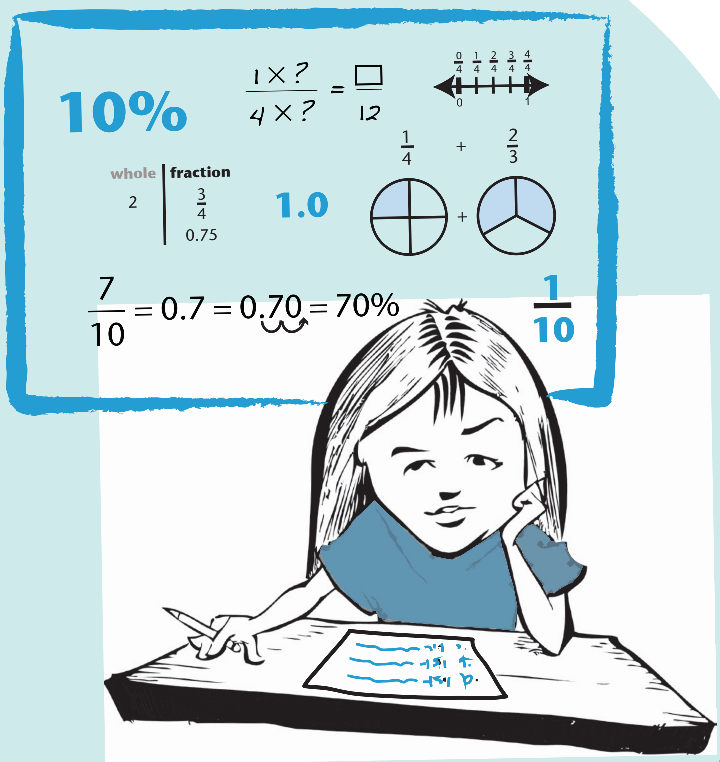
Module 2
Version 03
Sections 1–3

Math 7

Module 2

Fractions, Decimals, and Percent



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Course History
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Project Manager: Jennifer Riddel
Project Coordinator: Eleanor Liddy, Jennifer Riddel
Planning Team: Renee Gallant (South Island Distance Education School), Eleanor Liddy (Open School BC), Steve Lott (Vancouver Learning Network), Jennifer Riddel (Open School BC), Mike Sherman, Alan Taylor (Raven Research), Angela Voll (School District 79), Anne Williams (Fraser Valley Distance Education School)
Writers: Meghan Canil (Little Flower Academy), Shelley Moore (School District 38), Laurie Petrucci (School District 60), Angela Voll (School District 79)
Reviewers: Daniel Laidlaw, Steve Lott (Vancouver Learning Network), Angela Voll (School District 79)
Editor: Shannon Mitchell, Leanne Baugh-Peterson
Production Technician: Beverly Carstensen, Caitlin Flanders, Sean Owen
Media Coordinator: Christine Ramkeesoon
Graphics: Cal Jones

Module 2 Table of Contents

Course Overview

Welcome to Mathematics 7!

In this course you will continue your exploration of mathematics. You'll have a chance to practise and review the math skills you already have as you learn new concepts and skills. This course will focus on math in the world around you and help you to increase your ability to think mathematically.

Organization of the Course

The Mathematics 7 course is made up of seven modules. These modules are:

Module 1: Numbers and Operations

Module 2: Fractions, Decimals, and Percents

Module 3: Lines and Shapes

Module 4: Cartesian Plane

Module 5: Patterns

Module 6: Equations

Module 7: Statistics and Probability

Organization of the Modules

Each module has either two or three sections. The sections have the following features:

Pretest	This is for students who feel they already know the concepts in the section. It is divided by lesson, so you can get an idea of where you need to focus your attention within the section.
Section Challenge	This is a real-world application of the concepts and skills to be learned in the section. You may want to try the problem at the beginning of the section if you're feeling confident. If you're not sure how to solve the problem right away, don't worry—you'll learn all the skills you need as you complete the lessons. We'll return to the problem at the end of the section.

Each section is divided into lessons. Each lesson is made up of the following parts:

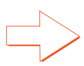
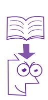
Student Inquiry	Inquiry questions are based on the concepts in each lesson. This activity will help you organize information and reflect on your learning.
Warm-up	This is a brief drill or review to get ready for the lesson.
Explore	This is the main teaching part of the lesson. Here you will explore new concepts and learn new skills.
Practice	These are activities for you to complete to solidify your new skills. Mark these activities using the answer key at the end of the module.

At the end of each module you will find:

Resources	Templates to pull out, cut, colour, or fold in order to complete specific activities. You will be directed to these as needed.
Glossary	This is a list of key terms and their definitions for the module.
Answer Key	This contains all of the solutions to the Pretests, Warm-ups and Practice activities.

Thinking Space

The column on the right hand side of the lesson pages is called the Thinking Space. Use this space to interact with the text using the strategies that are outlined in Module 1. Special icons in the Thinking Space will cue you to use specific strategies (see the table below). Remember, you don't have to wait for the cues—you can use this space whenever you want!

	Just Think It: Questions	Write down questions you have or things you want to come back to.
	Just Think It: Comments	Write down general comments about patterns or things you notice.
	Just Think It: Responses	Record your thoughts and ideas or respond to a question in the text.
	Sketch It Out	Draw a picture to help you understand the concept or problem.
	Word Attack	Identify important words or words that you don't understand.
	Making Connections	Connect what you are learning to things you already know.

More About the Pretest

There is a pretest at the beginning of each section. This pretest has questions for each lesson in the sections. Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in the section. Mark your answers using the key found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Materials and Resources

There is no textbook required for this course. All of the necessary materials and exercises are found in the modules.

In some cases, you will be referred to templates to pull out, cut, colour, or fold. These templates will always be found near the end of the module, just in front of the answer key.

You will need a calculator for some of the activities and a geometry set for Module 3 and Module 7.

If you have Internet access, you might want to do some exploring online. The Math 7 Course Website will be a good starting point. Go to:

http://www.openschool.bc.ca/courses/math/math7/mod2.html

and find the lesson that you're working on. You'll find relevant links to websites with games, activities, and extra practice. Note: access to the course website is not required to complete the course.

Icons

In addition to the thinking space icons, you will see a few icons used on the left-hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.

Explore Online

Warm-up

Explore

Practice

Answer Key

Use a Calculator

Module 2 Overview

Module 2 is all about parts! Fractions, decimals and percents are all ways of thinking about parts of a whole. In this module you'll explore each of these concepts and how they're related to each other.

You will have plenty of opportunities to review what you already know about these topics and to practise the new skills you learn. You'll try some hands-on activities and practice questions.

Section Overviews

Section 2.1: Fractions

In the first section of Module 2 you'll explore fractions in detail. You probably already know a lot about fractions from previous grades. You'll review a few concepts and then learn some new skills. Get ready to compare, simplify, add and subtract fractions!

Section 2.2: Decimals

How are decimals related to fractions? Can we compare decimals and fractions? What are the different types of decimals? How can we classify them?

By the end of this section you'll be able to answer all of these questions and more! You'll build on what you already know about fractions and decimals, and expand on this understanding by learning new terminology and skills.

Section 2.3: Percents

You probably know a lot about percents from previous grades. Using that knowledge, as well as your understanding of fractions and decimals, we'll explore how percents, fractions and decimals are related. We'll focus on problem solving in this section, so put on your thinking cap and get ready to figure out percent in the world around you!

Section 2.1: Fractions

Section 1

Contents at a Glance

Learning Outcomes

By the end of this section you will be better able to:

- compare and order positive fractions…by using benchmarks, place value, equivalent fractions.
- demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences).

Pretest 2.1

Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in this section. Mark your answers using the key found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson 2.1A

1. Reduce these fractions:

   a. $\frac{20}{35}$

   b. $\frac{28}{42}$

   c. $\frac{40}{50}$

2. For each of the lists below, circle the fraction that is NOT equivalent to the rest.

   a. $\frac{1}{2}, \frac{4}{8}, \frac{15}{10}, \frac{20}{40}$

   b. $\frac{3}{9}, \frac{5}{15}, \frac{7}{21}, \frac{18}{6}$

   c. $\frac{8}{10}, \frac{14}{21}, \frac{2}{3}, \frac{6}{9}$

Lesson 2.1B

1. Write equivalent fractions so the pairs of fractions below have common denominators.

   a. $\frac{1}{2}, \frac{2}{3}$

   b. $\frac{2}{5}, \frac{3}{4}$

   c. $\frac{5}{6}, \frac{1}{4}$

2. List the fractions below from least to greatest.

   a. $\frac{1}{2}, \frac{3}{5}, \frac{3}{4}, \frac{1}{5}$

   b. $\frac{2}{7}, \frac{1}{5}, \frac{5}{7}, \frac{6}{35}$

Lesson 2.1C

1. Add. Reduce your answers to simplest terms. Change improper fractions to mixed numerals.

   a. $\frac{2}{7}+\frac{4}{7}=$

   b. $\frac{3}{10}+\frac{2}{10}=$

c. $\frac{7}{9}+\frac{2}{9}=$

d. $\frac{1}{6}+\frac{3}{6}=$

e. $\frac{7}{10}+\frac{5}{10}=$

f. $\frac{2}{3}+\frac{1}{6}=$

g. $\frac{1}{4}+\frac{3}{8}=$

h. $\frac{1}{5}+\frac{9}{10}=$

i. $\frac{5}{6}+\frac{1}{2}=$

j. $\frac{2}{5}+\frac{1}{10}=$

2. A boy rides a bicycle $\frac{5}{10}$ of a kilometre. He then pushes the bicycle $\frac{2}{10}$ of a kilometre. How many kilometres did the boy travel with his bicycle?

3. Subtract. Reduce your answer to simplest terms. Change improper fractions to mixed numerals.

a. $\frac{7}{9} - \frac{2}{9} =$

b. $\frac{8}{10} - \frac{3}{10} =$

c. $\frac{7}{12} - \frac{3}{12} =$

d. $\frac{5}{6} - \frac{5}{6} =$

e. $\frac{5}{6} - \frac{3}{6} =$

Lesson 2.1D

1. Add. Reduce your answers to simplest terms. Change improper fractions to mixed numerals.

   a. $\frac{2}{3}+\frac{1}{6}=$

   b. $\frac{1}{4}+\frac{3}{8}=$

   c. $\frac{1}{5}+\frac{9}{10}=$

   d. $\frac{5}{6}+\frac{1}{2}=$

   e. $\frac{2}{5}+\frac{1}{10}=$

2. Subtract. Reduce your answer to simplest terms. Change improper fractions to mixed numerals.

a. $\frac{7}{9} - \frac{1}{3} =$

b. $\frac{8}{12} - \frac{1}{4} =$

c. $\frac{5}{10} - \frac{1}{2} =$

d. $\frac{9}{10} - \frac{2}{5} =$

e. $\frac{5}{8} - \frac{1}{4} =$

Lesson 2.1E

1. Express each improper fraction as a mixed number.

1. a. $\frac{13}{5}$

b. $\frac{14}{3}$

c. $\frac{12}{7}$

d. $\frac{22}{10}$

2. Add. Simplify your answer.

a. $1\frac{1}{3}+3\frac{1}{3}=$

b. $4\frac{1}{6}+2\frac{3}{6}=$

c. $8\frac{3}{4}+2\frac{1}{4}=$

d. $2\frac{3}{5}+1\frac{4}{5}=$

e. $2\frac{1}{5}+3\frac{7}{10}=$

f. $3\frac{1}{4}+2\frac{1}{2}=$

3. Subtract.

a. $2\frac{4}{5}-1\frac{2}{5}=$

b. $5\frac{9}{10}-1\frac{4}{10}=$

c. $6\frac{3}{4}-6\frac{1}{4}=$

d. $5\frac{2}{4}-3\frac{3}{4}=$

Turn to the Answer Key at the end of the Module and mark your answers.

Section Challenge

You are helping your mom prepare a special dinner this week. Relatives are coming from all over, and you have to prepare a meal for 10 people. Your mom has asked you to help write a grocery list. She gives you the three recipes below. Make sure you put all of the ingredients on the grocery list.

Celery Casserole

Serves 10 people

Ingredients

- $^1/_2$ cup butter
- $3^1/_2$ cups of celery
- $^3/_8$ cup flour
- $^3/_4$ cup milk
- $^2/_3$ cup mushrooms

Stuffing

Serves 5 people

Ingredients

- $^1/_8$ tsp poultry seasoning
- $^1/_4$ cup croutons
- $^1/_4$ cup water
- $^1/_3$ cup celery
- $^1/_4$ cup mushrooms

Apple Pie

Serves 5 people

Ingredients

- 6 apples
- $^3/_4$ cup butter
- $1^1/_8$ cup sugar
- 1 cup flour

Notice that some ingredients are in more than one recipe. How much of each ingredient will be on your mom's grocery list?

If you're not sure how to solve the problem now, don't worry. You'll learn all the skills you'll need to solve the problem in this section. Give it a try now, or wait until we go through it at the end of the section—it's up to you!

Lesson 2.1A: Exploring Fractions and Equivalent Fractions

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
What is an equivalent fraction?		answer
		example
How can I write equivalent fractions?		answer
		example
How can I simplify fractions?		answer
		example

Lesson 2.1A: Exploring Fractions and Equivalent Fractions

Thinking Space

Introduction

Fractions are everywhere! Think about recipes, sports statistics, and measuring distances. All of these things depend on parts; parts of something greater, which together make a whole.

Think of times in your day when you use fractions. You may not realize how much about fractions you already know.

When I think about fractions, I think about…

Some things I already know about fractions are…

Some places I have seen fractions in my everyday life are...

You already know a lot about fractions. The warm-up exercise in this section will be bigger than usual, so you can review what you need to know before we begin our lesson. After the review we'll look at equivalent fractions.

Explore Online

Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:
http://www.openschool.bc.ca/courses/math/math7/mod2.html
Look for *Lesson 2.1A: Eploring Fractions and Equivalent Fractions* and check out some of the links!

Warm-up

Let's see now if you remember some of the key terms that will help you in this lesson.

1. Use the list of words to fill in the blanks. The words can be used only once.

   fraction
   numerator
   denominator
   part
   whole
   common
   multiple
   equal
   mixed
   improper

   a. Fractions are ________ parts of a whole.

   b. The ___________tells you how many parts are counted.

   c. The _____________tells you how many parts are in total.

   d. The _________ of a fraction needs to be of equal size.

   e. Before you split something into its parts, it is a ________.

   f. When comparing fractions, it is helpful to find a __________ denominator.

   g. Sometimes a fraction includes a whole number; this is called a

      _________fraction.

   h. Another way of writing a mixed number is to turn it into an

      _________ fraction; a fraction that has a numerator bigger than the denominator.

Thinking Space

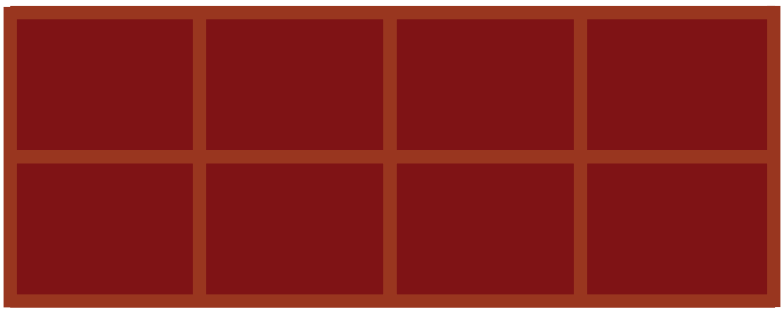
Thinking Space

Can you remember how to write a fraction?

Let's look at an example to refresh your memory.

This chocolate bar has 8 pieces. It can be written as $\frac{8}{8}$. The top number (the **numerator**) tells us how many pieces there are and the bottom number (the **denominator**) tells us how many pieces we started with.

$$\frac{\text{Numerator (parts of the whole)}}{\text{Denominator (total parts)}} = \frac{\text{Pieces in the chocolate bar} =}{\text{Pieces in total} =} \frac{8}{8}$$



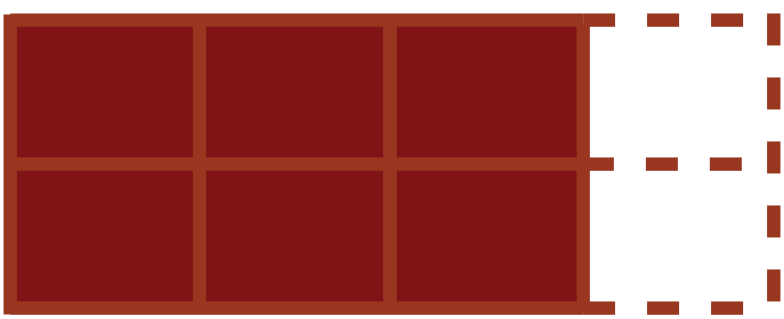
Notice that all the pieces are the same, or equal sizes.

Make a guess about how the numerator and the denominator might change as you eat the chocolate pieces.

As I eat the chocolate bar, the numerator will…

As I eat the chocolate bar the denominator will…

Let's try it and see. Let's say you just had dinner and so you want only 2 pieces. The chocolate bar looks like this now.



Imagine the space that the pieces you ate took up, and write a fraction for how much of the chocolate bar is left.

$$\frac{\text{Numerator (parts of the whole)}}{\text{Denominator (total parts)}} = \frac{\text{Pieces left}}{\text{Pieces in total}}$$

Thinking Space

Did you get $\frac{6}{8}$? Great job!!!

Did you notice the numerator getting smaller? Explain why you think that is.

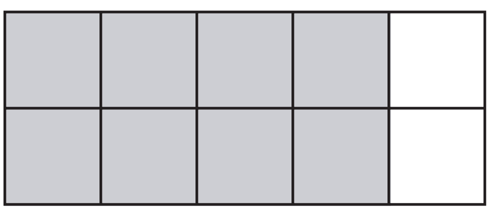
Did you notice the denominator staying the same? Explain why you think that is.

Now make another fraction, but this time write a fraction showing how many pieces you ate.

$$\frac{\text{Numerator (parts of the whole)}}{\text{Denominator (total parts)}} = \frac{\text{Pieces I ate}}{\text{Pieces in total}}$$

You should have written $\frac{2}{8}$. Let's try some questions to see if you really understand fractions. We can do the first question together.

2. Write the following as a fraction.

a. 

Fraction of shaded pieces: $\frac{8}{10}$

8 pieces out of a total of 10 pieces are shaded

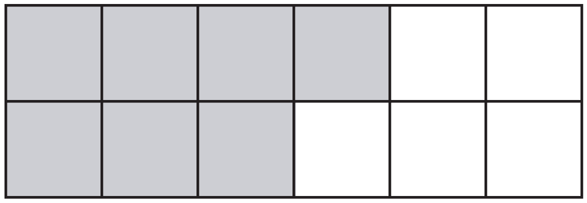
Fraction of unshaded pieces: $\frac{2}{10}$

2 pieces out of a total of 10 pieces are unshaded

Note: When you see the words "out of," think of a fraction.

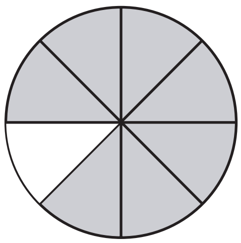
For example, 3 out of 4 is $\frac{3}{4}$.

Thinking Space

b. 

Fraction of shaded pieces:

Fraction of unshaded pieces:

c. 

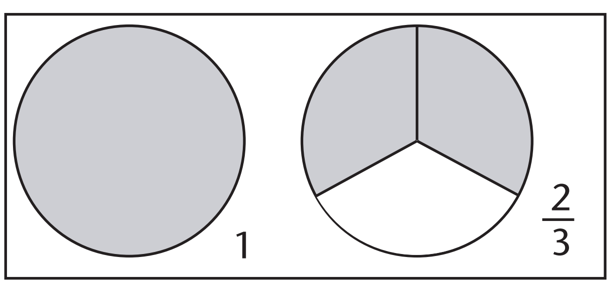
Fraction of shaded pieces:

Fraction of unshaded pieces:

The last thing we need to know for this lesson is how to convert between a **mixed number** and an **improper fraction**.

A mixed number has a whole number and a fraction.

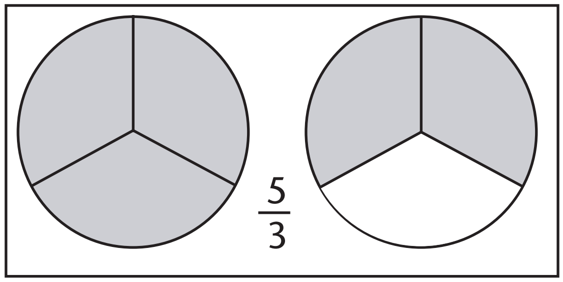
An example of a mixed number is $1\frac{2}{3}$.



This means you have one whole and two thirds.

An improper fraction has a numerator greater than or equal to the denominator.

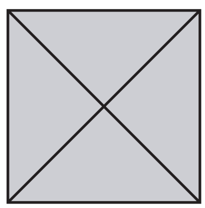
An example of an improper fraction is $\frac{5}{3}$.



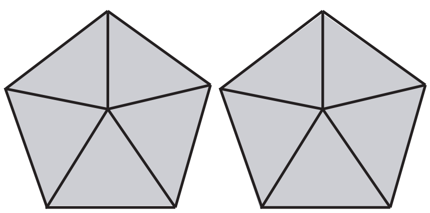
Look at the example. Do you see a similarity between $1\frac{2}{3}$ and $\frac{5}{3}$?

$1\frac{2}{3}$ and $\frac{5}{3}$ are the same!

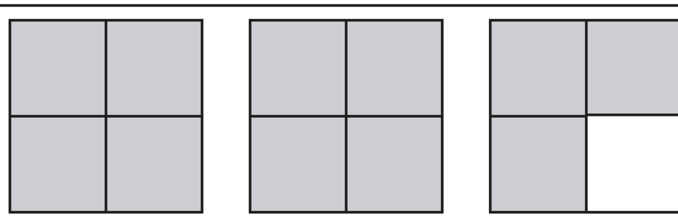
When an entire shape is filled this represents a whole number.



represents $\frac{4}{4}$ which is 1.



represents $\frac{10}{5}$ which is 2.



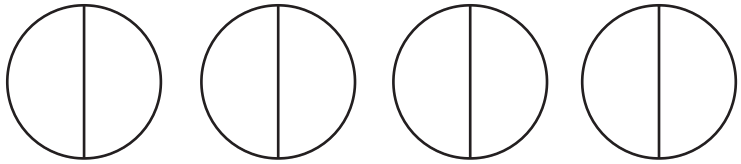
the shaded area represents $\frac{11}{4}$(improper fraction)

or $2\frac{3}{4}$(mixed number)

Now let's try some examples.

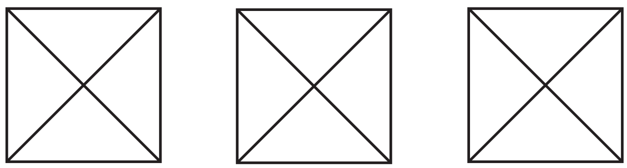
3. Shade in the amount shown by each mixed number. Then write an improper fraction for the amount shaded.

   a. $3\frac{1}{2}$

   Improper fraction =

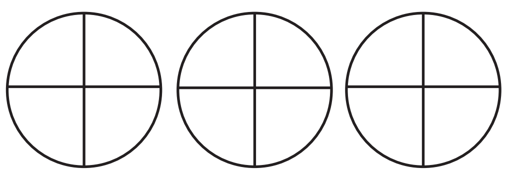
   b. $2\frac{3}{4}$

   Improper fraction =

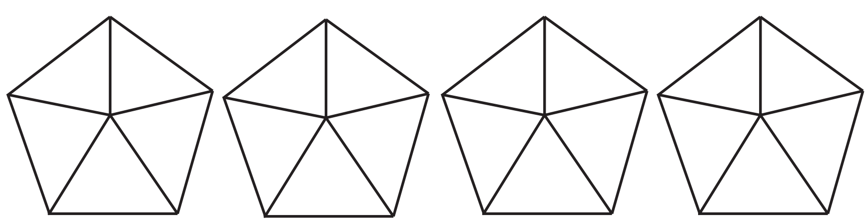
4. Now shade in the amount shown by each improper fraction. Then write a mixed number for the amount shaded.

   a. $\frac{11}{4}$

   Mixed Number =

   b. $\frac{19}{5}$

You can also convert between mixed numbers and improper fractions without pictures. You may remember doing this conversion in previous grades. If not, have a look at the steps shown below.

Here's a quick way to write a mixed number as an improper fraction:

1. To find the new numerator, multiply the whole number by the denominator.
2. Then add it to the numerator.
3. Keep the denominator the same.

Example:

$$4\frac{1}{3} = \frac{\overset{+}{4}\,1}{\times\,3}$$

$$= \frac{4 \times 3 + 1}{3}$$

$$= \frac{13}{3}$$

Now let's try converting the other way. Follow the steps to write an improper fraction as a mixed number:

1. Divide the numerator by the denominator.
2. The remainder is the numerator of the fraction.
3. Keep the denominator the same.

Example:

$$\frac{13}{3} = 13 \div 3$$

$$= 4r1$$

$$= 4\frac{1}{3}$$

These questions do not have drawings but feel free to make your own.

5. Convert the following mixed numbers into improper fractions.

a. $3\frac{1}{4}$

b. $4\frac{5}{6}$

c. $7\frac{4}{5}$

6. Convert the following improper fractions to mixed numbers.

a. $\frac{7}{5}$

b. $\frac{15}{4}$

c. $\frac{14}{3}$

Turn to the Answer Key at the end of the Module and mark your answers.

Practice 1

Let's begin with an activity to start our exploration of fractions.

For this activity, you'll need:

- circle template (you'll find this at the back of the module)
- scissors
- a pencil

Here's what to do:

- Cut out the four circles in the template along the dotted lines.
- Cut each circle into parts, following the dotted lines.
- Use your circle pieces to answer these questions:

1. Which is more, $\frac{1}{3}$ or $\frac{2}{6}$? Or are they the same?

2. How many $\frac{1}{4}$ pieces do you need to make the same shape as one $\frac{1}{2}$ piece?

3. How many $\frac{1}{6}$ pieces do you need to make the same shape as one $\frac{1}{2}$ piece?

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Equivalent is another word for "the same," or "equal." So, **equivalent fractions** are fractions that represent the same amount.

Let's talk about what you discovered in Practice 1.

Having two $\frac{1}{4}$ pieces means we have $\frac{2}{4}$ of the whole. Did you notice that $\frac{2}{4}$ and $\frac{1}{2}$ are the same size?

Remember: We call fractions that are the same, or equal, equivalent fractions.

So $\frac{2}{4}$ and $\frac{1}{2}$ are equivalent fractions, even though they have different numerators and denominators.

You probably also noticed that $\frac{3}{6}$ and $\frac{1}{2}$ were the same size.

$\frac{3}{6}$ and $\frac{1}{2}$ are equivalent fractions.

Similarly, $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent fractions.

Do you notice anything about the numerators of the equivalent fractions? The denominators?

In our example we said $\frac{1}{2} = \frac{2}{4}$.

Look at the numerators: 2 is a multiple of 1 (because 1 x 2 = 2).

Look at the denominators: 4 is a multiple of 2 (because 2 x 2 = 4).

It looks like we can create equivalent fractions by multiplying the numerator and the denominator by a number. Let's try:

Multiply the numerator and the denominator of $\frac{1}{2}$ by 2:

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Let's find another equivalent fraction for $\frac{1}{2}$.
This time we can multiply by 3.

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

Thinking Space

Is it possible to name every equivalent fraction of $\frac{1}{2}$?

Well, let's see. We can multiply the numerator and the denominator:

- by 2 to get $\frac{2}{4}$
- by 3 to get $\frac{3}{6}$
- by 4 to get $\frac{4}{8}$
- by 5 to get $\frac{5}{10}\ldots$

This list will never end. Although we can name a lot of fractions equivalent to $\frac{1}{2}$, there is always another number we can multiply our numerator and denominator by. So the answer is no, we can't name every equivalent fraction of $\frac{1}{2}$.

Now it's your turn to create some equivalent fractions by multiplying. Remember that you have to multiply the numerator **and** the denominator by the *same number*.

Practice 2

1. Please fill in the missing number to make these fractions equivalent.

   a. $\frac{3}{5} = \frac{\_\_\_}{10}$

   b. $\frac{3}{7} = \frac{12}{\_\_\_}$

   c. $\frac{1}{4} = \frac{\_\_\_}{16}$

2. Find equivalent fractions for the following:

   a. $\frac{1}{3}$

   b. $\frac{3}{5}$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Reducing Fractions

Thinking Space

You may have noticed in our previous examples that:

- the number in the numerator of each fraction got bigger.
- the number in the denominator of each fraction got bigger.

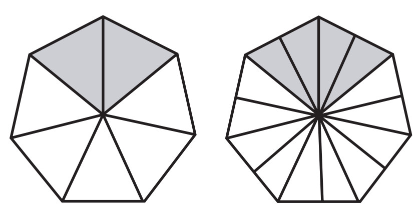
This is because we multiplied the top and bottom of our original fraction to get a new, equivalent fraction.

We can also use equivalent fractions to make numerators and denominators smaller. Can you guess how? By dividing, of course!

For example let's look at $\frac{4}{14}$. You may notice that 14 and 4 are both divisible by 2. Let's divide the numerator and the denominator by 2.

$$\frac{4}{14} = \frac{4 \div 2}{14 \div 2} = \frac{2}{7}$$

So $\frac{4}{14}$ is equivalent to $\frac{2}{7}$. Let's see what these fractions would look like in a diagram.



$\frac{2}{7} = \frac{4}{14}$

Since $\frac{2}{7}$ and $\frac{4}{14}$ both shade in the same amount of the shape, we can see these fractions are equivalent.

When we make equivalent fractions by dividing, we sometimes say we are *reducing* the fraction. To reduce fractions, you must find a **common factor** of the numerator and the denominator. In our example, that common factor was 2.

Thinking Space

Let's try another one.

Reduce $\frac{28}{32}$.

Both 28 and 32 are divisible by 2. Let's reduce!

$$\frac{28}{32} = \frac{28 \div 2}{32 \div 2} = \frac{14}{16}$$

So we know that $\frac{28}{32} = \frac{14}{16}$. But wait… 14 and 16 still have a common factor: 2.

Let's try reducing again.

$$\frac{14}{2} = \frac{14 \div 2}{16 \div 2} = \frac{7}{8}$$

So, $\frac{28}{32} = \frac{14}{16} = \frac{7}{8}$. Can you reduce $\frac{7}{8}$ any further? To answer this question, try looking for a common factor between 7 and 8. Are there any?

The numerator and denominator of $\frac{7}{8}$ have no common factors. This means we can't reduce $\frac{7}{8}$ any further. When a fraction can no longer be reduced, we say that the fraction is in *lowest terms* or *simplified*.

Could we have put $\frac{28}{32}$ into lowest terms in one step? Let's look at the factors of the numerator and the denominator.

Factors of numerator (28)	1	2	4	7	14	28
Factors of denominator (32)	1	2	4	8	16	32

When we reduced this fraction, we used the common factor 2. From the table, you can see that 4 is also a common factor. Actually, it is the *greatest common factor* (GCF). Let's see what happens when we reduce $\frac{28}{32}$ using the greatest common factor of the numerator and the denominator.

$$\frac{28}{32}=\frac{28\div 4}{32\div 4}=\frac{7}{8}$$

When we reduced by the GCF, we ended up with the most simplified equivalent fraction. In fact, this works all of the time. To get a fraction into lowest terms in one step, just divide the numerator and the denominator by their greatest common factor.

It's your turn to try a few.

Thinking Space

!

Practice 3

1. Please fill in the missing number to make these fractions equivalent.

   a. $\dfrac{12}{14} = \dfrac{6}{\quad}$

   b. $\dfrac{24}{26} = \dfrac{12}{\quad}$

   c. $\dfrac{9}{30} = \dfrac{\quad}{10}$

2. Reduce the following fractions to their lowest terms.

   a. $\dfrac{3}{15} =$

   b. $\dfrac{6}{9} =$

   c. $\dfrac{36}{40} =$

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 2.1B: Comparing Fractions

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
How can I order fractions from smallest to biggest or biggest to smallest?		answer
		example
Where do I place fractions on a number line?		answer
		example
What strategies can I use to solve questions with fractions?		answer
		example
How can I find a common denominator for fractions?		answer
		example

Lesson 2.1B: Comparing Fractions

Thinking Space

Introduction

In the last lesson, we explored equivalent fractions. You might be wondering, "why would I need to create equivalent fractions?" Well, being able to create fractions that have the same value but different denominators can be useful in a whole bunch of situations. Comparing fractions, as well as adding and subtracting fractions, are some times when you'll need to use equivalent fractions.

In this lesson we'll focus on comparing fractions. Can you think of times where you need to compare fractions?

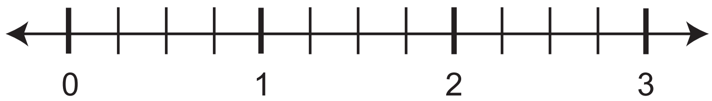
Let's say you LOVE chili. It is your favorite dinner; especially when it's loaded up with tomatoes! Your mom asks you to find a recipe on the internet for chili and you find two that look pretty good. One calls for $\frac{2}{3}$ of a can of tomatoes and the other asks for $\frac{1}{3}$ of a can. Which one would you choose?

In this lesson we'll look at examples like this one, where you need to compare two fractions. We'll also look at some examples where you have to put several fractions in order. Let's start with a warm-up.

Warm-up

For this lesson, we'll need to know how to place fractions on a number line. Let's try it with the following fractions:

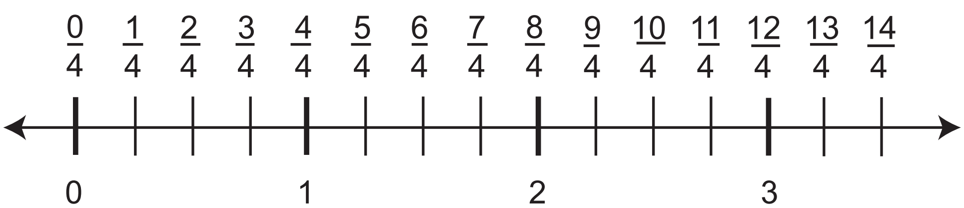
$$2\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{11}{4}, \frac{4}{4}$$



Thinking Space

Did you notice that every space on the number line represents $\frac{1}{4}$?

Let's write what number each line represents so we can find out where each fraction goes. Notice that the numerator determines the order of fractions on the number line.



Do you see any numbers we are trying to find?

Circle the $\frac{1}{4}$, $\frac{3}{4}$, $\frac{11}{4}$, and $\frac{4}{4}$. Notice that $\frac{4}{4}$ is the same as 1.

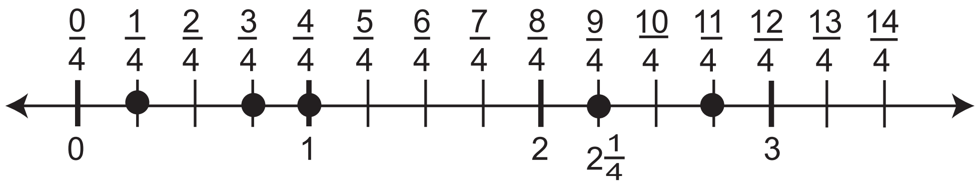
Can you find the $2\frac{1}{4}$? This one is a little more difficult to find because $2\frac{1}{4}$ is a mixed number, and all of the fractions we wrote are improper fractions.

Let's convert the $2\frac{1}{4}$ into an improper fraction.

$$2\frac{1}{4} \text{ is } \frac{9}{4}$$

Can you find $\frac{9}{4}$ on the number line? That's where $2\frac{1}{4}$ would go.

Here is our solution:



Notice that the smallest numbers are to the left, and the largest numbers are to the right. The number line makes it easy to see which numbers are bigger than others.

Let's write $2\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{11}{4}, \frac{4}{4}$ in **ascending** order (smallest to largest).

$$\frac{1}{4}, \frac{3}{4}, \frac{4}{4}, 2\frac{1}{4}, \frac{11}{4}$$

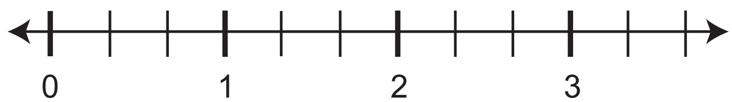
Notice that the denominators are the same. Also notice that the numerators go from smallest to largest (remember $2\frac{1}{4} = \frac{9}{4}$).

Now it's your turn.

1. Write $2\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{11}{4}, \frac{4}{4}$ in **descending** order (largest to smallest).

   Hint: use the example above to help you.

2. Place the following numbers on a number line: $\frac{1}{3}, \frac{6}{3}, 1\frac{2}{3}, \frac{4}{3}, 2\frac{1}{3}$.
   Then put them in ascending and descending order.



   a. $\frac{1}{3}, \frac{6}{3}, 1\frac{2}{3}, \frac{4}{3}, 2\frac{1}{3}$ in ascending order:

   b. $\frac{1}{3}, \frac{6}{3}, 1\frac{2}{3}, \frac{4}{3}, 2\frac{1}{3}$ in descending order:

I think we're ready to build on what you already know about fractions.

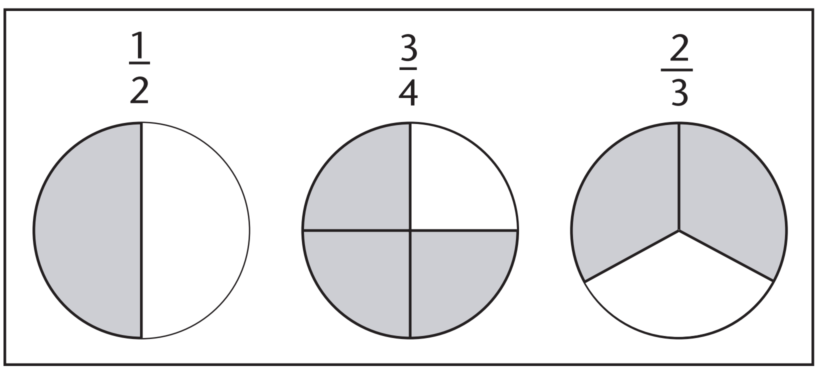
Explore
Comparing Fractions

Thinking Space

In the Warm-Up, you had a chance to put some groups of fractions in order. You used a number line to compare the fractions. Did you notice that, in each example, the fractions you were comparing had the same denominators? When two (or more) fractions have the same denominator, we say that they have **common denominators**.

What happens if we want to compare fractions that have different denominators? For example, let's say you wanted to make a pie that had the least sugar. If you find three recipes that call for $\frac{1}{2}$ cup of sugar, $\frac{3}{4}$ cup of sugar, and $\frac{2}{3}$ cup of sugar, which recipe would you use?

The fractions in the example have different denominators. Let's look at this visually. Each whole circle represents one cup. In each recipe, the cup has been divided into a different number of parts. Look at the table below.



So which recipe would you choose? From the picture, you can see that the recipe with $\frac{1}{2}$ cup of sugar has the least amount of sugar.

Is there a way to compare fractions without drawing pictures?

Remember that it was easy to place fractions with common denominators on a number line and compare them. Now if only there was a way that we could make the fractions all have the same denominator *without changing the values of each fraction...* wait! There is! That sounds like equivalent fractions!

Explore

Finding Common Denominators Using Equivalent Fractions

Thinking Space

Let's say we want to compare $\frac{2}{3}$ and $\frac{3}{4}$. Right now these fractions have different denominators. We need to convert each to an equivalent fraction. We want to choose our equivalent fractions carefully so that they have the same denominators.

$$\frac{2}{3} = \frac{\square}{\square} \qquad \frac{3}{4} = \frac{\square}{\square}$$

We want these to be the same.

Let's go through the process.

We need to multiply the numerator and the denominator of each fraction by some number so that we end up with two new fractions that have the same denominators. Let's look at the multiples of each denominator.

Multiples of 3	3	6	9	12	15	18	21	24…
Multiples of 4	4	8	12	16	20	24	28	32…

12 is a common multiple of 3 and 4. Let's use this as the denominator for each equivalent fraction.

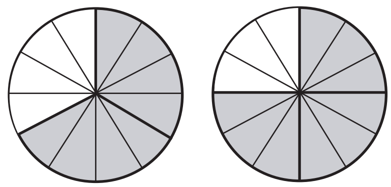
$$\frac{2}{3} = \frac{\square}{12} \qquad \frac{3}{4} = \frac{\square}{12}$$

Now think about what we need to multiply the denominator by in each case. We have to do the same thing to the numerator of each fraction.

$$\frac{2 \times 4}{3 \times 4} = \frac{\boxed{8}}{12} \qquad \frac{3 \times 3}{4 \times 3} = \frac{\boxed{9}}{12}$$

Thinking Space

Now that we have two fractions with common denominators, it's easy to compare them. You can see that $\frac{9}{12}$ is larger than $\frac{8}{12}$ (because 9 parts of a whole is more parts than 8 parts). Check out the picture below to make sure:



$\frac{8}{12}$ $\frac{9}{12}$

In our example, we chose 12 as the denominator. But if you look at the multiples of 3 and 4, we could have also picked 24. We picked 12 because it is the lowest common multiple. Why do you think it's easier to work with the lowest common multiple as the common denominator? By choosing the lowest common multiple, we ended up with the **lowest common denominator**.

Let's take a look at the words "lowest common denominator." Lowest means we want the smallest number. "Common" is another word for the same. So "lowest common denominator" really means the "smallest same denominator."

Let's try another example. Find equivalent fractions for $\frac{1}{4}$ and $\frac{1}{6}$. Make sure you choose the lowest common denominator.

If we want to find the lowest common denominator we need to find the lowest common multiple of 4 and 6.

Multiples of 4	4	8	12	16	20...
Multiples of 6	6	12	18	24	30...

Since the lowest common multiple of 4 and 6 is 12, it means the lowest common denominator of $\frac{1}{4}$ and $\frac{1}{6}$ is 12. So let's write equivalent fractions with denominator of 12.

$\frac{1 \times ?}{4 \times ?} = \frac{\square}{12}$

What do I multiply the 4 by to get 12?

Oh yeah!
$12 \div 4 = 3$
$4 \times 3 = 12$

$\frac{1}{4} = \frac{\square}{12}$	$\frac{1}{6} = \frac{\square}{12}$

$\frac{1 \times 3}{4 \times 3} = \frac{\boxed{3}}{12}$	$\frac{1 \times 2}{6 \times 2} = \frac{\boxed{2}}{12}$

Thinking Space

These fractions now have common denominators.

Practice 1

Now it's your turn.

1. What will be the lowest common denominator for the following pairs of fractions?

   a. $\frac{1}{2}, \frac{3}{5}$

   b. $\frac{2}{3}, \frac{1}{7}$

   c. $\frac{3}{4}, \frac{1}{8}$

   d. $\frac{5}{9}, \frac{1}{6}$

2. Write equivalent fractions so the pairs of fractions below have the same denominators.

   a. $\frac{1}{2}, \frac{3}{5}$

   b. $\frac{2}{3}, \frac{1}{7}$

   c. $\frac{3}{4}, \frac{1}{8}$

   d. $\frac{5}{9}, \frac{1}{6}$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore
Comparing Fractions Again

Thinking Space

Now that we know what lowest common denominators are, we should know why we use them. The reason we find the *lowest* common denominator is because it is easier to work with smaller numbers. Let's use our common-denominator-finding-skills to help us compare fractions.

Remember our pie example?

> Let's say you wanted to make a pie that had the least sugar. If you find three recipes which call for $\frac{1}{2}$ cup of sugar, $\frac{3}{4}$ cup of sugar and $\frac{2}{3}$ cup of sugar, which recipe would you use?

We looked at this example using pictures. Now let's try finding the common denominator to compare. The denominators are 2, 4 and 3. Let's look at the multiples of 2, 4, and 3.

Multiples of 2	2	4	6	8	10	12	14	16…
Multiples of 3	3	6	9	12	15	18	21	24…
Multiples of 4	4	8	12	16	20	24	28	32…

The lowest common multiple is 12, so let's find equivalent fractions with a denominator of 12.

$\frac{1}{2} = \frac{\square}{12}$	$\frac{2}{3} = \frac{\square}{12}$	$\frac{3}{4} = \frac{\square}{12}$

Think about what we need to multiply the denominator by in each case. Remember: we have to do the same thing to the numerator of each fraction.

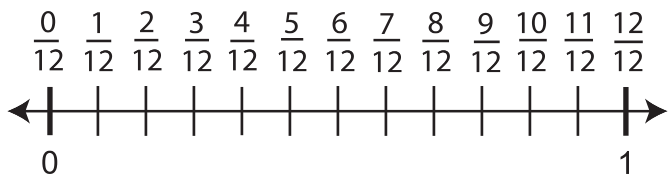
$\frac{1 \times ?}{6 \times ?} = \frac{\square}{12}$

What do I multiply the 6 by to get 12?

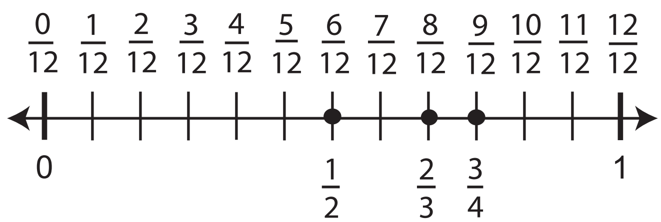
Oh yeah!
$12 \div 6 = 2$
$6 \times 2 = 12$

$\frac{1\times 6}{2\times 6} = \frac{\boxed{6}}{12}$	$\frac{2\times 4}{3\times 4} = \frac{\boxed{8}}{12}$	$\frac{3\times 3}{4\times 3} = \frac{\boxed{9}}{12}$

Now that our fractions have common denominators, we can place them on a number line. All of our numbers have 12 as the denominator, so let's make each space on the number line equal to $\frac{1}{12}$.



Now, you can find out where $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{3}$ are on the number line.



Let's write our fractions in ascending order: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.

So since you want less sugar in your pie, which recipe would you choose?

The one with $\frac{1}{2}$ cup of sugar.

Now it's time to try some on your own.

Thinking Space

?

Do you have any questions about this number line?

Practice 2

Write these sets of fractions below in ascending order. If you get stuck, you can shade in parts of shapes or create a number line. *Hint:* try to find the lowest common denominator to write your equivalent fractions.

1. $\frac{1}{3}, \frac{3}{5}, \frac{2}{5}, \frac{2}{3}$

   The fractions in ascending order are:

2. $\frac{3}{4}, \frac{1}{4}, \frac{2}{6}, \frac{3}{6}, \frac{5}{6}$

   The fractions in ascending order are:

3. $\frac{1}{3}, \frac{3}{2}, \frac{7}{6}, \frac{5}{4}, \frac{1}{2}, \frac{5}{6}$

The fractions in ascending order are:

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 2.1 C Adding and Subtracting Fractions with Common Denominators

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
How can I show the addition and subtraction of fractions using a number line or a diagram?		answer
		example
How do I find the sum of fractions with like denominators?		answer
		example
How do I find the difference of fractions with like denominators?		answer
		example
How can I simplify fractions?		answer
		example

Lesson 2.1C Adding and Subtracting Fractions with Common Denominators

Thinking Space

Introduction

Have you ever worked on an assignment two times in one day? If you got a quarter of the assignment done each time you worked on it, how complete is your assignment? This lesson will help you to answer questions like this by adding and subtracting fractions with the same denominators.

Explore Online

Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:
http://www.openschool.bc.ca/courses/math/math7/mod2.html
Look for *Lesson 2.1C: Adding and Subtracting Fractions with Common Denominators* and check out some of the links!

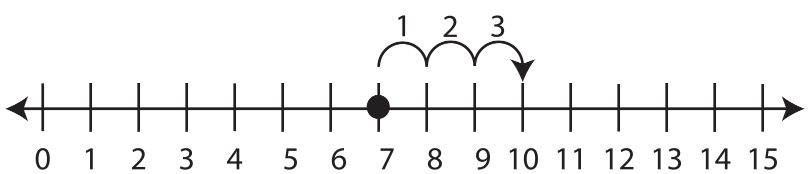
Warm-up

Thinking Space

Working with fractions is similar to working with whole numbers. So, before we add and subtract fractions, let's review how to add and subtract whole numbers. This may seem simple, but let's look closely at the process involved. When we add a whole number we can think of it as moving to the right on the number line.

For example 7 + 3

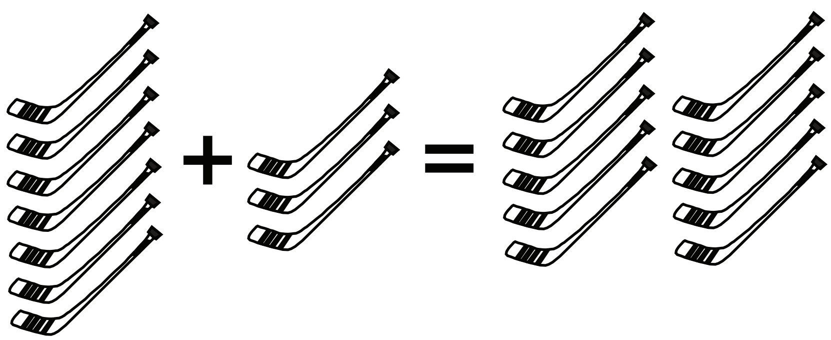
We start at 7 then move 3 spaces to the right:



When we move three to the right we see that 7 + 3 is 10.

We can also see how 7 + 3 = 10 with pictures.

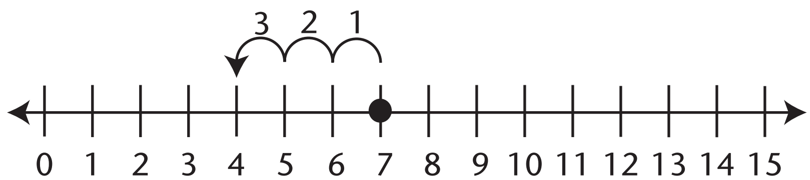
If we have 7 hockey sticks, then we get 3 more, we will have 10.



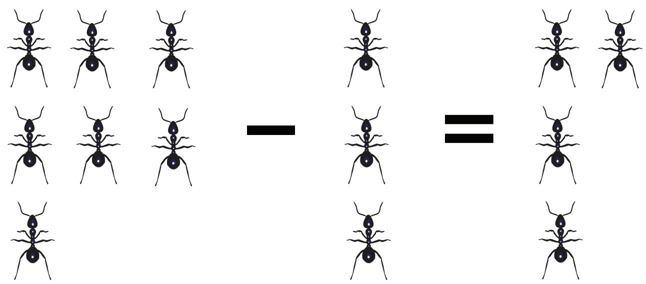
We can also subtract number using the number line and pictures.

For example, let's find 7 – 3.

When we subtract that means we move to the left on the number line. In this case we start at 7 then move 3 left. Moving 3 to the left brings us to 4. So 7 – 3 = 4.



We can also find the answer to 7 – 3 with pictures. If there were 7 ants, but 3 went away, then we are left with 4 ants.



Thinking Space

Please answer the following questions. Try using a number line or draw pictures to help you add or subtract.

1. 27 + 10 =

2. 15 + 12 =

3. 40 + 16=

4. $32 + 8 =$

5. $17 + 9 =$

6. $30 - 4 =$

7. $18 - 15 =$

8. $42 - 7 =$

9. $60 - 20 =$

10. $38 - 27 =$

Turn to the Answer Key at the end of the Module and mark your answers.

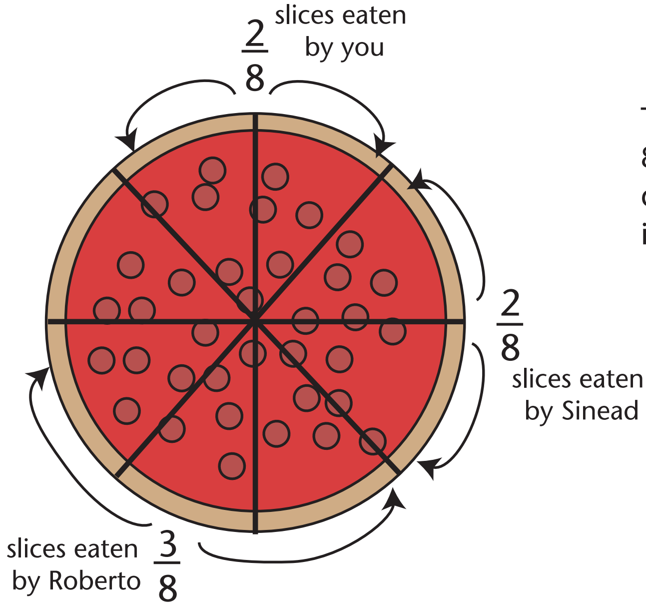
Explore

Thinking Space

Let's say that you and two of your friends, Sinead and Roberto, are about to share a pizza. The pizza has 8 slices.

Roberto was really hungry and ate 3 slices of the pizza. You and Sinead both ate 2 slices of pizza. What fraction of the pizza did you and your friends eat?

To find the answer to this question, let's look at writing the slices of pizza as fractions.



The pizza was cut into 8 slices, so 8 is our denominator. Each slice is $\frac{1}{8}$ of the whole pizza.

Roberto ate 3 slices, so 3 is our numerator for the first fraction. That's $\frac{3}{8}$ of the pizza.

Sinead ate 2 slices, so that's $\frac{2}{8}$ of the pizza.

You ate $\frac{2}{8}$ of the pizza.

Let's add up the fraction that each person ate to find the total amount eaten.

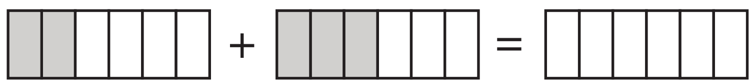
$$\frac{3}{8}+\frac{2}{8}+\frac{2}{8}$$

By looking at the diagram and counting the slices, we can see that 7 slices of the pizza were eaten. Written as a fraction, 7 slices is $\frac{7}{8}$ of the pizza. So that must mean that

$$\frac{3}{8}+\frac{2}{8}+\frac{2}{8}=\frac{7}{8}$$

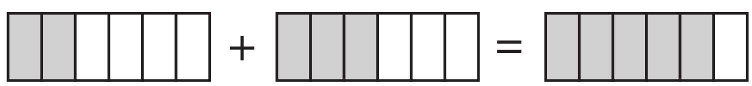
Thinking Space

Let's look at another example together.



Each rectangle is divided into 6 pieces. In the first rectangle, 2 pieces are shaded. In the second rectangle, 3 pieces are shaded. How many parts of a whole rectangle do we have in total?

Count up the shaded pieces. There are 5 shaded pieces—colour these 5 pieces on the blank rectangle.



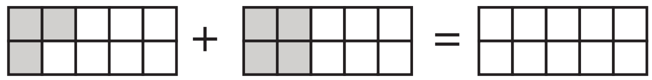
We can also write this using fractions:

$$\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$$

Practice 1

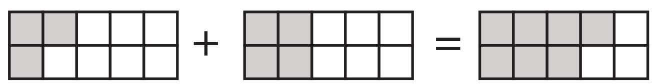
Let’s practise what we just learned. First show how much of the figure will be shaded, then write a fraction for that figure. We can complete the first one together.

$$\frac{3}{10} + \frac{4}{10} = \boxed{}$$



We have a drawing with 10 equal parts. Three pieces ($\frac{3}{10}$) of one drawing is coloured in, and 4 pieces ($\frac{4}{10}$) of the other drawing is coloured in. So that means we have a total of 3 + 4 = 7 pieces. Remember, we’re only looking at the shaded pieces. We have 7 shaded pieces out of the 10 pieces ($\frac{7}{10}$) of the final drawing.

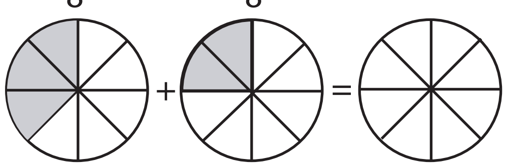
$$\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$$



Now it’s your turn to add the following fractions. Remember, first show how much of the figure will be shaded. Then write a fraction for that figure.

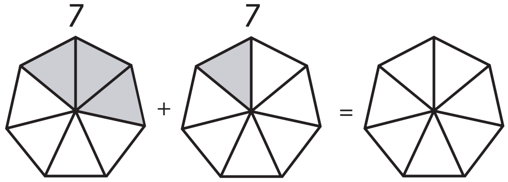
1.

$$\frac{3}{8} + \frac{2}{8} =$$



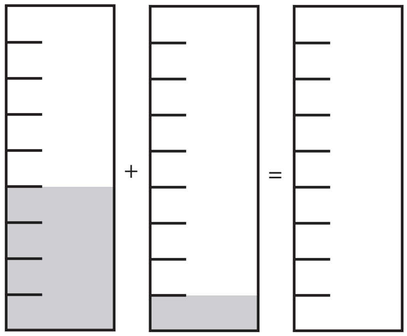
2.

$$\frac{3}{7} + \frac{1}{7} =$$



3.

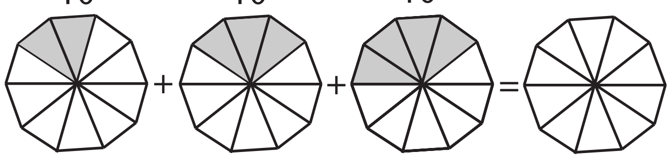
$\frac{4}{9} + \frac{1}{9} =$



This next question has you adding three fractions together. See if you can figure out how to add them all.

4.

$\frac{2}{10} + \frac{3}{10} + \frac{4}{10} =$



Now add these fractions. Draw pictures to help you find a solution.

5. $\frac{2}{5} + \frac{1}{5} =$

6. $\frac{4}{7}+\frac{2}{7}=$

7. $\frac{3}{11}+\frac{2}{11}+\frac{4}{11}=$

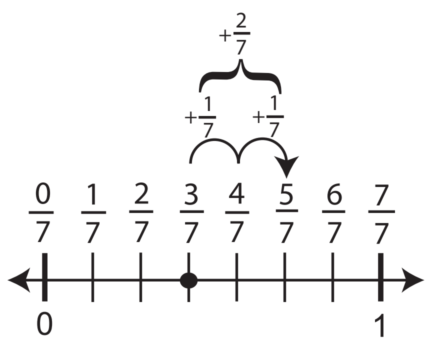
Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Recall your warm-up activity. We added whole numbers using a number line. We can also use a number line to add fractions.

Let's say we want to add $\frac{3}{7}$ and $\frac{2}{7}$. We start at $\frac{3}{7}$ on the number line, then move $\frac{2}{7}$ to the right. We can see what this would look like on the number line below:

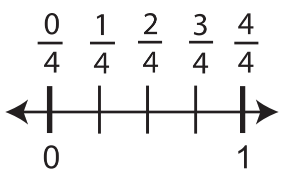


The number line above shows that $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$.

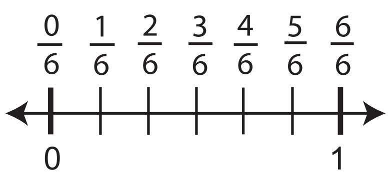
Practice 2

Use the number line provided to add the following fractions.

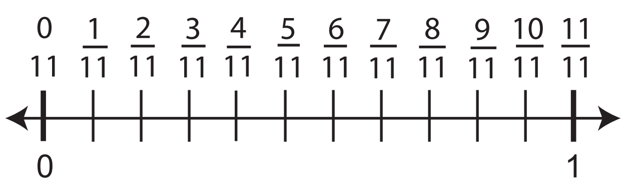
1. $\frac{2}{4} + \frac{1}{4} =$



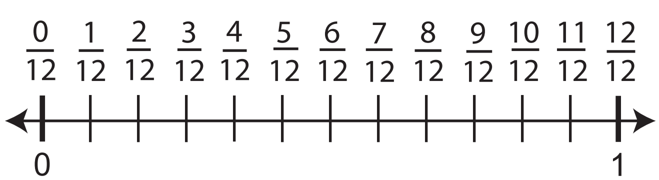
2. $\frac{3}{6} + \frac{2}{6} =$



3. $\frac{5}{11} + \frac{1}{11} =$



4. $\frac{3}{12} + \frac{4}{12} =$



Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Did you notice that the denominator does not change after the addition of fractions with common denominators? Why do you think the denominator stays the same?

Let's look at an example to help us understand why the denominator does not change. Say we run $\frac{1}{6}$ of the way home, and have a rest. Then we run another $\frac{2}{6}$. We've run $\frac{3}{6}$ of the total distance. These fractions tell us that the total distance is represented by the 6, and the parts we've run are the 1, 2, and 3.

Can you see how the numerator changes after the fractions are added?

We add up all of the parts of the whole to see how much we have.

> **Remember**: When you add fractions with common denominators, you add the numerators but keep the denominators the same.

For instance, $\frac{12}{15}+\frac{1}{15}=\frac{12+1}{15}=\frac{13}{15}$.

Thinking Space

Do you have any questions?

Practice 3

Let's practise adding fractions. If you get stuck, you can shade in parts of pictures or draw a number line to help you.

1. $\frac{2}{7}+\frac{3}{7}=$

2. $\frac{15}{30}+\frac{8}{30}=$

3. $\frac{24}{50}+\frac{15}{50}=$

4. $\frac{15}{79}+\frac{32}{79}=$

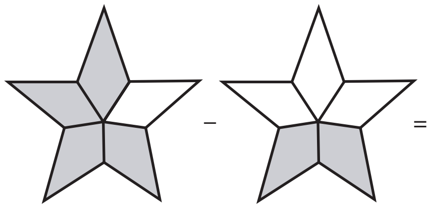
5. $\frac{37}{95}+\frac{28}{95}=$

Turn to the Answer Key at the end of the Module and mark your answers.

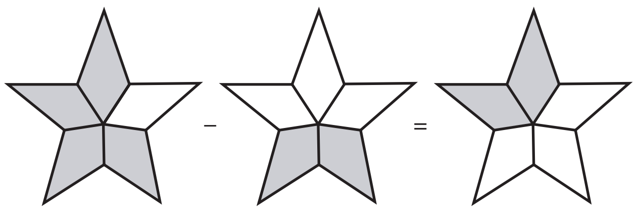
Explore

Thinking Space

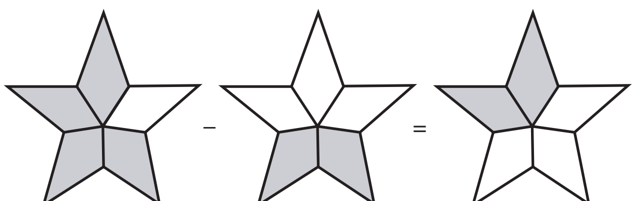
Subtracting fractions is very similar to adding fractions. Let's have a look at subtracting fractions using diagrams.



We have $\frac{4}{5}$ of a star but remove $\frac{2}{5}$. This leaves us with the following picture



Remember that in subtraction questions, we are *taking away* some of the shaded pieces. So



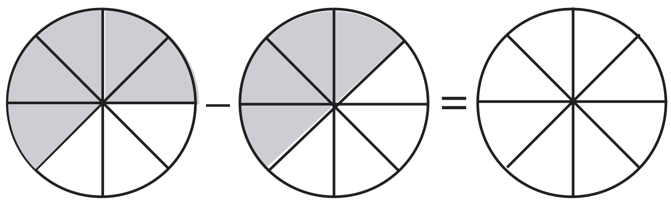
$$\frac{4}{5} - \frac{2}{5} = \frac{2}{5}$$

Practice 4

Now it's your turn. First show how much of the figure will be shaded. Then write a fraction for that figure.

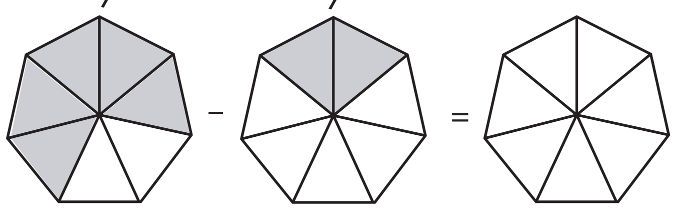
1.

$$\frac{5}{8} - \frac{4}{8} =$$



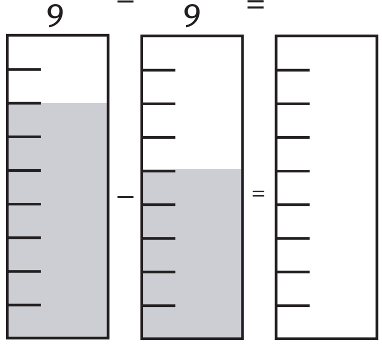
2.

$$\frac{5}{7} - \frac{2}{7} =$$



3.

$$\frac{7}{9} - \frac{5}{9} =$$

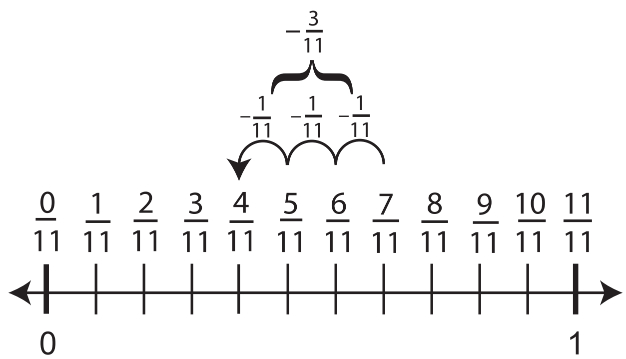


Turn to the Answer Key at the end of the Module and mark your answers.

Explore

We can also show how fractions are subtracted on a number line. Remember that when we subtract we start at one number and move *left* by the number being subtracted.

For example, let's find the difference of $\frac{7}{11}-\frac{3}{11}$.



So $\frac{7}{11}-\frac{3}{11}=\frac{4}{11}$.

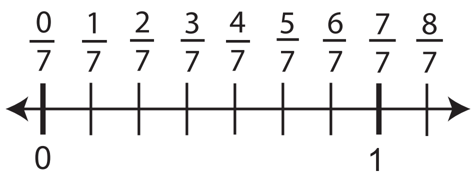
Thinking Space

Practice 5

Find the difference of the following fractions. Use the number line provided to help you.

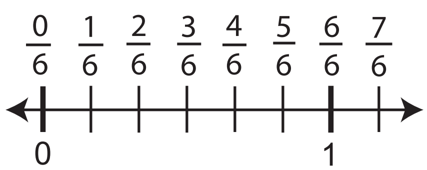
1.

$$\frac{6}{7} - \frac{1}{7} =$$



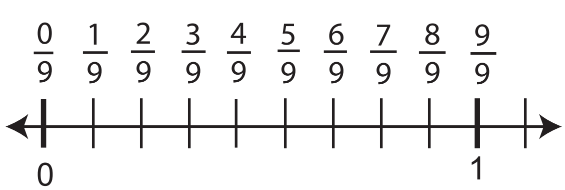
2.

$$\frac{5}{6} - \frac{4}{6} =$$



3.

$$\frac{6}{9} - \frac{4}{9} =$$



Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Do you see the pattern when subtracting fractions with common denominators?

Just like in addition, the denominator stays the same. Why do you think the denominator stays the same?

Can you see how the numerator changes after the fractions are subtracted?

If you are having trouble answering these questions, look back on what you wrote for the addition section.

> **Remember**: When you subtract fractions with common denominators, subtract the numerators but keep the denominators the same.

For example, $\frac{12}{15} - \frac{8}{15} = \frac{12-8}{15} = \frac{4}{15}$

Practice 6

Solve the following questions. If you get stuck, you can shade in parts of pictures or draw a number line to help you.

1. $\frac{3}{4}-\frac{2}{4}=$

2. $\frac{4}{7}-\frac{1}{7}=$

3. $\frac{9}{12}-\frac{4}{12}=$

4. $\frac{13}{15}-\frac{6}{15}=$

5. $\frac{25}{35}-\frac{21}{35}=$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

So far all of the answers to our addition and subtraction questions have been reduced fractions. However, if you get a fraction like $\frac{4}{8}$, you will need to reduce it to lowest terms in your final answer. This makes it so the answer looks as simple as possible.

For example:

$$\frac{5}{8}-\frac{1}{8}=\frac{4}{8}=\frac{4\div 4}{8\div 4}=\frac{1}{2}$$

Also, our answers so far have been smaller than one.

When we see a fraction like $1\frac{1}{2}$ it means that we have one whole and one half: $1\frac{1}{2}=1+\frac{1}{2}$.

Let's pretend your friend had two chocolate bars He ate half of one and offered the rest to you. Would your friend say "would you like $\frac{3}{2}$ chocolate bars?" It doesn't sound right, does it? Your friend would probably say "would you like $1\frac{1}{2}$ chocolate bars?" That sounds much better. Just like your friend used mixed numbers, so should you. If you get an answer greater than 1, you will need to rewrite it as a mixed number for your final answer.

For example:

$$\frac{4}{2}-\frac{1}{2}=\frac{3}{2}=1\frac{1}{2}$$

Practice 7

Answer the following addition and subtraction questions. Make sure that your answers are reduced and written as mixed numbers when necessary.

1. $\frac{1}{4} + \frac{1}{4} =$

2. $\frac{2}{9} + \frac{1}{9} =$

3. $\frac{3}{8} + \frac{5}{8} =$

4. $\frac{2}{4} + \frac{3}{4} =$

5. $\frac{6}{10} + \frac{2}{10} =$

6. $\frac{9}{30} + \frac{11}{30} =$

7. $\frac{7}{9} - \frac{4}{9} =$

8. $\frac{18}{6} - \frac{9}{6} =$

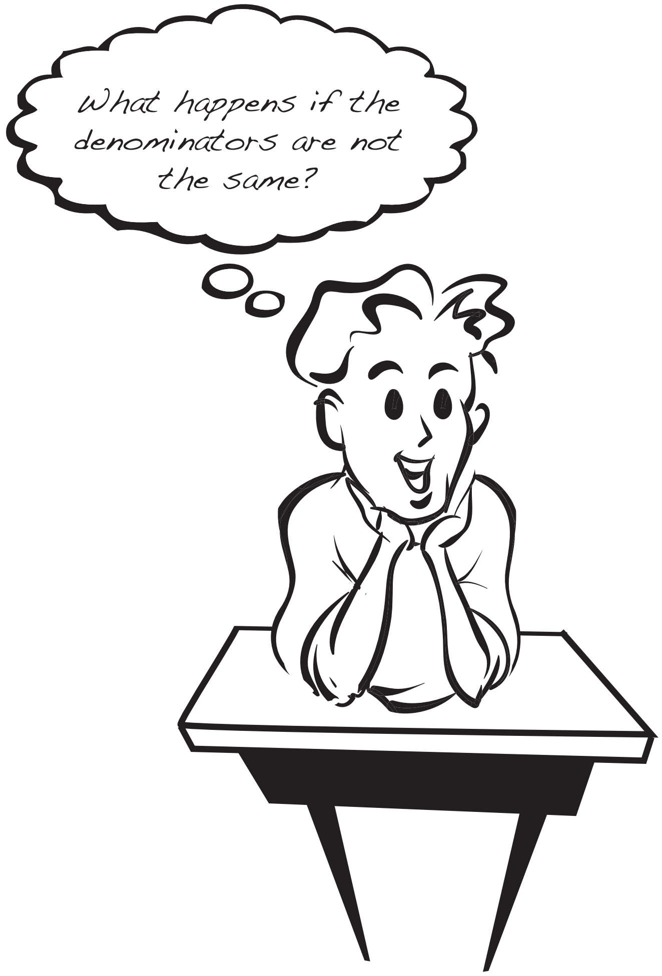
9. $\frac{20}{15} - \frac{3}{15} =$

10. $\frac{17}{12} - \frac{3}{12} =$

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 2.1D: Adding and Subtracting Fractions With Unlike Denominators

Student Inquiry



This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
How can I find a common denominator of fractions?		answer
		example
How do I find the sum of fractions with unlike denominators?		answer
		example
How do I find the difference of fractions with unlike denominators?		answer
		example
How can I simplify a fraction?		answer
		example

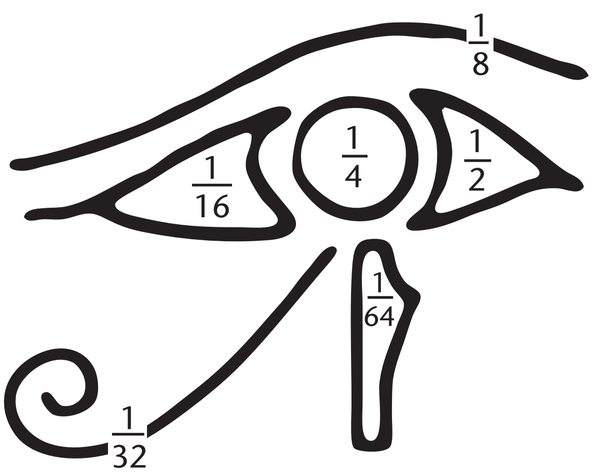
Lesson 2.1D: Adding and Subtracting Fractions with Unlike Denominators

Thinking Space

Introduction

In the last lesson, we looked at adding and subtracting fractions. But what happens if the denominators are not the same? This isn't the first time someone has asked this question. Do you know about the ancient Egyptians? They were the people who built majestic pyramids and mummified their kings. These ancient Egyptians also believed in a god called Horus. The Egyptians labelled each part of the Eye of Horus with a fraction. The Eye was only a notation system (a way of writing fractions down) so the part labeled $\frac{1}{2}$ was not physically $\frac{1}{2}$ of the whole eye.

They used the Eye of Horus fraction system to keep track of trading. The Egyptians also believed that all of the fractions in the Eye of Horus added up to 1.



Do the fractions add up to exactly 1?

You will be able be able to answer this question by the end of this lesson on adding and subtracting fractions with unlike denominators.

Explore Online

Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:
http://www.openschool.bc.ca/courses/math/math7/mod2.html
Look for *Lesson 2.ID: Adding and Subtracting Fractions with Unlike Denominators* and check out some of the links!

Warm-up

Thinking Space

Before we learn how to add fractions with unlike denominators, we need to review a few concepts.

Do you remember what a **factor** is?

A factor is a whole number that divides into another number without a remainder.

For example, factors of 10 are 1, 2, 5, and 10 since $\frac{10}{1} = 10$, $\frac{10}{2} = 5$, $\frac{10}{5} = 2$, $\frac{10}{10} = 1$.

3 is not a factor of 10 since $\frac{10}{3} = 3.333333....$

Do you see how the factors pair up?

$1 \times 10 = 10$ so 1 and 10 are factors

$2 \times 5 = 10$ so 2 and 5 are factors

Pairing up the factors helps you to make sure you do not forget to include any factors.

However, sometimes a factor is paired with itself.

For example, factors of 4 are 1, 2, and 4. We pair up 1 and 4 ($1 \times 4 = 4$) and 2 pairs up with itself ($2 \times 2 = 4$)

Can you find the factors of 6?

Since 1, 2, 3, and 6 all go evenly into 6, they are called factors of 6.

A **common factor** is a factor that divides into two numbers.

For example let's find common factors of 6 and 10. We can start by listing all of the factors. Then we can find the factors that are found in both lists:

6	10
1	1
2	2
3	5
6	10

Thinking Space

!

Since 1 and 2 are factors of 6 and 10, we can say that 1 and 2 are common factors of 6 and 10.

We can also find a *greatest common factor*.

As you might have figured out by its name, the greatest common factor is the common factor with the largest value.

In our example we found out that 1 and 2 are common factors of 6 and 10, but which one is the greatest?

Since 2 has the largest value, 2 is the greatest common factor.

We can find the greatest common factor of more than 2 numbers.

Let's find the greatest common factor of 16, 12, and 8. The first step is to list all of the factors.

Factors of 16: 1, 2, 4, 8, 16

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 8: 1, 2, 4, 8

Now can you find the greatest common factor?

1, 2, and 4 are all factors, but 4 is the greatest common factor since it has the largest value.

For the following questions, let's list all of the factors of the given numbers then find the greatest common factor. The first example is done for you.

1.

Find the greatest common factor of:	Factors	Greatest Common Factor
14 42	1, 2, 7, 14 1, 2, 6, 7, 21, 42	7
10 12		
30 20		
12 16 36		

Can you remember what a multiple is? Here's a hint: What operation (addition, subtraction, multiplication or division) does the word multiple remind you of?

Thinking Space

Multiples look a lot like multiplication. There is a reason for this. A **multiple** is a number that has been multiplied.

For example:

Multiples of 5 are: 5 (5 x 1), 10 (5 x 2), 15 (5 x 3) and so on.

A **common multiple** is a multiple of two or more numbers.

For example:

Multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 100, ...

Multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, ...

Common multiples of 12 and 8 are 24, 48, 72, 96, ...

We can find a LOT of common multiples, but there is one called the *lowest common multiple*. This is the multiple with the smallest value. In the case of 12 and 8, the lowest common multiple is 24.

For the following questions, list a few multiples of the given numbers, then find the lowest common multiple. The first example is done for you.

2.

Find the lowest common multiple of:	Multiples	Lowest Common Multiple
4 14	4, 8, 12, 16, 20, 24, 28, ... 14, 28, ...	28
6 8		
12 15		
16 40		

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

For this activity, you'll need:

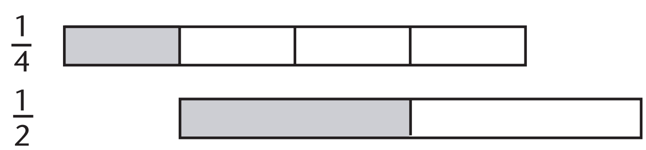
- fraction strip template (you'll find this at the back of this module)
- scissors
- a pencil

The rectangles on the template are called fraction strips.

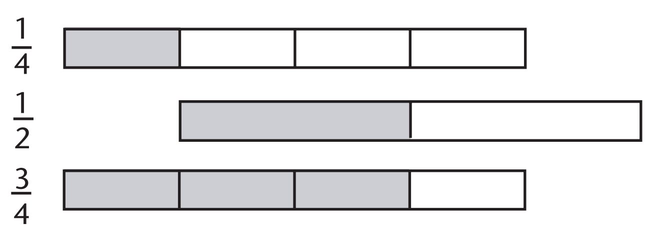
Here is what you need to do:

1. Cut all five rectangles along the DOTTED lines. (Do NOT cut along the solid lines).
2. Fold along the solid lines on each fraction strip.
3. Use the fraction strips to work through the examples below:

Let's add $\frac{1}{2}$ and $\frac{1}{4}$ using the fraction strips. To add, place the $\frac{1}{2}$ strip at the end of the shaded part of the $\frac{1}{4}$ strip as shown below.



Can you find the strip that matches the combined shaded area?



Notice that the shaded area of $\frac{1}{4}$ and $\frac{1}{2}$ is the same length of $\frac{3}{4}$.

This means $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

Practice 1

Now it's your turn. Use fraction strips to find the sum of these fractions:

1. $\frac{1}{3} + \frac{2}{4}$

2. $\frac{2}{3} + \frac{1}{6}$

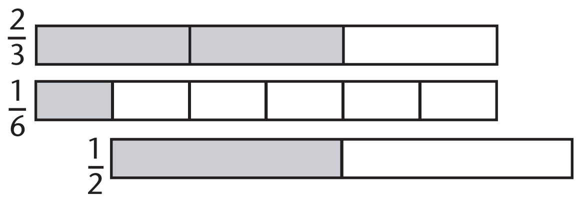
Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Now let's use fraction strips to find the difference between fractions.

For example, let's find the solution to $\frac{2}{3} - \frac{1}{6}$.

To find the solution, place $\frac{2}{3}$ and $\frac{1}{6}$ side by side. Can you find the strip that matches the left over shaded area?



So we know that $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$.

Practice 2

Now it's your turn. Use fraction strips to answer:

1. $\frac{3}{4} - \frac{1}{2}$

2. $\frac{1}{1} - \frac{4}{6}$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Fraction strips are great because we can easily see the solution. However, they are not always the best way to solve fraction questions. Can you imagine making a fraction strip for $\frac{17}{50}$!?! Let's learn other ways to solve these types of questions.

How do we add fractions with unlike denominators? Can we just add the numerators? Let's look at an example to find out.

If you bought 2 cartons of eggs, but you already had 5 eggs at home, how many eggs are there in total? Remember, there are 12 eggs in a carton.

We can write this as a type of fraction:

$$\frac{2}{\text{Cartons of Eggs}} + \frac{5}{\text{Eggs}}$$

It may look like we have 7 eggs, but remember we bought two whole cartons of eggs. We know that there are 12 eggs in each carton, so let's rewrite our fractions.

$$\frac{2}{\text{Cartons of Eggs}} + \frac{5}{\text{Eggs}}$$

$$= \frac{24}{\text{Eggs}} + \frac{5}{\text{Eggs}}$$

$$= \frac{29}{\text{Eggs}}$$

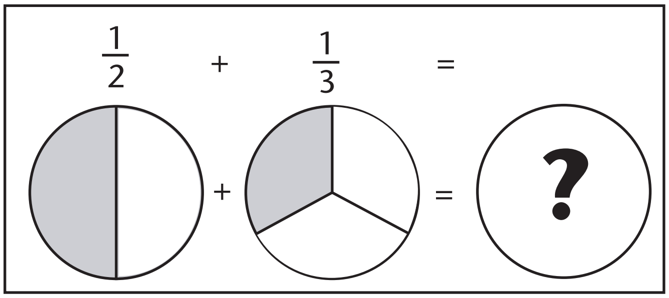
When we are just looking at the number of eggs, it is much easier to find the total number of eggs.

So when we work with fractions it is much easier to have the same denominator.

If we look at $\frac{1}{2} + \frac{1}{3}$ we can't just add the numerators because the denominators are not the same. You may be wondering why the denominators need to be the same. Let's look at an example that describes these fractions.

Thinking Space

Let's say it's your best friend's birthday, and you made a birthday cake. You and your friends ate $\frac{1}{2}$ of the cake at the party. Later, your friend's brother ate $\frac{1}{3}$ of the cake. How much of the whole cake was eaten? To help us answer this question, let's look at a diagram.

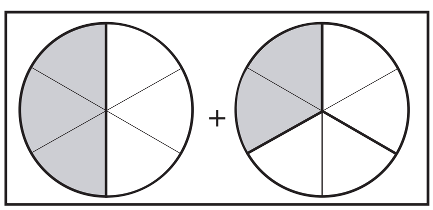


We can't just count the pieces, because the pieces aren't the same size.

To shade in the total amount eaten, draw more lines so both circles have the same number of pieces. How many pieces should there be?

Multiples of 2:	2,	4,	**6**,	8,	10, ...
Multiples of 3:	3,	**6**,	9,	12,	...

The lowest common multiple of 2 and 3 is 6, so we should make each circle have 6 equal pieces.

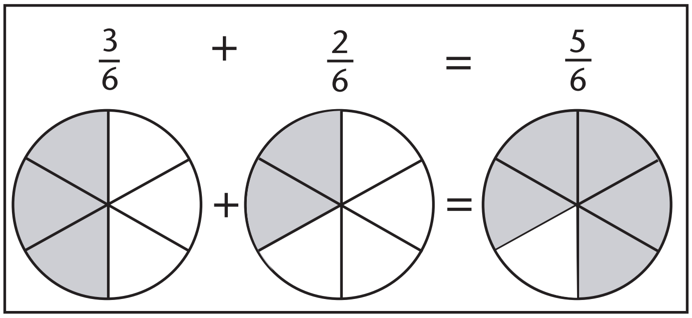


Remember that the denominator (bottom number) tells us how many pieces there are in total. The numerator (top number) tells us how many out of the pieces we have (or in this case, have eaten). When we found we could divide both circles into 6 equal parts, we made both of the denominators 6. Now the denominators are **lowest common denominators**.

Thinking Space

We have turned $\frac{1}{2}$ into an equivalent fraction of $\frac{3}{6}$. We made $\frac{1}{3}$ into its equivalent fraction of $\frac{2}{6}$.

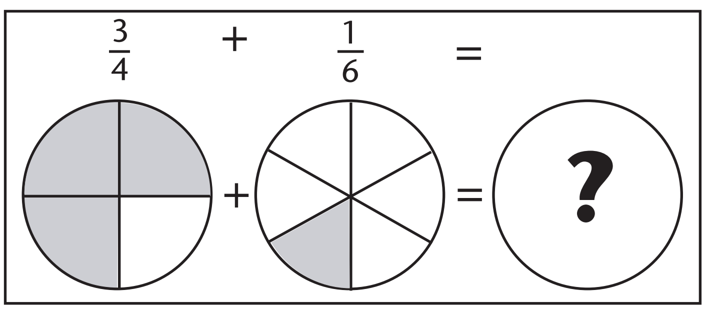
Now we have $\frac{3}{6} + \frac{2}{6}$.



We know that $\frac{5}{6}$ of the cake was eaten.

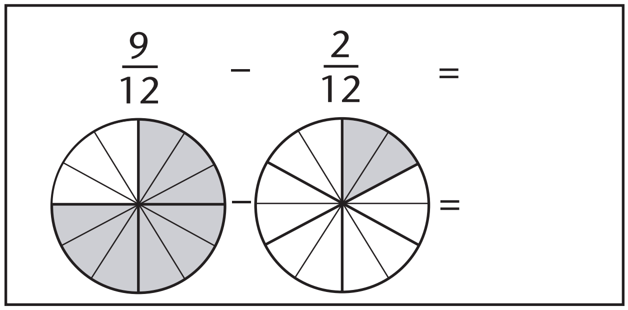
We can also use this type of fraction circle to subtract fractions.

Let's find the difference of $\frac{3}{4} - \frac{1}{6}$

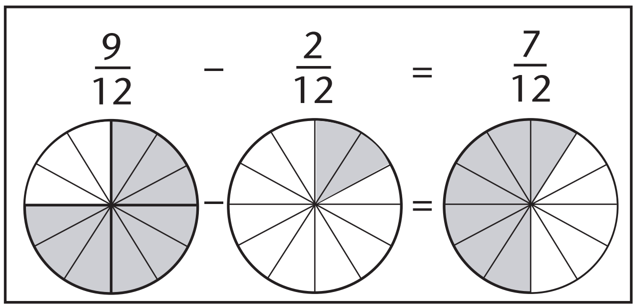


What is the lowest common denominator of $\frac{3}{4}$ and $\frac{1}{6}$?

Since the lowest common multiple of 4 and 6 is 12, we know that 12 is the lowest common denominator. So let's make each circle have 12 pieces. This makes equivalent fractions with denominator of 12.

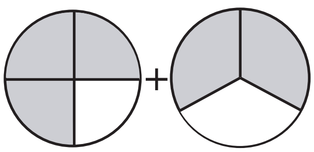
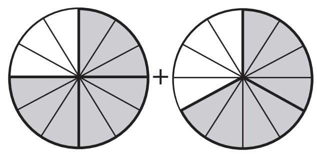
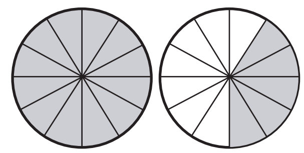


Subtracting the two fractions gives us:



Practice 3

Solve the following addition and subtraction questions using diagrams like the fraction circles we just saw. The first question is solved for you. Do not forget to leave your answer in lowest terms. Write your answer as a mixed number if needed.

Question	Question with lowest common denominators	Solution
$\frac{3}{4}+\frac{2}{3}$ 	$\frac{9}{12}+\frac{8}{12}$ 	$\frac{17}{12}=1\frac{5}{12}$ 
1. $\frac{1}{2}+\frac{2}{3}$		
2. $\frac{4}{5}+\frac{1}{2}$		
3. $\frac{5}{6}-\frac{1}{3}$		

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Sometimes you might want to work with fractions without drawing pictures or using fraction strips. A quick way to add or subtract fractions is to write equivalent fractions.

To add or subtract fractions with unlike denominators, follow these steps:

1. Find the lowest common denominator.
2. Change the fractions into equivalent fractions with the lowest common denominator.
3. Add or subtract.
4. Leave your answer in lowest terms. Write your answer as a mixed number if needed.

For example, we can find the difference between $\frac{2}{3}$ and $\frac{1}{5}$

$$\frac{2}{3} - \frac{1}{5}$$

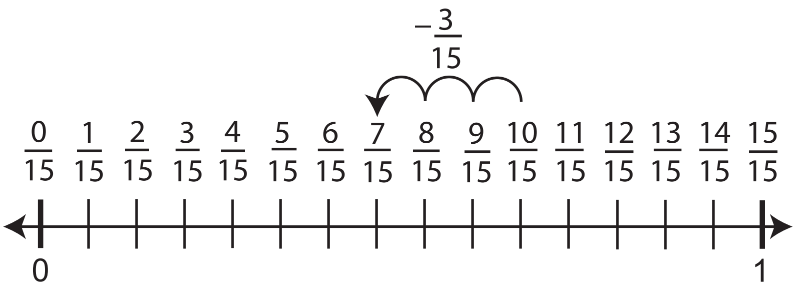
The lowest common denominator is 15.

Rewrite as equivalent fractions.

$$\frac{10}{15} - \frac{3}{15} = \frac{7}{15}$$

If you get stuck while subtracting, you may want to use a number line.

$\frac{10}{15} - \frac{3}{15}$



Practice 4

Let's solve some addition and subtraction questions by writing equivalent fractions. The first question is completed for you. Don't forget to leave your answer in lowest terms. Write your answer as a mixed number if needed.

Question	Equivalent fractions	Solution
$\frac{7}{10} + \frac{3}{4}$	$\frac{14}{20} + \frac{15}{20}$	$\frac{29}{20} = 1\frac{9}{20}$
1. $\frac{2}{7} + \frac{1}{2}$		
2. $\frac{1}{5} + \frac{5}{6}$		
3. $\frac{5}{7} - \frac{1}{3}$		
4. $\frac{3}{5} - \frac{2}{7}$		

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 2.1E: Adding and Subtracting Mixed Numbers

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

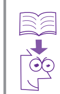
When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
How can I show the addition and subtraction of mixed numbers using diagrams?		answer
		example
How do I add and subtract mixed numbers with unlike denominators?		answer
		example
How can I check to see if my answer is reasonable?		answer
		example

Lesson 2.1E: Adding and Subtracting Mixed Numbers

Thinking Space

Introduction



Have you ever used a recipe to bake a birthday cake or to cook dinner? If you have then you have probably already seen mixed fractions. This lesson will give you the skills to add or subtract mixed numbers.

Explore Online

Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:
http://www.openschool.bc.ca/courses/math/math7/mod2.html
Look for *Lesson 2.1E: Adding and Subtracting Mixed Numbers* and check out some of the links!

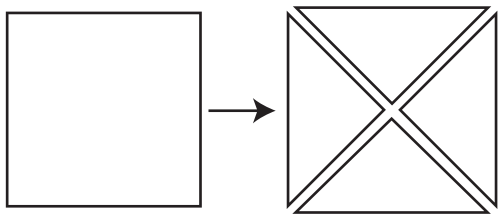
Warm-up

For this activity, you'll need:

- square template (you'll find this at the back of the module)

Here's what to do:

- Cut out all six squares along the dotted lines.
- Cut each square into four equal pieces, following the dotted lines.
- Use these pieces to work through the examples below:

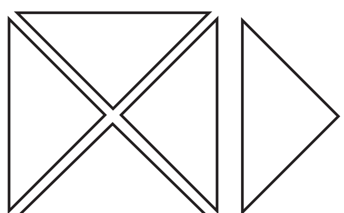


Square before you cut

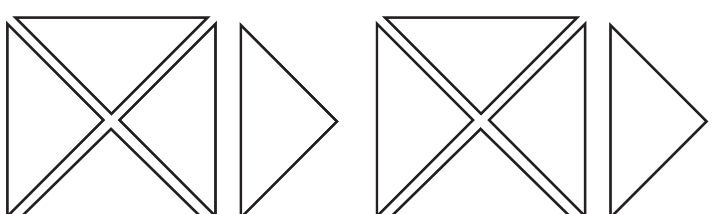
Square after you cut it into 4 equal pieces

Make a total of 5 squares that are cut into 4 equal triangles.

Let's say you have a total of 5 pieces of a square (that's 5 triangles). How many squares can you make? When you put your triangles together, you should get something like the picture below:

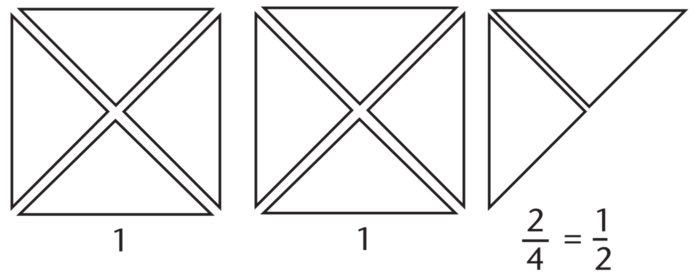


Now let's say you had two sets of these $1\frac{1}{4}$ pieces:



How many squares do you have in total?

We can solve this by placing our whole squares next to each other, and then we can group our $\frac{1}{4}$ pieces together.

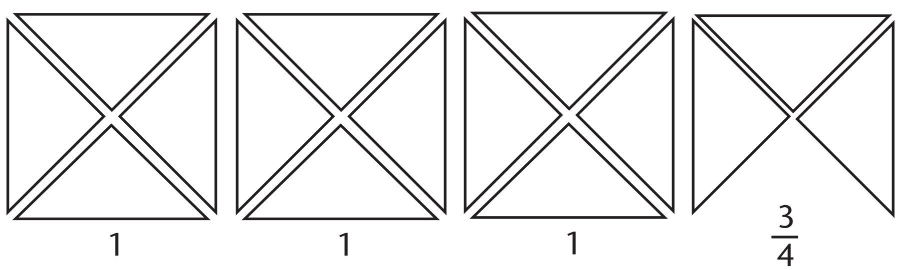


This gives us a total of $2\frac{1}{2}$ squares.

Let's try another example.

If you have a total of 15 triangles, how many squares do you have?

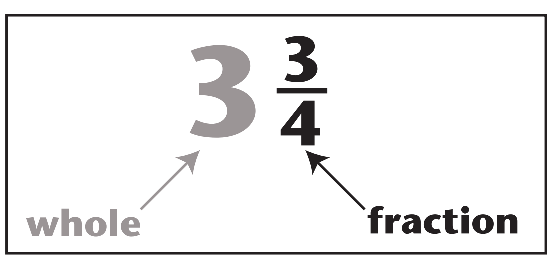
Let's make as many full squares as we can, and group these together. Then we can put the leftover pieces together.



This gives us $3\frac{3}{4}$ squares.

Do you remember the different parts of a mixed number?

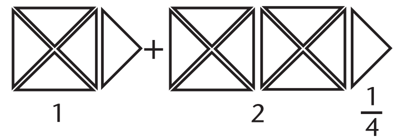
There is a whole number and a fraction.



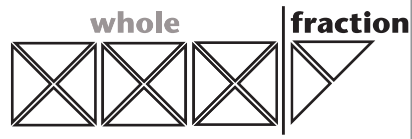
We are going to use the idea of grouping the wholes and grouping the parts to solve addition and subtraction questions in the next section. Hang on to your triangle pieces.

Explore

Let's use the triangles you made in the warmup section to add $1\frac{1}{4}+2\frac{1}{4}$.



Let's separate the wholes and the fractions:



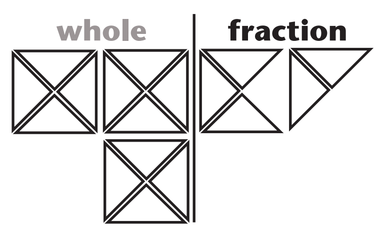
We have a total of 3 whole squares. In the fraction part, the $\frac{1}{4}$ and $\frac{1}{4}$ equal $\frac{1}{2}$ a square.

$$\mathbf{1\frac{1}{4}+2\frac{1}{4}=3\frac{1}{2}}$$

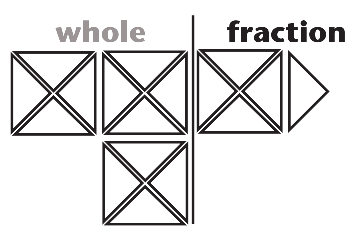
Let's look at another example.

Add $1\frac{3}{4}$ and $2\frac{2}{4}$.

Let's put the 1 and 2 in the "whole" section and the $\frac{3}{4}$ and $\frac{2}{4}$ in the "fraction" section.

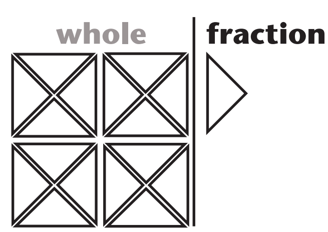


Putting the fraction pieces together gives us:



Now we have a whole in the "fraction" section.

Move that whole square to the "whole column."



Thinking Space

The denominators are the same.

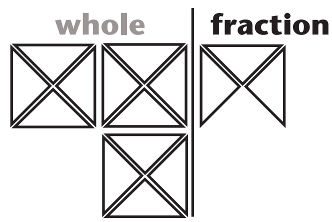
Thinking Space

This gives us 4 and $\frac{1}{4}$. So $1\frac{3}{4} + 2\frac{1}{4} = 4\frac{1}{4}$.

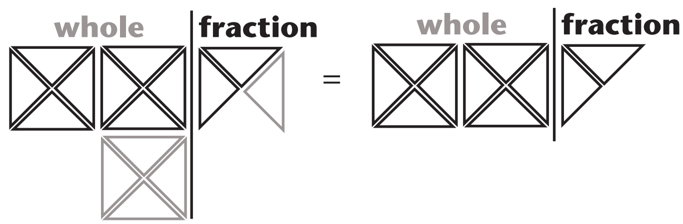
We can also subtract mixed numbers using these fraction pieces.

Let's find the solution to $3\frac{3}{4} - 1\frac{1}{4}$.

$3\frac{3}{4}$ tell us we start with three whole squares and 3 out of 4 pieces of a square:



Subtracting $1\frac{1}{4}$ means we need to take away 1 whole and $\frac{1}{4}$ piece.

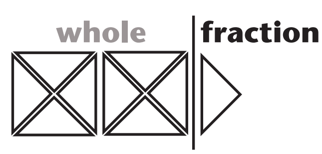


This tells us that $3\frac{1}{4} - 1\frac{1}{4} = 2\frac{2}{4}$ which simplifies to $2\frac{1}{2}$.

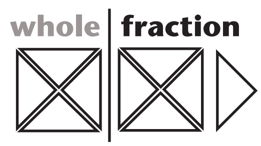
Let's try one more example.

Find the solution to $2\frac{1}{4} - \frac{3}{4}$.

We start with 2 wholes and our fraction is $\frac{1}{4}$.

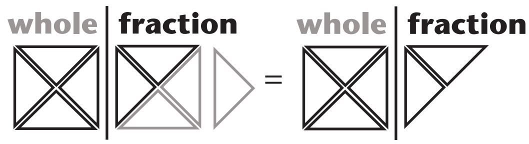


Now we need to subtract $\frac{3}{4}$. But there are not enough fraction pieces to remove $\frac{3}{4}$ in the fraction column! We can take one of the wholes and put it into our fraction section.



Thinking Space

Now let's take away $\frac{3}{4}$.



Now we know that $2\frac{1}{4} - \frac{3}{4} = 1\frac{2}{4}$ which simplifies to $1\frac{1}{2}$.

Practice 1

Use your triangles to solve the following questions.

1. $\frac{3}{4}+\frac{2}{4}=$

2. $1\frac{3}{4}-\frac{2}{4}=$

3. $2\frac{3}{4}+1\frac{3}{4}=$

4. $3\frac{2}{4}-\frac{3}{4}=$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

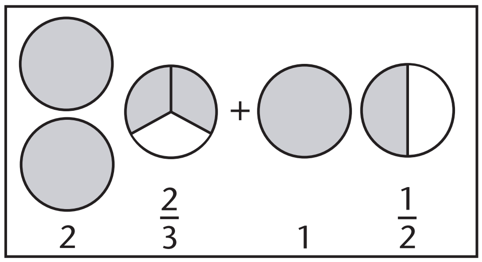
Thinking Space

We can also solve mixed number questions with different denominators. Let's use fraction circles to see how this works.

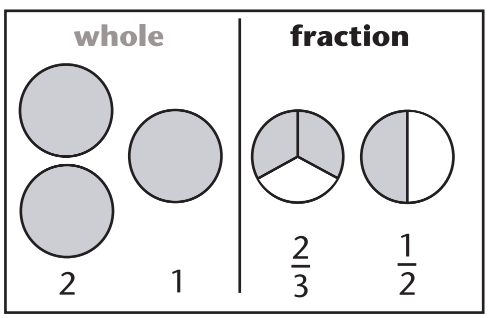
Addition

Let's find the solution to $2\frac{2}{3}+1\frac{1}{2}$:

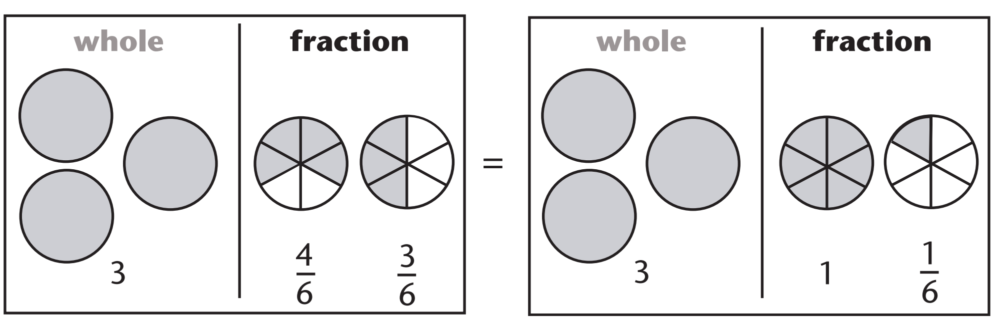
Draw these in fraction circles.



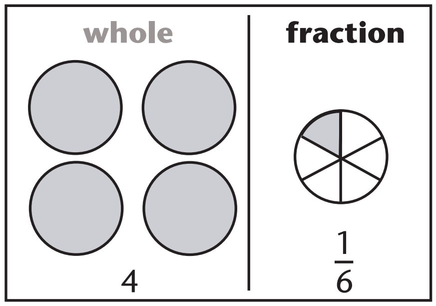
Let's put the wholes together and the fractions together.



We can add the whole parts pretty easily, but we need to have the same number of pieces in each circle to add the fractions. The lowest common denominator is 6. Write the equivalent fractions. Now we can easily add the fraction pieces.



We have a whole circle in the "fraction" section, so let's move it to the "whole" section.

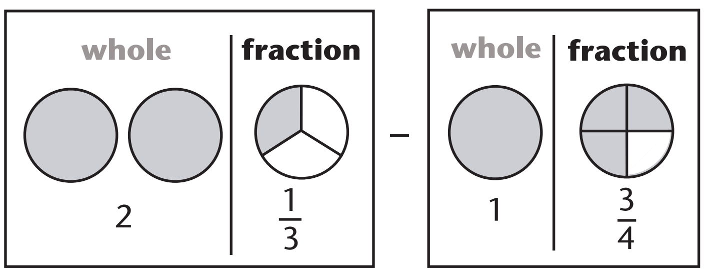


$$2\frac{2}{3}+1\frac{1}{2}=4\frac{1}{6}$$

Subtraction

Now let's try a subtraction question. Simplify $2\frac{1}{3}-1\frac{3}{4}$.

We'll start by drawing the fraction circles.



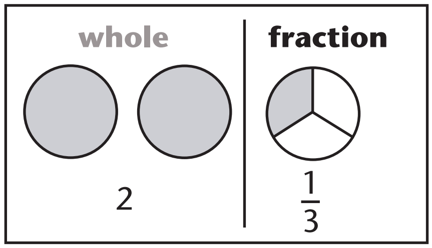
What do you notice about the fractions?

Before we subtract we need to:

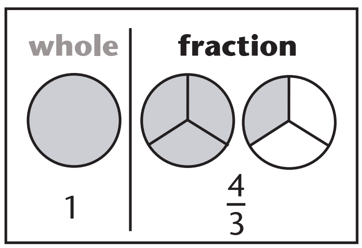
- borrow from the "wholes" so we have anough pieces to take away $\frac{3}{4}$.
- cut the fractions up so they have the same number of pieces.

Let's try!

First let's draw $2\frac{1}{3}$ in fraction circles and place the circles in the "whole" and "fraction" sections.

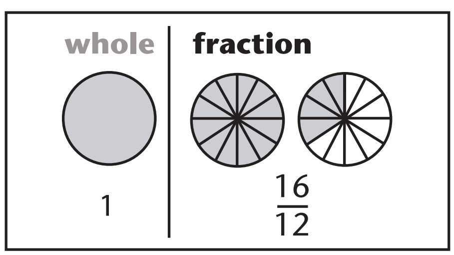


$\frac{3}{4}$ is more than $\frac{1}{3}$, so we will need to move a circle from the "whole" section over.



We borrowed a whole to change $2\frac{1}{3}$ to $1\frac{4}{3}$.

Also, we need to have the same number of pieces in each circle so our fraction circles will need to have 12 parts.

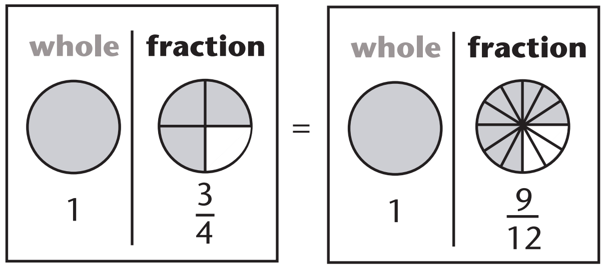


Now we can see that $1\frac{4}{3} = 1\frac{16}{12}$.

Thinking Space

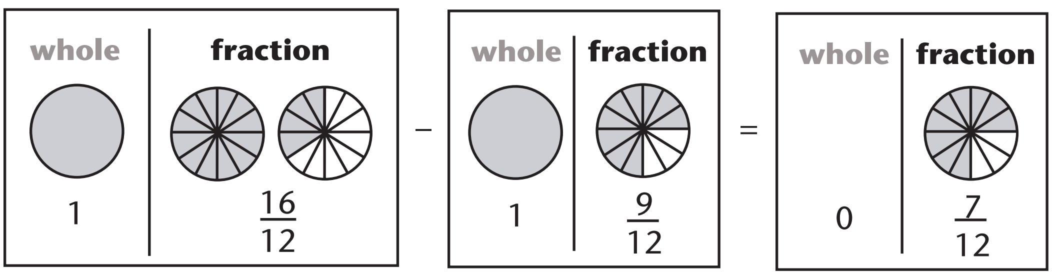
Thinking Space

Now we need to make the $\frac{3}{4}$ fraction circle have 12 pieces:



You can see that $1\frac{3}{4} = 1\frac{9}{12}$.

Now that we've done all our conversions, let's subtract!



What we converted:

$2\frac{1}{3} = 1\frac{16}{12}$

$1\frac{3}{4} = 1\frac{9}{12}$

When we subtract the wholes we get zero.

When we subtract the fractions we get $\frac{7}{12}$.

Now we can answer the original question: $2\frac{1}{3} - 1\frac{3}{4} = \frac{7}{12}$.

Practice 2

Borrow from the whole number to make the numerator in the fraction larger. The first question is done for you.

1. $2\frac{3}{4} = 1 + \frac{4}{4} + \frac{3}{4} = 1\frac{7}{4}$

2. $5\frac{1}{2}$

3. $2\frac{1}{3}$

4. $3\frac{3}{5}$

5. $4\frac{5}{6}$

Draw fraction circles to solve these questions:

6. $2\frac{1}{3}+1\frac{1}{4}=$

7. $2\frac{3}{5}+\frac{1}{2}=$

8. $3\frac{1}{2}-1\frac{1}{3}=$

9. $2\frac{1}{6}-1\frac{1}{2}=$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

There are two ways to think about adding mixed numbers. Let's explore both ways to answer the following question:

Trevor buys apple pies and cherry pies to share with his friends. By the end of the day he has $2\frac{1}{2}$ apple pies and $3\frac{2}{3}$ cherry pies left over. How many pies does Trevor have left over?

Estimation: Trevor has about 3 apple pies and about 4 cherry pies. This means he would have close to 7 pies. Notice that we rounded up for both pies, so our estimate may be a little high.

First way: **convert the improper fractions to a mixed number**.

To solve a mixed fraction question, we can:

1. Convert the mixed numbers into improper fractions
2. Add the improper fractions.
3. Convert the final answer into a mixed number if the answer is larger than 1.

Example:

$$2\frac{1}{2}+3\frac{2}{3}=\frac{5}{2}+\frac{11}{3}$$

$$\frac{5}{2}+\frac{11}{3}=\frac{15}{6}+\frac{22}{6}=\frac{37}{6}$$

$$\frac{37}{6}=6\frac{1}{6}$$

Second way: **add the fraction parts, then add the whole parts.**

To solve a mixed fraction question, we can:

	Example: $2\frac{1}{2}+3\frac{2}{3}$
1. Add the fraction part of the mixed number:	Fraction parts: $\frac{1}{2}$ and $\frac{2}{3}$ Whole parts: 2 and 3 $\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$
2. Convert the answer to a mixed number if it is larger than 1.	$\frac{7}{6}=1\frac{1}{6}$ Fraction part: $\frac{1}{6}$ Whole part: 1
3. Add the whole number part of the mixed numbers.	2 + 3 + 1 = 6
4. Add the whole number and fraction parts together.	$6+\frac{1}{6}=6\frac{1}{6}$

Both methods show us that $2\frac{1}{2}+3\frac{2}{3}=6\frac{1}{6}$. So Trevor has $6\frac{1}{6}$ pies left over.

Check: $6\frac{1}{6}$ is between 6 and 7. Our estimation was a little less than 7, so we know that $6\frac{1}{6}$ is a reasonable answer.

Make sure you leave your answer simplified and as a mixed number if it is larger than 1. Remember, you would say "Trevor has $6\frac{1}{6}$ pies" not "Trevor has $\frac{37}{6}$ pies."

Thinking Space

Which method do you prefer? Why?

Now let's try both of these methods with subtraction to answer this question:

There are $4\frac{1}{3}$ pizzas left over at a party. Your friend says that she will take $2\frac{1}{2}$ pizzas home for her family and you can have the rest. How much pizza will you get to take home?

Estimation: $4\frac{1}{3} - 2\frac{1}{2}$ is close to $4 - 3 = 1$

Look at how we rounded each fraction. Will our estimate be a little high or a little low?

First way: **convert the improper fractions to a mixed number**.

To solve a mixed fraction question, we can:

	Example
1. Convert the mixed numbers into improper fractions.	$4\frac{1}{3} - 2\frac{1}{2} = \frac{13}{3} - \frac{5}{2}$
2. Subtract the improper fractions.	$\frac{13}{3} - \frac{5}{2} = \frac{26}{6} - \frac{15}{6} = \frac{11}{6}$
3. Convert the final answer into a mixed number if the answer is larger than 1.	$\frac{11}{6} = 1\frac{5}{6}$

Thinking Space

What do you like/ dislike about this method?

Second way: **subtract the wholes and subtract the fractions**

To solve mixed fraction questions, we can:

1. Convert the mixed numbers into equivalent fractions with common denominators.

2. If the first numerator is smaller than the second numerator, then transfer from the whole to make the first numerator larger.

3. Subtract the wholes and the fractions.

Example

$4\frac{1}{3}-2\frac{1}{2}=4\frac{2}{6}-2\frac{3}{6}$

$\frac{2}{6}$ is smaller than $\frac{3}{6}$ so we need to make $\frac{2}{6}$ larger.

We do this by borrowing one whole number from the 4 and converting it into a fraction:

Change $4\frac{2}{6}$ into $3\frac{8}{6}$ by transferring a whole ($\frac{6}{6}$) from the 4.

$4\frac{2}{6}-2\frac{3}{6}=3\frac{8}{6}-2\frac{3}{6}$

$=1\frac{5}{6}$

Thinking Space

What do you like/ dislike about this method?

Both methods show us that $4\frac{1}{3} - 2\frac{1}{2} = 1\frac{5}{6}$. You get to take $1\frac{5}{6}$ pizzas home with you.

Thinking Space

Check: Let's make sure this answer is reasonable.

$1\frac{5}{6}$ is between 1 and 2. Our estimate was 1, so do you think $1\frac{5}{6}$ is a reasonable answer?

Remember that we rounded one fraction down and the other fraction up. Our estimate was a little low, but it was still useful in helping us to decide if our answer was reasonable.

Don't forget to simplify your answer, and if it is larger than one, write it as a mixed number. Remember you would not say "I should take $\frac{11}{6}$ pizzas."

This doesn't mean that $\frac{11}{6}$ is wrong, but in this case we would prefer to use $1\frac{5}{6}$. There will be times when improper fractions are more useful than mixed fractions. But we'll leave our final answers as mixed fractions for these lessons.

Practice 3

Use your knowledge of fractions to solve the following questions. If you get stuck on a question you can draw fraction circles. Make sure you check if your answer is reasonable.

For each question, think carefully about whether you should *add* or *subtract*.

1. Jasmine is baking a cake. The recipe calls for a total of $2\frac{1}{3}$ cups of flour. She already added $1\frac{2}{3}$ cups of flour. How much flour is still needed?

2. Kelly ran $3\frac{1}{4}$ kilometres in an hour. Sebastian ran $2\frac{3}{4}$ kilometres in an hour. How much further did Kelly run?

3. Navjeet just finished a triathlon. She swam for $\frac{3}{4}$ of an hour, ran for $\frac{1}{6}$ of an hour, and rode her bike for $1\frac{2}{3}$ of an hour.

   a. How long did it take Navjeet to complete the swim and the bike ride?

   b. How long did it take Navjeet to complete the entire triathlon?

4. The movie "You Can't Eat Pi!" is $2\frac{1}{3}$ hours long. Min has been watching the movie for $1\frac{3}{5}$ of an hour. How much longer will the movie run?

Turn to the Answer Key at the end of the Module and mark your answers.

Section Summary

1. To compare fractions, write equivalent fractions with the same denominator, and then place them in order.

   Example

   Write the following in ascending order:

   $$\frac{2}{3}, \frac{5}{8}, \frac{1}{2}, \frac{5}{6} = \frac{16}{24}, \frac{15}{24}, \frac{12}{24}, \frac{20}{24}$$

   Ascending order:

   $$\frac{12}{24}, \frac{15}{24}, \frac{16}{24}, \frac{20}{24} = \frac{1}{2}, \frac{5}{8}, \frac{2}{3}, \frac{5}{6}$$

2. When you add (or subtract) fractions with common denominators, you add (or subtract) the numerators but keep the denominators the same.

   Example

   $$\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$$

   $$\frac{5}{7} - \frac{4}{7} = \frac{5-4}{7} = \frac{1}{7}$$

3. The least common denominator of two fractions is the lowest common multiple of the two denominators.

4. To add (or subtract) fractions with different denominators, write equivalent fractions with the same denominators then add (or subtract).

   Example

   $$\frac{3}{5} + \frac{2}{3} = \frac{9}{15} + \frac{10}{15} = \frac{19}{15} = 1\frac{4}{15}$$

5. To add or subtract mixed numbers you can:

   a. Rewrite the fractions as improper fractions, then add or subtract:

      Example

      $$3\frac{1}{3}-1\frac{2}{3}=\frac{10}{3}-\frac{5}{3}=\frac{5}{3}=1\frac{2}{3}$$

   b. Add or subtract the fraction part, then add or subtract the whole part. You may need to borrow.

      $$3\frac{1}{3}-1\frac{2}{3}=2\frac{4}{3}-1\frac{2}{3}$$

      Example

      $$3\frac{1}{3}-1\frac{2}{3}=\frac{10}{3}-\frac{5}{3}=\frac{5}{3}=1\frac{2}{3}$$

      Fraction part: $\frac{4}{3}-\frac{2}{3}$

      Whole part: 2 – 1 = 1

      $$=1\frac{2}{3}$$

6. Always reduce your final answer.

   Example

   $$\frac{4}{6}=\frac{4\div 2}{6\div 2}=\frac{2}{3}$$

7. If your answer is greater than 1, write it as a mixed number.

   Example

   $$\frac{13}{6}=2\frac{1}{6}$$

Section Challenge

If you have not already solved the Section Challenge, use your new skills to solve it now.

You are helping your mom prepare Thanksgiving dinner this year. Relatives are coming from all over and you have to prepare a meal for 10 people. Your mom has given you the job of making the grocery list. She gives you the following recipe ingredient lists:

Celery Casserole

Serves 10 people

Ingredients

$\frac{1}{2}$ cup butter
$3\frac{1}{2}$ cups of celery
$\frac{3}{8}$ cup flour
$\frac{3}{4}$ cup milk
$\frac{2}{3}$ cup mushrooms

Stuffing

Serves 5 people

Ingredients

$\frac{1}{8}$ tsp poultry seasoning
$\frac{1}{4}$ cup croutons
$\frac{1}{4}$ cup water
$\frac{1}{3}$ cup celery
$\frac{1}{4}$ cup mushrooms

Apple Pie

Serves 5 people

Ingredients

6 apples
$\frac{3}{4}$ cup butter
$1\frac{1}{8}$ cup sugar
1 cup flour

How much of each ingredient will be on your mom's grocery list? Did you notice that you will be making two batches of stuffing and two apple pies?

Fill in the chart on the next page to find how much of each ingredient you will need. You already have water, croutons, and apples at home so you don't need to have these on your grocery list.

Ingredient	Needed for the recipe	Amount needed for 1 batch	Amount needed for 10 people
Butter	Celery casserole Apple pie	$^1/_2$ cup $^3/_4$ cup	$^1/_2$ cup $^3/_4 + {}^3/_4 = {}^6/_4 = 1^2/_4 = 1^1/_2$ Total $= {}^1/_2 = 1^1/_2 = 2$ cups

Turn to the Answer Key at the end of the Module and mark your answers.

Section 2.2: Decimals

Section
2

Contents at a Glance

Learning Outcomes

By the end of this section you will be better able to:

- demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.
- compare and order positive fractions, positive decimals (to thousandths), and whole numbers by using benchmarks, place value, equivalent fractions and/or decimals.

Pretest 2.2

Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in this section. Mark your answers using the key found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson 2.2A

1. Express each of the following as a terminating decimal.

   a. $\frac{3}{10}$

   b. $\frac{21}{100}$

   c. $\frac{5}{100}$

   d. $\frac{6}{1000}$

   e. $\frac{2}{5}$

   f. $\frac{5}{8}$

   g. $\frac{3}{20}$

2. Write each fraction in decimal form. Do not use a calculator.

   a. $1\frac{1}{2}$

   b. $2\frac{3}{4}$

   c. $3\frac{2}{5}$

   d. $3\frac{23}{25}$

3. Write each fraction in decimal form. You may use a calculator.

   a. $\frac{3}{11}$

   b. $\frac{2}{3}$

   c. $\frac{5}{8}$

Lesson 2.2B

1. Write each decimal as a fraction in lowest terms.

   a. 0.7

   b. 0.23

   c. 0.85

d. 0.101

e. 0.246

2. Circle each terminating decimal

   $0.349,\ 0.\overline{324},\ 0.\overline{87},\ 0.38926$

3. Convert the following fractions into a decimal without using a calculator.

   a. $\frac{4}{5}$

   b. $\frac{13}{20}$

   c. $\frac{18}{25}$

Lesson 2.2C

1. Write the next five digits in each decimal.

   a. $0.\overline{45}$

   b. $8.3\overline{56}$

   c. $5.4\overline{0}$

   d. $2.\overline{615}$

2. Convert each fraction into a decimal. Do not use a calculator.

   a. $\frac{1}{9}$

   b. $\frac{32}{99}$

   c. $\frac{765}{999}$

3. Write each decimal as a fraction.

   a. $0.\dot{4}$

   b. $0.\dot{8}$

   c. $0.\overline{67}$

Lesson 2.2D

1. Write the correct inequality sign (< or >) in the space provided.

   a. 0.39 ___ $\frac{1}{2}$

   b. 0.83 ___ $\frac{4}{5}$

   c. $\frac{3}{10}$ ___ 0.03

2. Write the following in ascending order: 0.5, 1.7, $\frac{3}{4}$, 1, $1\frac{1}{5}$

3. Circle the number between the following two numbers.

   a. $\frac{1}{4}$, 0.6 The number between is: 0.2, $\frac{3}{4}$, or $\frac{2}{5}$.

   b. 0.3, $\frac{1}{2}$ The number between is: $\frac{1}{3}$, 0.7, or 0.54.

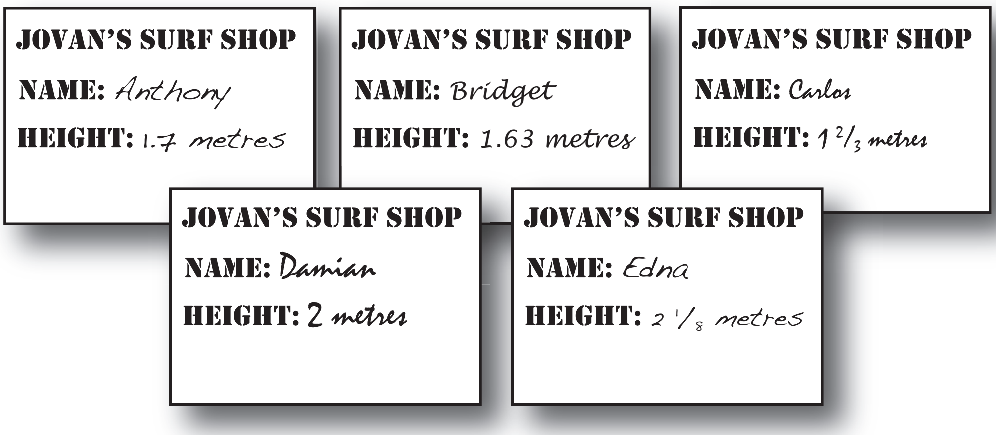
   c. $1\frac{2}{5}$, 1.5 The number in between is: $1\frac{1}{4}$, $1\frac{2}{3}$, or 1.45.

Turn to the Answer Key at the end of the Module and mark your answers.

Section Challenge

Anthony, Bridget, Carlos, Damian, and Edna all want to go surfing. They go to a surf shop to rent surfboards. There are 5 surfboards. The surfboards are ordered by height from longest to shortest: red, orange, green, blue, then purple.

Jovan, the surf shop owner, asks each person to fill out a form which includes their name and height. Here is what each person wrote:

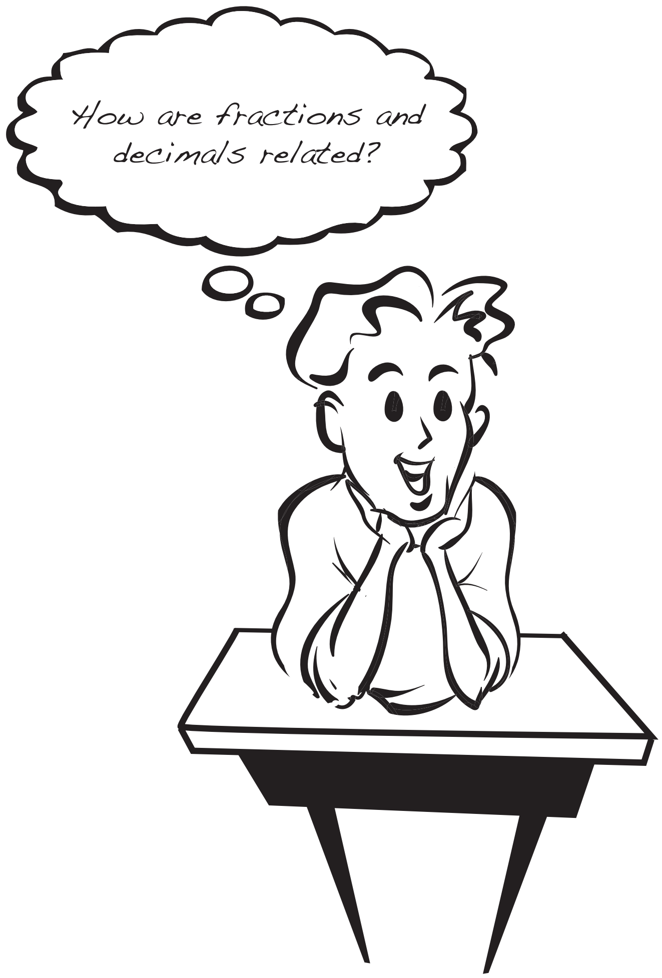


The tallest person should get the longest surfboard, the second tallest should get the second longest surfboard, and so on.

Which surfboard should Jovan rent to each person?

Lesson 2.2A: Making Connections—Fractions and Decimals

Student Inquiry



This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
How do you match a fraction with its decimal?		answer
		example
How can I write a fraction as a decimal?		answer
		example
How can I write a decimal as a fraction?		answer
		example

Lesson 2.2A: Making Connections—Fractions and Decimals

Thinking Space

Introduction

You have already worked with fractions and decimals, but how are they related? We can make decimals into fractions and fraction into decimals. Why would we want to convert between decimals and fractions?

One reason we convert fractions into decimals is to simplify calculations with lots of fractions. For instance, it is easier to add 0.75 + 0.33 + 0.73 than it is to add $\frac{3}{4}+\frac{1}{3}+\frac{8}{11}$.

Sometimes we want to convert decimals into fractions. For instance, if a solution is a very long decimal and we do not want to round it we can leave the answer as a fraction. Instead of working with 0.22222222... you can rewrite it as $\frac{2}{9}$.
This lesson will give you the tools to make these conversions.

Explore Online

Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:
http://www.openschool.bc.ca/courses/math/math7/mod2.html
Look for *Lesson 2.2A: Making Connections: Fractions and Decimals* and check out some of the links!

Warm-up

Do you remember the place values of decimals?

Here's a reminder in case you forget.

1.	2	3	4
ones	tenths	hundredths	thousandths

1. Find the digit in the tenths and ones place value.

   a. 3.947 digit in the tenths place:

   digit in the ones place:

   b. 9.875 digit in the tenths place:

   digit in the ones place:

   c. 0.21 digit in the tenths place:

   digit in the ones place:

Now that we remember the place values, let's practice dividing whole numbers.

2. Please complete the following calculations.

   a. 50 ÷ 5 =

   b. 28 ÷ 4 =

   c. 100 ÷ 20 =

   d. 88 ÷ 2 =

   e. 246 ÷ 3 =

We will be dividing numbers by 10 in this lesson, so let's get some practice.

3. Divide the following numbers. The first one is done for you.

   a. $3 \div 10 = 0.3$

   b. $7 \div 10 =$

   c. $9 \div 10 =$

   d. $1 \div 10 =$

   e. $2 \div 10 =$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

In the number 0.6 the 6 is in the tenths place. Why do you think that place value is called tenths?

Well, let's figure this out together. Have you ever noticed that the word "tenths" starts with "ten"? This is not a coincidence.

If someone says they have six tenths we could write this as a fraction:

$$\frac{6}{10} \text{ or as a decimal: } 0.6$$

This means $\frac{6}{10} = 0.6$.

So if we have "5 tenths," then we have $\frac{5}{10}$ or 0.5.

Notice that the number in the tenths place can be written as that number over ten.

For example: $0.4 = \frac{4}{10}$

We can also make a fraction with ten as the denominator into a decimal.

For example: $\frac{2}{10} = 0.2$

Have you ever noticed that the division symbol "÷" looks a lot like a fraction?

$\frac{1}{2}$	$\div$
fraction	division symbol

Do you know why the division symbol looks like a fraction?

It's because the fraction line means divide.

Let's double check $\frac{2}{10} = 0.2$. Divide 2 by 10: $\begin{array}{r} .2 \\ 10\overline{)2.0} \end{array}$

$\frac{2}{10} = 0.2$

It works!

Practice 1

1. Rewrite the following as a fraction and as a decimal. The first question is done for you. Remember to write your fraction in lowest terms.

	Fraction	**Decimal**
Three tenths	$\frac{3}{10}$	0.3
a. Seven tenths		
b. Eight tenths		
c. Four tenths		

2. Rewrite the following fractions as decimals. An example is done for you.

Decimal	**Fraction**
$\frac{72}{100}$	0.72
a. $\frac{23}{100}$	
b. $\frac{97}{100}$	
c. $\frac{62}{100}$	

3. Rewrite the following decimals as fractions. An example is done for you. Remember to write your fraction in lowest terms.

Decimal	Fraction
0.232	$\frac{232}{1000} = \frac{116}{500} = \frac{29}{125}$
a. 0.564	
b. 0.597	
c. 0.280	

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

How do you think we might convert fractions with denominators other than ten into decimals?

Remember that the fraction line means divide. Let's look at an example.

You, Jordan, Zamie, Cleo and Amanda are walking home from the park. You see that your neighbour has a car full of groceries and is about to bring them into her house. You all decide to help her unload the groceries. Your neighbour is so happy you helped that she gives you two dollars. You all want to split the money evenly. How much does each person get?

Two dollars split among five people is 2 divided by 5. Each person gets *two fifths* of the total. Saying two fifths is correct, but it isn't really the best way to describe how much money each person gets. Let's see if a decimal works better.

$$\frac{2}{5} = 2 \div 5$$

$$5\overline{)2.00}^{\;.40}$$

So $\frac{2}{5} = 0.40$.

Each of you receives \$0.40—in other words, 40 cents.

We're going to look at another way to write a fraction as a decimal in the next lesson. Can you think of another way?

Practice 2

Now it's your turn. Convert the following fractions into decimals. You may use a calculator to check your answers.

Fraction	Show Work	Decimal
a. $\frac{2}{50}$		
b. $\frac{4}{25}$		
c. $\frac{32}{160}$		
d. $\frac{45}{75}$		

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

What happens if we want to convert a mixed number to a fraction? Well, let's have a look at an example.

Let's convert $2\frac{3}{4}$ into a decimal.

First let's separate the whole and the fraction of $2\frac{3}{4}$.

whole	fraction
2	$\frac{3}{4}$

Let's rewrite $\frac{3}{4}$ as a decimal.

whole	fraction
2	$\frac{3}{4}$
	0.75

Now let's put our whole and our fraction together: $2 + 0.75 = 2.75$.

Let's try another example:

Convert $45\frac{7}{25}$ into a decimal.

$\frac{7}{25} = 0.28$ so $45\frac{7}{25} = 45 + 0.28 = 45.28$

Practice 3

Convert the following mixed numbers into decimals. (How could you use a calculator to check your answers for these questions?)

Mixed Number	Show Work	Decimal
1. $2\frac{7}{10}$		
2. $9\frac{8}{40}$		
3. $8\frac{6}{25}$		
4. $3\frac{7}{40}$		
5. $6\frac{35}{80}$		

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 2.2B: Classifying Decimals

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
What is a terminating decimal?		answer
		example
What is a repeating decimal?		answer
		example
How can I write a fraction as a terminating decimal?		answer
		example
How can I write a terminating decimal as a fraction?		answer
		example

Lesson 2.2B: Classifying Decimals

Thinking Space

Introduction

Scientists classify animals into different groups, like mammals, birds, and fish, based on their characteristics. This helps scientists to organize information about these creatures, compare them with one another, and study them to gain more information. Humans have many classification systems to help us understand the world around us.

Mathematicians classify numbers into different groups based on their characteristics. This lesson will help us to get a better understanding of decimals by classifying them. Also, we will learn more about how to express a fraction as a decimal.

Explore Online

Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:
http://www.openschool.bc.ca/courses/math/math7/mod2.html
Look for *Lesson 2.2B: Classifying Decimals* and check out some of the links!

Warm-up

1. Please divide the following numbers. Do not use a calculator. The first one is done for you.

   Hint: When a smaller number is divided by a larger number we need to have a decimal in our answer.

   a. $3 \div 5 = \begin{array}{r} .6 \\ 5\overline{)3.0} \end{array} = 0.6$

   b. $1 \div 2 =$

   c. $5 \div 25 =$

   d. $1 \div 20 =$

   e. $8 \div 10 =$

If you need to divide larger numbers you may want to use a calculator.

2. Divide the following numbers using a calculator.

   a. $15 \div 75 = 0.2$

   b. $90 \div 720 =$

   c. $27 \div 540 =$

   d. $14 \div 56 =$

   e. $65 \div 1040 =$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

How do you think scientists classify animals into groups like mammals, birds, reptiles, and so on? What characteristics do all mammals have in common? What are some differences between mammals and the animals in other groups?

With these ideas in mind, let's try a sorting activity with numbers.

Decimal Sorting

First, convert the following fractions into decimals. Remember that the fraction line means divide. To speed up your work, you may want to use your calculator to divide the numerator by the denominator.

1. $\frac{7}{10}$
2. $\frac{5}{9}$
3. $\frac{8}{11}$
4. $\frac{38}{100}$
5. $\frac{5}{15}$
6. $\frac{36}{60}$
7. $\frac{75}{99}$
8. $\frac{12}{32}$
9. $\frac{77}{111}$
10. $\frac{456}{999}$

Check your answers on the next page before you continue the activity.

Answers:

1. 0.7
2. 0.555555...
3. 0.727272....
4. 0.38
5. 0.333333...
6. 0.6
7. 0.757575...
8. 0.375
9. 0.693693...
10. 0.456456...

Thinking Space

How can we sort these decimal answers? Think about what each number has in common with the other numbers. What differences are there between numbers? Use the thinking space to make some observations about these decimals.

Use the chart below to separate these decimals into two groups. Once you've grouped the decimals, choose a name for each group. The name should describe the decimals that are in that group.

Name of First Group	Name of Second Group

Explore
Terminating and Repeating Decimals

Mathematicians have classified decimals just like you just have. They decided to classify these decimals as "terminating" and " repeating."

Repeating decimals show that converting fractions into decimals doesn't always result in a nice clean answer—in fact, it often doesn't. When converting fractions to decimals, repeating decimals can present a problem. For example, $\frac{2}{3} + \frac{4}{9} = 0.666\ldots + 0.444\ldots$ How will you add two decimal numbers that don't have an endpoint? In this section we will review how to simplify repeating decimal numbers.

Terminating means ending. A **terminating decimal** has an end.

Here are some examples:

$$\frac{1}{2} = 0.5$$

$$\frac{2}{10} = 0.2$$

$$\frac{223}{100} = 2.23$$

Repeating decimals have a pattern that keeps repeating (they never end).

Here are some examples:

$$\frac{1}{3} = 0.33333333333\ldots$$

The 3 repeats, so we write $\frac{1}{3} = 0.\dot{3}$.

A dot on top of a digit means that digit repeats.

$$\frac{9}{7} = 1.285714285714285714\ldots.$$

The 285714 repeats, so we write $\frac{9}{7} = 1.\overline{285714}$.

$$\frac{5}{99} = 0.0505050505\ldots.$$

The 05 repeats, so we write $\frac{5}{99} = 0.\overline{05}$.

Thinking Space

?

Did You Know ...?
There is another type of decimal that doesn't terminate or repeat: an irrational decimal. The most famous example of this is Pi (π) 3.141592654... It goes on and on without repeating!

!

!

A line on top of digits means those digits repeat.

Look at the ten decimal numbers that you have already classified. Did you classify the decimals as terminating or repeating decimals? If you did, then great!

If you classified the decimals in another way, then you should be thrilled with your creativity. To make sure you understand how to classify decimals as repeating or terminating, have a look at the chart below:

Repeating	Terminating
$\frac{7}{10}$	$\frac{5}{9}$
$\frac{38}{100}$	$\frac{8}{11}$
$\frac{36}{60}$	$\frac{5}{15}$
$\frac{12}{32}$	$\frac{75}{99}$
	$\frac{77}{111}$
	$\frac{456}{999}$

Thinking Space

!

Practice 1

Classify each decimal as terminating or repeating by writing "terminating" or "repeating" to the right of the decimal.

1. 34.3728 ________________

2. 0.83838383838... ________________

3. 0.943 ________________

4. 0.55555555... ________________

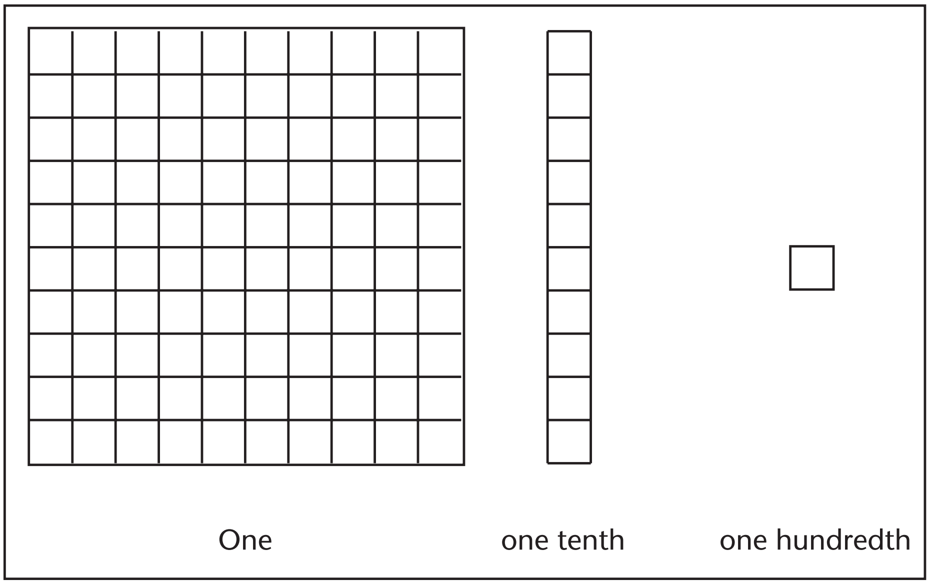
Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

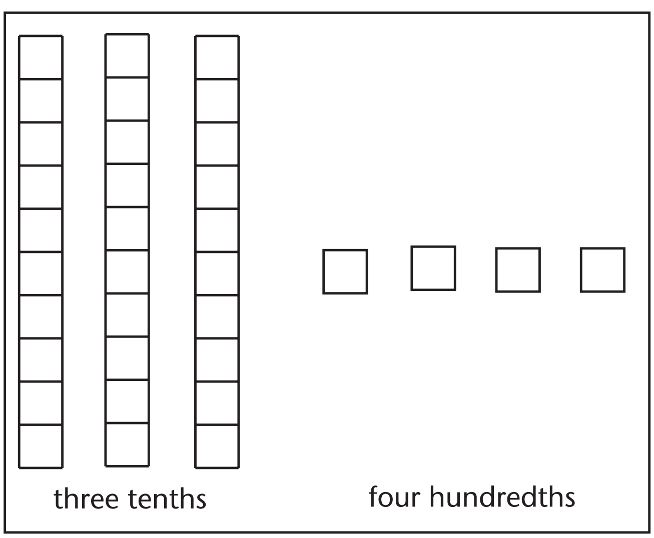
Now that we know about terminating and repeating decimals, we're going to learn a bit more about how decimals and fractions are related. This lesson will focus on terminating decimals. We'll look at repeating decimals in Lesson 2.2 C. We will be using several different ways of working with fractions and decimals. Try out the different methods out, then choose the one that works best for you.

Let's have a look at a hundredths block to help us better understand decimals and fractions.



You can see that the big square represents one whole. It has been divided into 100 small squares. Each of these small squares represents one hundredth. Ten of these small squares together represent one tenth.

If we had three tenths and four hundredths it would look like this:



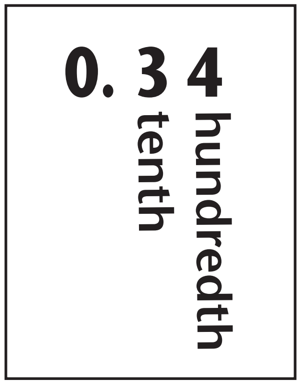
Let's write this as a fraction:

Three tenths = $\frac{3}{10}$

Four hundredths = $\frac{4}{100}$

So three tenths and four hundredths = $\frac{3}{10}+\frac{4}{100}=\frac{30}{100}+\frac{4}{100}=\frac{34}{100}$

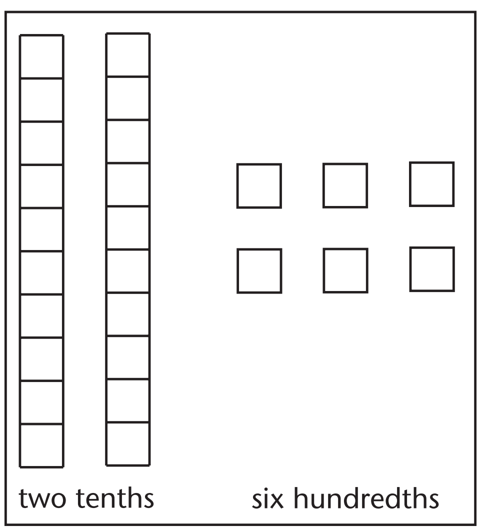
We can also write this as a decimal:



So we can see that $\frac{34}{100}=0.34$.

Let's try another question.

Write 2 tenths and 6 hundredths as a fraction and into a decimal.



Fraction:

$$\frac{2}{10}+\frac{6}{100}=\frac{20}{100}+\frac{6}{100}=\frac{26}{100}$$

Decimal:

0. 2 6

tenth

hundredth

So $\frac{26}{100}=0.26$

Let's try one last example.

Write 9 tenths, 2 hundredths, and 8 thousandths as a fraction and as a decimal.

Fraction:

$$\begin{aligned}&\frac{9}{10}+\frac{2}{100}+\frac{8}{1000}\\&=\frac{900}{1000}+\frac{20}{1000}+\frac{8}{1000}\\&=\frac{928}{1000}\end{aligned}$$

Decimal:

0.9 2 8

tenth

hundredth

thousandth

So $\frac{928}{1000}=0.982$.

Thinking Space

Practice 2

Write the following as a fraction and a decimal.

	Fraction	Decimal
1. 4 tenths, 9 hundredths		
2. 7 tenths, 3 hundredths, 5 thousandths		
3. 6 tenths, 4 hundredths, 2 thousandths		
4. 3 tenths, 2 hundredths, 1 thousandth		

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

A **power** is a number that had been multiplied by itself. Examples of powers of 10 are 10, 100, 1000, and 10 000.

You might have noticed that a terminating decimal can be written as a fraction with a denominator that is a power of 10. The number of place values in the decimal tells us how many 0's are needed in the denominator of the fraction.

Look at the examples below and notice the pattern:

$$0.3 = \frac{3}{10}$$
$$0.36 = \frac{36}{100}$$
$$0.361 = \frac{361}{1000}$$

Just like when we were working with fractions before, we often want to simplify our final answer. This means reducing fractions to their lowest terms. Can we simplify any of the fraction answers above? Let's check.

$0.3 = \frac{3}{10}$

3 and 10 don't have any common factors. This fraction is already in lowest terms.

$0.36 = \frac{36}{100}$

36 and 100 can both be divided by 4—that means we can reduce this fraction.

$$\frac{36}{100} = \frac{(36 \div 4)}{(100 \div 4)} = \frac{9}{25}$$

9 and 25 don't have any common factors, so this fraction is simplified.

$0.361 = \frac{361}{1000}$

Now our numbers are getting bigger. Do 361 and 1000 have any common factors? (*Hint:* you could try using your divisibility rules!) Nope! This fraction is already in lowest terms.

Thinking Space

"I remember learning about divisibility rules in Lesson 1.3B!"

Thinking Space

To summarize:

$$0.3 = \frac{3}{10}$$
$$0.36 = \frac{36}{100} = \frac{9}{25}$$
$$0.361 = \frac{361}{1000}$$

So what happens when you see fractions like $\frac{1}{2}$ and $\frac{7}{25}$? They can be written as terminating decimals, but they don't have denominators that are powers of 10.

This is because $\frac{1}{2}$ and $\frac{7}{25}$ are written as reduced fractions. We can rewrite them as **equivalent fractions** with denominators that are powers of 10.

Let's try it.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$$
$$\frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 0.28$$

Here's another example. Let's convert $\frac{1}{4}$ into a fraction with a denominator of 100 to find the equivalent decimal.

Writing fractions as decimals—we did this another way in Lesson 2.2A.

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100}$$ Since $\frac{25}{100} = 0.25$ we know that $\frac{1}{4} = 0.25$.

Can you think of some reasons that it might be useful to rewrite a fraction with a denominator of 100? (*Hint:* we'll look at this in the last section of this module.)

Practice 3

1. Rewrite each of the following as a fraction with the denominator as a power of 10. Then write the fraction as a decimal. The first one is done for you.

Reduced Fraction	Equivalent Fraction With a Power of 10 as Its Denominator	Decimal
$\frac{3}{5}$	$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$	0.6
a. $\frac{2}{5}$		
b. $\frac{16}{25}$		
c. $\frac{7}{20}$		
d. $\frac{123}{500}$		

2. a. Write down 5 reduced fractions that you think will have a terminating decimal.

   b. Why did you choose these fractions?

c. Check to see if your fractions have equivalent terminating decimals. If any of your fractions did not have a terminating decimal, why do you think this is so?

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 2.2C: Repeating Decimals as Fractions

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
How can I write a fraction as a repeating decimal?		answer
		example
How can I write a repeating decimal as a fraction?		answer
		example

Lesson 2.2C: Repeating Decimals as Fractions

Thinking Space

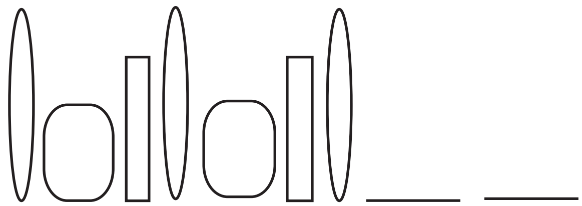
Introduction

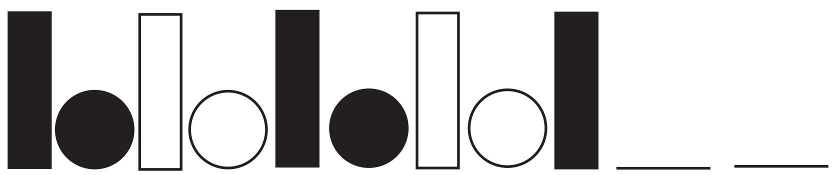
We already know how to convert between fractions and terminating decimals. What about repeating decimals? This lesson will show you how to write a fraction as a repeating decimal and a repeating decimal as a fraction.

Warm-up

In this lesson we will need to find patterns. Let's practice.

1. Draw the next two shapes.

   a. 

   b. 

2. Find the missing numbers.

   a. 2, 4, 6, ___, ___, ___

   b. 1, 3, 5, 7, ___, ___, ___

   c. $\dfrac{1}{11} = 0.090909\ldots$

   $\dfrac{1}{111} = 0.009009\ldots$

   $\dfrac{1}{1111} = 0.00090009\ldots$

   $\dfrac{1}{1111} =$

Sometimes we need to round an answer. Remember, we round up if the digit to the right of where we are rounding is 5, 6, 7, 8, or 9.

3. Round the following decimals to 2 decimal places (hundredths place value). The first one is done for you.

   a. 0.2342: Since 4 is less than 5 we round down, so 0.2342 rounds to 0.23.

   b. 0.8790

   c. 0.3802

   d. 4.1924

   e. 8.9268

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Remember: to write fractions as decimals, divide the numerator by the denominator.

Example

$$\frac{1}{3} = 1 \div 3 = 0.3333.... = 0.\overline{3}$$

Now, look at the pattern below. Without using a calculator, can you write $\frac{4}{9}$ as a decimal?

$$\frac{1}{9} = 0.11111....$$
$$\frac{2}{9} = 0.2222.....$$
$$\frac{3}{9} = 0.3333.....$$
$$\frac{4}{9} =$$

Write a rule for converting repeating decimals like 0.777777.... into a fraction.

What happens if the denominator is 99? Let's find out.

$$\frac{1}{99} = 0.0101010101....$$
$$\frac{2}{99} = 0.02020202.....$$
$$\frac{3}{99} = 0.03030303.....$$
$$\frac{15}{99} = 0.15151515.....$$
$$\frac{83}{99} = 0.8383838.....$$
$$\frac{65}{99} = 0.65656565...$$

Thinking Space

Practice 1

1. Using these patterns write what decimal these fractions are equal to. Do not use a calculator.

   a. $\frac{8}{99} =$

   b. $\frac{35}{99} =$

   c. $\frac{15}{99} =$

   d. $\frac{84}{99} =$

2. Write a rule for converting repeating decimals where two digits repeat like 0.67676767.... into a fraction.

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Have you figured out the pattern? How do you write a repeating decimal as a fraction?

Here are the steps:

	Example
1. Find out which digits repeat.	In 0.123123123123...., 123 repeats.
2. Write the repeating part as the numerator.	$\frac{123}{?}$
3. Count how many digits repeat; this will tell you how many 9s will be needed in the denominator.	123 repeats (this is three digits,) so 999 will be the denominator $\frac{123}{999}$
4. Be sure to reduce your fraction.	$\frac{123}{999} = \frac{123 \div 3}{999 \div 3} = \frac{41}{333}$

Thinking Space

Practice 2

1. Convert the following repeating decimals to fractions. Make sure your fractions are reduced.

   a. 0.5555...

   b. 0.343434...

   c. 0.789789789....

   d. 0.24682468...

   e. 0.015015015.....

2. Use a calculator to write the following fractions as repeating decimals.

   a. $\frac{1}{7} =$

   b. $\frac{2}{7} =$

   c. $\frac{3}{7} =$

3. Describe the pattern of your answers in question 2.

4. Without using a calculator, predict the decimal form of the fraction $\frac{4}{7}$.

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Can you think of an example where we would want to round our decimal answer?

A store is selling 3 apples for $0.98. You only want one apple. How much will the apple cost?

$$\frac{\$0.98}{3} = \$0.3266666...$$

You need to round your answer to the nearest penny.

One apple would cost $0.33.

Make sure you always round your answer to two decimal places when working with money.

Practice 3

Let's convert fractions into decimals. This time our decimal number will represent money.

Fraction	Decimal Not Rounded	Rounded
$\frac{10}{33}$	0.30303030...	\$0.30
a. $\frac{16}{25}$		\$
b. $\frac{80}{120}$		\$
c. $\frac{17}{200}$		\$
d. $\frac{125}{300}$		\$

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 2.2D: Comparing Fractions and Decimals

Student Inquiry

This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON	AFTER THE LESSON
	What I already know about this question:	What I thought at the end: My final answer, and examples:
How can I order fractions, decimals, and whole numbers?		answer
		example
How can I find a number whose value is between two given numbers?		answer
		example

Lesson 2.2D: Comparing Fractions and Decimals

Thinking Space

Introduction

All around us we can see fractions, decimals, and whole numbers. These numbers are in newspapers, stores, and even on TV. It is important for us to understand the values of these numbers and how they relate to each other. Can you think of some reasons it might be important to compare the values of fractions, decimals, and whole numbers?

This lesson will describe ways of ordering numbers and even finding values between two different numbers.

Warm-up

Let's review **equivalent fractions**. Remember that equivalent means "the same." Equivalent fractions have equal value, but have different numerators and denominators.

Equivalent fractions can be found by either:

- multiplying the numerator AND denominator of a fraction by the same number

OR

- dividing the numerator AND denominator of a fraction by a common factor

In this example, we'll multiply the top and bottom of the fraction by 5.

$$\boxed{\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}}$$

1. What would the equivalent fraction of $\frac{1}{2}$ be if you multiplied by 4 instead?

Thinking Space

2. Match each of the ten fractions with an equivalent fraction. The first one is done for you.

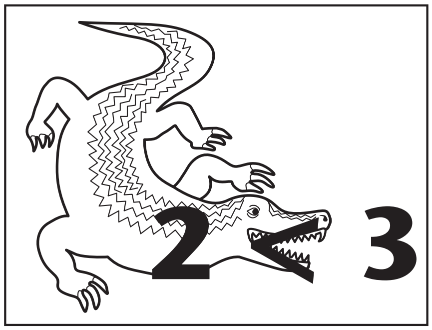
$\frac{1}{10}$	$\frac{9}{15}$
$\frac{2}{10}$	$\frac{24}{80}$
$\frac{3}{10}$	$\frac{18}{20}$
$\frac{4}{10}$	$\frac{35}{50}$
$\frac{5}{10}$	$\frac{7}{7}$
$\frac{6}{10}$	$\frac{7}{70}$
$\frac{7}{10}$	$\frac{1}{5}$
$\frac{8}{10}$	$\frac{2}{5}$
$\frac{9}{10}$	$\frac{1}{2}$
$\frac{10}{10}$	$\frac{5}{4}$

We will be comparing numbers in this lesson, so let's review the inequality signs.

One way to remember which way the inequality goes is to think of the sign as a crocodile and the number as an amount of fish.

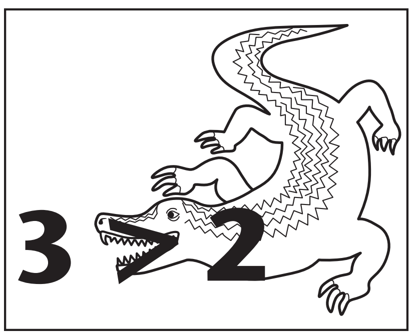
< means less than

For example, 2 < 3.



> means greater than

For example, 3 > 2.



If you were a hungry crocodile, would you want to eat 5 fish or 10 fish?

You would want to eat 10 fish. So 5 < 10

Another way to look at < is think of it as an L. The L stands for "less than." So we know 5 < 10 means 5 is less than 10.

When comparing decimal numbers that are less than 1, make sure you compare digits from left to right. Compare the tenths values; if they are the same, then compare the hundredths, and then continue moving until a digit is different. Don't be fooled. Just because a number is longer does not mean that it is larger. For instance, 0.1000001 is longer than 0.1001 but its value is less.

Compare 0.5 and 0.4. 5 is greater than 4. So, 0.5 > 0.4

Thinking Space

!

!

?

Do you have any questions?

Compare 0.54 and 0.5. There is a 5 in both tenths places, so let's look at the hundredths. 4 is bigger than nothing (0), so 0.54 > 0.5.

Let's practice using inequalities.

3. Write the inequality for each pair of numbers below. The first one is done for you

   a. 4 < 9

   b. 9 __ 8

   c. 21 __ 86

   d. 12 __ 65

   e. 12 __ 43

   f. 0.5 __ 0.8

   g. 0.12 __ 0.36

   h. 0.123 __ 0.2

   i. 0.49 __ 0.4

   j. 0.55 __ 0.551

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

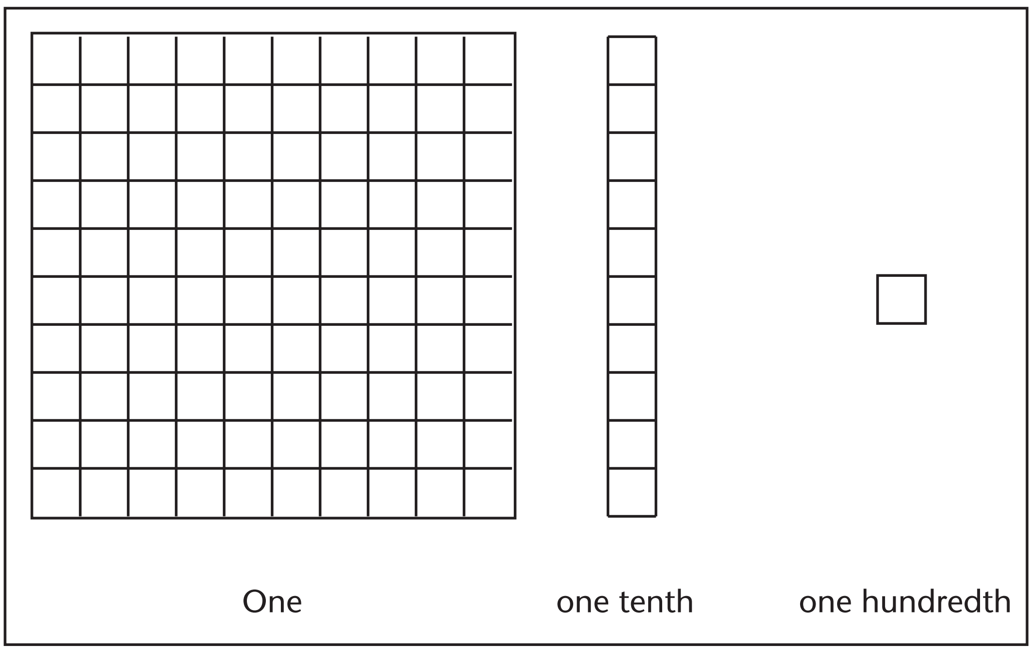
Thinking Space

There are several ways to compare decimals, fractions, and whole numbers. The first method we'll look at is drawing pictures.

Pictures can be useful tools to help us "see" the numbers we are comparing.

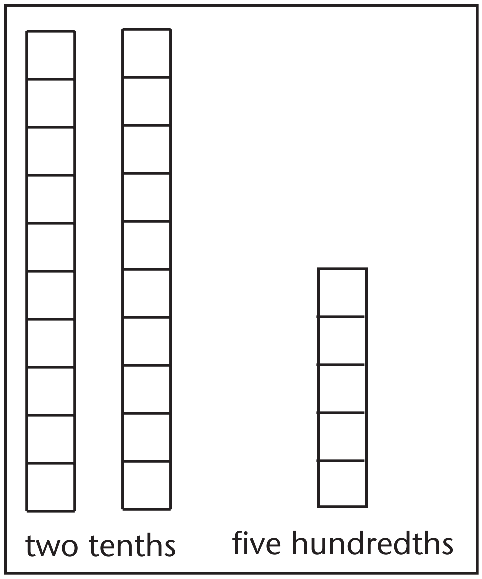
For instance, which is greater: 0.25 or $\frac{1}{5}$? Pictures can make it easier to visualize how big these numbers are..

We can represent both with hundredths blocks. In case you forgot what they looked like, here they are below.



Thinking Space

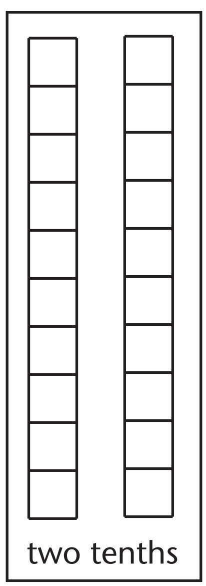
0.25 has two tenths and five hundredths.



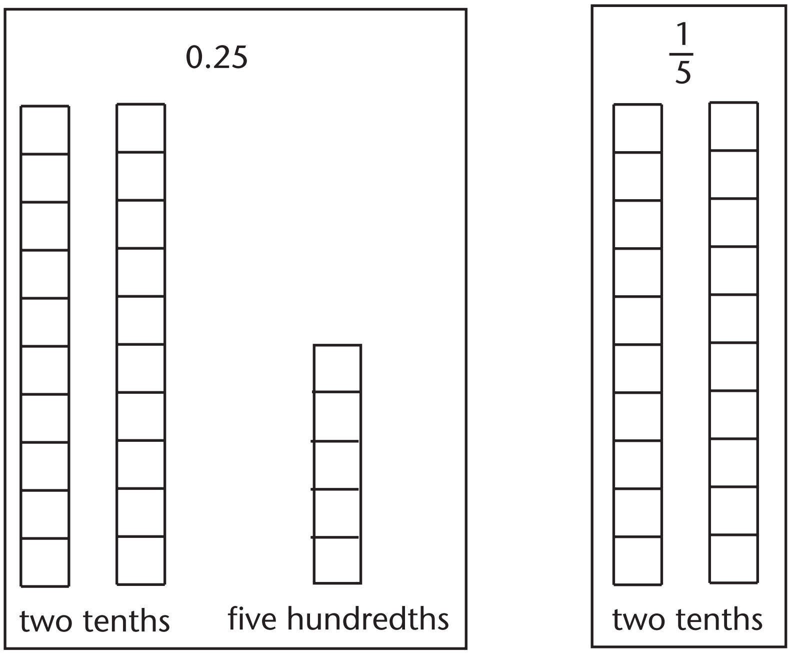
Now let's rewrite $\frac{1}{5}$ into a fraction over ten so we can use the hundredths blocks.

$$\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

We have two tenths.



Now let's see which one is larger.



0.25 has 5 extra hundredths. 0.25 is greater.

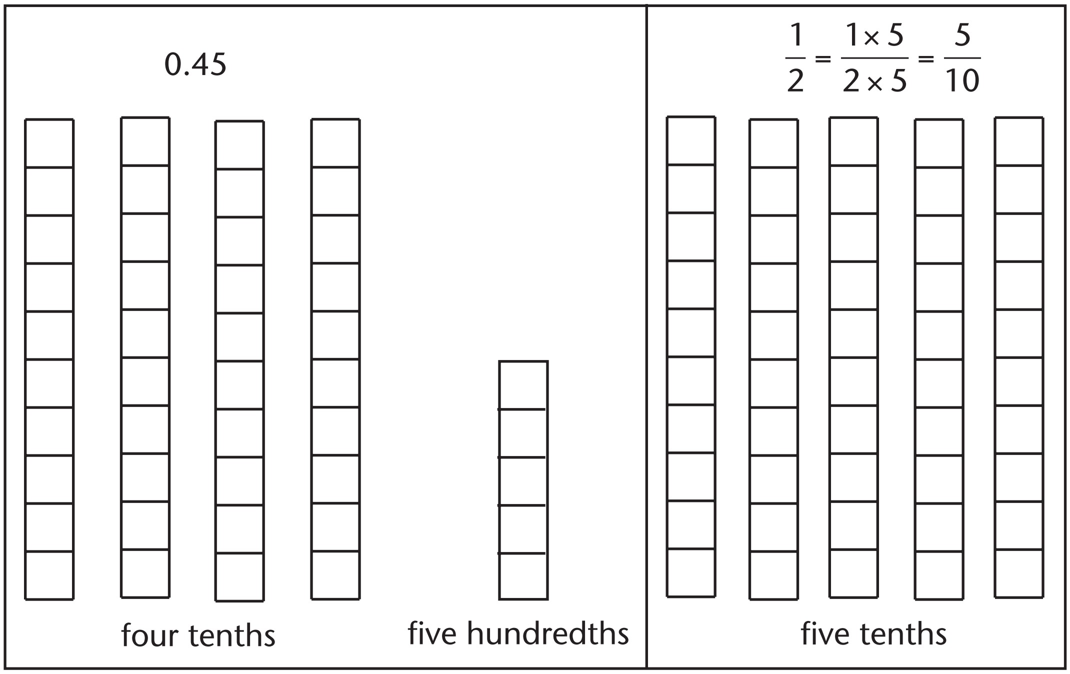
$$0.25 > \frac{1}{5}$$

Thinking Space

Practice 1

Choose the correct inequality. You can draw a picture to help you compare. The first one is done for you.

1. $0.45 \underline{\ <\ } \frac{1}{2}$



2. $0.5 \ \_\_\_ \ \frac{1}{5}$

3. $0.8 \_\_\_ \frac{17}{20}$

4. $0.17 \_\_\_ \frac{3}{4}$

5. $0.36 \_\_\_ \frac{3}{6}$

6. $0.93 \_\_\_ \frac{23}{25}$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Another way to compare fractions and decimals is to put the numbers into the same form. You can either:

- convert all the numbers to decimals

OR

- convert all the numbers to fractions

And then compare them.

Comparing Decimals

First, let's try changing everything into a decimal.

To compare 0.36 and $\frac{23}{25}$ we can change $\frac{23}{25}$ into a decimal.

Method 1: using a common factor:

$$\frac{23}{25} = \frac{23 \times 4}{25 \times 4} = \frac{92}{100} = 0.92$$

or

Method 2: dividing the numerator by the denominator:

$$\frac{23}{25} = 23 \div 25 = 0.92$$

Which is smaller? 0.36 or 0.92? Since 0.36 is smaller than 0.92, we know $0.36 < \frac{23}{25}$.

Comparing Fractions

Now let's compare these same numbers, but this time let's convert everything into a fraction.

$0.36 = \frac{36}{100}$. To compare these two fractions we need both of them over the same denominator. 100 is the lowest common denominator of $\frac{36}{100}$ and $\frac{23}{25}$. So let's change $\frac{23}{25}$ into its equivalent fraction of $\frac{92}{100}$.

Which is smaller? Since $\frac{36}{100}$ is smaller than $\frac{92}{100}$ we know $0.36 < \frac{23}{25}$.

Thinking Space

Which method do you like to use to convert fractions into decimals?

Do you prefer comparing two fractions or two decimals?

Practice 2

Now it's your turn.

1. Convert all of the fractions into decimals and then circle the LOWEST value.

   a. 1.34, $\frac{27}{20}$ $\frac{27}{20} = 27 \div 20 = 1.35$

   b. 0.78, $\frac{3}{5}$

   c. 0.96, $\frac{19}{20}$

   d. 1.27, $\frac{38}{25}$

   e. 3.45, $\frac{162}{50}$

2. Convert all of the decimals into fractions and then circle the LOWEST value.

   a. 0.82, $\frac{4}{5}$ $0.82 = \frac{82}{100}$ $\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100}$ $\frac{82}{100} > \frac{80}{100}$ so 0.82

   b. 0.2, $\frac{6}{25}$

   c. 0.4, $\frac{7}{20}$

   d. 0.78, $\frac{37}{50}$

   e. 0.93, $\frac{19}{20}$

3. Which method did you like using best? Did one method take longer than another method?

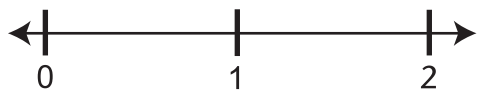
Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

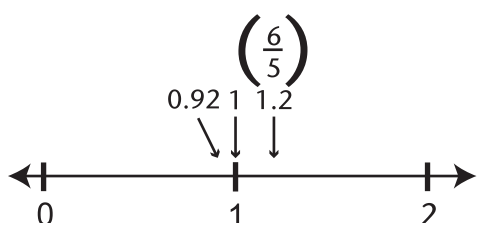
Now that we know how to compare fractions and decimals we can place them on the number line. We can convert all of the numbers into decimals, or into fractions with common denominators. In this lesson we'll convert everything into decimals to compare and place on a number line. We'll use this method since it often takes longer to change everything into common fractions with the same denominators. If you prefer another method, you can use it in your own work.

Let's compare the numbers on a number line like the one below.



We can convert $\frac{6}{5}$ into a decimal $\frac{6}{5} = 6 \div 5 = 1.2$.

Now let's place 1.2, 0.92, 1 on the number line.



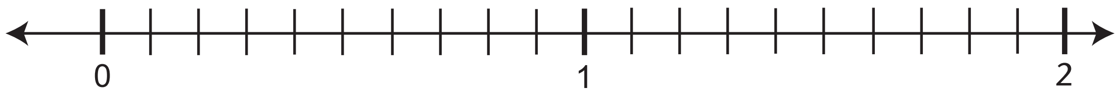
Now that the numbers are on the number line, you can compare them easily:

- 0.92 is furthest left, so it's the smallest number in the set.
- 1.2 (or $\frac{6}{5}$) is furthest right, so it's the largest number in the set.

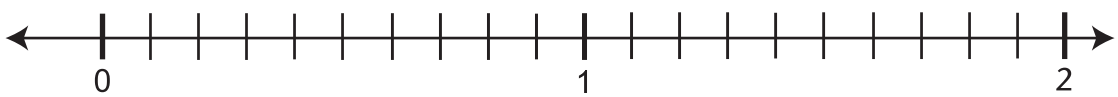
Practice 3

Convert these lists of numbers into decimals, and then place them on the number line.

1. 1.3, 0.2, 1.5, $\frac{3}{4}$, 1, $1\frac{4}{5}$



2. 1.75, 0.4, $\frac{3}{5}$, 0.9, $\frac{7}{10}$, $\frac{1}{2}$

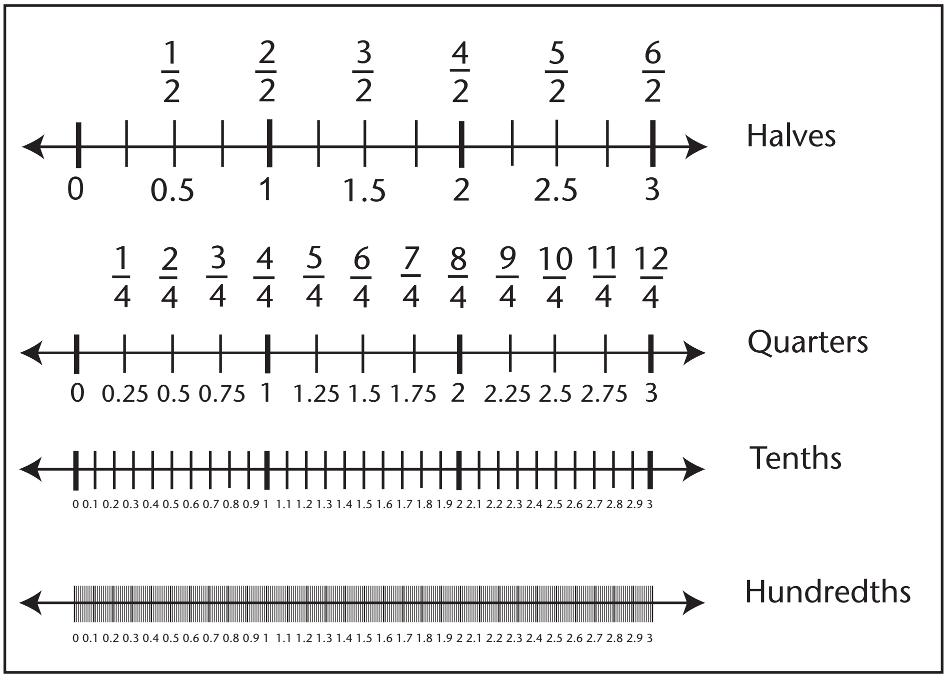


Turn to the Answer Key at the end of the Module and mark your answers.

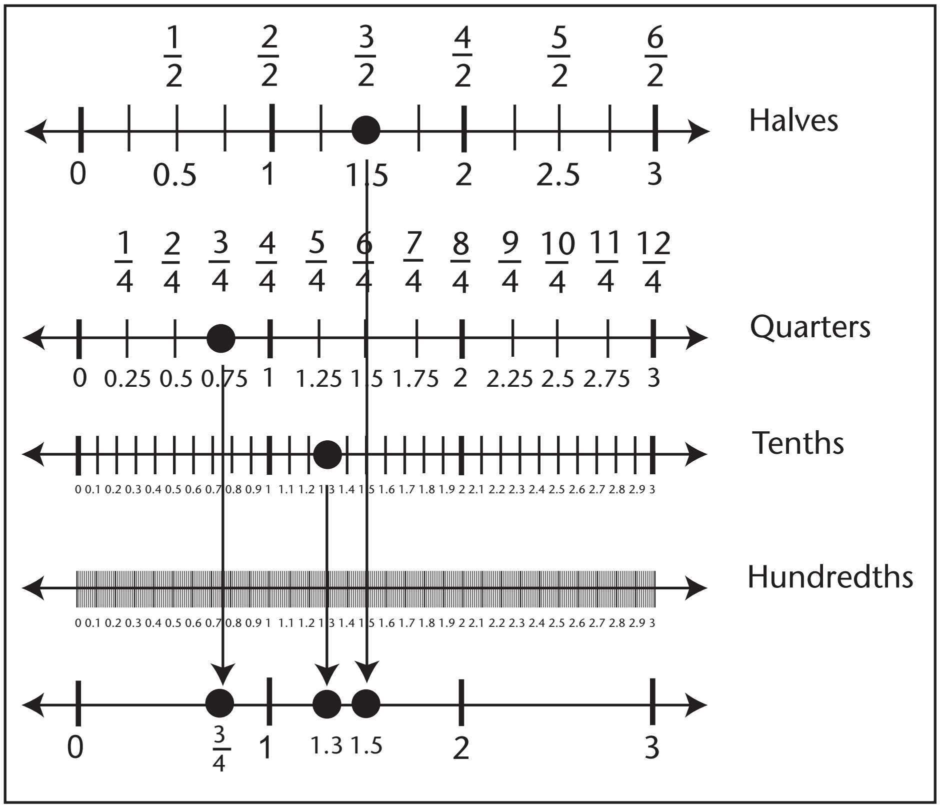
Explore

Thinking Space

We can also order fractions on a number line without converting the number into decimals. We do this by having lots of number lines, each divided into different sized increments like the ones below. (An *increment* is the spacing between the markings on the number line.)



Let's compare 1.5, $\frac{3}{4}$, and 1.3. Using the number lines above, we can see where each number lies.



We can use the number lines to put the set of numbers in *ascending* or *descending* order:

- ascending order is $\frac{3}{4}$, 1.3, 1.5.
- descending order is 1.5, 1,3, $\frac{3}{4}$

Thinking Space

This is a very visual method.

"Ascending" means least to greatest

"Descending" means greatest to least

Practice 4

Use the number lines to find the ascending (smallest to largest) order of the following fractions.

1. $\frac{1}{2}$, $\frac{7}{4}$, 0.7, $\frac{11}{4}$

2. 2.13, $\frac{3}{10}$, 0.7, $\frac{9}{4}$

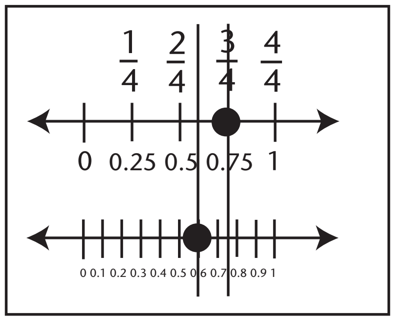
3. $\frac{1}{4}$, 0.82, $\frac{5}{4}$, 2.14

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Number lines can also help us find values between two different numbers. For instance, if we wanted to find a number between 0.6 and $\frac{3}{4}$ we can place these numbers on a number line and then find a number in between.



The number 0.7 is between those two lines that we drew, so we know that 0.7 is between 0.6 and $\frac{3}{4}$. Are there other numbers between 0.6 and $\frac{3}{4}$? What about 0.65? There are many numbers that fall between 0.6 and $\frac{3}{4}$. You could pick any one of them.

Practice 5

Find one number between the following pairs of numbers. Use the number line to help.

1. $\frac{1}{2}$, 0.9 a number in between is:

2. $\frac{1}{4}$, $\frac{1}{2}$ a number in between is:

3. 0.4, 0.7 a number in between is:

4. $\frac{3}{4}$, 1 a number in between is:

5. 0.3, $\frac{1}{2}$ a number in between is:

Turn to the Answer Key at the end of the Module and mark your answers.

Section Summary

In these lessons we saw how decimals and fractions are related. Let's review some of the key points:

1. We learned about two kinds of decimals: terminating and repeating.

   Example

   A terminating decimal is 0.5.

   A repeating decimal is 0.33333....

2. A terminating decimal can be written as a fraction with a power of 10 as the denominator. Make sure the fraction is simplified.

   Example

   $$0.5 = \frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

3. A repeating decimal can be written as a fraction with 9s in the denominator. The number of digits that repeat tell you how many 9s are needed in the denominator. Make sure the fraction is simplified.

   Example

   $$0.\dot{3} = \frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$$

4. Fractions can be written as decimals. Divide the numerator by the denominator, or write the fraction with a power of 10 to help you find the decimal.

   Example

   $$\frac{3}{4} = 3 \div 4 = 0.75$$ or $$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$$

5. To order fractions, decimals, and whole numbers you can use a number line.

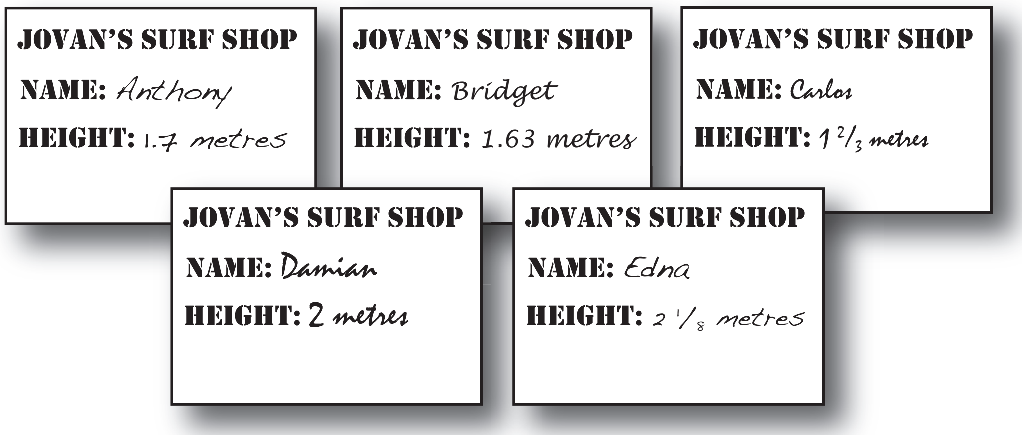
6. Plotting fractions, decimals, and whole numbers on a number line can help you to find values between two numbers.

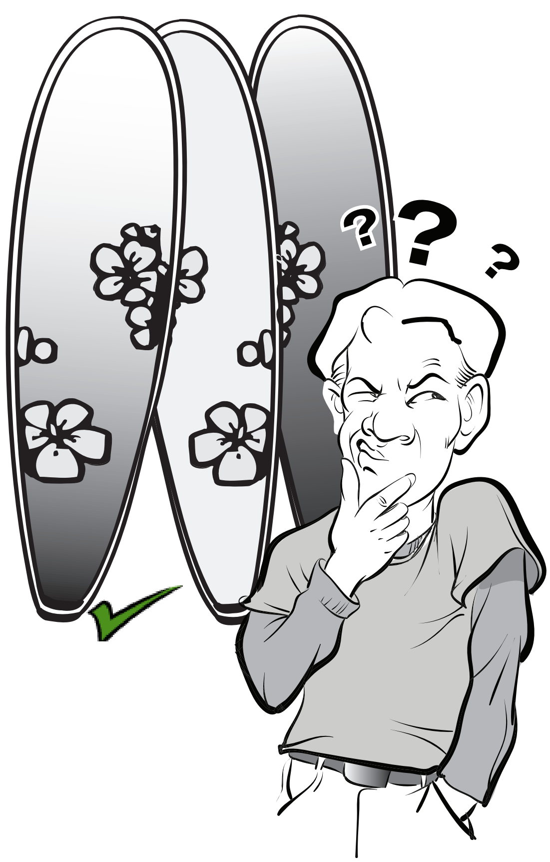
Section Challenge

Let's look back at our challenge.

Anthony, Bridget, Carlos, Damian, and Edna all want to go surfing. They go to a surf shop to rent surfboards. There are 5 surfboards. The surfboards are ordered by height from longest to shortest: red, orange, green, blue, then purple.

Jovan, the surf shop owner, asks each person to fill out a form which includes their name and height. Here is what each person wrote:





The tallest person should get the longest surfboard, the second tallest should get the second longest surfboard, and so on.

Which surfboard should Jovan rent to each person?

Section 2.3: Percent

Section
3

Contents at a Glance

Learning Outcomes

By the end of this section you will be better able to:

- solve problems involving percents from 1% to 100%.

Pretest 2.3

Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in this section. Mark your answers using the key found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson 2.3A

1. Write each fraction as a percent.

   a. $\frac{70}{100}$

   b. $\frac{6}{10}$

   c. $\frac{3}{4}$

   d. $\frac{4}{5}$

2. Write each decimal as a percent.

   a. 0.6

   b. 0.54

   c. 0.03

   d. 0.25

3. Write each percent as a fraction in lowest terms.

   a. 20%

   b. 9%

   c. 15%

4. Write each percent as a decimal.

   a. 19%

   b. 36%

   c. 6%

5. Follow each direction carefully.

   a. 0.85 to a percent

   b. 0.7 to a percent

   c. 46% to a fraction

   d. 5% to a fraction

   e. 80% to a decimal

   f. 5% to a decimal

Lesson 2.3B

1. Calculate.

   a. 10% of 100

   b. 20% of 200

   c. 10% of 250

   d. 20% of 400

2. Which is more money? Circle the answer.

a. 10% of $200

OR

b. 50% of $100

3. Write the correct number in the space provided.

a. 40% of 30 is _____

b. ____% of 50 is 35

c. ____% of 20 is 12

d. 35% of ____ is 14

e. 82% of ____ is 73.8

4. Out of 32 students, 24 attended the school play. What percent of the students went to the play?

5. There are 30 people in an office. 60% walk to work. How many walk to work?

6. A test had 40 questions. John got 34 right. What percent of the questions did he get right?

7. The regular price of a dress was $85. It was on sale for 40% off.

   a. What was the discount?

   b. What was the sale price on the dress?

8. A coat was sold for $89. The sales tax was 6%.

   a. How much money is the sales tax?

   b. What was the total cost of the coat?

Turn to the Answer Key at the end of the Module and mark your answers.

Section Challenge

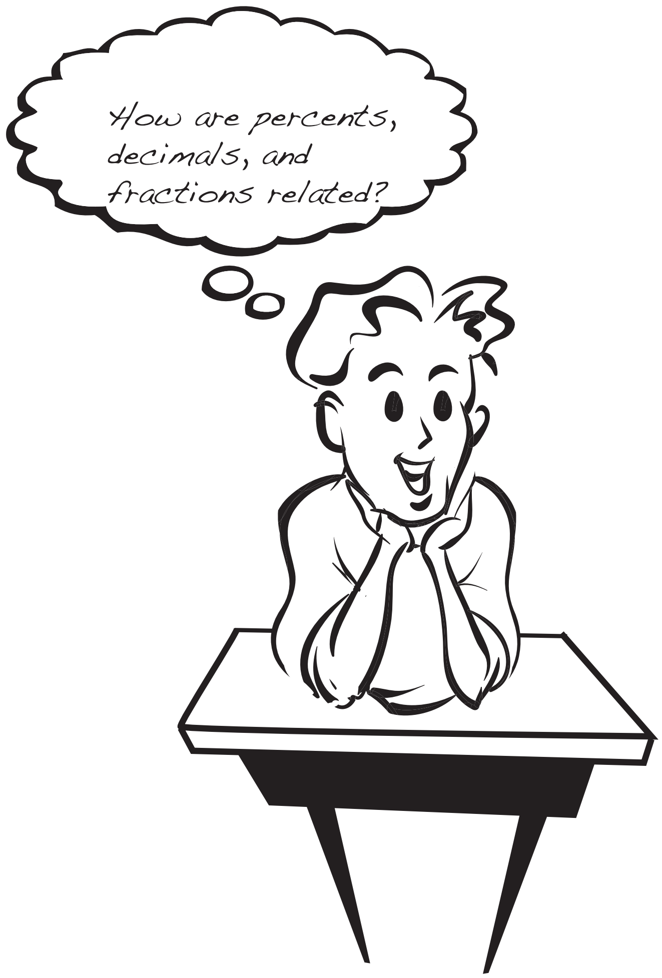
Moira received $\frac{7}{10}$ on her integer quiz. She divided 7 by 10 and got 0.7, but she does not know what that means. Can you help Moira find what the 0.7 means on her integer quiz?

On the fraction quiz she earned 80%. She received 24 marks, but cannot remember the total number of marks on that quiz. Can you help her find out what her fraction quiz was out of?

You can try to solve this question now or at the end of this section. It's up to you.

Lesson 2.3A: Making Connections—Decimals, Fractions and Percent

Student Inquiry



This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
How can I write a percent as a decimal?		answer
		example
How can I write a decimal as a percent?		answer
		example
How can I write a percent as a fraction?		answer
		example
How can I write a fraction as a percent?		answer
		example

Lesson 2.3A: Making Connections—Decimals, Fractions and Percent

Thinking Space

Introduction

Take out a newspaper and have a look through it. Do you see any fractions? How about decimals? Can you find any percents? You will probably find all three in the sports section. The batting averages in baseball are decimals, basketball free throw averages are usually in percents, and fractions are found when comparing games won and total games played. You will build on your understanding how percents, decimals and fractions are all related in this lesson. After we review some skills, we'll focus on using those skills to solve problems.

Explore Online

Looking for more practice or just want to play some fun games?
If you have internet access, go to the Math 7 website at:
http://www.openschool.bc.ca/courses/math/math7/mod2.html
Look for *Lesson 2.3A: Making Connections—Decimals, Fractions and Percent* and check out some of the links!

Warm-up

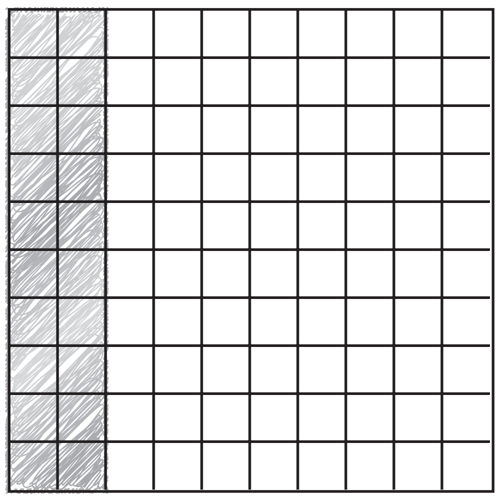
Do you remember what percent means?

Let's separate percent into two words: "per" and "cent."

When somebody says that they are driving at 50 kilometres per hour, we can write this as 50 km/h. We can see that "per" is written as a fraction, so whenever you see the word "per" you can think of a fraction.

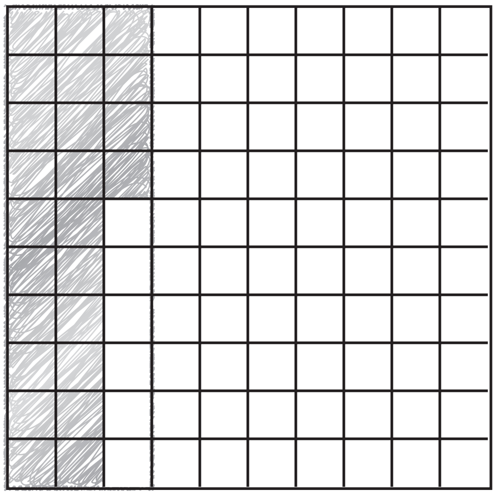
Now "cent" is French for 100. So whenever you see percent think of "out of 100." This is a fraction with a denominator of 100.

For example, 20% is $= \dfrac{20}{100}$



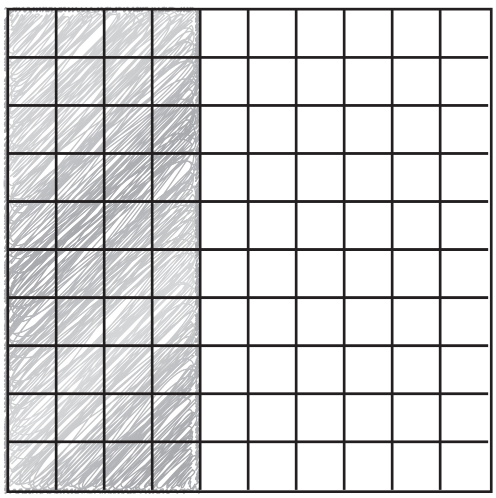
1. Here are some 10 by 10 grids. Find the percent that is shaded in each.

a.



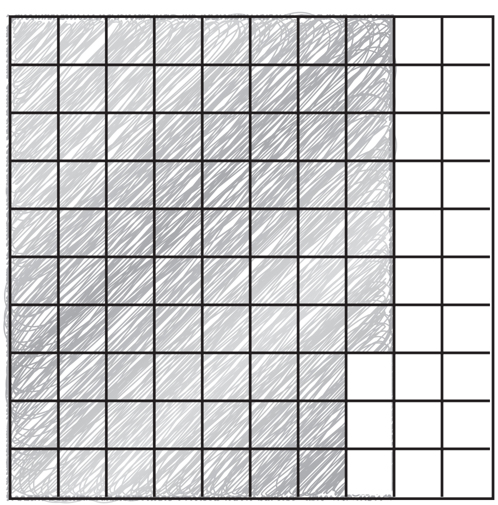
= ____ %

b.

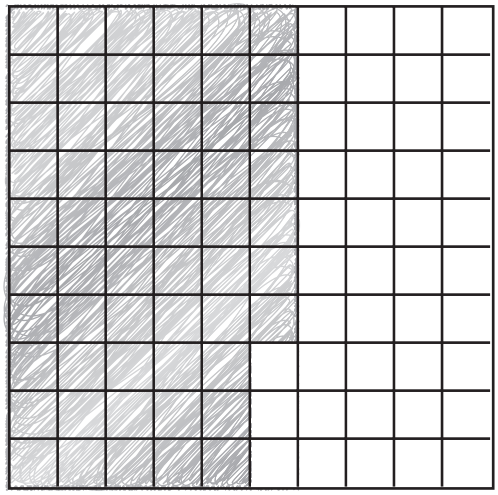


= ____ %

c.



= ____ %

d.  = ____ %

We can also write percents as fractions. For instance, 72% is $\frac{72}{100}$.

Remember, we need to simplify fractions.

$$\frac{72}{100} = \frac{72 \div 4}{100 \div 4} = \frac{18}{25}$$

2. Write the following percents as reduced fractions.

   a. 30% =

   b. 85% =

   c. 46% =

   d. 55% =

   e. 28% =

Let's review one more thing. Percents can also be written as decimals. Since a percent can be written as a fraction over 100, we can easily write it as a decimal. Let's take a look at an example.

$$25\% = \frac{25}{100} = 0.25$$

$$33\% = \frac{33}{100} = 0.33$$

Here are a couple of helpful hints!

The two 00s in the 100 remind us that there are two decimal points when changing a % to a decimal. For instance 20% = 0.20.

The two 00s in the % symbol also can remind us that % is "**o**ut **o**f" 100. For example, 15% is 15 "out of 100."

3. Now change these percents to decimals.

   a. 15%

   b. 65%

   c. 87%

   d. 59%

   e. 49%

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

When looking through the newspaper, do you see fractions, decimals, or percents used the most?

When looking at a shampoo bottle ad, why do you think it would say 33% more instead of 0.33 more or $\frac{1}{3}$ more?

Maybe it's because 33% *seems* bigger than 0.33 or $\frac{1}{3}$, even though they actually have the same value. People may think they are getting more when the advertisement uses percents rather than decimals. Or maybe not everyone understands fractions and decimals. Either way, let's make sure you have a great understanding of all three.

Before we get into the details of converting between fractions, decimals, and percents, let's look at some reasons WHY you'd want to convert. What about situations other than the shampoo ad we talked about? Complete the following practice activity to explore some real world scenarios.

Practice 1

Choose a decimal, fraction, or percent to fill in the blanks with the type of measurement that you think is appropriate.

	Example
Test scores	Naomi took a test and had 9 out of 10 correct. Her final mark is ______.
Money	Jane has 3 loonies and a quarter. Jane has a total of $________.
Metric measurements	Rajah took out his centimetre ruler and measured the length of an ant. The ant measures _____ cm long?
Baking measurements	A recipe calls for two and _____ cups of raisins.
Interest payments	Mike has put some money in a savings account. He receives _____ interest each month.
Comparing sizes	A juice box says that it has _____ more real fruit than other juices.
Very small numbers	The lead in the pencil measures ____mm.
Discounts	A shirt is on sale for _____ off.

Now fill in the chart below to show which type of number is the best to use in each situation. The first one is done for you.

	Decimal	Percent	Fraction
Test scores		Best	
Money			
Metric measurements			
Baking measurements			
Interest payments			
Comparing sizes			
Very small numbers			
Discounts			

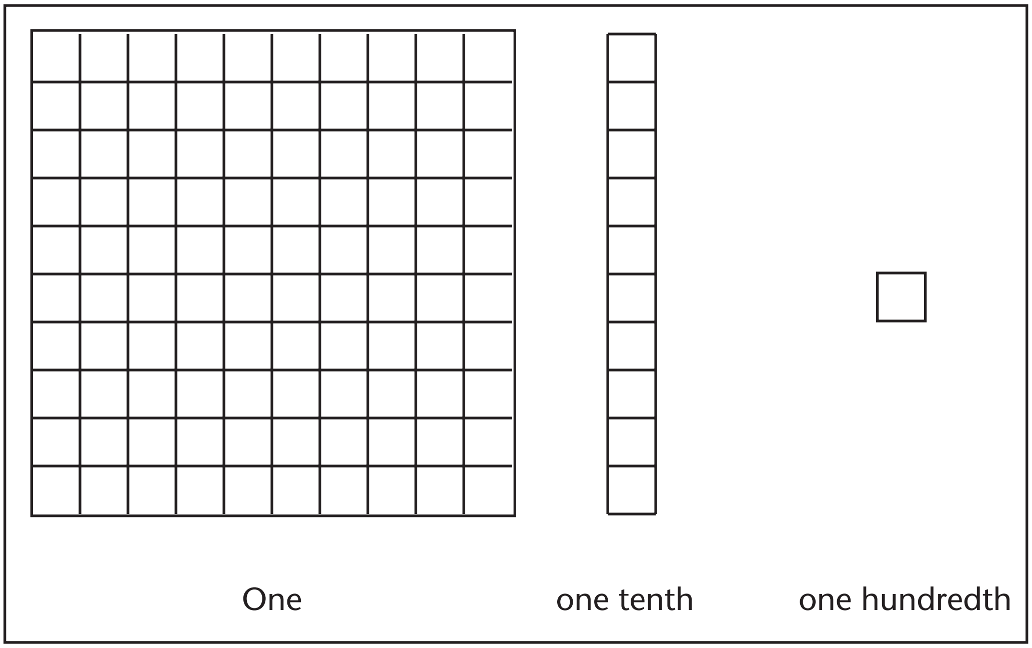
Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

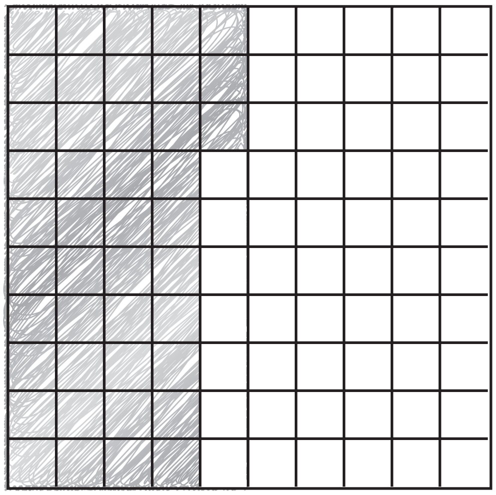
Now that we've thought of some reasons we might want to convert between fractions, decimals, and percents, HOW do we do it? You probably have some experience with this skill. In fact, you used the skills you already have in the warm-up part of this lesson.

Let's start by looking at our 10 by 10 grids:

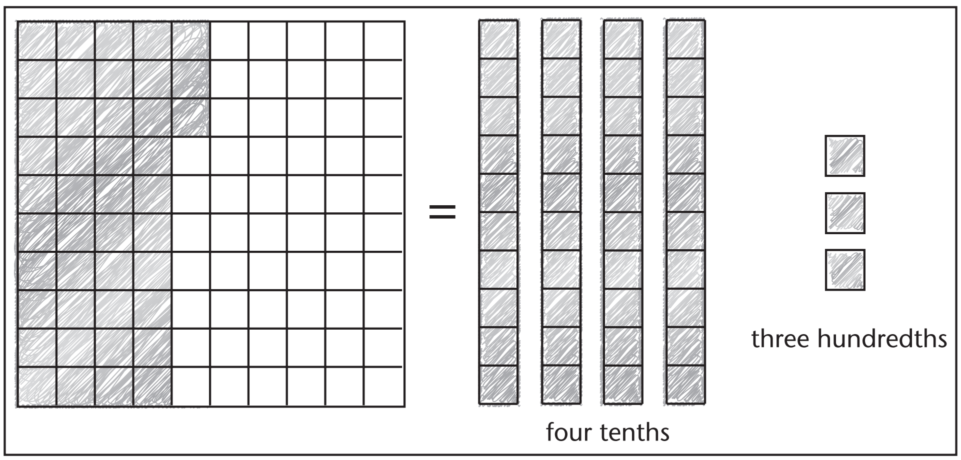


These blocks can help us to move between fraction, decimal, and percent form.

Let's look at an example.



We can see that 43 pieces are shaded; this means we have $\frac{43}{100}$ or 43% shaded. But we can also see that there are 4 tenths and 3 hundredths:



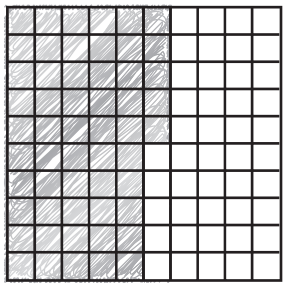
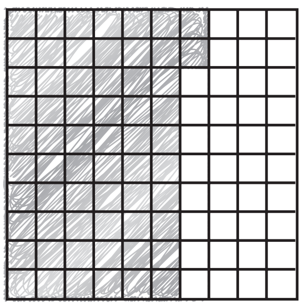
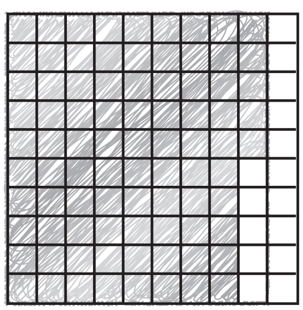
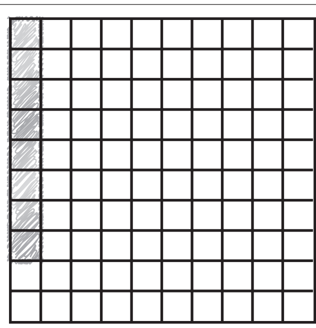
This shows us we have 0.43.

So $\frac{43}{100} = 43\% = 0.43$

Thinking Space

Practice 2

Write the amount shaded in the 10 by 10 grid as a fraction (in lowest terms), percent, and decimal

10 by 10 grid Tenths and hundredths	Fraction (in lowest terms)	Percent	Decimal
1. 			
2. 			
3. 			
4. 			

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Drawing a picture can be helpful, but it's not always the fastest method. What other methods have you used in the past?

Let's look at some other ways to make these conversions.

In Section 2.2, you became an expert at converting between fractions and decimals! Let's review with some examples.

Fraction to decimal:

$$\frac{7}{20} = 7 \div 20 = 0.35 \quad \text{OR} \quad \frac{7}{20} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100} = 0.35$$

Decimal to fraction:

$$0.35 = \frac{35}{100} = \frac{35 \div 5}{100 \div 5} = \frac{7}{20}$$

Practice 3

1. Josephine is a baseball player. She hit the ball 8 out of the last 10 times she was at bat. What is her batting average? *Note:* batting average is written as a decimal.

2. Miranda's batting average is 0.7. How many times would she probably hit the ball if she was at bat 20 times?

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

Percent to Fraction

Let's look at how to change from percents to fractions. Remember, a percent means over 100. So all we need to do is remove the % sign and make it over 100. Remember to simplify.

For example: $35\% = \frac{35}{100} = \frac{35 \div 5}{100 \div 5} = \frac{7}{20}$

Fraction to Percent

To change from a fraction to a percent, find an equivalent fraction with a denominator of 100. Then multiply by 100 to find the percent.

%

Remember: when you put the % symbol at the end, you're actually saying "out of 100", so you don't need the 100 as a denominator anymore.

For example: $\frac{7}{20} = \frac{7 \times 5}{7 \times 5} = \frac{35}{100} = 35\%$

Practice 4

1. Samuel received 75% on his last test. What fraction of the test did he get right?

2. Norio received a quiz back. He received a mark of 3/5. What percent did he get on the quiz?

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Decimal to Percent

To change a percent to a decimal, divide by 100%. This means we need to move the decimal twice to the left and remove the percent sign.

For example: $35\% = 0.35$

Percent to Decimal

To change a decimal to a percent, multiply by 100%. This means we need to move the decimal twice to the right and add a percent sign.

For example: $0.35 = 35\%$

Thinking Space

Practice 5

1. Jessica chose a page out of the newspaper and saw that ads took up one quarter of the page. What percent of the page is covered in ads?

2. Pierre chose a page out of the newspaper and saw that ads took up two thirds of the page. How can this be written as a decimal?

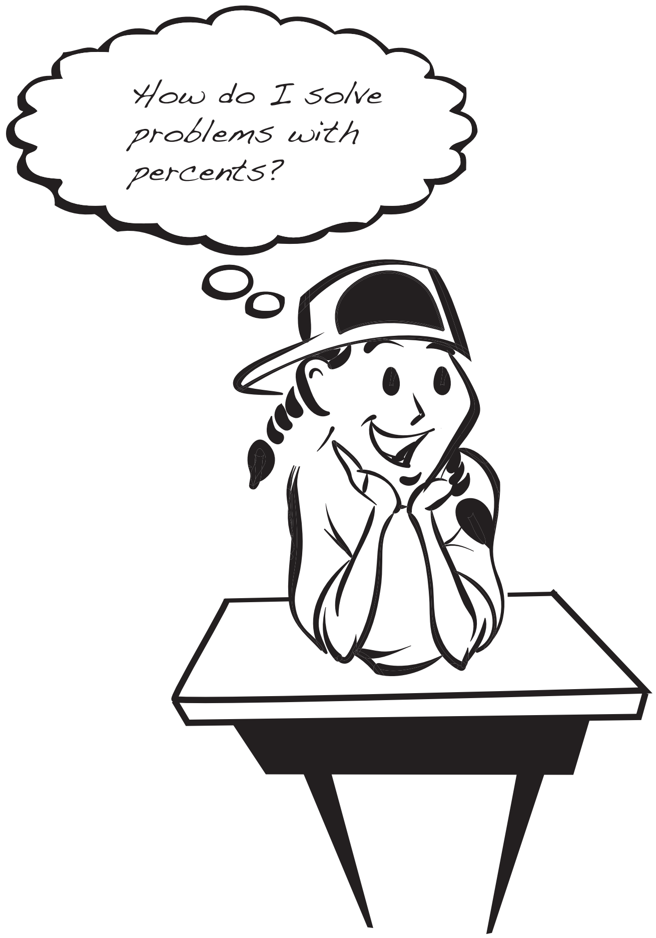
3. Fill in the chart below. The first one is done for you.

Fraction (in lowest terms)	Decimal	Percent
$\frac{1}{4}$	0.25	25%
a. $\frac{19}{20}$		
b. $\frac{43}{50}$		
c.	0.32	
d.	0.74	
e.		92%
f.		8%

Turn to the Answer Key at the end of the Module and mark your answers.

Lesson 2.3B: Problem Solving with Percent

Student Inquiry



This activity will help you get ready for, learn, and review the information in the upcoming lesson.

When you turn this page over, you will find a chart containing the inquiry outcomes for this lesson. You may be able to answer some of these questions already! Start by writing down your thoughts before the lesson.

When you finish the lesson, answer each question and give an example.

Student Inquiries	BEFORE THE LESSON What I already know about this question:	AFTER THE LESSON What I thought at the end: My final answer, and examples:
How do I solve problems with percents?		answer
		example
When should I round an answer for a percent problem?		answer
		example

Lesson 2.3B: Problem Solving with Percent

Thinking Space

Introduction

We know how percents, decimals, and fractions are related. We have also worked closely with fractions and decimals.

What about percents?

We need to know how to work with percents to answer questions like "what percent did I get on that last test?" and "how much is that sweater going to cost with the discount?" You will be able to answer questions just like these once we go through the steps to solve problems with percents.

Warm-up

Let's have a look at some problems involving percents:

1. Richard has 50 songs on his iPod. Twelve of these songs are rock songs. What percent of his songs are rock songs?

2. Barbara has a collection of 20 DVDs. She has not seen 3 of the DVDs yet. What percent of the DVDs has Barbara not seen?

3. Caroline's team won 19 of the last 25 basketball games. What percent of the games has Caroline's team won?

Let's also review how to multiply decimals.

When multiplying decimals:

	Example
1. Multiply the numbers as if they were both whole numbers.	0.5×60.2 Ignoring the decimals we have $5 \times 602 = 3010$
2. Count how many digits are to the right of the decimal in the original numbers.	0.5 has one digit and 60.2 has one digit to the right of the decimal. We have a total of 2 digits to the right of the decimal.
3. Place the decimal in your answer so it has the same number of digits to the right of the decimal as you found in step 2.	From step 2 we know we need two digits after the decimal. 3010 becomes 30.10. So $0.5 \times 60.2 = 30.10$, which equals 30.1.

Here's another example: 0.25×40

Let's follow the steps above to solve:

1. $25 \times 40 = 1000$
2. 0.25 has 2 decimal places, 40 has none.
3. 1000 with 2 decimal places is $10.00 = 10$.

So $0.25 \times 40 = 10$

4. Use the above steps to find the following products:

   a. $0.75 \times 80 =$

   b. $0.8 \times 50 =$

c. $0.35 \times 200 =$

d. $0.9 \times 1.5 =$

e. $0.63 \times 5.5 =$

Turn to the Answer Key at the end of the Module and mark your answers.

Explore

Thinking Space

All of the word problems we are going to work with use the following equation:

(percent)	×	(original number)	=	(amount)
P	×	**O**	=	A

Here's what you need to know to use this equation:

Percent (P) = the percent in DECIMAL form

Original number (**O**) = the number after the word "of"

Amount (A) = percent of a number

You can think of the *O* as the "of number" or the "big number."

You can think of this as the "little number."

50% of 20 is 10, so let's see what this will look like in the equation:

(percent)	×	(original number)	=	(amount)
(0.5)	×	(20)	=	(10)

Notice that the word "of" means multiply.

We will be looking at problems where the percent, original number (of number), or the amount is missing.

Let's take a closer look at the equation: P × **O** = A

This equation is solved for A. But what if we want to find P?

You will learn more about ways to solve equations like this later in Math 7. For now, here is a short explanation.

To have P on its own, we need to move the **O**. Since the P and **O** are being multiplied we need to "un-multiply" them. Un-multiplying is the same dividing. We "un-multiply" by dividing both sides by **O.**

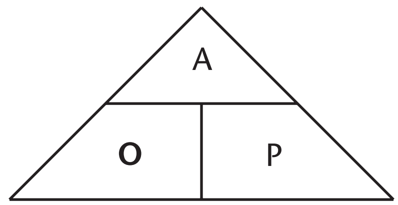
$$\frac{P \times \mathbf{O}}{\mathbf{O}} = \frac{A}{\mathbf{O}} \rightarrow P = \frac{A}{\mathbf{O}}$$

"un-multiply" isn't a real math word, but it helps us understand what's happening.

We can use the same "un-multiply" method to solve for **O**. We can isolate **O** by dividing both sides by P. $O = \frac{A}{P}$.

It is important to understand how to isolate variables. But in case you have trouble understanding this right away, you can use a handy trick. Rewrite the equation in a triangle.

Thinking Space



A = amount

O = original number (of number)

P = percent in decimal form

Now we will see how to use this triangle by going through examples.

Example 1

20% of 40 is what number?

We know:

The percent is 20% = 0.20 and the number after "of" is 40 so this is our original number (of number).

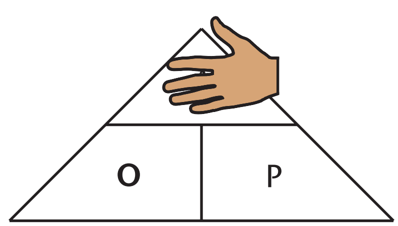
P = 0.20

O = 40

A = ?

To find the amount, we cover the A in the triangle and see what we have:

We always cover the letter that we're trying to find.



Since the **O** and the P are next to each other, we multiply them.

Thinking Space

This is the same as our original equation:

(percent) × (original number) = (amount)

$A = \mathbf{O} \times P$

$A = 0.20 \times 40 = 8$

So we know the amount is 8.

Example 2

What percent of 45 is 27?

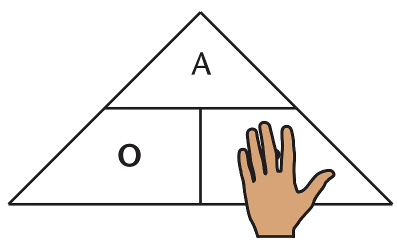
We do not know the percent. The number after "of" is 45, so this is our original number (of number) and our amount is 27.

$P = ?$

$\mathbf{O} = 45$

$A = 27$

We do not know P, so let's cover the P in our triangle to see what we need to do:



Since the A is over the **O**, we know $P=\frac{A}{O}$. This is the same equation we found earlier.

$$P = \frac{A}{O}$$

$$P = \frac{27}{45} = 0.6 = 60\%$$

Example 3

Thinking Space

80% of what number is 29.6?

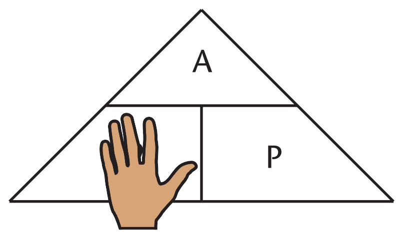
The percent is 80% = 0.80, we do not know the original number (of number), and our amount is 29.6.

P = 0.80

O = ?

A = 29.6

We do not know *O*, so let's cover the *O* in our triangle to see what we need to do:



A is over the P so we need to divide.

$$O = \frac{A}{P}$$

$$O = \frac{29.6}{0.80} = 37$$

Our original number is 37.

Explore

Some simple steps can make problem solving easier. In this lesson we'll focus on the following:

1. What is the question asking for?	Read through the problem and identify what you are trying to find. Knowing what sort of answer you're looking for can help you solve the problem.
2. Estimate an answer.	If you have an estimate, then you'll know if your answer is reasonable.
3. Find an answer.	Do the appropriate calculations, and come up with an answer.
4. Make sure the answer is reasonable.	This is a good way to check your work and can help you identify if you've made an error. Compare your answer with your estimate. Is it close? Why or why not? Remember that just because your answer isn't the same as your estimate, doesn't make it wrong. Look carefully at how you rounded the numbers to make your estimate. Stop to think about how your answer compares to your estimate, and decide if your solution is reasonable.

Here's an example problem. We'll use the steps outlined above to solve it.

Scott just ate dinner at a restaurant. His bill came to $23.54. He wants to leave his waiter a 15% tip. How much money should Scott leave for the tip?

Let's follow the four steps:

1. Write what this question is asking (e.g. find 15% of $23.54).
2. Estimate an answer.
3. Find an answer.
4. Make sure the answer is reasonable.

Thinking Space

Think about the steps you used to solve problems in Module 1.

The table shown here simplifies those steps.

1. What is the question asking for?	What is 15% of $23.54?
2. Estimate an answer.	The bill is about $24. 10% of $24 is $2.40. 15% is 1½ times as much as a 10%. So, $2.40 + $1.20 = $3.60.
3. Find an answer.	The percent is 15% = 0.15, the original number (of number) is 23.54, and we want to find the amount. P = 0.15 **O** = 23.54 A = ? A = **O** × P A = 23.54 × 0.15 = 3.531 = $3.53 The tip should be $3.53.
4. Make sure the answer is reasonable. If not then check over your work.	$3.53 is very close to our estimate, so our answer is reasonable.

Don't forget to answer the question in a sentence.

Scott should leave $3.53 for a tip.

Here are some definitions you may need to review:

Discount = how much money is taken off the original price?

Example: A jacket is originally $100. It is on sale at 25% off.

The discount = 100 × 0.25 = $25

Sales Price = original price – discount.

Example: A jacket is originally $100 and the discount is $25.

The sale price = $100 – $25 = $75

Thinking Space

How do you like these simplified steps?

Do you think most people tip exactly 15%? Or are most people more likely to round off the tip?

Practice

Answer the following questions by following four steps.

1. Write what this question is asking.
2. Estimate an answer. (To estimate, round off the numbers. Then use these rounded numbers to find an estimated answer.)
3. Find an answer.
4. Make sure the answer is reasonable. (Make sure your answer in step 3 is close to your estimate in step 2.)

A chart is provided to guide you for the first few questions. Remember to round to 2 decimal places when working with money.

1. A shirt is regularly priced at $37.49. It is discounted at 30% off. How much is the discount?

1. What is the question asking for?	
2. Estimate an answer.	
3. Find an answer.	
4. Make sure the answer is reasonable. If not then check over your work.	

2. Bruce just received a mark of 59 out of 70 on his math test. What percent score did he get on the test?

1. What is the question asking for?	
2. Estimate an answer.	
3. Find an answer.	
4. Make sure the answer is reasonable. If not then check over your work.	

3. William bought a shirt 30% off. It was discounted at $11.27 less than the original price. What was the original price?

1. What is the question asking for?	
2. Estimate an answer.	

3. Find an answer.	
4. Make sure the answer is reasonable. If not then check over your work.	

4. Terri wants to buy cereal with the highest percent of iron. The amount of daily required iron in one serving of each type of cereal is:

Crunchy Cereal

Nutrition Facts Per 125 mL (87 g)	
Amount	**% DV***
Calories 80	
Fat 0.5 g	**1** %
Saturated 0 g + Trans 0 g	**0** %
Cholesterol 0 mg	
Sodium 0 mg	**0** %
Carbohydrate 18 g	**6** %
Fibre 2 g	**8** %
Sugars 2 g	
Protein 3 g	
Vitamin A	2 %
Vitamin C	10 %
Calcium	0 %
Iron	25 %
* DV = Daily Value	

Original Oats

Nutrition Facts Per 125 mL (87 g)	
Amount	**% DV***
Calories 80	
Fat 0.5 g	**1** %
Saturated 0 g + Trans 0 g	**0** %
Cholesterol 0 mg	
Sodium 0 mg	**0** %
Carbohydrate 18 g	**6** %
Fibre 2 g	**8** %
Sugars 2 g	
Protein 3 g	
Vitamin A	2
Vitamin C	10
Calcium	0
Iron	0.3
* DV = Daily Value	

Green Flake

Nutrition Facts Per 125 mL (87 g)	
Amount	**% DV***
Calories 80	
Fat 0.5 g	**1** %
Saturated 0 g + Trans 0 g	**0** %
Cholesterol 0 mg	
Sodium 0 mg	**0** %
Carbohydrate 18 g	**6** %
Fibre 2 g	**8** %
Sugars 2 g	
Protein 3 g	
Vitamin A	2
Vitamin C	10
Calcium	0
Iron	1/5
* DV = Daily Value	

Which cereal should Terri buy?

5. Two years ago Bert was 120 cm tall. He has grown 20% since then. What is Bert's current height?

Turn to the Answer Key at the end of the Module and mark your answers.

Section Summary

Here is a quick review of what we have learned in this section:

1. A percent can be written as a fraction or a decimal.

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$50\% = 0.50 = 0.5$$

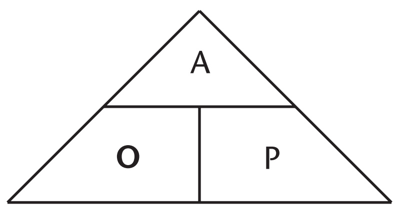
2. We can solve problems with percents by using this equation.

$$\boxed{\begin{array}{ccccc}(\text{percent}) & \times & (\text{original number}) & = & (\text{amount}) \\ \text{P} & \times & \mathbf{O} & = & \text{A}\end{array}}$$

This equation can be rearranged to solve for P and *O*:

$$\text{P}=\frac{\text{A}}{\text{O}} \text{ and } \text{O}=\frac{\text{A}}{\text{P}}$$

This equation can also be rewritten in a triangle:



Example

What percent of 50 is 10?

P = ?

O = 50

A = 10

$$\text{P} = \frac{\text{A}}{\mathbf{O}}$$

$$= \frac{10}{50} = \frac{20}{100} = 20\%$$

Section Challenge

Moira received $\frac{7}{10}$ on her integer quiz. She divided 7 by 10 and got 0.7, but does not know what that means. Can you help Moira find what the 0.7 means on her integer quiz?

On the fraction quiz, she earned 80%. She received a 24, but cannot remember how many total marks were on that quiz. Can you help her find out what her fraction quiz was out of?

If you've already solved the problem, then you should solve the problem now.

Turn to the Answer Key at the end of the Module and mark your answers.

Answer Key Table of Contents

Answer to Pretest 2.1

Lesson 2.1A

1. a. $\frac{4}{7}$

   b. $\frac{2}{3}$

   c. $\frac{4}{5}$

2.

   a. $\frac{1}{2}, \frac{4}{8}, \frac{15}{10}, \frac{20}{40}$

   b. $\frac{3}{9}, \frac{5}{15}, \frac{7}{21}, \frac{18}{6}$

   c. $\frac{8}{10}, \frac{14}{21}, \frac{2}{3}, \frac{6}{9}$

Lesson 2.1B

1. a. $\frac{3}{6}, \frac{4}{6}$

   b. $\frac{8}{20}, \frac{15}{20}$

   c. $\frac{10}{12}, \frac{3}{12}$

2. a. $\frac{1}{5}, \frac{1}{2}, \frac{3}{5}, \frac{3}{4}$

   b. $\frac{6}{35}, \frac{1}{5}, \frac{2}{7}, \frac{5}{7}$

Lesson 2.1C

1. a. $\frac{2}{7}+\frac{4}{7}=\mathbf{\frac{6}{7}}$

   b. $\frac{3}{10}+\frac{2}{10}=\mathbf{\frac{5}{10}}=\mathbf{\frac{1}{2}}$

   c. $\frac{7}{9}+\frac{2}{9}=\mathbf{\frac{9}{9}}=\mathbf{1}$

   d. $\frac{1}{6}+\frac{3}{6}=\mathbf{\frac{4}{6}}=\mathbf{\frac{2}{3}}$

   e. $\frac{7}{10}+\frac{5}{10}=\mathbf{\frac{12}{10}}=\mathbf{1\frac{2}{10}}=\mathbf{1\frac{1}{5}}$

   f. $\frac{2}{3}+\frac{1}{6}=\mathbf{\frac{4}{6}}+\mathbf{\frac{1}{6}}=\mathbf{\frac{5}{6}}$

   g. $\frac{1}{4}+\frac{3}{8}=\mathbf{\frac{2}{8}}+\mathbf{\frac{3}{8}}=\mathbf{\frac{5}{8}}$

   h. $\frac{1}{5}+\frac{9}{10}=\mathbf{\frac{2}{10}}+\mathbf{\frac{9}{10}}=\mathbf{\frac{11}{10}}=\mathbf{1\frac{1}{10}}$

   i. $\frac{5}{6}+\frac{1}{2}=\mathbf{\frac{5}{6}}+\mathbf{\frac{3}{6}}=\mathbf{\frac{8}{6}}=\mathbf{1\frac{2}{6}}=\mathbf{1\frac{1}{3}}$

   j. $\frac{2}{5}+\frac{1}{10}=\mathbf{\frac{4}{10}}+\mathbf{\frac{1}{10}}=\mathbf{\frac{5}{10}}=\mathbf{\frac{1}{2}}$

2. $\mathbf{\frac{5}{10}}+\mathbf{\frac{2}{10}}=\mathbf{\frac{7}{10}}$ **of a kilometre**

3. a. $\frac{7}{9}-\frac{2}{9}=\mathbf{\frac{5}{9}}$

   b. $\frac{8}{10}-\frac{3}{10}=\mathbf{\frac{5}{10}}=\mathbf{\frac{1}{2}}$

   c. $\frac{7}{12}-\frac{3}{12}=\mathbf{\frac{4}{12}}=\mathbf{\frac{1}{3}}$

   d. $\frac{5}{6}-\frac{5}{6}=\mathbf{0}$

   e. $\frac{5}{6}-\frac{3}{6}=\mathbf{\frac{2}{6}}=\mathbf{\frac{1}{3}}$

Lesson 2.1D

1. a. $\frac{2}{3}+\frac{1}{6}=\mathbf{\frac{4}{6}}+\mathbf{\frac{1}{6}}=\mathbf{\frac{5}{6}}$

   b. $\frac{1}{4}+\frac{3}{8}=\mathbf{\frac{2}{8}}+\mathbf{\frac{3}{8}}=\mathbf{\frac{5}{8}}$

   c. $\frac{1}{5}+\frac{9}{10}=\frac{2}{10}+\frac{9}{10}=\mathbf{\frac{11}{10}}=\mathbf{1\frac{1}{10}}$

   d. $\frac{5}{6}+\frac{1}{2}=\mathbf{\frac{5}{6}}+\mathbf{\frac{3}{6}}=\mathbf{\frac{8}{6}}=\mathbf{1\frac{2}{6}}=\mathbf{1\frac{1}{3}}$

   e. $\frac{2}{5}+\frac{1}{10}=\mathbf{\frac{4}{6}}+\mathbf{\frac{1}{6}}=\mathbf{\frac{5}{6}}$

2. a. $\frac{7}{9}-\frac{1}{3}=\mathbf{\frac{7}{9}}-\mathbf{\frac{3}{9}}=\mathbf{\frac{4}{9}}$

   b. $\frac{8}{12}-\frac{1}{4}=\mathbf{\frac{8}{12}}-\mathbf{\frac{3}{12}}=\mathbf{\frac{5}{12}}$

   c. $\frac{5}{10}-\frac{1}{2}=\mathbf{\frac{5}{10}}-\mathbf{\frac{5}{10}}=\mathbf{0}$

   d. $\frac{9}{10}-\frac{2}{5}=\mathbf{\frac{9}{10}}-\mathbf{\frac{4}{10}}=\mathbf{\frac{5}{10}}=\mathbf{\frac{1}{2}}$

   e. $\frac{5}{8}-\frac{1}{4}=\mathbf{\frac{5}{8}}-\mathbf{\frac{2}{8}}=\mathbf{\frac{3}{8}}$

Lesson 2.1E

1. a. $\frac{13}{5} = \mathbf{2\frac{3}{5}}$

   b. $\frac{14}{3} = \mathbf{4\frac{2}{3}}$

   c. $\frac{12}{7} = \mathbf{1\frac{5}{7}}$

   d. $\frac{22}{10} = \mathbf{2\frac{2}{10}} = \mathbf{2\frac{1}{5}}$

2. a. $1\frac{1}{3} + 3\frac{1}{3} = \mathbf{4\frac{2}{3}}$

   b. $4\frac{1}{6} + 2\frac{3}{6} = \mathbf{6\frac{4}{6}} = \mathbf{6\frac{2}{3}}$

   c. $8\frac{3}{4} + 2\frac{1}{4} = \mathbf{10\frac{4}{4}} = \mathbf{11}$

   d. $2\frac{3}{5} + 1\frac{4}{5} = \mathbf{3\frac{7}{5}} = \mathbf{4\frac{2}{5}}$

   e. $2\frac{1}{5} + 3\frac{7}{10} = \mathbf{5\frac{9}{10}}$

   f. $3\frac{1}{4} + 2\frac{1}{2} = \mathbf{3\frac{1}{4}} + \mathbf{2\frac{2}{4}} = \mathbf{5\frac{3}{4}}$

3. a. $2\frac{4}{5} - 1\frac{2}{5} = \mathbf{1\frac{2}{5}}$

   b. $5\frac{9}{10} - 1\frac{4}{10} = \mathbf{4\frac{1}{2}}$

   c. $6\frac{3}{4} - 6\frac{1}{4} = \mathbf{\frac{2}{4}} = \mathbf{\frac{1}{2}}$

   d. $5\frac{2}{4} - 3\frac{3}{4} = \mathbf{1\frac{3}{4}}$

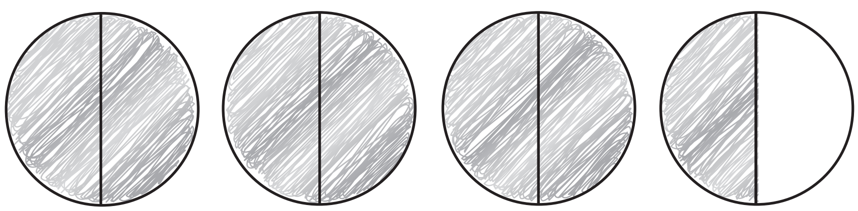
Answer to Lesson 2.1A Warm-up

1. a. equal
   b. numerator
   c. denominator
   d. part
   e. whole
   f. common
   g. mixed
   h. improper

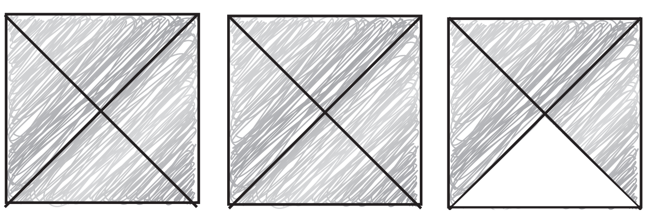
2. b. shaded: $\frac{7}{12}$, unshaded: $\frac{5}{12}$

   c. shaded: $\frac{7}{8}$, unshaded: $\frac{1}{8}$

3.

a. 

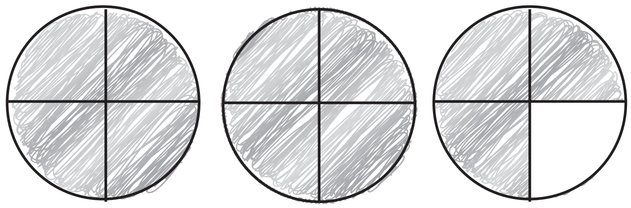
$\frac{7}{2}$

b. 

$\frac{11}{4}$

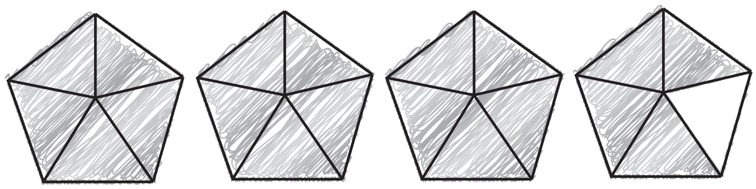
4.

a.



$2\frac{3}{4}$

b.



$3\frac{4}{5}$

5. a. $\frac{13}{4}$

   b. $\frac{29}{6}$

   c. $\frac{39}{5}$

6. a. $1\frac{2}{5}$

   b. $3\frac{3}{4}$

   c. $4\frac{2}{3}$

Answer to Lesson 2.1A Practice 1

1. same
2. 2
3. 3

Answer to Lesson 2.1A Practice 2

1. a. 6

   b. 28

   c. 4

2. a. Answers include $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}\ldots$

   b. Answers include $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}\ldots$

Answer to Lesson 2.1A Practice 3

1. a. 7

   b. 13

   c. 3

2. a. $\frac{1}{5}$

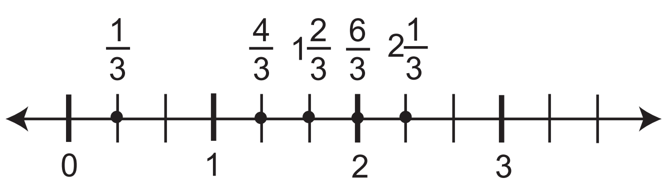
   b. $\frac{2}{3}$

   c. $\frac{9}{10}$

Answer to Lesson 2.1B Warm-up

1. $\frac{11}{4}, 2\frac{1}{4}, \frac{4}{4}, \frac{3}{4}, \frac{1}{4}$

2.



   a. $\frac{1}{3}, \frac{4}{3}, 1\frac{2}{3}, \frac{6}{3}, 2\frac{1}{3}$

   b. $2\frac{1}{3}, \frac{6}{3}, 1\frac{2}{3}, \frac{4}{3}, \frac{1}{3}$

Answer to Lesson 2.1B Practice 1

1. a. 10

   b. 21

   c. 8

   d. 18

2. a. $\frac{5}{10}, \frac{6}{10}$

   b. $\frac{14}{21}, \frac{3}{21}$

   c. $\frac{6}{8}, \frac{1}{8}$

   d. $\frac{10}{18}, \frac{3}{18}$

Answer to Lesson 2.1B Practice 2

1. $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{2}{3}$

2. $\frac{1}{4}, \frac{2}{6}, \frac{3}{6}, \frac{3}{4}, \frac{5}{6}$

3. $\frac{1}{3}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{5}{4}, \frac{3}{2}$

Answer to Lesson 2.1C Warm-up

1. 37
2. 27
3. 56
4. 40
5. 26
6. 26
7. 3

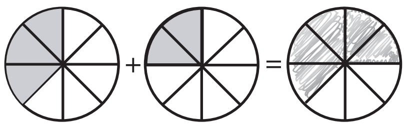
8. 35

9. 40

10. 11

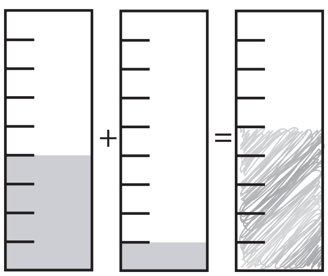
Answer to Lesson 2.1C Practice 1

1. $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

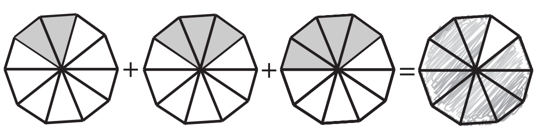


2. $\frac{3}{7} + \frac{1}{7} = \frac{4}{7}$

3. $\frac{4}{9} + \frac{1}{9} = \frac{5}{9}$



4. $\frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{9}{10}$



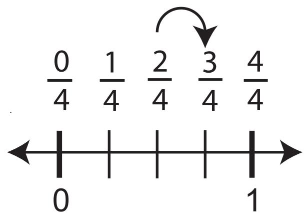
5. $\frac{3}{5}$

6. $\frac{6}{7}$

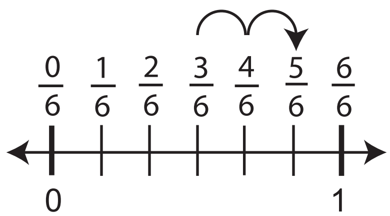
7. $\frac{9}{11}$

Answer to Lesson 2.1C Practice 2

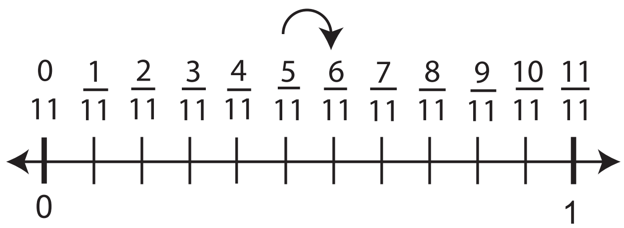
1. $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$



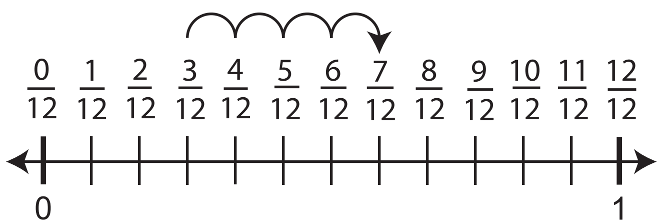
2. $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$



3. $\frac{5}{11} + \frac{1}{11} = \frac{6}{11}$



4. $\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$

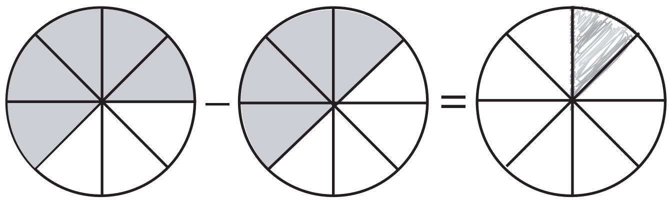


Answer to Lesson 2.1C Practice 3

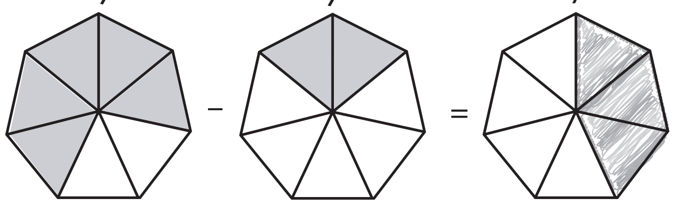
1. $\frac{5}{7}$
2. $\frac{23}{30}$
3. $\frac{39}{50}$
4. $\frac{47}{79}$
5. $\frac{65}{95}$

Answer to Lesson 2.1C Practice 4

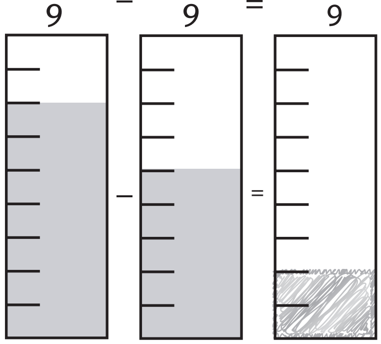
1. $\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$



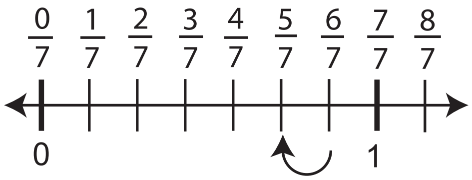
2. $\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$



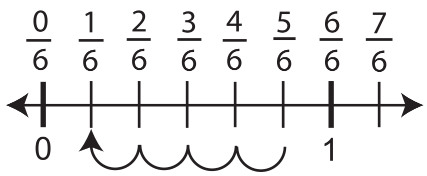
3. $\frac{7}{9} - \frac{5}{9} = \frac{2}{9}$



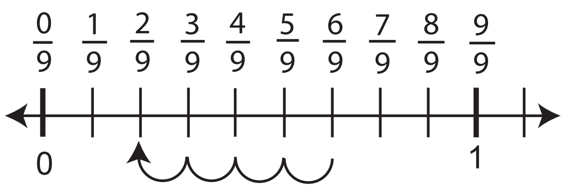
Answer to Lesson 2.1C Practice 5

1. 

$\frac{6}{7} - \frac{1}{7} = \frac{5}{7}$

2. 

$\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

3. 

$\frac{6}{9} - \frac{4}{9} = \frac{2}{9}$

Answer to Lesson 2.1C Practice 6

1. $\frac{1}{4}$

2. $\frac{3}{7}$

3. $\frac{5}{12}$

4. $\frac{7}{15}$

5. $\frac{4}{35}$

Answer to Lesson 2.1C Practice 7

1. $\frac{2}{4} = \frac{1}{2}$

2. $\frac{3}{9} = \frac{1}{3}$

3. $\frac{8}{8} = 1$

4. $\frac{5}{4} = 1\frac{1}{4}$

5. $\frac{8}{10} = \frac{4}{5}$

6. $\frac{20}{30} = \frac{2}{3}$

7. $\frac{3}{9} = \frac{1}{3}$

8. $\frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$

9. $\frac{17}{15} = 1\frac{2}{15}$

10. $\frac{14}{12} = \frac{7}{6} = 1\frac{1}{6}$

Answer to Lesson 2.1D Warm-up

1.

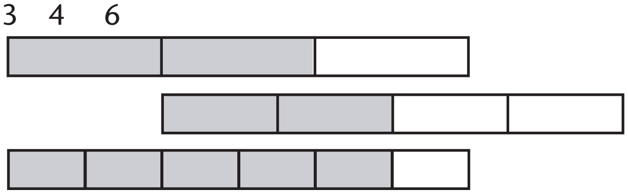
Find the greatest common factor of:	Factors	Greatest Common Factor
14 42	1, 2, 7, 14 1, 2, 6, 7, 21, 42	7
10 12	1, 2, 5, 10 1, 2, 3, 4, 6, 12	2
30 20	1, 2, 3, 5, 6, 10, 15, 30 1, 2, 4, 5, 10, 20	10
12 16 36	1, 2, 3, 4, 6, 12 1, 2, 4, 8, 16 1, 2, 3, 4, 6, 9, 12, 18, 36	4

2.

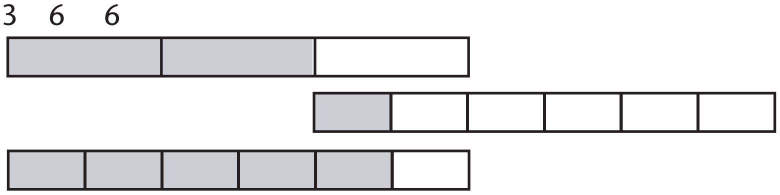
Find the lowest common multiple of:	Multiples	Lowest Common Multiple
4 14	4, 8, 12, 16, 20, 24, 28, ... 14, 28, ...	28
6 8	6, 12, 18, 24, ... 8, 16, 24, ...	24
12 15	12, 24, 36, 48, 60, ... 15, 30, 45, 60, ...	60
16 40	32, 48, 64, 80, ... 40, 80, ...	80

Answer to Lesson 2.1D Practice 1

1. $\frac{1}{3}+\frac{2}{4}=\frac{5}{6}$



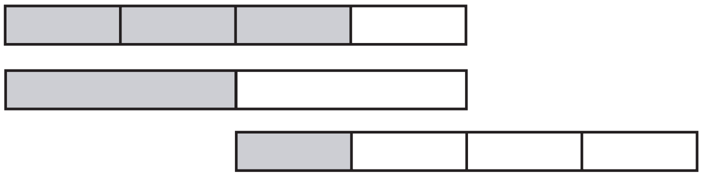
2. $\frac{2}{3}+\frac{1}{6}=\frac{5}{6}$



Answer to Lesson 2.1D Practice 2

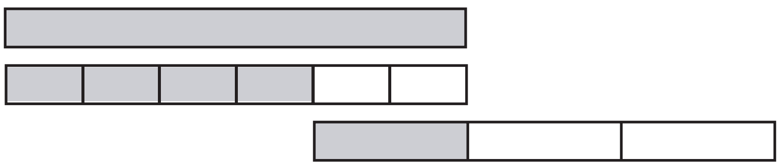
1.

$$\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$$

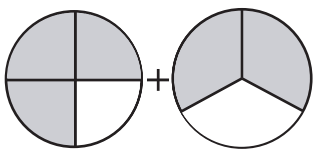
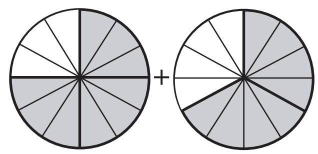
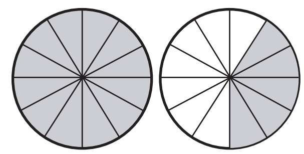
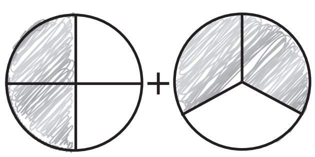
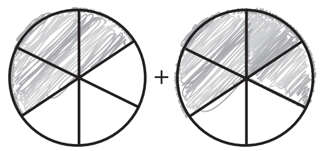
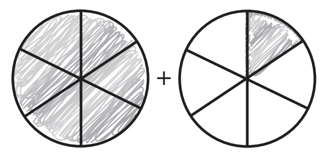
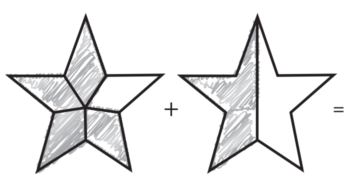
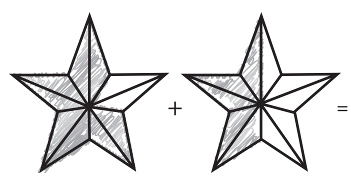
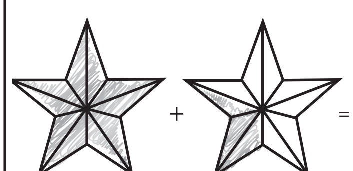
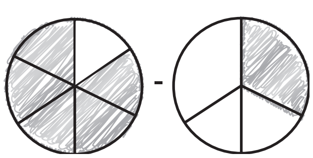
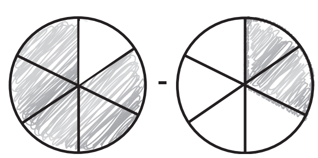
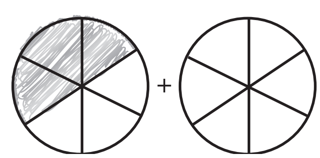


2.

$$\frac{1}{1} - \frac{4}{6} = \frac{1}{3}$$



Answer to Lesson 2.1D Practice 3

Question	Question with equal number pieces in all circles (equivalent fractions with lowest common denominators)	Solution
$\frac{3}{4}+\frac{2}{3}$ 	$\frac{9}{12}+\frac{8}{12}$ 	$\frac{17}{12}=1\frac{5}{12}$ 
1. $\frac{1}{2}+\frac{2}{3}$ 	$\frac{3}{6}+\frac{4}{6}$ 	$\frac{7}{6}=1\frac{1}{6}$ 
2. $\frac{4}{5}+\frac{1}{2}$ 	$\frac{8}{10}+\frac{5}{10}$ 	$\frac{13}{10}=1\frac{3}{10}$ 
3. $\frac{5}{6}-\frac{1}{3}$ 	$\frac{5}{6}-\frac{2}{6}$ 	$\frac{3}{6}=\frac{1}{2}$ 

Answer to Lesson 2.1D Practice 4

Question	Equivalent fractions	Solution
$\frac{7}{10} + \frac{3}{4}$	$\frac{14}{20} + \frac{15}{20}$	$\frac{29}{20} = 1\frac{9}{20}$
1. $\frac{2}{7} + \frac{1}{2}$	$\frac{4}{14} + \frac{7}{14}$	$\frac{11}{14}$
2. $\frac{1}{5} + \frac{5}{6}$	$\frac{6}{30} + \frac{25}{30}$	$\frac{31}{30} = 1\frac{1}{30}$
3. $\frac{5}{7} - \frac{1}{3}$	$\frac{15}{21} - \frac{7}{21}$	$\frac{8}{21}$
4. $\frac{5}{6} - \frac{3}{4}$	$\frac{10}{12} - \frac{9}{12}$	$\frac{1}{12}$

Answer to Lesson 2.1E Practice 1

1. $1\frac{1}{4}$
2. $1\frac{1}{4}$
3. $4\frac{1}{2}$
4. $2\frac{3}{4}$

Answer to Lesson 2.1E Practice 2

1. $2\frac{3}{4} = 1 + \frac{4}{4} + \frac{3}{4} = 1\frac{7}{4}$
2. $4\frac{3}{2}$
3. $1\frac{4}{3}$
4. $2\frac{8}{5}$
5. $3\frac{11}{6}$

6. $2\frac{4}{12}+1\frac{3}{12}=3\frac{7}{12}$

7. $2\frac{6}{10}+\frac{5}{10}=2\frac{11}{10}=3\frac{1}{10}$

8. $3\frac{3}{6}-1\frac{2}{6}=2\frac{1}{6}$

9. $2\frac{1}{6}-1\frac{3}{6}=1\frac{7}{6}-1\frac{3}{6}=\frac{4}{6}=\frac{2}{3}$

Answer to Lesson 2.1E Practice 3

1. $2\frac{1}{3}-1\frac{2}{3}=\frac{2}{3}$ of a cup

2. $3\frac{1}{4}-2\frac{3}{4}=\frac{1}{2}$ of a kilometre

3. a. $\frac{3}{4}+1\frac{2}{3}=2\frac{5}{12}$

   b. $2\frac{5}{12}+\frac{1}{6}=2\frac{7}{12}$

4. $2\frac{1}{3}-1\frac{3}{5}=2\frac{5}{15}-1\frac{9}{15}=1\frac{20}{15}=1\frac{9}{15}=\frac{11}{15}$

Answer to Section 2.1 Challenge

Ingredient	Needed for the recipe	Amount needed for 1 batch	Amount needed for 10 people
Butter	Celery casserole Apple pie	$\frac{1}{2}$ cup $\frac{3}{4}$ cup	$\frac{1}{2}$ cup $\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$ Total $= \frac{1}{2} = 1\frac{1}{2} = 2$ cups
Celery	Celery casserole Stuffing	$3\frac{1}{2}$ cups $\frac{1}{3}$ cup	$3\frac{1}{2}$ cups $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ Total: $3\frac{1}{2} + \frac{2}{3} = 3\frac{3}{6} + \frac{4}{6} = 3\frac{7}{6} = 4\frac{1}{6}$ cups
Flour	Celery casserole Apple pie	$\frac{3}{8}$ cup 1 cup	$\frac{3}{8}$ cup 1+1=2 Total: $\frac{3}{8} + 2 = 2\frac{3}{8}$ cups
Milk	Celery casserole	$\frac{3}{4}$ cup	$\frac{3}{4}$ cup
Mushrooms	Celery casserole Stuffing	$\frac{2}{3}$ cup $\frac{1}{4}$ cup	$\frac{2}{3}$ cup $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ cup Total: $\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$ cups
Poultry Seasoning	Stuffing	$\frac{1}{8}$ tsp	$\frac{1}{8}+\frac{1}{8}=\frac{2}{8}=\frac{1}{4}$ tsp
Sugar	Apple pie	$1\frac{1}{8}$ cup	$1\frac{1}{8}+1\frac{1}{8}=2\frac{2}{8}=2\frac{1}{4}$ cups

Answer to Pretest 2.2

Lesson 2.2A

1. a. 0.3
   b. 0.21
   c. 0.05
   d. 0.006
   e. 0.4
   f. 0.625
   g. 0.15

2. a. 1.5
   b. 2.75
   c. 3.4
   d. 3.92

3. a. $\frac{3}{11} = \mathbf{0.\overline{27}}$
   b. $\frac{2}{3} = \mathbf{0.\overline{6}}$
   c. $\frac{5}{8} = \mathbf{0.625}$

Lesson 2.2B

1. a. $0.7 = \mathbf{\frac{7}{10}}$
   b. $0.23 = \mathbf{\frac{23}{100}}$
   c. $0.85 = \mathbf{\frac{85}{100}} = \mathbf{\frac{17}{20}}$
   d. $0.101 = \mathbf{\frac{101}{1000}}$
   e. $0.246 = \mathbf{\frac{246}{1000}} = \mathbf{\frac{123}{500}}$

2. 0.349, $0.\overline{324}$, $0.\overline{87}$, 0.38926

3. a. 0.8

   b. 0.65

   c. 0.72

Lesson 2.2C

1. a. 45454

   b. 56565

   c. 00000

   d. 61561

2. a. $\frac{1}{9} = \mathbf{0.\overline{1}}$

   b. $\frac{32}{99} = \mathbf{0.\overline{32}}$

   c. $\frac{765}{999} = \mathbf{0.\overline{765}}$

3. a. $0.\overline{4} = \mathbf{\frac{4}{9}}$

   b. $0.\overline{8} = \mathbf{\frac{8}{9}}$

   c. $0.\overline{67} = \mathbf{\frac{67}{99}}$

Lesson 2.2D

1. a. $0.39 < \frac{1}{2}$

   b. $0.83 > \frac{4}{5}$

   c. $\frac{3}{10} > 0.03$

2. $0.5, \frac{3}{4}, 1, 1\frac{1}{5}$ 1.7

3. a. $\frac{1}{4}$, 0.6 The number between is: 0.2, $\frac{3}{4}$, or $\frac{2}{5}$.

   b. 0.3, $\frac{1}{2}$ The number between is: $\frac{1}{3}$, 0.7, or 0.54.

   c. $1\frac{2}{5}$, 1.5 The number in between is: $1\frac{1}{4}$, $1\frac{2}{3}$, or 1.45.

Answer to Lesson 2.2A Warm-up

1. a. tenths place: 9, ones place:3

   b. tenths place: 8, ones place:9

   c. tenths place: 2, ones place:0

2. a. 10

   b. 7

   c. 5

   d. 44

   e. 82

3. a. $3 \div 10 = 0.3$

   b. $7 \div 10 = 0.7$

   c. $9 \div 10 = 0.9$

   d. $1 \div 10 = 0.1$

   e. $2 \div 10 = 0.2$

Answer to Lesson 2.2A Practice 1

1. a. $\frac{7}{10}$, 0.7

   b. $\frac{8}{10} = \frac{4}{5}$, 0.8

   c. $\frac{4}{10} = \frac{2}{5}$, 0.4

2. a. 0.23

   b. 0.97

   c. 0.62

3. a. $\frac{564}{1000} = \frac{282}{500} = \frac{141}{250}$

   b. $\frac{597}{1000}$

   c. $\frac{280}{1000} = \frac{28}{100} = \frac{7}{25}$

Answer to Lesson 2.2A Practice 2

1. a. 0.04

   b. 0.16

   c. 0.2

   d. 0.6

Answer to Lesson 2.2A Practice 3

1. 2.7
2. 9.2
3. 8.24
4. 3.175
5. 6.4375

Answer to Lesson 2.2B Warm-up

1. b. 0.5
   c. 0.2
   d. 0.05
   e. 0.8
2. b. 0.125
   c. 0.05
   d. 0.25
   e. 0.0625

Answer to Lesson 2.2B Practice 1

1. terminating
2. repeating
3. terminating
4. repeating

Answer to Lesson 2.2B Practice 2

	Fraction	Decimal
1. 4 tenths, 9 hundredths	$\frac{49}{100}$	0.49
2. 7 tenths, 3 hundredths, 5 thousandths	$\frac{735}{1000} = \frac{147}{200}$	0.735
3. 6 tenths, 4 hundredths, 2 thousandths	$\frac{642}{1000} = \frac{321}{500}$	0.642
4. 3 tenths, 2 hundredths, 1 thousandth	$\frac{321}{1000}$	0.321

Answer to Lesson 2.2B Practice 3

1. a. $\frac{4}{10}$, 0.4

   b. $\frac{64}{100}$, 0.64

   c. $\frac{35}{100}$, 0.35

   d. $\frac{246}{1000}$, 0.246

2. a. answers will vary

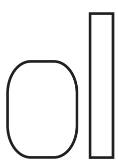
   b. answers will vary

   c. If a fraction did not equal a terminating decimal, it is because it could not be written as a fraction with a denominator with a power of 10 (10, 100,1000, ...).

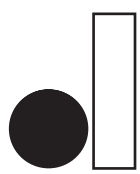
Answer to Lesson 2.2C Warm-up

1.

   a.



   b.



2. a. 8, 10, 12

   b. 9, 11, 13

   c. 0.0000900009...

3. b. 0.88

   c. 0.38

   d. 4.19

   e. 8.93

Answer to Lesson 2.2C Practice 1

1. a. 0.080808...

   b. 0.353535...

   c. 0.151515...

   d. 0.848484...

2. The two repeating digits are the numerator.

   99 is the denominator.

Answer to Lesson 2.2C Practice 2

1. a. $\frac{5}{9}$

   b. $\frac{34}{99}$

   c. $\frac{789}{999} = \frac{263}{333}$

   d. $\frac{2468}{9999}$

   e. $\frac{15}{999} = \frac{5}{333}$

2. a. 0.142857142857...

   b. 0.285714285714...

   c. 0.428571428571...

3. The pattern of 142857 repeats in each case. If the numerator is a 1, the first digit after the decimal is the smallest digit in the repeating pattern (1). If the numerator is a 2, the first digit after the decimal is the 2nd smallest (2). If the numerator is a 3, the first digit after the decimal is the 3rd smallest (4).

4. Since we have $\frac{4}{7}$ we need to find the 4th smallest digit in 142857 which is 5. So $\frac{4}{7}$ = 0.571428571428....

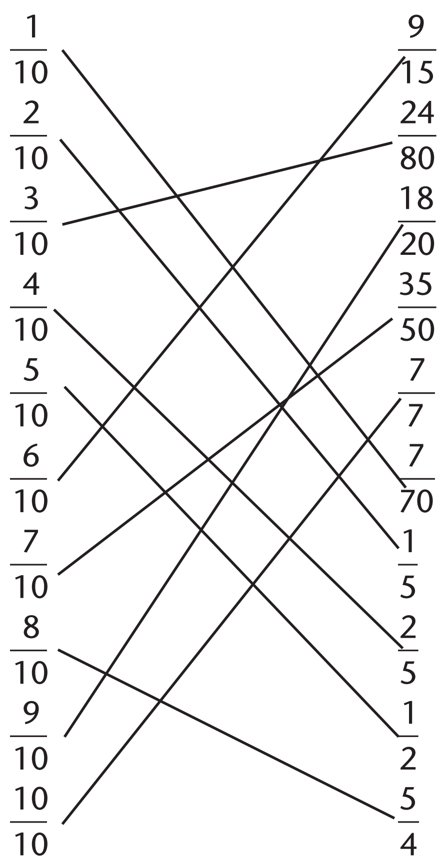
Answer to Lesson 2.2C Practice 3

Fraction	Decimal Not Rounded	Rounded
$\frac{10}{33}$	0.30303030...	$0.30
a. $\frac{16}{25}$	0.64	$0.64
b. $\frac{80}{120}$	0.6666..	$0.67
c. $\frac{17}{200}$	0.085	$0.09
d. $\frac{125}{300}$	0.416666...	0.$42

Answer to Lesson 2.2D Warm-up

1. $\frac{1}{2} = \frac{(1 \times 4)}{(2 \times 4)} = \frac{4}{8}$

2.



3. a. $4 < 9$
   b. $9 > 8$
   c. $21 < 86$
   d. $12 < 65$
   e. $12 < 43$
   f. $0.5 < 0.8$
   g. $0.12 < 0.36$
   h. $0.123 < 0.2$
   i. $0.49 > 0.4$
   j. $0.55 < 0.551$

Answer to Lesson 2.2D Practice 1

2. $>$
3. $<$
4. $<$
5. $<$
6. $>$

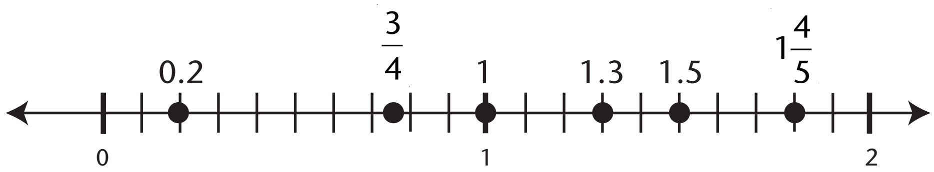
Answer to Lesson 2.2D Practice 2

1. b. $\frac{3}{5}$
   c. $\frac{19}{20}$
   d. 1.27
   e. $\frac{162}{50}$

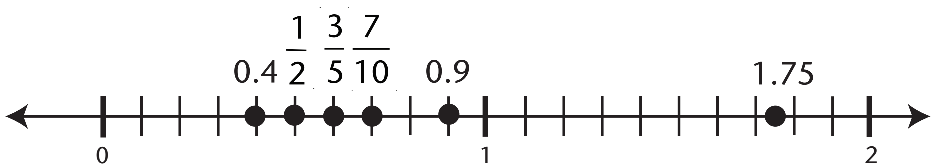
2. b. 0.2
   c. $\frac{7}{20}$
   d. $\frac{37}{50}$
   e. 0.93

Answer to Lesson 2.2D Practice 3

1.



2.



Answer to Lesson 2.2D Practice 4

1. $\frac{1}{2}$, 0.7, $\frac{7}{4}$, $\frac{11}{4}$

2. $\frac{3}{10}$, 0.7, 2.13, $\frac{9}{4}$

3. $\frac{1}{4}$, 0.82, $\frac{5}{4}$, 2.14

Answer to Lesson 2.2D Practice 5

Your answer might be different. Some examples of possible answers are given here.

1. $0.55, 0.6, 0.7, 0.8, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}$

2. $0.3, 0.4, \frac{1}{3}, \frac{2}{5}$

3. $0.5, 0.55, 0.6, \frac{1}{2}, \frac{3}{5}$

4. $0.8, 0.9, \frac{7}{8}$

5. $0.35, 0.4, 0.35, \frac{1}{3}, \frac{2}{5}$

Answer to Section 2.2 Challenge

The surfers from tallest to shortest are:

Edna: $2\frac{1}{8}$ metres = 2.125 metres	Edna gets the red surfboard
Damian: 2 metres	Damian gets the orange surfboard
Anthony: 1.7 metres	Anthony gets the green surfboard
Carlos: $1\frac{2}{3}$ metres = $1.6\dot{6}$ metres	Carlos gets the blue surfboard
Bridget: 1.63 metres	Bridget gets the purple surfboard

Answer to Pretest 2.3

Lesson 2.3A

1. a. 70%

   b. 60%

   c. 75%

   d. 80%

2. a. 60%

   b. 54%

   c. 3%

   d. 25%

3. a. $20\% = \mathbf{\frac{1}{5}}$

   b. $9\% = \mathbf{\frac{9}{100}}$

   c. $15\% = \mathbf{\frac{3}{20}}$

4. a. 19% = **0.19**

   b. 36% = **0.36**

   c. 6% = **0.06**

5. a. 0.85 to a percent = **85%**

   b. 0.7 to a percent = **70%**

   c. 46% to a fraction = $\mathbf{\frac{23}{50}}$

   d. 5% to a fraction = $\mathbf{\frac{1}{20}}$

   e. 80% to a decimal = **0.8**

   f. 5% to a decimal = **0.05**

Lesson 2.3B

1. a. 10% of 100 = **10**

   b. 20% of 200 = **40**

   c. 10% of 250 = **25**

   d. 20% of 400 = **80**

2. a. 10% of \$200 = **\$20**

   b. 50% of \$100 = **\$50**

   50% of \$100 is more money

3. a. 40% of 30 is **12**

   b. **70**% of 50 is 35

   c. **60**% of 20 is 12

   d. 35% of **40** is 14

   e. 82% of **90** is 73.8

4. $\mathbf{\frac{24}{32} = 75\%}$

5. $\mathbf{0.6 \times 30 = 18}$

6. $\mathbf{\frac{34}{40} = 85\%}$

7. a. $\mathbf{0.4 \times 85 = \$34}$

   b. **85 – 34 = \$51**

8. a. $\mathbf{0.06 \times 89 = \$5.34}$

   b. **\$89 + \$5.34 = \$94.34**

Answer to Lesson 2.3A Warm-up

1. a. 24
   b. 40
   c. 77
   d. 57

2. a. $30\% = \frac{30}{100} = \frac{3}{10}$
   b. $85\% = \frac{85}{100} = \frac{17}{20}$
   c. $46\% = \frac{46}{100} = \frac{23}{50}$
   d. $55\% = \frac{55}{100} = \frac{11}{20}$
   e. $28\% = \frac{28}{100} = \frac{7}{25}$

3. a. 0.15
   b. 0.65
   c. 0.87
   d. 0.59
   e. 0.49

Answer to Lesson 2.3A Practice 1

Answers will vary. Possible answers are:

Test scores	Naomi took a test and had 9 out of 10 correct. Her final mark is **90%**.
Money	Jane has 3 loonies and a quarter. Jane has a total of **$3.25**.
Metric measurements	Rajah took out his centimetre ruler and measured the length of an ant. The ant measures **0.5 cm** long?
Baking measurements	A recipe calls for two and $^1/_2$ **cups** of raisins.
Interest payments	Mike has put some money in a savings account. He receives **2%** interest each month.

Comparing sizes	A juice box says that it has **25%** more real fruit than other juices.
Very small numbers	The lead in the pencil measures **0.4 mm**.
Discounts	A shirt is on sale for **20%** off.

	Decimal	Percent	Fraction
Test scores		Best	
Money	Best		
Metric measurements	Best		
Baking measurements			Best
Interest payments		Best	
Comparing sizes		Best	
Very small numbers	Best		
Discounts		Best	

Answer to Lesson 2.3A Practice 2

1. $\frac{55}{100} = \frac{11}{20}$, 55%, 0.55
2. $\frac{62}{100} = \frac{31}{50}$, 62%, 0.62
3. $\frac{85}{100} = \frac{17}{20}$, 85%, 0.85
4. $\frac{8}{100} = \frac{2}{25}$, 8%, 0.08

Answer to Lesson 2.3A Practice 3

1. $\frac{8}{10} = 0.8$. Josephine's batting average is 0.8.
2. $0.7 = \frac{7}{10} = \frac{14}{20}$. Miranda would probably hit the ball 14 times.

Answer to Lesson 2.3A Practice 4

1. $75\% = \frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$

2. $\frac{3}{5} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60\%$

Answer to Lesson 2.3A Practice 5

1. 25%

2. 0.67

3.

Fraction (in lowest terms)	Decimal	Percent
$\frac{1}{4}$	0.25	25%
a. $\frac{19}{20}$	**0.95**	**95%**
b. $\frac{43}{50}$	**0.86**	**86%**
c. $\frac{32}{100} = \frac{8}{25}$	0.32	**32%**
d. $\frac{74}{100} = \frac{37}{50}$	0.74	**74%**
e. $\frac{92}{100} = \frac{23}{25}$	**0.92**	92%
f. $\frac{8}{100} = \frac{2}{25}$	**0.08**	8%

Answer to Lesson 2.3B Warm-up

1. $\frac{12}{50} = \frac{24}{100}$ or $\frac{12}{50} = 12 \div 50 = 0.24$ 24% of the songs are rock songs.

2. $\frac{3}{20} = \frac{15}{100}$ or $\frac{3}{20} = 0.15$ Barbara has not seen 15% of the DVDs.

3. $\frac{19}{25} = \frac{76}{100}$ or $\frac{19}{25} = 19 \div 25 = 0.76$ Caroline's team won 76% of their games.

4. a. 60
   b. 40
   c. 70
   d. 1.35
   e. 3.465

Answer to Lesson 2.3B Practice

1.

1. What is the question asking for?	What is 30% of $37.49?
2. Estimate an answer.	Estimate 30% of $40.00. 10% of $40.00 is $4.00. So, 30% of $40.00 = $12.00.
3. Find an answer.	P = 0.30 A = **O** × P **O** = $37.49 A = $37.49 × 0.30 A = ? A = $11.25 The discount is $11.25.
4. Make sure the answer is reasonable. If not then check over your work.	$11.25 is close to $12.00. The answer is reasonable.

2.

1. What is the question asking for?	Write $\frac{59}{70}$ as a percent.
2. Estimate an answer.	59 is close to 60. 70 is close to 75. $\frac{60}{75} = \frac{20}{25} = \frac{80}{100} = 80\%$
3. Find an answer.	$\frac{59}{70} = 0.8428.... = 84\%$
4. Make sure the answer is reasonable. If not then check over your work.	The answer is close to estimate.

3.

1. What is the question asking for?	\$11.27 is 30% of what?
2. Estimate an answer.	P = 0.30 **O** = ? A = \$12.00 $\mathbf{O} = \frac{A}{P}$ $= \frac{12}{.3} = \frac{120}{3} = 40$
3. Find an answer.	P = 0.30 **O** = ? A = \$11.27 $\mathbf{O} = \frac{A}{P}$ $= \frac{11.27}{3}$ $= \$37.57$ The original price is \$37.57.
4. Make sure the answer is reasonable. If not then check over your work.	The answer is close to estimate.

4. Crunchy Cereal 25%

Original Oats $0.3 = 30\%$

Green Flake $\frac{1}{5} = \frac{20}{100} = 20\%$

Original Oats has the most iron.

5. 20% of 120 cm

   $0.20 \times 120 = 24$ cm

   Bert grew 24 cm. His current height is 144 cm

Answer to Section 2.3 Challenge

Integer quiz:

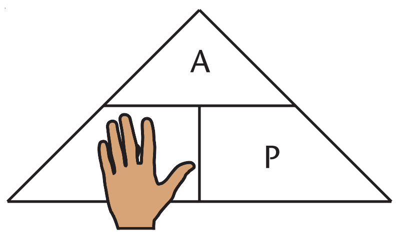
$$\frac{7}{10} = 0.7 = 0.70 = 70\%$$

Moira got 70% on her Integer quiz.

Fraction quiz:

80% of what number is 24?

We know: $80\% = \frac{80}{100} = 0.80$



A = 24

O = ?

P = 0.80

$$\mathbf{O} = \frac{\mathbf{A}}{\mathbf{P}} = \frac{24}{0.80} = 30$$

Moira's Fraction quiz was out of 30 marks.

Module 2 Glossary

Ascending

From least to greatest

Common Denominator

Common multiple of denominators

Common Factor

A number that divides two or more numbers evenly

Common Multiples

A multiple of two or more numbers

Denominator

The bottom of a fraction. The denominator is the total number of parts in a whole

Descending

From greatest to least

Equivalent

Another word for equal

Equivalent Fractions

Fractions that have different denominators but equal in value

Factor

A number that divides another number evenly

Fraction Strip

A strip of paper that represents the length of a fraction.

Fraction

A part of a whole. A fraction is written with a numerator and denominator.

Greater Than

A number larger than another. Symbol: >

Improper Fraction

A number in which the numerator is greater than the denominator

Inequality

Unequal in value

Less Than

A number smaller than another. Symbol: <

Mixed Number

A number made up of a whole number and a fraction

Multiple

A number multiplied by a natural number (1, 2, 3, ...)

Numerator

The top of a fraction. The numerator is the number of parts of a whole.

Part

A section of a whole

Percent

A number out of 100

Reduce

Rewrite in lowest terms

Repeating Decimal

A decimal number in which a digit or group of digits repeat forever

Simplify

Write in simplest form

Terminal Decimal

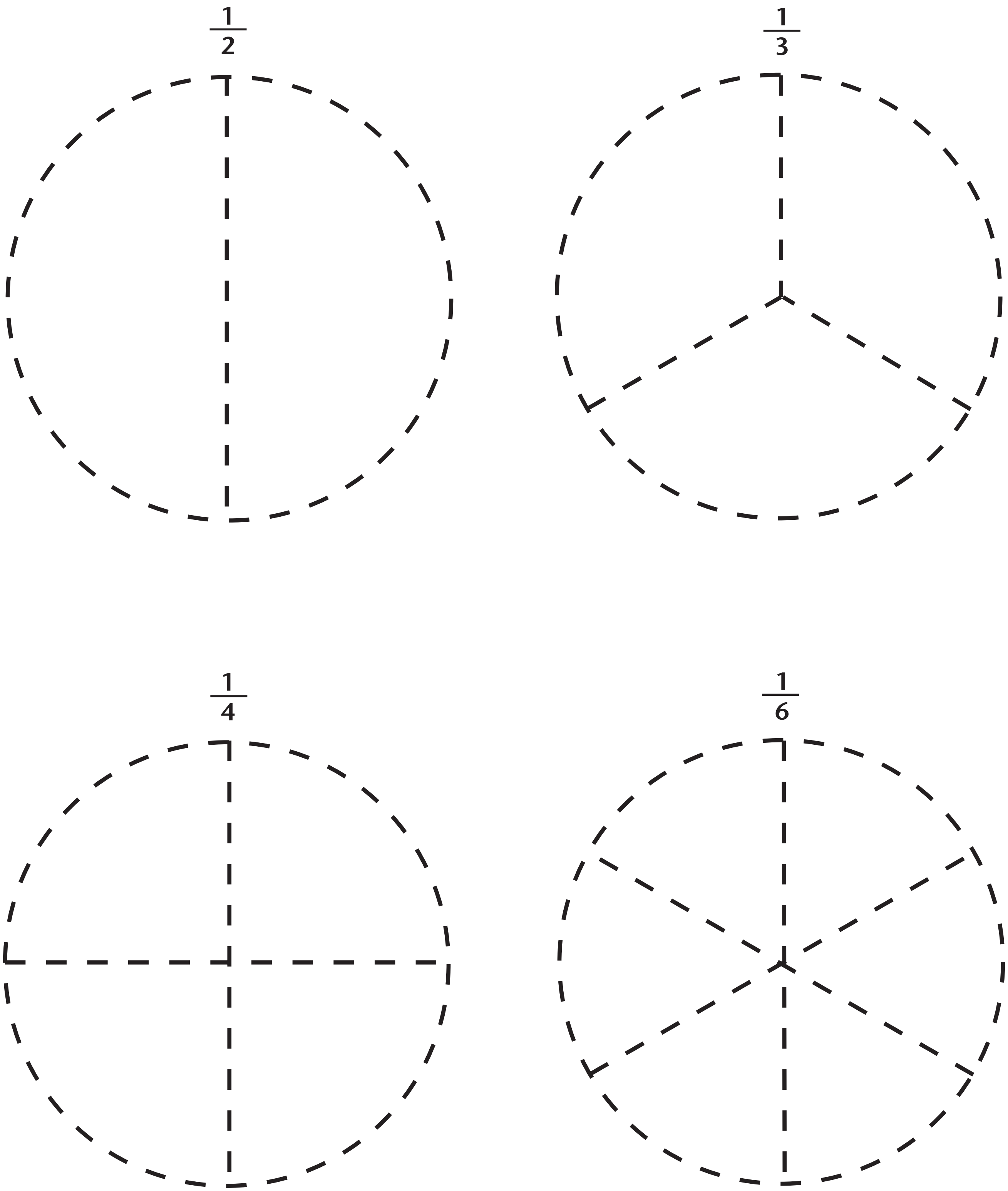
A decimal number in which the digits stop

Whole

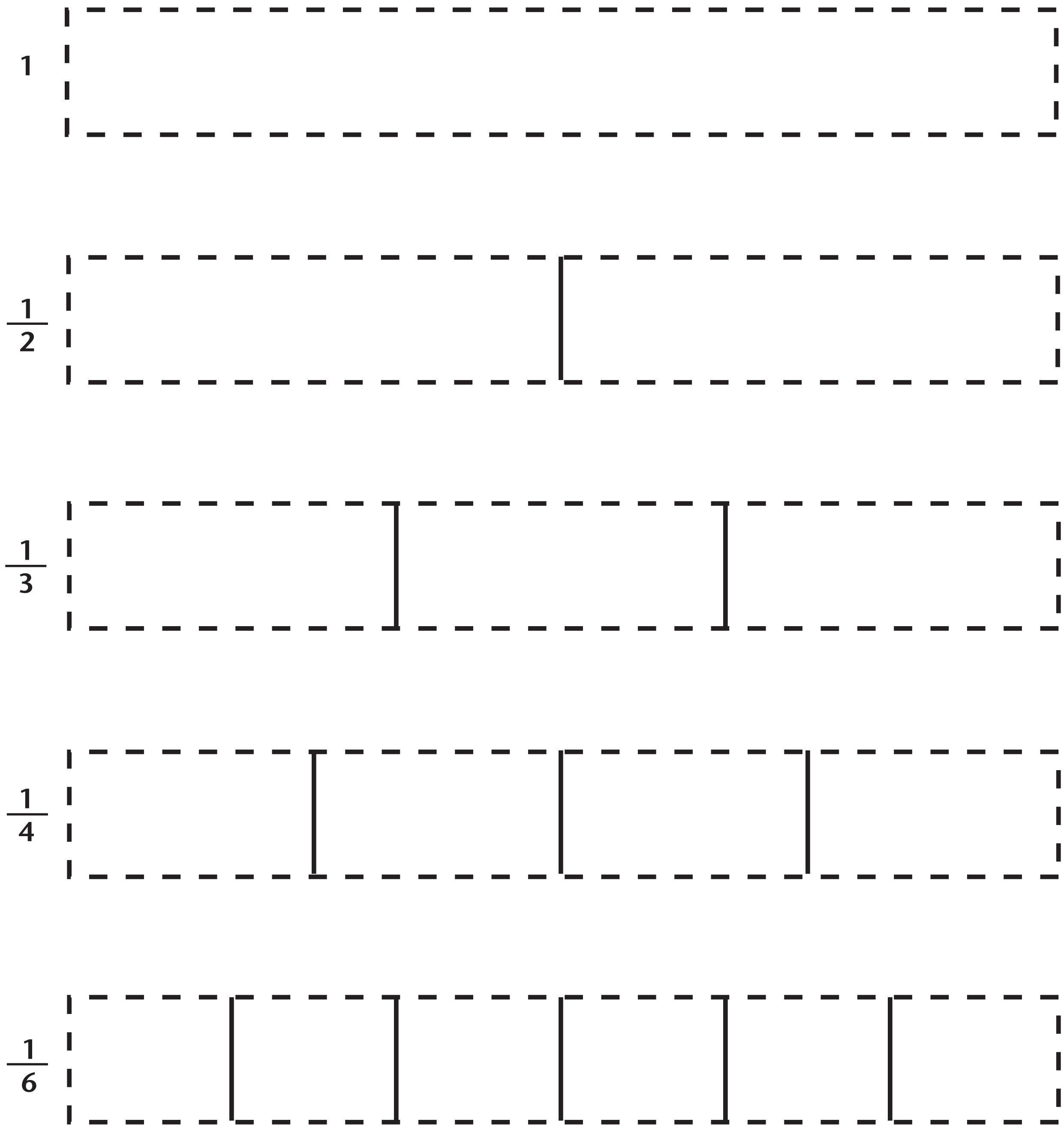
All of the parts put together

Module 2 Templates

Template for Lesson 2.1A Practice 1: Circles



Template for Lesson 2.1D: Fraction Strips



Template for Lesson 2.1E Warm-Up: Square