products are 36% of 3205 students.

   Students who ate milk products are 26% of 3205 students.

   Students who ate fruits or vegetables are 14% of 3205 students.

   Students who ate meat products are 12% of 3205 students.

   Students who did not have breakfast are 5% of 3205 students.

   Students who had other breakfast are 7% of 3205 students.

2. Step 1: 250 students

   Step 2: This graph measures the eye colour of 250 students.

   Step 3: Students with brown eyes are 44% of 250 students.

   Students with blue eyes are 29% of 250 students.

   Students with green eyes are 16% of 250 students.

   Students with another eye colour are 11% of 250 students.

Answer to Lesson 7.1B Practice 2

1. a. right

   b. 8% of 578

   $0.08 \times 578 = 46.24$

   Approximately 46 students out of the 578 surveyed are ambidextrous.

2. a. 4–6 people

   b. 20% of 33 620 = 6724

   c. 5% + 74% + 20% = 99% of students have less than 10 people in their household

   99% of 33 620 = 33 283.6

   Approximately 33 284 students have less than 10 people in their household.

Answer to Lesson 7.1C Warm-up

1. a. 45%

   b. 50%

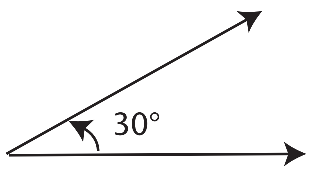
   c. 40%

2. a. 30°

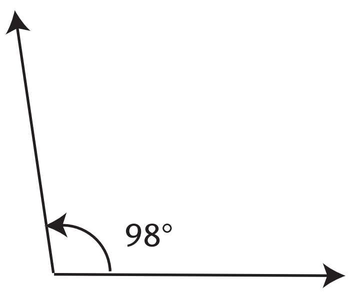
   b. 85°

   c. 190°

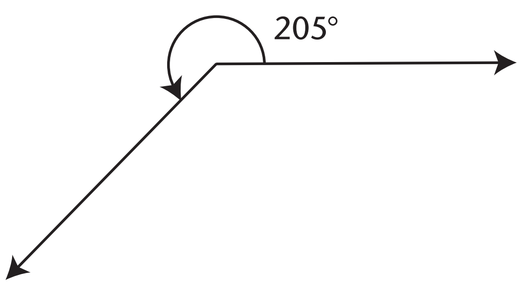
3. a.



   b.



   c.



Answer to Lesson 7.1C Practice 1

1. Step 1: 25 656 students

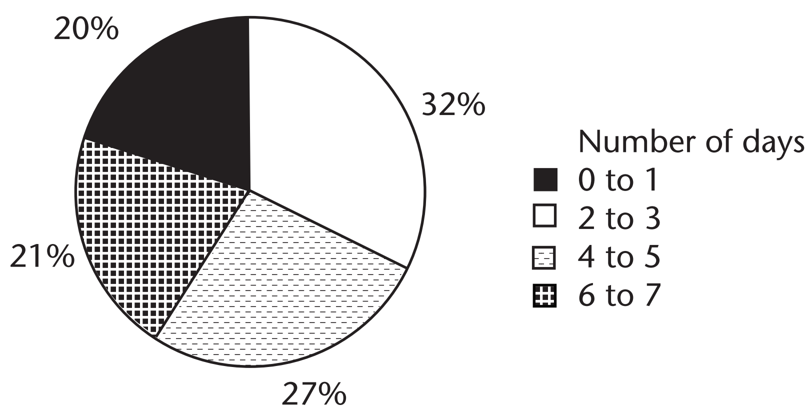
Sector Label	Number of Students	Percent of Total
0 to 1	**5131**	**20%**
2 to 3	**8210**	**32%**
4 to 5	**6927**	**27%**
6 to 7	**5388**	**21%**

Step 2: Draw the radius.

Step 3: Draw the sectors.

Step 4: Label the circle graph and add a title and a legend.

How many days last week did you do an intense physical activity?
(25 656 Canadian elementary students surveyed)



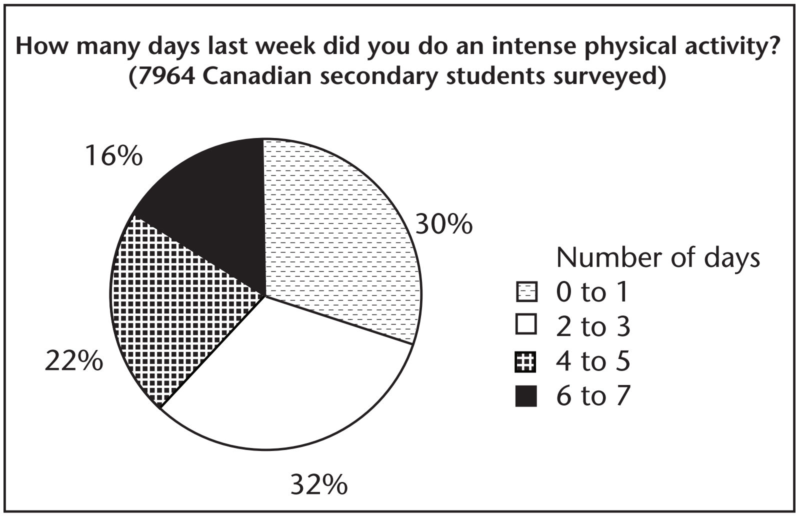
2. Step 1: 7964 students

Sector Label	Number of Students	Percent of Total
0 to 1	**2389**	**30%**
2 to 3	**2549**	**32%**
4 to 5	**1752**	**22%**
6 to 7	**1274**	**16%**

Step 2: Draw the radius.

Step 3: Draw the sectors.

Step 4: Label the circle graph and add a title and a legend.



3. a. From the statistics given, elementary students exercise more frequently.

   b. Answers will vary. Possible answers include:

   - Many elementary students have Physical Education class everyday in school, while not all secondary students (especially after grade 10) have these requirements.
   - Elementary students may have more free time after school to be physically active. Secondary students may not have as much time since many have part-time jobs (and may have more school work to complete at home).
   - Consider the number of students surveyed in each group. A larger population of elementary students was surveyed than secondary students. The data for the secondary students may not be as representative of reality because the sample size was small.

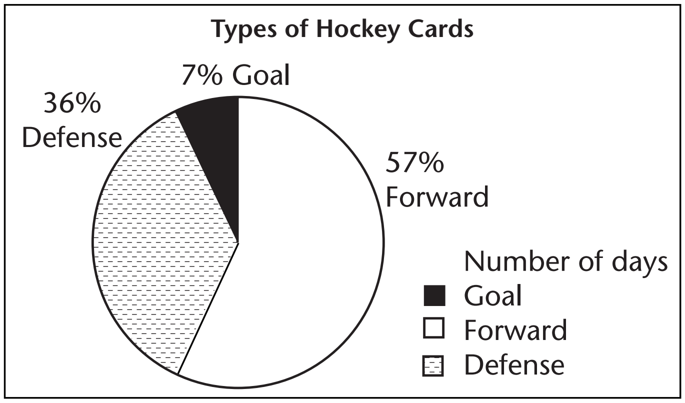
4. The question asks how many times you did a physical activity last week. The responses that were given by students represent physical activity during a specific week, not an overall average. Many factors may have influenced the responses: time of year (weather, what sports are in season), whether or not elementary students were surveyed during the same week as secondary students, etc.

Answer to Lesson 7.1C Practice 2

1.

Player's Position	Number of Hockey Cards	Percent of Total	Decimal Value	Central Angle
Forward	25	**57%**	**0.57**	**205 degrees**
Defence	16	**36%**	**0.36**	**130 degrees**
Goal	3	**7%**	**0.07**	**25 degrees**
Totals	**44**	**100%**	**1.0**	**360 degrees**

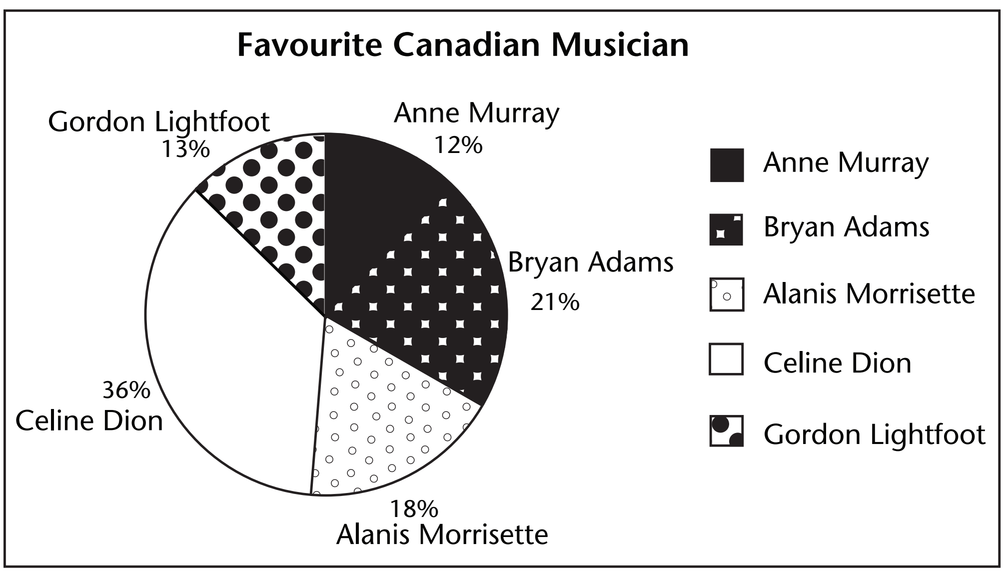
Source: Statistics Canada, Census at School, 2006/2007.



2.

Canadian Music Artist	Number of People	Percentage	Measurement of Central Angle
Anne Murray	31	**12%**	**43 degrees**
Bryan Adams	52	**21%**	**76 degrees**
Alanis Morissette	45	**18%**	**65 degrees**
Céline Dion	90	**36%**	**130 degrees**
Gordon Lightfoot	32	**13%**	**46 degrees**
Total	**250**	**100%**	**360 degrees**

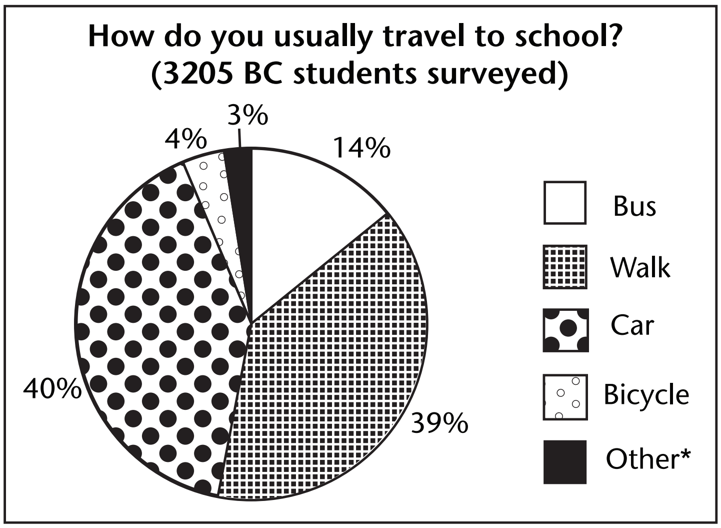
Source: Statistics Canada, Census at School, 2006/2007.



3.

Mode of Transportation	Number of Students	Percent of Total	Central Angle
Bus	452	**14.1**	**51**
Walk	1247	**38.9**	**140**
Car	1298	**40.5**	**146**
Bicycle	125	**3.9**	**14**
Other*	83	**2.6**	**9**
Totals	3205	**100**	**360**

Source: Statistics Canada, Census at School, 2006/2007.



4. a. 39%

   b. 17%

   c. Answers will vary. Some possible answers are:

   - weather (the most populated parts of Quebec are colder than the populated parts of BC)
   - many more students ride the bus in Quebec than in BC—maybe more schools have school buses that students ride so fewer students walk

Answer to Section Challenge 7.1

1. a.

1981	Married with children at home, married without children at home, lone-parent families, common-law without children at home, common-law with children at home
2001	Married with children at home, married without children at home, lone-parent families, common-law without children at home, common-law with children at home

The two lists are the same. Even though the percentages are different, the order is the same.

b.

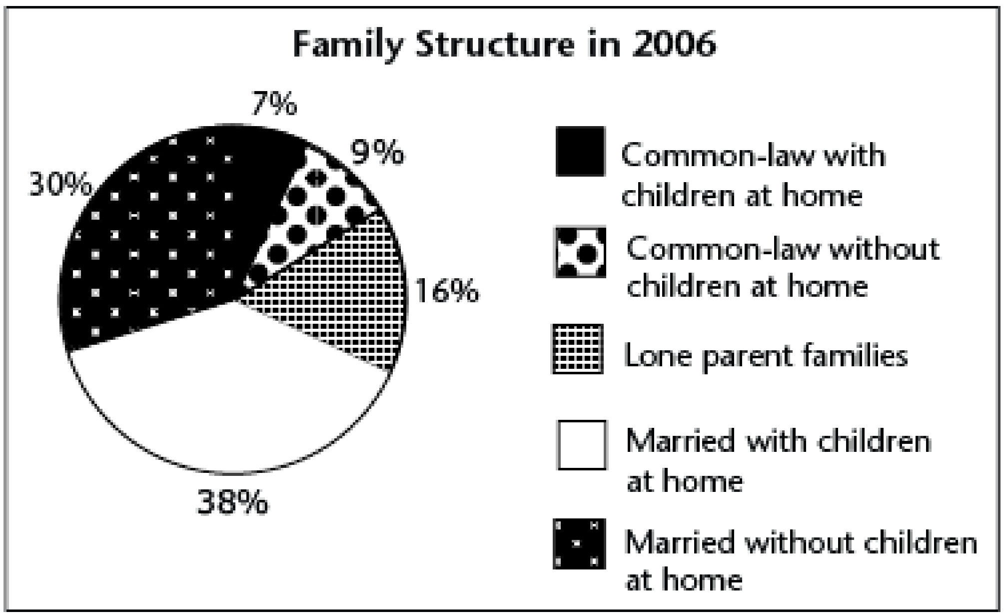
Family Type	Differences from 1981 to 2001 (Description)	Differences from 1981 to 2001 (Percentage)
Married with children at home	decreased	55% to 41%
Married without children at home	stayed close to the same	29% to 28%
Lone-parent families	Increased	11% to 16%
Common-law without children at home	increased	4% to 8%.
Common-law with children at home	increased	2% to 6%

c. If the trends in the data continue, the 2006 data will likely show:

- the same general trend with family types in the same order of frequency
- married with children at home will show continued decrease in percentage, married without children at home will stay approximately the same, while the other family types will increase

2.

Family Structure	Number of Families in 2006	Percentage of Families in 2006
Common-law with children at home	618 150	**7.0 %**
Common-law without children at home	758 715	**8.5 %**
Lone-parent families	1 414 060	**15.9 %**
Married with children at home	3 443 775	**38.7 %**
Married without children at home	2 662 135	**29.9 %**
Total	8 896 840	**100 %**



3. Answers will vary. Students should support or contradict their predictions from 1c with evidence from the graph.

Answer to Pretest 7.2

Lesson 7.2A

1. 52
2. 762

Lesson 7.2B

1. median 4 mode 4
2. median 106.5 mode 115

Lesson 7.2C

1. mean 102 mode 92   median 96
2. Answers may vary. Possible answers:

   Mean is the best. Half of Coco's scores are above it, and half are below.

   Median is the best. Half of Coco's scores are above it, and half are below.

   Mode is the best. This is the score that Coco got most often.

Lesson 7.2D

1. The outlier is the person who was 67. It would be a better representation of the data if the outlier was not included in the data set.
2. With the outlier:

   mode: 4, 15   mean: 23   median: 23

   Without the outlier:

   mode: 4, 15   mean: 19   median: 20

Answer to Lesson 7.2A Warm-up

s	a	a	r	g	s	i	m	u
c	i	r	t	t	e	s	m	a
i	o	e	n	a	i	o	d	a
t	h	s	m	a	d	u	g	t
s	p	i	h	e	a	t	a	n
i	r	e	p	m	d	l	i	s
t	i	g	a	e	g	i	l	n
a	v	e	r	a	g	e	a	a
t	d	e	g	n	a	r	i	n
s	a	a	u	r	e	s	g	c

Answer to Lesson 7.2A Practice 1

1. 0.242
2. Answers may vary. The difference between the highest value and the lowest value is 15.

   30, 21, 27, 16, 19, 20, 20, 15, 26, 29
3. a. 8

   b. 55

   c. 9.6

Answer to Lesson 7.2B Warm-up

1. 21, 34, 52, 53, 61, 68
2. 1, 3, 18, 22, 24, 27, 38, 41
3. 1, 3, 6, 6, 10, 11, 11, 35, 37, 48, 62, 63, 63, 74]

Answer to Lesson 7.2B Practice 1

1. a. mode: 4 median: 4

   b. mode: 21 median: 19

   c. mode: 8 median: 8

2. a. mode: 32
   median: 45

   b. median because the mode is the lowest value in the set

Answer to Lesson 7.2C Warm-up

91 + 64 = 155	89 + 65 = 154	92 + 47 = 139
57 + 43 = 100	87 + 19 = 106	49 + 37 = 86
43 + 76 = 119	78 + 40 = 118	22 + 39 = 61
91 + 66 = 157	97 + 20 = 117	55 + 43 = 98
53 + 24 = 77	24 + 29 = 53	94 + 61 = 155
61 + 50 = 111	87 + 10 = 97	35 + 37 = 72
32 + 99 = 131	79 + 61 = 140	83 + 64 = 147

Answer to Lesson 7.2C Practice 1

1. a. 2 536 617

   b. Eight of the 13 provinces and territories have populations less than the mean, therefore the mean is not a typical or central population value.

2. 155.1 cm

3. 7

Answer to Lesson 7.2D Warm-up

56 – 23= 33	43 – 12 = 31	26 – 11 = 15	98 – 23 = 75
12 – 4 = 8	56 – 25 = 31	99 – 24 = 75	65 – 7 = 58
32 – 18 = 14	90 – 64 = 26	101 – 23 = 78	87 – 19 = 68
29 – 19 = 10	63 – 59 = 4	82 – 59 = 23	134 – 76 = 58

Answer to Lesson 7.2D Practice 1

1.

Pittsburgh	Team Played	Number of Games	Goals Scored	Average Goals Scored per Game
Round 1	Ottawa	4	16	**4**
Round 2	NY Rangers	5	15	**3**
Round 3	Philadelphia	5	24	**4.8**
Round 4	Detroit	6	10	**1.7**

Detroit	Team Played	Number of Games	Goals Scored	Average Goals Scored per Game
Round 1	Nashville	6	17	**2.8**
Round 2	Colorado	4	21	**5.3**
Round 3	Dallas	6	17	**2.8**
Round 4	Pittsburg	6	17	**2.8**

2.

Pittsburgh (Goals scored per game)		
Mean	$\frac{(4+3+4.8+1.7)}{4} = \frac{13.5}{4} = \frac{3}{4}$	3.4
Mode	No repeating values	none
Median	~~1.7~~, 3, 4, ~~4.8~~ $\frac{(3+4)}{2} = 3.5$	3.5

Detroit (Goals scored per game)		
Mean	$\frac{(2.8+5.3+2.8+2.8)}{4} = \frac{13.65}{4} = \frac{3}{4}$	3.4
Mode	2.8 repeats 3 times	2.8
Median	~~2.8~~, 2.8, 2.8, ~~5.3~~ $\frac{(2.8+2.8)}{2} = 2.8$	2.8

3. Answers may vary. Use the statistics you calculated to justify your choice.

4. In the first data set, 1.7 is an outlier.
   In the second data set, 5.3 is an outlier.

Answer to Section Challenge 7.2

1. The range of fuel efficiency for the cars Elan researched is 4.0 to 9.2, which is 5.2 L/100 km.

2. Mode: 7.0 L/100 km
   Median: 7.0 L/100 km
   Mean: 6.88 L/100 km

3. Using this logic, she should just take the three most fuel efficient cars for a test drive. Calculating these measures of central tendency hasn't helped us at all.

4. Calculating measures of central tendency has nothing to do with seeing which cars are the most fuel efficient.

Answer to Pretest 7.3

Lesson 7.3A

1. a. $\frac{3}{10}$

   b. It is less likely that it will rain tomorrow.

Lesson 7.3B

1. a. The spins are independent events because one spin does not affect the results of the second spin.

   b. Rainy, sunny, and cloudy are not equally likely because the probability of spinning rain is $\frac{2}{4}$ or 50%, and the probability of spinning sun and clouds is $\frac{1}{4}$ or 25%.

Lesson 7.3C

	Coin	
Spinner	**Heads**	**Tails**
Blue	BH	BT
Blue	BH	BT
Red	RH	RT
Yellow	YH	YT

Lesson 7.3D

1.

	Die 2					
Die 1	**1**	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

2.

	Die 2					
Die 1	**1**	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

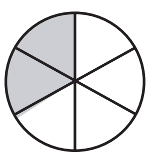
3. P(doubles) $= \frac{6}{36} = \frac{1}{6}$

   Answers will vary. P(doubles) $= \dfrac{\text{number of times you rolled doubles}}{36}$

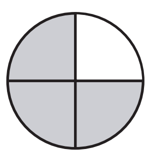
Answer to Lesson 7.3A Warm-up

2. Your pictures might be different.

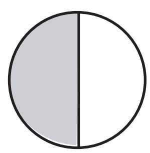
   a. $\frac{2}{6}$



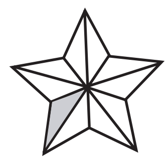
   b. $\frac{3}{4}$



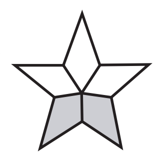
   c. $\frac{1}{2}$



   d. $\frac{1}{10}$



   e. $\frac{1}{5}$



3. a. 25%
   b. 20%
   c. 75%
   d. 20%
   e. 50%

Answer to Lesson 7.3A Practice 1

1. The possible rolls of a die are 1, 2, 3, 4, 5, and 6. There are 6 possible outcomes.
   a. 2, 4, and 6 are even numbers. There are 3 favourable outcomes.

      fraction: P $= \frac{3}{6} = \frac{1}{2}$

      ratio: 1:2

      percent: 50%

b. There is 1 favourable outcome.

fraction: $P = \frac{1}{6}$

ratio: 1:6

percent: 17%

c. There are 2 favourable outcomes.

fraction: $P = \frac{2}{6} = \frac{1}{3}$

ratio: 1:3

percent: 33%

2. a. $\frac{1}{4}$, 25%

   b. $\frac{1}{4}$, 25%

   c. $\frac{2}{4} = \frac{1}{2}$, 50%

Answer to Lesson 7.3B Warm-up

1. a. 50%

   b. 17%

   c. 25%

   d. 100%

   e. 0%

Answer to Lesson 7.3B Practice 1

1. a. no

   b. yes

2. These events are independent because regardless of what the result of either event, it does not affect the outcome of the other.

3. Answers vary, examples provided below:

   a. flipping a coin, rolling one die

   b. doing any of the above events together

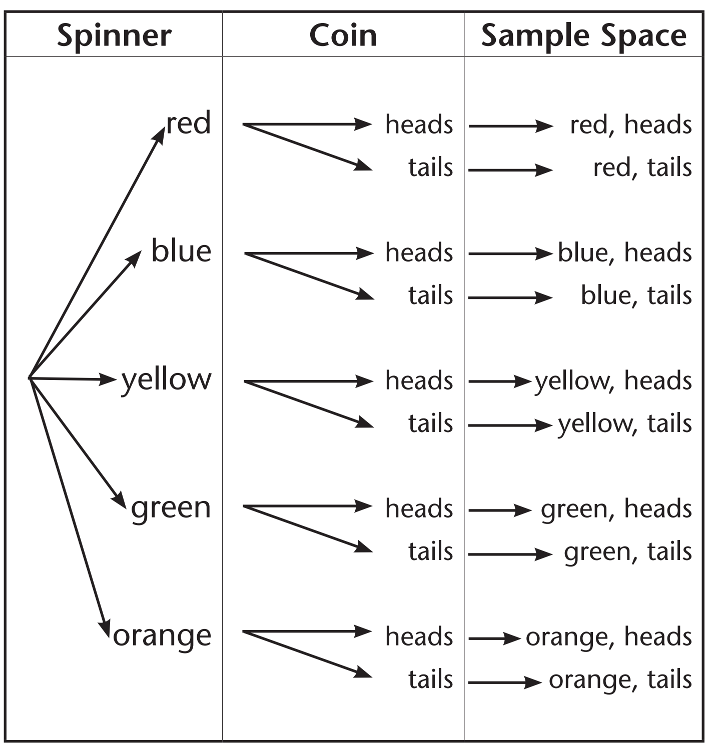
Answer to Lesson 7.3C Warm-up

1. PROBABILITY
2. EXPERIMENTAL
3. THEORETICAL
4. IMPOSSIBLE
5. INDEPENDANT EVENTS
6. SAMPLE SPACE
7. TREE DIAGRAM
8. ORGANIZED LIST

Answer to Lesson 7.3C Practice 1

1.

		Spinner				
		red	**blue**	**yellow**	**green**	**orange**
Coin	**tails**	RT	BT	YT	GT	OT
	heads	RH	BH	YH	GH	OH



2.

		Dinner		
		Pizza	Sushi	Burger
Movie	comedy	Pizza, comedy	Sushi, comedy	Burger, comedy
	drama	Pizza, drama	Sushi, drama	Burger, drama
	action	Pizza, action	Sushi, action	Burger, action
	TV	Pizza, TV	Sushi, TV	Burger, TV

Answer to Lesson 7.3D Warm-up

1. 3 pairs of good socks
   5 pairs of socks

   $$P = \frac{3}{5} = 60\%$$

2. a. $\frac{1}{6} = 17\%$

   b. $\frac{3}{6} = \frac{1}{2} = 50\%$

   c. $\frac{0}{6} = 0\%$

Answer to Lesson 7.3D Practice 1

1. a. There are 5 combinations which add up to 8 when rolling 2 dice out of a possible 36 combinations, therefore:

   $$\text{Probability of rolling an 8} = \frac{\text{favourable } (6)}{\text{possible } (36)} \qquad P = 22\%$$

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b. This time we are looking for the sum of 7.

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

There are 6 combinations which have a sum of 7 when rolling 2 dice out of a possible 36 combinations, therefore:

Probability of rolling an 8 = $\frac{\text{favourable (6)}}{\text{possible (36)}}$ P = 16%

c. In this example, we are finding the difference, not the sum, so we will need to create a new organized list.

Organized List of Differences

		Die 1					
		1	2	3	4	5	6
Die 2	1	0	1	2	3	4	5
	2	1	0	1	2	3	6
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

There are 6 combinations which have a difference of 3 when rolling 2 dice out of a possible 36 combinations, therefore:

Probability of rolling an 8 = $\frac{\text{favourable (6)}}{\text{possible (36)}}$ P = 16%

2. a. There are 4 possible outcomes when Duncan chooses a job from the jar. The sample space is:

   walk dog
   take out garbage
   load dishwasher
   make bed

   b. $\frac{1}{4} = 25\%$

   c. $\frac{2}{7} = 29\%$

Answer to Section Challenge 7.3

1.

	Meal Items			
Side Items	**grilled chicken**	**hamburger**	**steak**	**veggie burger**
fries	gc-f	h-f	s-f	vb-f
baked potato	gc-bp	h-bp	s-bp	vb-bp
salad	gc-s	h-s	s-s	vb-s
soup	gc-sp	h-sp	s-sp	vb-sp

gc-f, gc-bp, gc-s, gc-sp, h-f, h-bp, h-s, h-sp, s-f, h-bp, h-s, h-sp, hd-f, hd-bp, hd-s, hd-sp

Note: The student may have used a tree diagram instead of an organized list. See the example on the last page of the key

2.

$$P = \frac{\text{favourable}}{\text{possible}}$$

There are 12 combination combos without French fries out of a possible 16.

probability $\frac{12}{16} = \frac{3}{4}$ $P = 75\%$

	Meal Items			
Side Items	**grilled chicken**	**hamburger**	**steak**	**veggie burger**
fries	gc-f	h-f	s-f	vb-f
baked potato	gc-bp	h-bp	s-bp	vb-bp
salad	gc-s	h-s	s-s	vb-s
soup	gc-sp	h-sp	s-sp	vb-sp

3.

$$P = \frac{\text{favourable}}{\text{possible}}$$

There is 1 combination combo with steak and a baked potato out of a possible 16.

probability $\frac{1}{16}$ P = 6%

	Meal Items			
Side Items	grilled chicken	hamburger	steak	veggie burger
fries	gc-f	h-f	s-f	vb-f
baked potato	gc-bp	h-bp	s-bp	vb-bp
salad	gc-s	h-s	s-s	vb-s
soup	gc-sp	h-sp	s-sp	vb-sp

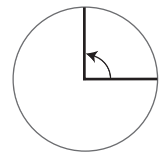
4.

probability $\frac{1}{16}$ P = 6%

Module 7 Glossary

Central angle

An angle formed by two radii of a circle. The vertex of the angle is at the centre of the circle

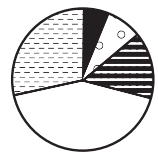


Circle

A set of points that are all the same distance from a fixed point called the centre

Circle graph

A graph that represents data using sections of a circle



Experimental probability

The probability of an event occurring based on experimental results.

Favorable outcome

A successful result in a probability experiment

Independent events

Two events are independent if the outcome of one event has no effect on the outcome of another event

Mean

The sum of a set of values divided by the number of values in the set. Sometimes the mean is called the average.

Measure of central tendency

A value that represents the centre of a set of data. It can be the mean, median or mode.

Median

The middle number in a set of data after the data has been arranged in order.

Mode

The most frequently occurring number in a set of data. There can be more than one mode

Outcome

One possible result of a probability experiment

Probability

The likelihood or chance of an event occurring. Probability can be expressed as a ratio, fraction, or percent

Sample space

All possible outcomes of an experiment

Sector

The section of a circle formed by two radii and an arc of the circle connecting the radii

Theoretical probability

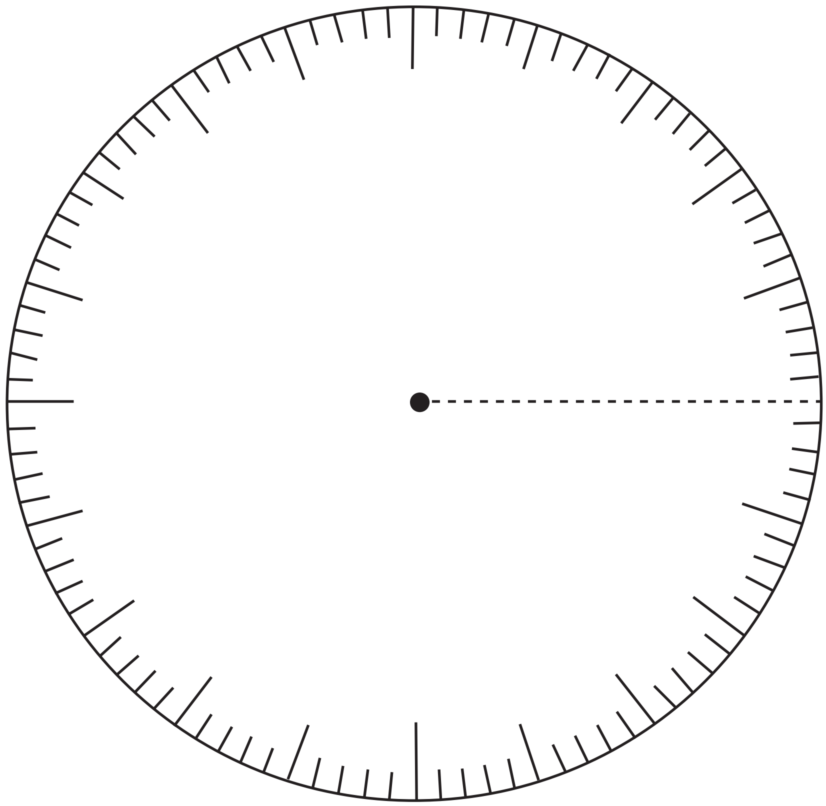
The expected probability of an event occurring

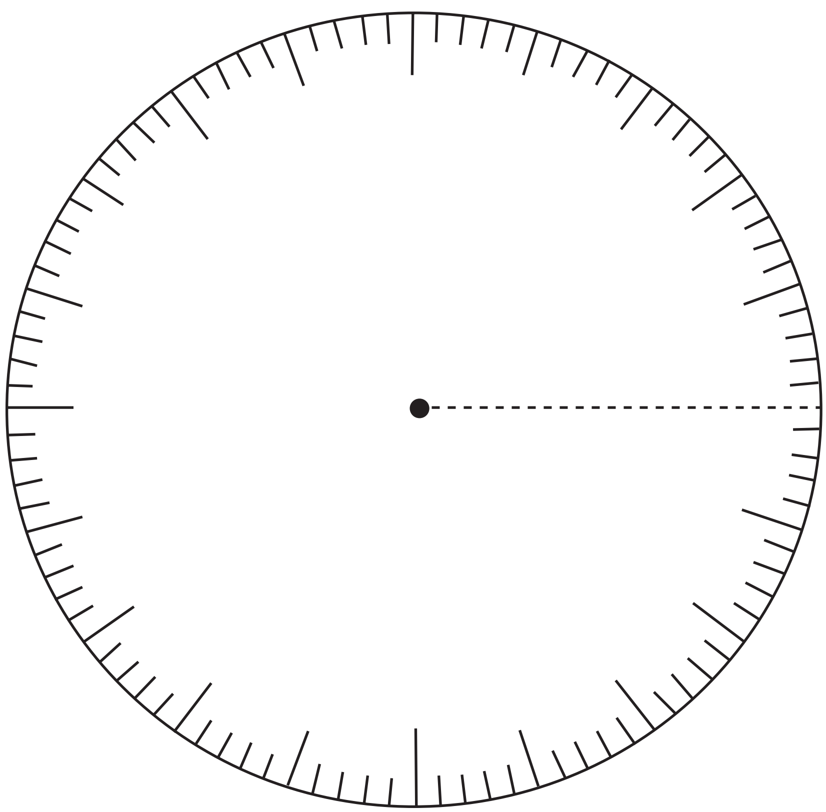
Tree diagram

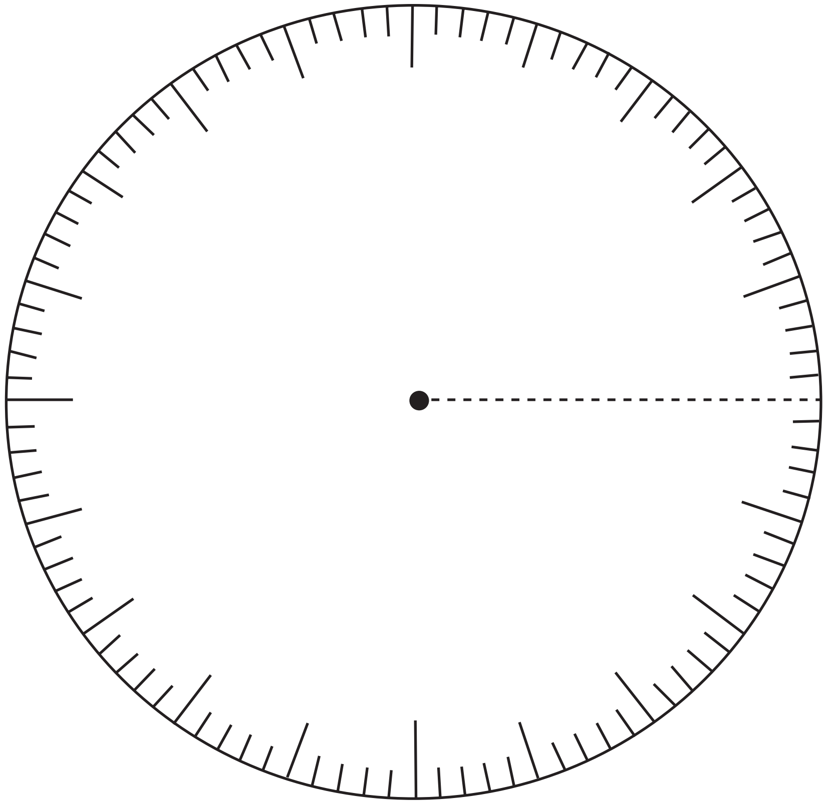
A diagram with a branch for each possible outcome of an event

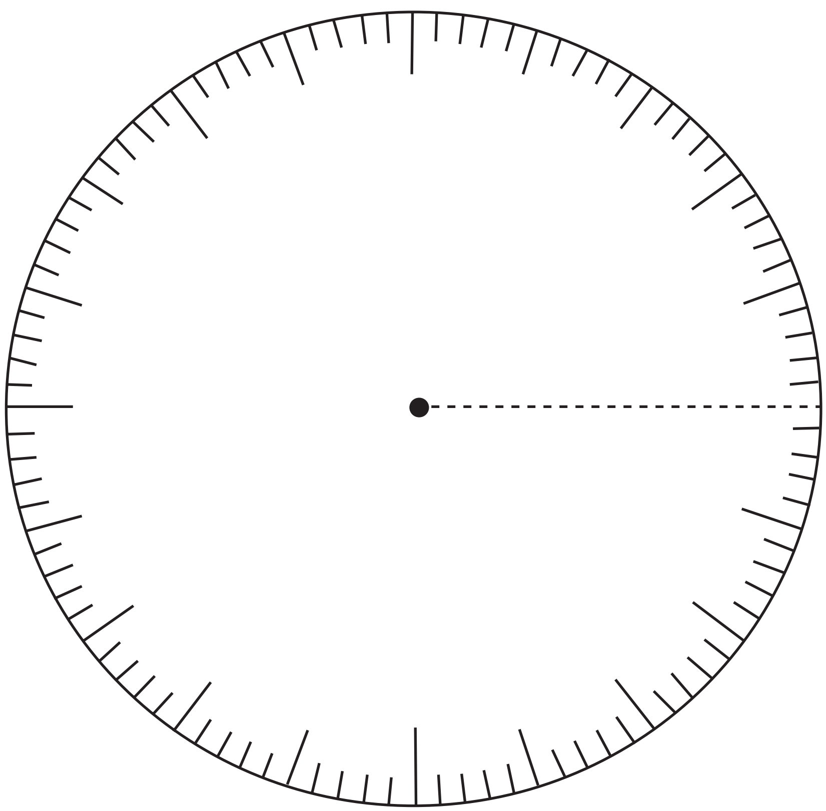
Module 7 Templates

Template for Lesson 7.1C Practice 1: Circle Template

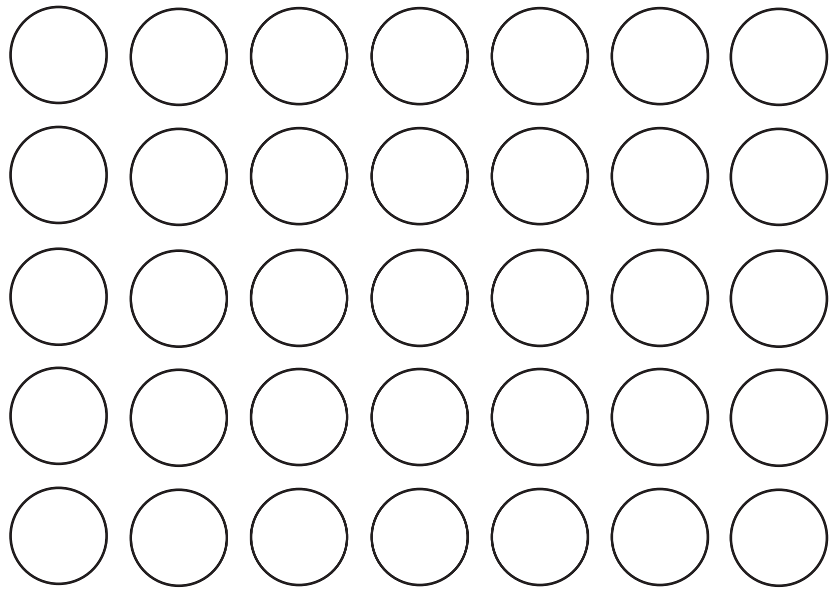


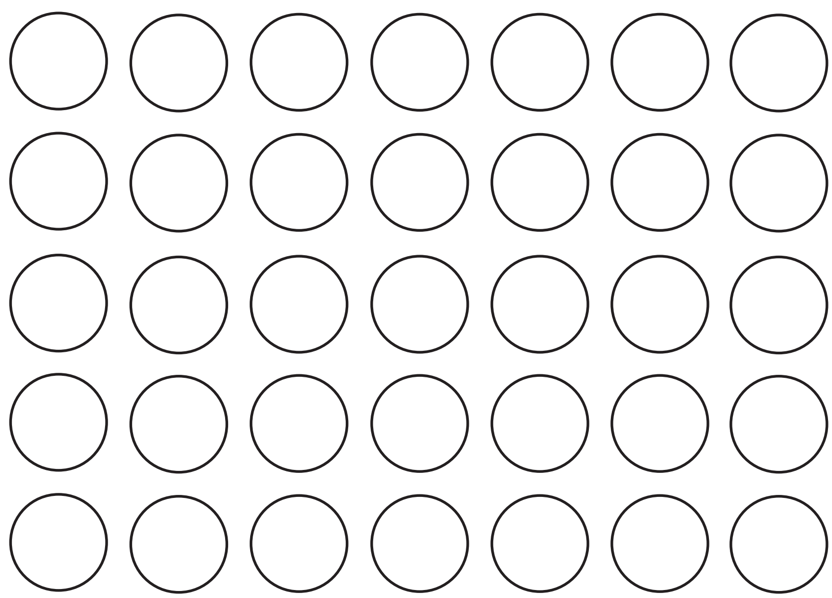




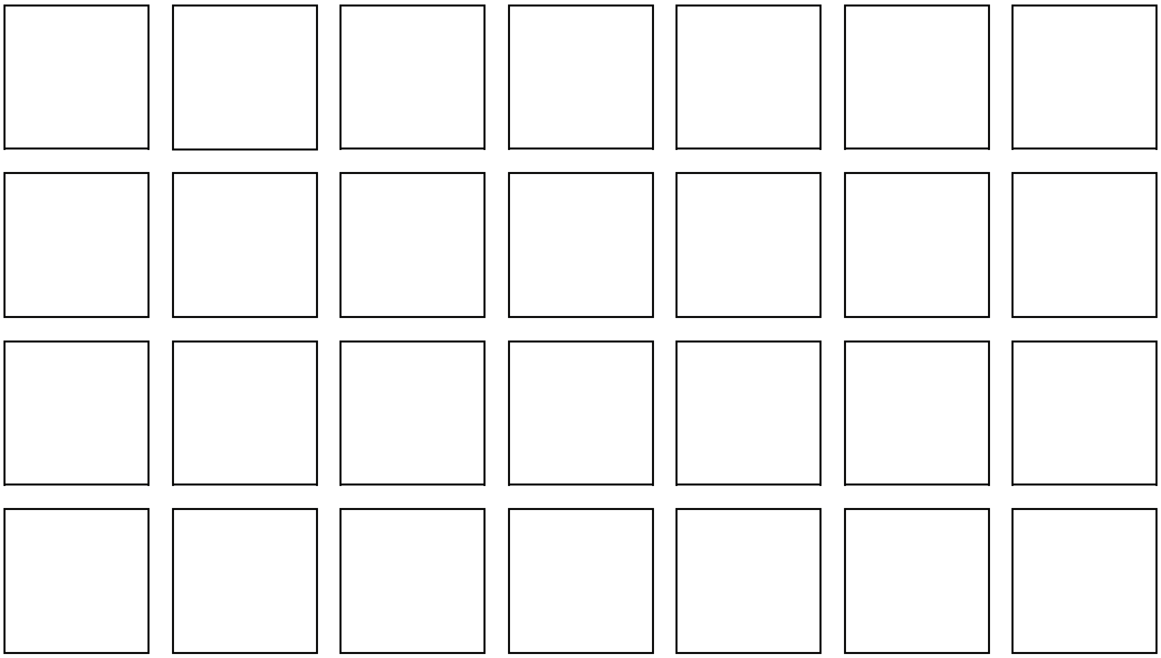


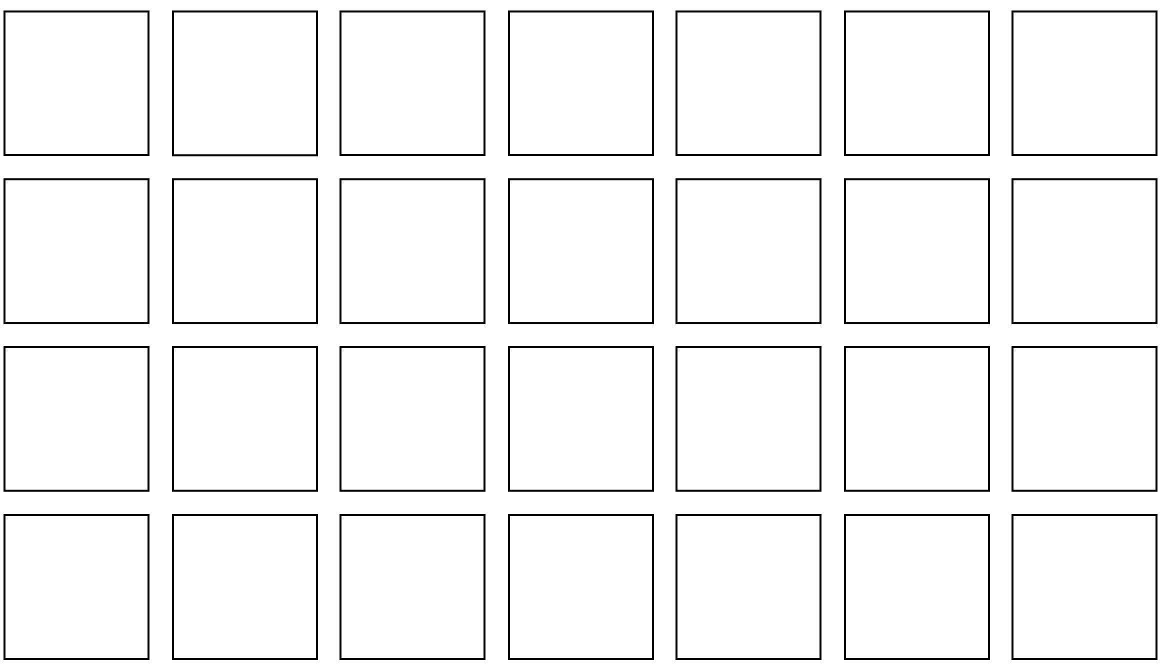
Template for Lesson 7.2C: Counters





Template for Lesson 7.3D: Chips





Template for Lesson 7.3D: Trial Charts

	Trial									
	1	2	3	4	5	6	7	8	9	10
Chip										
Die										

	Trial									
	11	12	13	14	15	16	17	18	19	20
Chip										
Die										

	Trial									
	1	2	3	4	5	6	7	8	9	10
Chip										
Die										

	Trial									
	11	12	13	14	15	16	17	18	19	20
Chip										
Die										

	Trial									
	1	2	3	4	5	6	7	8	9	10
Chip										
Die										

	Trial									
	11	12	13	14	15	16	17	18	19	20
Chip										
Die										

	Trial									
	21	22	23	24	25	26	27	28	29	30
Chip										
Die										

	Trial									
	31	32	33	34	35	36	37	38	39	40
Chip										
Die										

	Trial									
	41	42	43	44	45	46	47	48	49	50
Chip										
Die										

	Trial									
	1	2	3	4	5	6	7	8	9	10
Chip										
Die										

	Trial									
	11	12	13	14	15	16	17	18	19	20
Chip										
Die										

	Trial									
	21	22	23	24	25	26	27	28	29	30
Chip										
Die										

	Trial									
	31	32	33	34	35	36	37	38	39	40
Chip										
Die										

	Trial									
	41	42	43	44	45	46	47	48	49	50
Chip										
Die										