LITERACY FOUNDATIONS MATH

x	y
–2	–4
0	–2
3	1
6	

LEVEL 6

PATTERNS AND RELATIONS

To the Student

This resource covers topics from the British Columbia Ministry of Education's Literacy Foundations Math Level 6. You may find this resource useful if you're a Literacy Foundations Math student, or a K-12 student in grades 7 – 9.

We have provided learning material, exercises, and answers for the exercises, which are located at the back of each set of related lessons. We hope you find it helpful.

Literacy Foundations Math Prescribed Learning Outcomes

Literacy Foundations Mathematics Level 6 follows two pathways: Math Foundations (MF) and Apprenticeship and Workplace (AW). The Prescribed Learning Outcomes (PLOs) are grouped into four areas: Number (A), Patterns and Relations (B), Shape and Space (C), and Statistics and Probability (D). For a complete list of the PLOs in Level 6, go to the BC Ministry of Education's website and search for Literacy Foundations Math curriculum.

PLOs Represented in This Resource

The PLOs represented in this Level 5 resource are as follows:

Number

MF: All topics, A1 – A6, with the exception of a portion of A3, roots of fractions, and a portion of A4, word problems with powers

AW: All topics, A1 – A9, with the exception of a portion of A3, roots of fractions, and A6, word problems with powers

Patterns and Relations

MF: B3 and B4, with the exception of a portion of B4, rational coefficients with variables in more than one term

AW: B3 and B4

Shape and Space

MF: C1 and C3, with the exception of a portion of C3, composite objects
AW: C1 – C3, C5, C6, with the exception of a portion of C5, composite objects

PLOs Not Represented in This Resource

The PLOs for which no material is included in this resource are as follows:

Number

MF: portions of A3, roots of fractions and a portion of A4, word problems with powers
AW: portions of A3, roots of fractions and a portion of A6, word problems with powers

Patterns and Relations

MF: B1, linear relationships; B2, patterns to linear equations; portions of B4, rational coefficients and variables in more than one term; B5, single-variable linear inequalities; B6 – B8, polynomials

AW: B1, linear relationships; B2, patterns to linear equations; and B5 – B8, polynomials

Shape and Space

MF: C2, polygons and polyhedra; and portions of C3, composite objects and line and rotation symmetry

AW: C4, polygons and polyhedra; and portions of C5, composite objects; and C7, line and rotation symmetry

Acknowledgements and Copyright

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Lesson 1
Graphing

Learning Outcomes

By the end of this section you will be better able to:

- recognize linear equations
- identify terms, variables, coefficients and constants in linear equations
- evaluate expressions to complete a table of values
- determine whether a given point satisfies an equation
- describe a linear relationship shown in a given graph or table of values

Plotting Points on the Plane

René Descartes was home sick in bed in the early 1600s. He watched a fly crawl around on the ceiling. René noticed that he could describe the fly's position no matter where it was by giving its distance from the corner of the room in two directions.

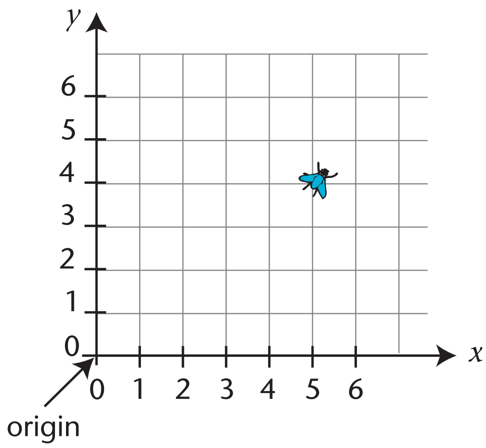
There are many situations where we need to clearly describe the location of an object. Video game designers, architects, and your GPS system all use René Descartes' bug-finding idea to precisely describe information about location.

A flat surface is called a plane. We call René's bug-finder the **Cartesian plane**.

The corner of the room is the **origin**. That just means the place where we start. All of our descriptions of distances will be measured from this spot.

The horizontal direction is called ***x***. The horizontal number line is called the ***x*-axis**.

The vertical direction is called ***y***. The vertical number line is called the ***y*-axis**.



The fly is called a **point**.

To describe the location of the fly, we ALWAYS give the distance in the ***x*** direction first. This fly is located 5 units to the right of the origin and 4 units above the origin. The fly is at (5,4).

The first number describes the distance in the *x* direction. This number is called the ***x*-coordinate**. The ***x***-coordinate of the location of the fly is 5.

The second number describes the distance in the *y* direction. This number is called the ***y*-coordinate**. The *y*-coordinate of the location of the fly is 4.

When we write the two coordinates together, they are ALWAYS in round brackets. The two numbers are separated by a comma. The **coordinates** of the location of the fly are (5,4).

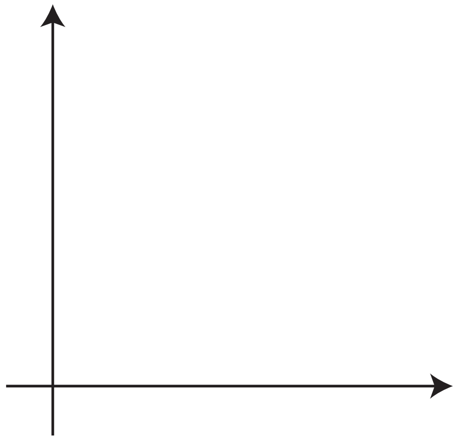
Sometimes we call coordinates a coordinate pair or an **ordered pair**. The Cartesian plane is just one example of a **coordinate system**.

Exercises 1.1

1. Label the origin.

   Label the ***x***-axis.

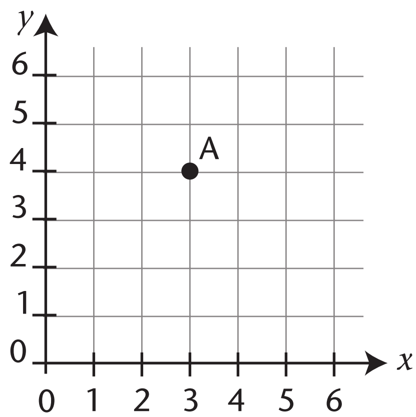
   Label the ***y***-axis.



2 a. What is the x-coordinate of point A?

  b. What is the y-coordinate of point A?

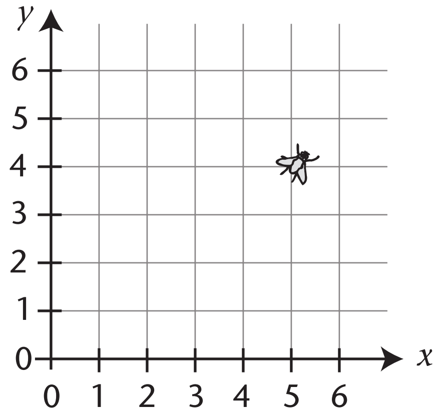
  c. What are the coordinates of point A?



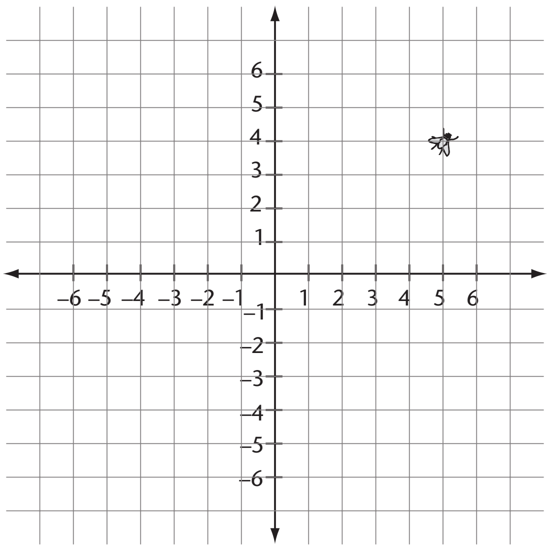
3. The Cartesian plane is an example of a ______________________________ system.

Turn to the Answer Key at the end of the module to check your work.

Until now, we have been using only half of a number line for each axis: *x* and y.



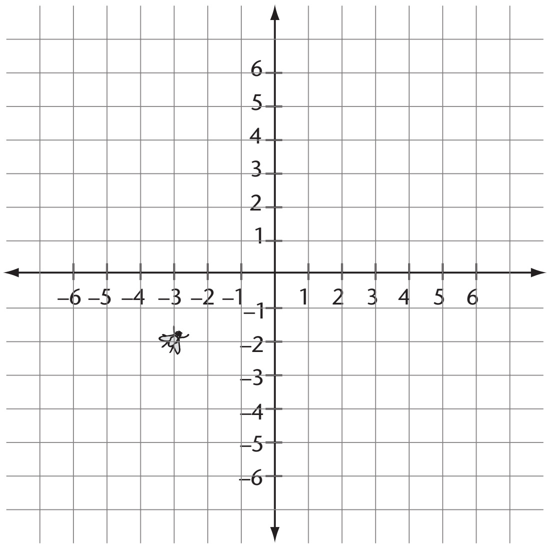
It's time to stretch out. Let's use a complete number line for each axis.



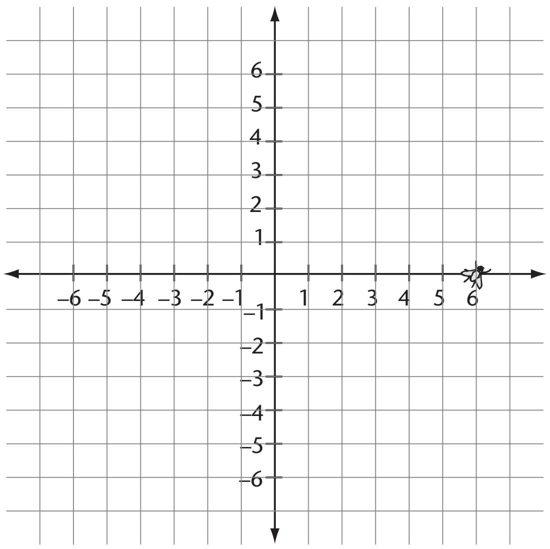
Now our fly isn't stuck walking around on René's bedroom ceiling anymore. It can go as far as it likes in any direction.

The positive numbers on the *x*-axis describe distances to the right of the origin, just like you have already seen. If the *x*-coordinate is 2, we know that the point is 2 units to the right of the origin. When we want to describe distances to the left of the origin, we use the negative numbers on the *x*-axis. If the *x*-coordinate is –2, we know that the point is 2 units to the left of the origin.

The *y*-axis works in a similar way. The *positive* numbers on the y-axis describe distances *above* the origin. If the *y*-coordinate is 5, we know that the point is 5 units above the origin. When we want to describe distances *below* the origin, we use the *negative* numbers on the *y*-axis. If the *y*-coordinate is –5 we know that the point is 5 units below the origin.



The fly moved. Can you describe its new location? It is 3 units to the LEFT of the origin. The *x*-coordinate is now –3. It is 2 units BELOW the origin. The *y*-coordinate is –2. The fly is at coordinates (–3,–2).

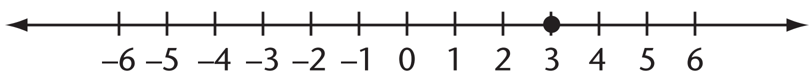


When the fly has landed on an axis, we still have to describe its position. This fly is 6 units to the right of the origin. The *x*-coordinate is 6. However, the fly is neither above nor below the origin. The *y*-coordinate is 0. The fly is at coordinates (6, 0).

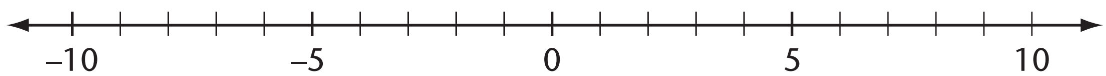
Exercises 1.2

René’s bug-finder is like two number lines stuck together.

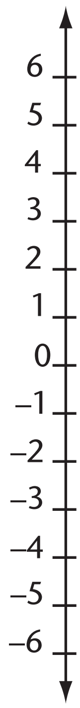
1. There’s already a point on 3. Put a point at each of the following locations.
   a. –2
   b. 5
   c. 0



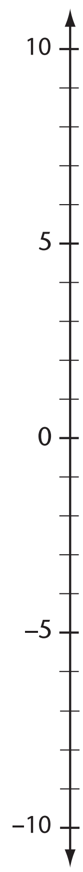
2. Put a point at:
   a. –3
   b. 7
   c. –6



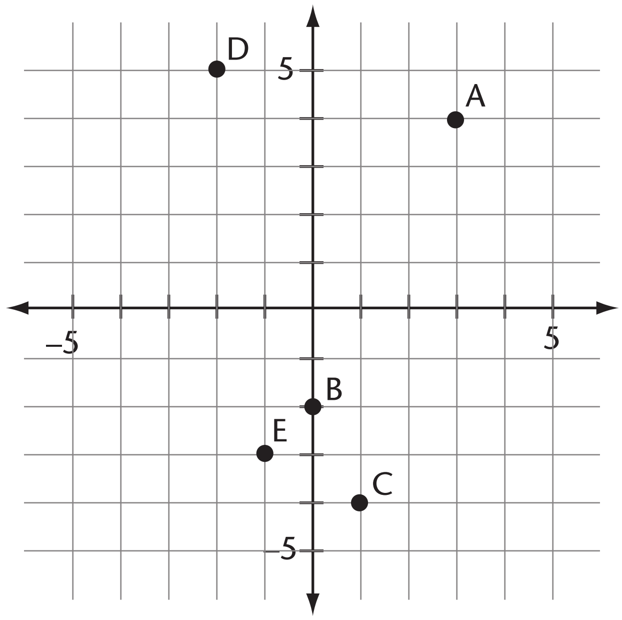
3. Put a point at:
   a. –3
   b. 4
   c. 1



4. Put a point at:
   a. –7
   b. 9
   c. –4



5. Give the coordinates of each point.



A (,)

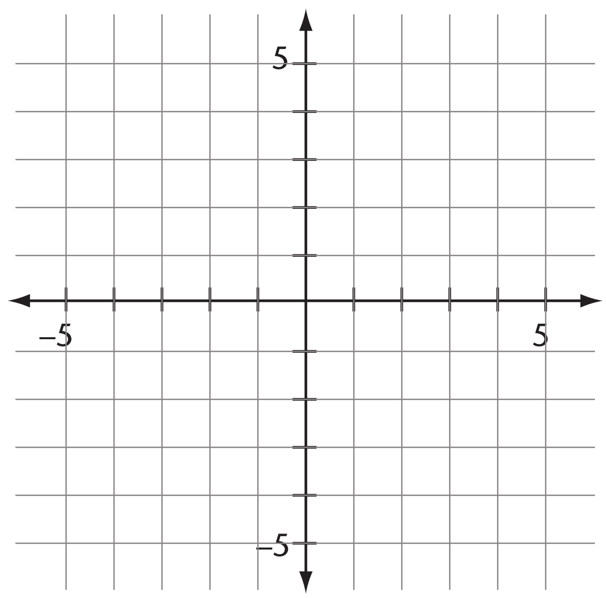
B (,)

C (,)

D (,)

E (,)

6. Plot each point on the Cartesian plane.



F (–3, 1)

G (–1, 4)

H (–2 , –4)

I (0, –2)

J (2, 1)

Turn to the Answer Key at the end of the module to check your work.

Equations, Graphs, and Points

An equation can be used to describe a group of points on a graph. A *linear* equation describes a group of points that line up. (Note the "line" in the word *linear*). These are examples of linear equations:

$y = 3x$ $\qquad$ $y = -4x + 12$ $\qquad$ $y = 5x + 2$

Linear equations have the form

$y =$ _____ $x +$ _____

where a number multiplies the x and some other number is added (or subtracted) at the end. You can see in the first example ($y = 3x$) that sometimes the last number is zero, and then we don't write it.

Linear equations can also look like the following, but we won't be dealing with them in this math course:

$x + y = 7$ $\qquad$ $x = -2y + 5$ $\qquad$ $3x - 5y = 21$

The following equations are NOT linear equations—they form curved patterns when graphed:

$y = \dfrac{1}{x}$ $\qquad$ $y = 3x^2 + 8$ $\qquad$ $x^2 + y^2 = 16$

How can a simple equation represent a bunch of points? The x and y variables in the equation stand for pairs of numbers called ordered pairs. When you plot the pairs of numbers that make the linear equation into a true statement, they will line up together as points on the graph.

Ordered Pairs

Let's find some ordered pairs for a linear equation.

$y = 2x - 1$

We can find ordered pairs by choosing a number for x, substituting that number into the equation, and then calculating the matching y-value.

For example, let's start with $x = 1$ and substitute it into the equation:

$y = 2x - 1$
$y = 2(1) - 1$

Now solve this equation to find the value of y.

$y = 2(1) - 1$
$y = 2 - 1$
$y = 1$

So an ordered pair for this equation is $x = 1$ and $y = 1$ or (1, 1).

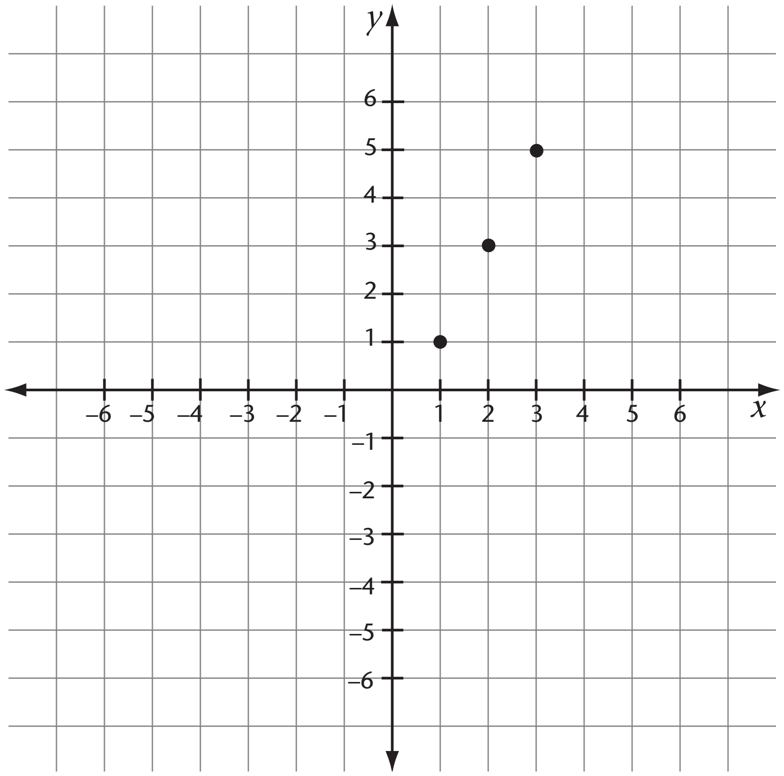
Let's do it again with an x value of 2. Substitute 2 for x in the equation and do the math:

$y = 2x - 1$
$y = 2(2) - 1$
$y = 4 - 1$
$y = 3$

So when $x = 2$, $y = 3$ and our next ordered pair is (2, 3).

We'll do a third one: use $x = 3$.
$y = 2x - 1$
$y = 2(3) - 1$
$y = 6 - 1$
$y = 5$
So when $x = 3$, $y = 5$ and our third ordered pair is (3, 5).

If we plot all 3 points on a grid, we should see them line up nicely:



Exercises 1.3

1. For each equation calculate the matching y-values for the given x-values. Then list the ordered pairs. The first question is partly done.

   a. $y = x - 5$

   $x = 0$

   $x = -3$

   $y = x - 5$

   $y = (-3) - 5$

   $y = -8$

   $x = 2$

   $x = 7$

   Ordered pairs: (–3, –8), (0,____), (2,____), (7,____)

   b. $y = -3x + 2$

   $x = -5$

   $x = 0$

   $x = 1$

   $x = 3$

   Ordered pairs:

Turn to the Answer Key at the end of the module to check your work.

Vocabulary

Important words to know when dealing with linear relations are **coefficient**, **constant**, **term**, and **variable.**

variables

$$\underline{y} = \underline{3x} + \underline{7} \longleftarrow \text{constant}$$

coefficient

The terms in this equation are underlined.

> ### coefficient
> A coefficient is a number that multiplies a variable in a mathematical expression.
>
> ### constant/constant term
> A constant is a number in a mathematical expression that has no variable attached to it. The number can't be changed.
>
> ### term
> A term is an item in an expression that is a constant, variable, or coefficient-and-variable combination. An expression is built by adding (or subtracting) terms.
>
> ### variable
> A variable is a value that is unknown or that could change. It is often represented in an expression by a letter such as x, but could be represented by a word or other symbol.

$t = \frac{r}{5} + 2$ This can be rewritten as $t = \frac{1}{5}r + 2$.

Its terms are t, $\frac{1}{5}r$, and 2. The variables are t and r, and the r has a coefficient of $\frac{1}{5}$. The constant term is 2.

$y = -3x$ This equation has no constant term (or we could say that the constant is 0). Its variables are x and y, and the terms are y and $-3x$. The co-efficient of x is -3.

Exercises 1.4

For the following equations, identify the variables, constant, coefficient and terms. Separate the variables and terms with commas. The first one has been done for you.

Equation	Variables	Constant	Coefficient	Terms
$y = -2x + 3$	y, x	3	–2	$y, -2x, 3$
$a = \frac{1}{2}p + 8$				
$c = 4q - 6$				
$m = \frac{n}{3}$				
$s = -4t - \frac{1}{2}$				

Turn to the Answer Key at the end of the module to check your work.

Satisfying Equations

Ordered pairs (x, y) can be substituted for the variables of a linear equation. For the equation $y = 8x - 9$, let's try substituting (3, 7) for the x- and y-values, and check to see if it makes the equation true.

We'll use the left side (LS) and right side (RS) method. The line down the middle represents the equal sign (=). We substitute the numbers into the equation, and put the parts to the left and right of the equal sign in their correct places.

Since the y-value is the only thing on the left side, we won't have to do any calculations there.

Example 1

Does the point (3, 7) satisfy $y = 8x - 9$?

$$y = 8x - 9$$
$$7 = 8(3) - 9$$

Left Side (LS)	Right Side (RS)
7	$8(3) - 9$
	$24 - 9$
	15

LS ≠ RS so the point (3, 7) does **not** satisfy the equation $y = 8x - 9$.

Example 2

Does the point (3, 15) satisfy $y = 8x - 9$?

$$y = 8x - 9$$
$$15 = 8(3) - 9$$

Left Side (LS)	Right Side (RS)
15	$8(3) - 9$
	$24 - 9$
	15

LS = RS so the point (3, 15) **satisfies** the equation $y = 8x - 9$.

Exercises 1.5

1. Use the Left Side (LS) and Right Side (RS) method to decide if the point satisfies the equation. The first one is set up for you.

   a. (2, 11)

   $$y = 2x + 8$$

   $$11 = 2(2) + 8$$

Left Side (LS)	Right Side (RS)

   Does the LS = RS? ___________

   Does the point (2, 11) satisfy the equation $y = 2x + 8$? ___________

   b. (15, 3)

   $$y = \frac{1}{5}x$$

   $\frac{1}{5}x$ is the same as $\frac{x}{5}$ or $x \div 5$.

Left Side (LS)	Right Side (RS)

   Does the LS = RS? ___________

   Does the point (15, 3) satisfy the equation $y = \frac{1}{5}x$? ___________

c. (–2, 8)

$y = -3x + 2$

Left Side (LS)	Right Side (RS)

Does the LS = RS? ___________

Does the point (2, 11) satisfy the equation $y = -3x + 2$? ___________

2. Which ordered pairs satisfy which equation? Underline or draw a circle or box around the ordered pairs that match each equation. One has been done for you.

$y = \frac{1}{2}x + 4$

$\underline{y = 3x - 2}$

$\boxed{y = -x - 1}$

(–2, 3) (4, 10) (4, 6) (2, 4) (0, –1) (–3, 2) (2, –3) (–4, 2) (–3, –11)

Turn to the Answer Key at the end of the module to check your work.

Table of Values

A list of several ordered pairs that satisfy an equation is called a table of values. You've seen tables of values already, so we'll just review how to fill them out.

We'll work with the equation $y = \frac{1}{2}x + 6$.

Choose a number to substitute for x, such as 2.

Because we have to take half of the number, we'll start out with even numbers so it's easier to calculate.

$y = \frac{1}{2}(2) + 6$

One half of 2 is 1.

Calculate using the order of operations.

$y = 1 + 6$

$y = 7$

Now you have an entry for the table of values.

(2, 7) →

x	y
2	7

This process should be repeated to make at least three entries for the table. Four or five entries are even better.

Here are two more points:

Choose $x = -4$

$$y = \frac{1}{2}x + 6$$
$$y = \frac{1}{2}(-4) + 6$$
$$y = -2 + 6$$
$$y = 4$$

$\frac{1}{2}(4) = 2$

$\frac{1}{2}(-4) = -2$

Choose $x = 5$

$$y = \frac{1}{2}x + 6$$
$$y = \frac{1}{2}(5) + 6$$
$$y = 2\frac{1}{2} + 6$$
$$y = 8\frac{1}{2}$$

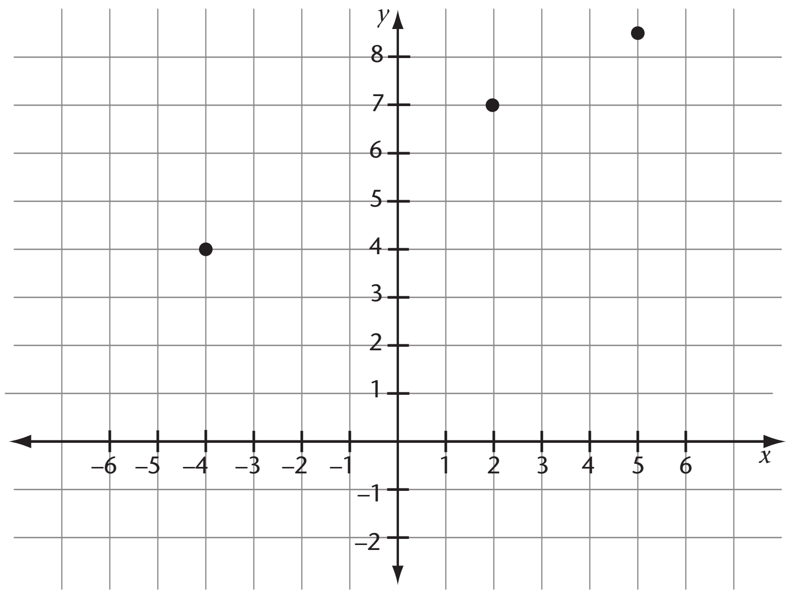
$\frac{1}{2}(5) = 2\frac{1}{2}$
try 0.5 x 5
on your calculator
if it helps

We can add these numbers to the table of values. Tables of values can be either vertical or horizontal.

x	-4	2	5
y	4	7	$8\frac{1}{2}$

The value that goes on the horizontal axis of the graph is listed first in the table of values.

Once you have the table of values, you can use the ordered pairs that you made to plot the points on a graph.



If you hold a ruler up to your points, you will see that they form a straight line. If one of your points isn't part of the line, go back and check that you have calculated and plotted the point correctly.

Costs For a Lemonade Stand

Sebastian is going to sell lemonade from a stand in his yard. To help him calculate how much the lemonade will cost to make, we'll create a table of values and draw a graph.

For a day of selling lemonade, Sebastian will have to buy one package of paper cups for $4. His mom will charge him $0.20 for the ingredients (lemon juice and sugar) for each glass of lemonade that he makes.

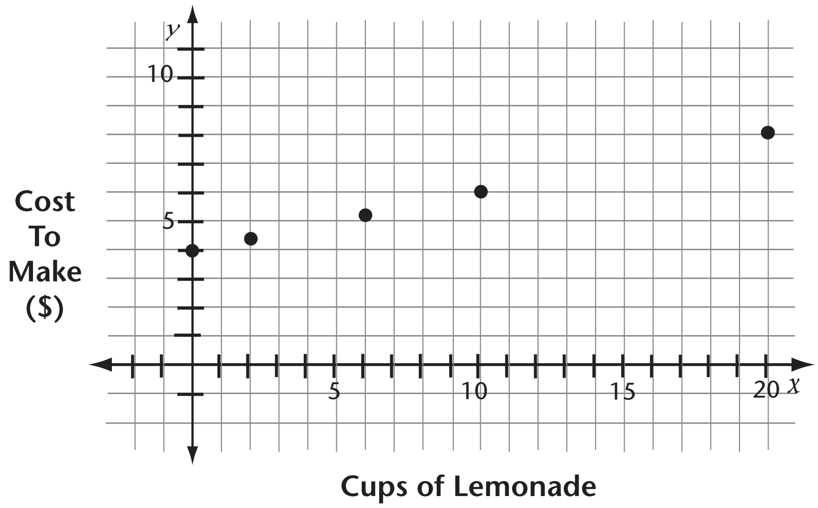
This means that his fixed costs are $4 (the constant) and his variable costs are $0.20 per cup. From this information we can make a table of values that shows the number of cups and the cost.

Here is the calculation for two cups of lemonade:

cost = price for cups + $0.20 for each cup of lemonade
cost = $4.00 + $0.20 (2)
cost = $4.00 + $0.40
cost = $4.40

Cups of lemonade	0	2	6	10	20
Cost	$4.00	$4.40	$5.20	$6.00	$8.00

Then we'll plot the graph to see Sebastian's costs.

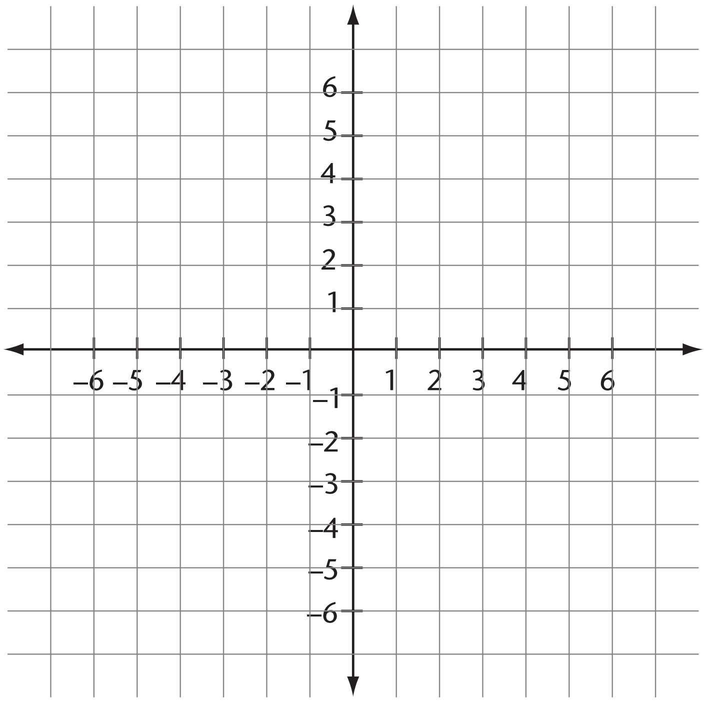


Exercises 1.6

1. For the following linear equations, fill in the table of values using the given *x-value*s. Then plot the graph.

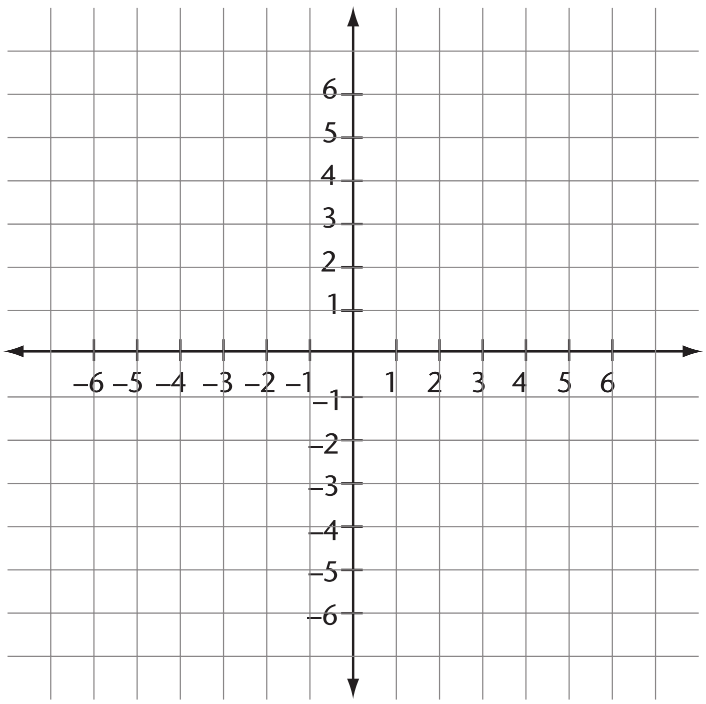
a. $y = x - 2$

x	y
–2	
0	
3	
6	



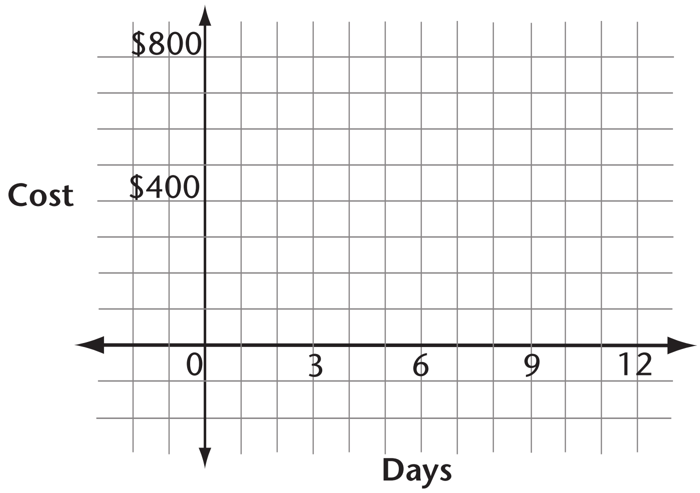
b. $y = -x$

x	y
–4	
–1	
2	
5	



2. Katie is getting a "value pass" for a local ski resort. She'll pay $250 plus $30 per day of skiing. Fill in the table to show her skiing costs based on how many days she skis. Show the cost for 6 days, 10 days, and two more numbers of your choice. Then plot the graph of the costs.

Days				
Cost				



Turn to the Answer Key at the end of the module to check your work.

Graphing on a Spreadsheet

If you have a computer with spreadsheet software, you can make excellent graphs very easily. For this example, we'll graph $y = \frac{1}{2}x + 3$

Here are the steps:

Open your spreadsheet program.

Enter the *x*-values into column A by doing the following:

Type –5 in A1 (the top left cell) and press the Enter or Return key.

Continue typing values, pressing the Enter or Return key after each one:

in A2, type –4

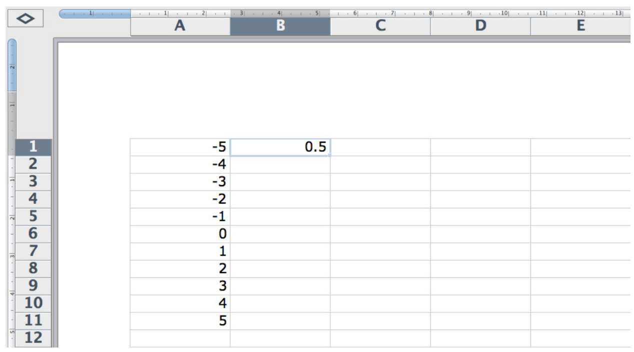
in A3, type –3… and so on, until you finish typing 5 into A11 (don't forget 0).

Enter the formula (linear equation) into column B. Click in cell B1 and type the following:

=(1/2)*A1+3

Press the Enter key.

You should see 0.5 appear in cell B1.



Copy this formula to cells B2 to B11 by doing the following:

Click on cell B1.

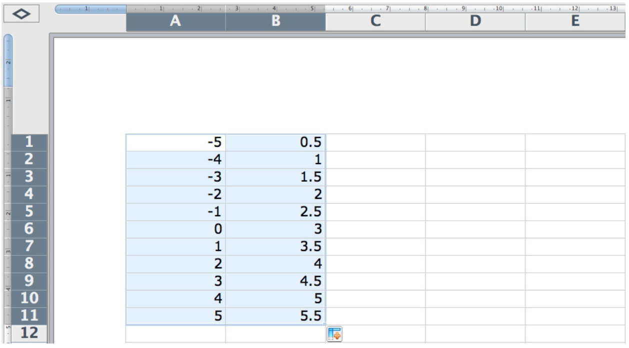
Use your mouse to go to the Edit menu and choose Copy.

Use your mouse to drag down through cells B2 to B11 to highlight them.

Use your mouse to go to the Edit menu and choose Paste.

(You may have to press the Esc key to deselect cell B1.)

Compare the resulting numbers with the following:



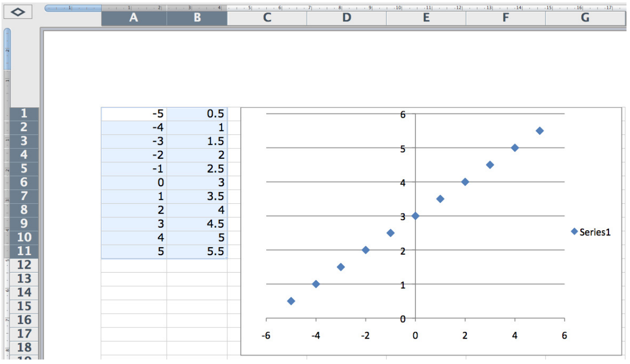
	A	B
1	-5	0.5
2	-4	1
3	-3	1.5
4	-2	2
5	-1	2.5
6	0	3
7	1	3.5
8	2	4
9	3	4.5
10	4	5
11	5	5.5

Use your mouse to drag through cells A1 to B11 (both columns) to highlight them.

Find the command for “Insert Chart” in your spreadsheet program. It may show as a Chart Wizard on the tool bar, or you may find it in the Insert menu.

Choose an XY Scatter plot with no lines and just click the Next button through the rest of the steps.

You should see something like this:



If you like, you can customize your graph by adding titles and labels for the axes.

Describing Relationships: Start With the Table of Values

Linear relationships might sometimes be given by a table of values.

x	y
2	1
4	2
6	3
8	4

If we look at the values, we can discover the relationship between the two variables.

Look at the first row in the table of values (2, 1). Two possibilities for the relationships are:

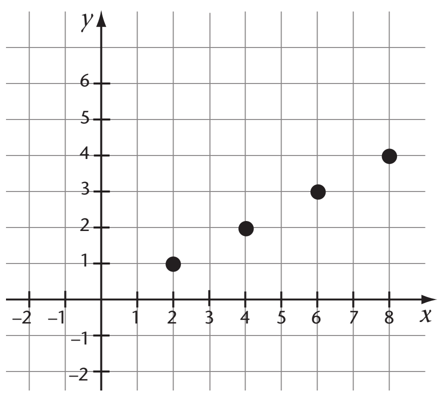
1. Take away 1 from x to get y $(2 - 1 = 1)$ OR
2. Divide x by 2 to get y $(2 \div 2 = 1)$.

We won't know which one it is until we test another point.

Look at the point (8, 4). We can confirm that you divide x by 2 to get y, not subtract $(8 - 1$ is not $4)$. So we can say that x is twice as big as y, or the other way around, that y is one–half the value of x.

Be sure to check it against the other points that you have listed in order to confirm your work.

Here's the graph:



x	y
2	1
4	2
6	3
8	4

We can describe the graph by saying that for every one that y increases, x increases by 2.

The following table shows Corey's costs when he went to the summer fun fair.

Number of Rides	0	1	2	5	8	10
Cost	\$8	\$10	\$12	\$18	\$24	\$28

Looking at the table of values, you can see the following:

When he went on one ride, it cost him \$2 more than going on no rides (\$10 – \$8 = \$2).

When he went on two rides, it cost him \$2 more than going on one ride (\$12 – \$10 = \$2).

It looks as though each ride cost him \$2.

You can also see from the table that it cost Corey \$8 even when he didn't go on any rides. It probably cost him \$8 just to get into the fairgrounds.

You can test this out by calculating what it would cost Corey to go on 10 rides, and then compare your answer to the one given in the table of values.

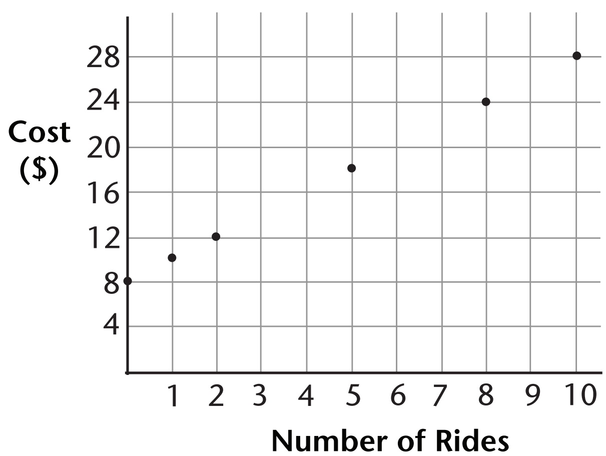
Our guess: admission to the fairgrounds is \$8, and each ride is \$2

For 10 rides: cost = admission + amount for 10 rides
cost = \$8 + 10 (\$2)
cost = \$8 + \$20
cost = \$28

This matches the value given in the table of values, so our description of the cost is probably accurate. To be sure, you could check it against more points.

You can also check it against the graph.

To describe this graph, we would say that it starts at \$8 on the *y*–axis. Then for every one it goes to the right, it goes up two.



Exercises 1.7

Look at the following tables of values. Describe the relationships between the variables.

1.

x	y
2	–2
8	4
10	6
12	8

Relationship: ______________________________

2.

Number of Snacks Consumed at Dance	0	1	3	5
Cost to Attend Dance	\$5	\$6	\$8	\$10

Relationship: ______________________________

3.

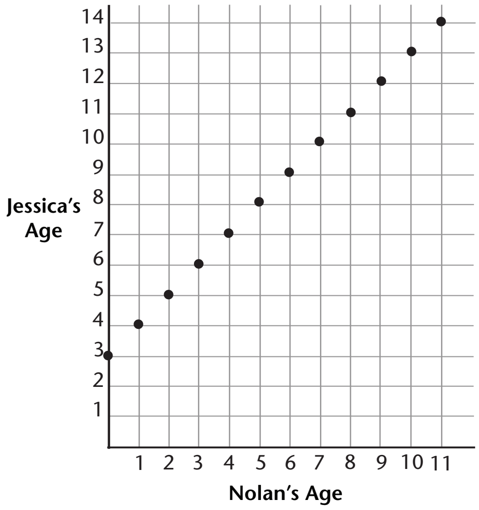
r	t
–3	9
–1	3
0	0
4	–12

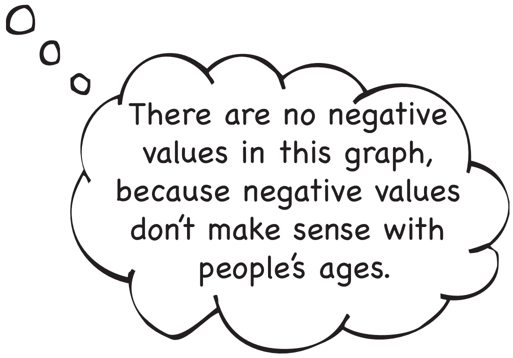
Relationship: ______________________________

Turn to the Answer Key at the end of the module to check your work.

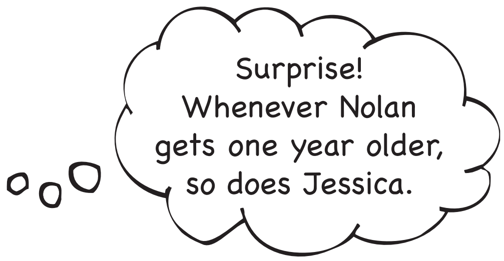
Describing Relationships: Start With the Graph

Here's a graph of the relationship between Jessica's age and the age of her little brother, Nolan.





This graph starts at three on the *y*–axis. Following the points from left to right, you can see that every time you move over one space, you also move up one space.



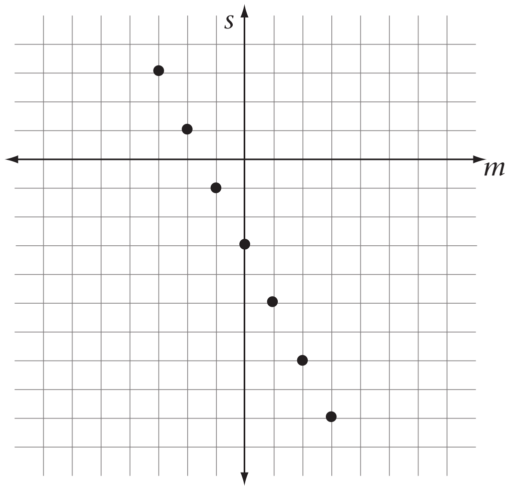
When you're given only the graph, it's a good idea to make a table of values containing at least three ordered pairs. These points can give you a better idea how these two numbers are related.

Nolan's Age	0	2	11
Jessica's Age	3	5	14

This means that when Nolan was 0 (just born), Jessica was three years old. Then when Nolan was two, Jessica was five. And now Nolan is 11, and Jessica is 14.

You may have already noticed that Jessica is three years older than her brother Nolan. To put it into more algebraic terms, Jessica's age is Nolan's age plus three years.

Here is another graph. We don't know what *m* and *s* represent, but it is clear that there is a linear relationship between them.



This graph touches the vertical axis at –3. Looking at the points from left to right, you can see that for every one space the graph goes to the right, it goes down two spaces.

Find some ordered pairs from the graph and list them in a table of values:

m	0	1	2	3
s	–3	–5	–7	–9

You can see from the table of values that for every one that *m* increases, *s* decreases by two.

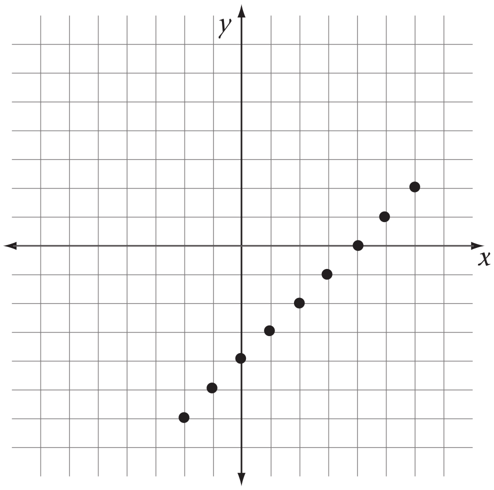
Exercises 1.8

Examine the graphs and describe each one by explaining:

- at which point it touches the y–axis
- how the position of the points change as you move from left to right

Then list four ordered pairs in the table of values, and describe the relationship between the two variables.

1.



This graph touches the y–axis at ______________.

Moving from left to right: ____

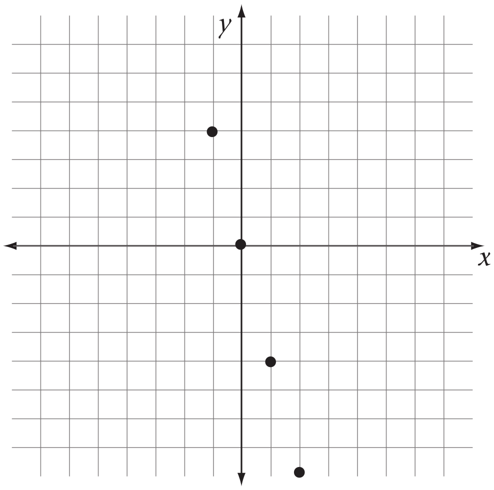
____

x	–2	0	2	4
y				

Describe the relationship from the table of values: ________________

____

2.



This graph touches the y–axis at ____________.

Moving from left to right: ____________________________

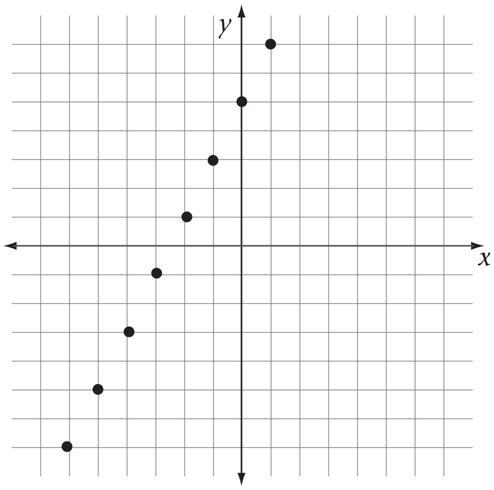
____

x	–1	0	1	2
y				

Describe the relationship from the table of values: ________________

____

3.



This graph touches the y–axis at ______________.

Moving from left to right: ____

____

x	–2	–1	0	1
y				

Describe the relationship from the table of values: __________________

____

Turn to the Answer Key at the end of the module to check your work.

Lesson 2
Algebra

Learning Outcomes

By the end of this section you will be better able to:

- solve one and two step algebra equations
- use substitution to check your solution to an algebra equation
- combine like terms
- solve equations that have variable terms on both sides
- use the distributive property to simplify equations

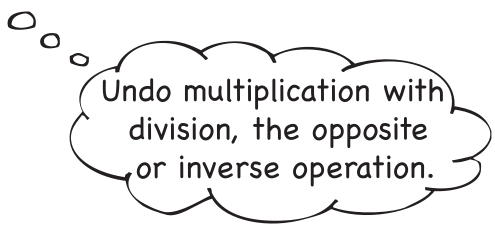
Solving Equations by "Undoing" Them

It can be awkward to solve some equations using models. This is why we usually use algebra for complex equations. We'll examine each equation to learn what is being done to the variable, and then we'll "undo" what we see happening. Our goal is to get the variable by itself on one side of the equals sign. This is known as isolating the variable.

Example 1:

$$4x = 24$$

We see that something's happening to x: it's being multiplied by 4. To solve an equation we must *undo* everything that's happening to x, so we'll undo the multiplication by 4 with another operation—division by 4.



We do the opposite action, and of course we must do that operation on both sides to keep the equation balanced.

$$\frac{4x}{4} = \frac{24}{4}$$

$$\frac{4}{4} = 1$$
$$1x = x$$

Now let's write the results of our operation. On the left side, the result is that x is no longer multiplied by 4; it's just plain x, all alone. On the right, 24 is divided by 4, resulting in 6.

$$x = 6$$

When x is isolated on one side of the equation, we've solved the equation.

Example 2

$$\frac{x}{5} = 3$$

We see that x is being divided by 5. We'll undo the division by 5 with another operation—multiplication by 5 (the inverse operation). Of course, we must do that same operation on both sides to keep the equation balanced.

$$\frac{(5)x}{5} = 3(5)$$

$$\frac{5}{5} = 1$$
$$1x = x$$

Now let's write down the results of our operation. On the left, x is no longer divided by 5; it's just a plain x all alone. On the right 3 is multiplied by 5, resulting in 15.

$$x = 15$$

When x is isolated on one side of the equation, we've solved the equation.

Exercises 2.1

1. For the expression or equations in the following table, answer the following questions:
   - What is happening to x?
   - What's the opposite action?

   The first one is done for you.

Expression or Equation	What's happening to x?	What's the opposite?
$3x$	multiplied by 3	divided by 3
$x - 5$		
$\frac{x}{7}$		
$-2x$		
$14 + x$		
$-5x = 10$		
$\frac{x}{-4} = 8$		

2. Perform the "undoing" actions to each side of the equations to solve them:

   a. $3x = 21$ b. $x + 16 = 8$ c. $x - 9 = 8$

3. Solve. Be careful with the negative numbers.

   a. $\frac{x}{8} = 2$ b. $5x = -35$ c. $x - 16 = -3$

Turn to the Answer Key at the end of the module to check your work.

Solving Two-step Equations

Often more than one thing is happening to x. In these situations, we must do more than one "undoing" action, and we have to do them in *reverse-BEDMAS* order. This is because we're not performing calculations, we're *reversing* the calculations that are in the equation.

Brackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction

Look at $4x - 6 = 2$. x is being multiplied by 4, and then 6 is subtracted. BEDMAS tells us to do multiplication before subtraction. To go in reverse BEDMAS order, we'll first undo the subtraction by adding 6, and then undo the multiplication by dividing by 4.

Note: It's very important to show your steps on the line below the equation, then write the new equation that results from your steps, like we've shown here.

$$\begin{array}{c} 4x - 6 = 2 \\ (+6) \quad (+6) \end{array}$$

Add 6 to the left side, then add 6 to the right side to keep it balanced.

$$\frac{4x}{4} = \frac{8}{4}$$

Divide the left side by 4 to get x by itself, and then divide the right side by 4 to keep it balanced.

$$x = 2$$

Here's another example:

$$\begin{aligned} 19 + 7x - 6 &= 40 \\ (-19) \qquad\qquad & \quad (-19) \\ 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$

Exercises 2.2

1. For the two solved equations below, fill in the blanks to explain how they were solved.

a.

$\frac{x}{5} - 8 = 2$

x is being ________ by 5 and then _____ is subtracted.

$(+8) \quad (+8)$

To undo the "subtract 8," add 8 to the left side, and then add _____ to the ________ side to keep it ________.

$\frac{x}{5} = 10$

$(5)\frac{x}{5} = 10(5)$

$x = 50$

To undo the "divided by 5," multiply the left side by _____, and then ________ the right side by _____ to keep it balanced.

b. $14 = -3x + 20$

x is multiplied by _____ and then _____ is ________.

$(-20) \quad (-20)$

Undo the "plus 20" by ________ 20 from the _____ side, and then ____________ 20 from the ____________ side to keep it balanced.

$-6 = -3x$

$\frac{-6}{(-3)} = \frac{-3x}{(-3)}$

Undo the "multiplied by –3" by ____________ by –3 on the right side, and then __________ by _____ on the left side to keep it ____________.

$2 = x$

$x = 2$

2. Using the same procedure as above, solve these equations.

a. $-14 = 3x + 4$

b. $-14x + 1 = 43$

c. $\frac{y}{9} - 5 = -3$

d. $13 = 3 + 5x$

Turn to the Answer Key at the end of the module to check your work.

Just Checking

Once we've solved a linear equation, it's a good idea to check the answer. To do this, we can simply substitute the solution into the original equation, then do the equation's arithmetic according to BEDMAS.

When we solve

$$\begin{aligned} -14 &= 7x + 7 \\ (-7) & \quad\;\; (-7) \\ \frac{-21}{(7)} &= \frac{7x}{(7)} \\ -3 &= x \end{aligned}$$

We get $x = -3$

To check the answer, substitute –3 in place of x in the original equation, and do the math. Here we'll use a Left Side (LS) and Right Side (RS) format.

LS	RS	
–14	7(–3) + 7	
–14	–21 + 7	
–14	–14	LS = RS

The left side equals the right side, so we know our solution was correct. If the two sides had different numbers, we'd have to go over our work to find the error.

In solving the following equation, we've made an error. Can you spot it? We'll use the check that follows to show how we can catch the mistake.

$$\begin{aligned} -2x + 7 &= -3 \\ -7 & \quad\; -7 \\ -2x &= 4 \\ \frac{-2x}{-2} &= \frac{4}{-2} \\ x &= -2 \end{aligned}$$

Check:

LS	RS	
$-2x + 7$	–3	
–2(–2) + 7	–3	
4 + 7	–3	
11	–3	LS ≠ RS, so our answer is not correct.

Let's go back and solve it properly.

$$\begin{aligned} -2x + 7 &= -3 \\ \scriptstyle -7 \quad & \quad \scriptstyle -7 \\ -2x &= -10 \\ \frac{-2x}{-2} &= \frac{-10}{-2} \\ x &= 5 \end{aligned}$$

$-3 + (-7) \neq 4$
Last time we must have added +7, not −7.

Check:

LS	RS
$-2x + 7$	-3
$-2(5) + 7$	-3
$-10 + 7$	-3
-3	-3

LS = RS, so our answer is correct.

Exercises 2.3

Answers are provided for each of the equations below. Some are correct, and some are not. Start by providing a check for each equation. If the provided answer is correct, place a check mark beside the equation. If the answer is not correct, solve the equation for the correct answer.

1. $6m + 4 = 28$
   $m = 3$

   Check:

LS	RS
$6m + 4$	28

2. $-8 = -24 - 4x$
   $x = -4$

   Check:

LS	RS
-8	$-24 - 4x$

3. $-5 - 9s = 13$
   $s = -2$

   Check:

LS	RS
$-5 - 9s$	13

4. $7 = 4x - 13$
$x = -5$

Check:

LS	RS
7	$4x - 13$

Turn to the Answer Key at the end of the module to check your work.

Combining Like Terms

Things must be alike in order for us to add or subtract them, and the "units" stay the same in the answer as they were in the question.

8 goats plus 5 goats is 13 goats, not 13 of something new. If you have 8 goats and 5 dollars, you don't have 13 of anything. Apply the same thinking to expressions with variables in them:

$8x + 5x$ has to equal $13x$, not 13 of something else.

$7y - 10y$ is $-3y$

$10q + (-2q) = 8q$ (just add the integer coefficients)

$9x + 12y$ is not possible. We can't group unlike terms together.

When we see more than one variable term in an equation, we have to group them before we can solve it, so they'd better be alike! Look at the following example:

$$5x + 6 - 2x = 15$$

The variable x is in two places, so we'll rearrange the terms so that the ones with x in them appear together. Then it will be easier to combine them. Just remember to keep them on the correct side of the equals sign.

Terms are separated by + or −.

Rewrite $-2x$ to $+ (-2x)$ (Instead of subtracting, we're adding the opposite.)

$$5x + 6 - 2x = 15$$
$$5x + 6 + (-2x) = 15$$

Rearrange to put like terms together.

$$5x + 6 + (-2x) = 15$$
$$5x + (-2x) + 6 = 15$$

$5x + (-2x) = 3x$, so we're left with $3x$ and the constant 6 on the left.

$$3x + 6 = 15$$

Now that it looks familiar, we can go through the usual steps to solve it.

$$\begin{aligned} 3x + 6 &= 15 \\ -6 \quad & \quad -6 \\ 3x &= 9 \\ \frac{3x}{3} &= \frac{9}{3} \\ x &= 3 \end{aligned}$$

Check:

$$\begin{aligned} 5x + 6 - 2x &\stackrel{?}{=} 15 \\ 5(3) + 6 - 2(3) &\stackrel{?}{=} 15 \\ 15 + 6 - 6 &\stackrel{?}{=} 15 \\ 15 &= 15 \quad \checkmark \end{aligned}$$

Let's try another one.

$$-4x - 7 + 7x - 9 = 32$$

Change subtractions to additions: $-4x + (-7) + 7x + (-9) = 32$

Rearrange to put like terms together: $-4x + 7x + (-7) + (-9) = 32$

Simplify: $3x + (-16) = 32$

Rewrite:

$$\begin{aligned} 3x - 16 &= 32 \\ +16 \quad & \quad +16 \end{aligned}$$

Solve:

$$\begin{aligned} 3x &= 48 \\ \frac{3x}{3} &= \frac{48}{3} \\ x &= 16 \end{aligned}$$

Check:

$$\begin{aligned} -4x - 7 + 7x - 9 &= 32 \\ -4(16) - 7 + 7(16) - 9 &\stackrel{?}{=} 32 \\ -64 - 7 + 112 - 9 &\stackrel{?}{=} 32 \\ -71 + 112 - 9 &\stackrel{?}{=} 32 \\ 41 - 9 &\stackrel{?}{=} 32 \\ 32 &= 32 \quad \checkmark \end{aligned}$$

Exercises 2.4

Rearrange these equations so that all of the similar terms are grouped together, and then simplify them. Solve the equation and check your answers. The first one has been done for you.

1. $$\begin{aligned} 2x + 3 + x &= 15 \\ 2x + x + 3 &= 15 \\ 3x + 3 &= 15 \\ -3 \quad & \;\; -3 \\ \frac{3x}{3} &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

   Check :

   $$\begin{aligned} 2(4) + 3 + (4) &= 15 \\ 8 + 3 + 4 &= 15 \\ 15 &= 15 \checkmark \end{aligned}$$

2. $3x - 2 - x = 6$

3. $3x + 7 + 6x = 61$

4. $-8a + 5 - 2a + 1 = -44$

5. $4x - 1 - x - 3 = 5$

6. $-4g + 3 - 7g = 25$

Turn to the Answer Key at the end of the module to check your work.

Variable Terms on Both Sides of the Equation

What if an x-term appears on both sides of the equation? Are we allowed to just remove some of them?

It turns out we are allowed, if we remember the principle of balance. We must remove an equal number of the variable from the other side as well.

We can remove $4x$ from each side of this equation.

Then it becomes a simple 2-step equation to solve.

$$\begin{aligned} 6x - 8 &= 4x + 12 \\ {\scriptstyle -4x} \quad & \quad {\scriptstyle -4x} \\ 2x - 8 &= 12 \\ {\scriptstyle +8} \quad & \quad {\scriptstyle +8} \\ 2x &= 20 \\ \frac{2x}{2} &= \frac{20}{2} \\ x &= 10 \end{aligned}$$

Check:

$$\begin{aligned} 6(10) - 8 &\stackrel{?}{=} 4(10) + 12 \\ 60 - 8 &\stackrel{?}{=} 40 + 12 \\ 52 &= 52 \quad ✔ \end{aligned}$$

Another example:

It's our choice: subtract $7x$ from both sides, or add $8x$ to both sides. Let's try the second option.

$$-8x - 20 = 7x - 5$$

$$\begin{aligned} -8x - 20 &= 7x - 5 \\ {\scriptstyle +8x} \quad & \quad {\scriptstyle +8x} \\ -20 &= 15x - 5 \\ {\scriptstyle +5} \quad & \quad {\scriptstyle +5} \\ \frac{-15}{15} &= \frac{15x}{15} \\ -1 &= x \\ \textit{or} \quad x &= -1 \end{aligned}$$

Check:

$$\begin{aligned} -8(-1) - 20 &\stackrel{?}{=} 7(-1) - 5 \\ 8 - 20 &\stackrel{?}{=} -7 - 5 \\ -12 &= -12 \quad ✔ \end{aligned}$$

We'll do the same example again, but this time we'll start by subtracting $7x$ from both sides:

$$
\begin{aligned}
-8x - 20 &= 7x - 5 \\
\scriptstyle -7x \qquad &\qquad \scriptstyle -7x \\
-15x - 20 &= -5 \\
\scriptstyle +20 &\quad \scriptstyle +20 \\
-15x &= 15 \\
\frac{-15x}{-15} &= \frac{15}{-15} \\
x &= -1
\end{aligned}
$$

So it works out the same, no matter which option we choose!

Exercises 2.5

Solve these equations by working with the variable term first. Check your answers. To check, you can use either the Left Side—Right Side method, or put a question mark over the equals sign.

1. $6h + 19 = 4h + 29$
2. $-5x + 20 = 8x - 6$
3. $-9x - 13 = 11x + 7$

Turn to the Answer Key at the end of the module to check your work.

Distribution

Some expressions and some equations have brackets in them. The order of operations tells us to do what's in the brackets first, but if the brackets contain unlike terms, we can't!

$7(x+2)=3x-6$ Help! We can't add x and 2.

So we'll use another technique. The **distributive property** allows us to rewrite expressions that have brackets in them so they don't have brackets anymore. It works like this:

$7(x+2)$ means 7 times $(x+2)$. We'll *distribute* the 7 to the items inside the brackets: 7 times x and 7 times 2. Then we won't need the brackets anymore.

$7(x + 2)$
$= 7(x) + 7(2)$
$= 7x + 14$

When you finish multiplying, there are no brackets.

Here are a few more examples of the distributive property:

Remember: $(-4)(-5)=+20$

$-4(m - 5)$
$= -4m + 20$

$3(s - 12)$
$= 3s - 36$

$-5(2a + 3)$
$= -10a - 15$

Here's an example where we use the distributive property to help us solve a linear equation:

$7(x + 2) = 3x - 6$	Distribute the 7 over the x and the 2.
$7x + 14 = 3x - 6$	
$\underset{-3x}{7x} + 14 = \underset{-3x}{3x} - 6$	Subtract $3x$ from each side.
$4x + 14 = -6$	
$4x + \underset{-14}{14} = \underset{-14}{-6}$	Subtract 14 from each side.
$4x = -20$	
$\frac{4x}{4} = \frac{-20}{4}$	Divide each side by 4.
$x = -5$	

Remember, the number in front of the brackets has to multiply all the items within the brackets.

Here's another example:

$$2(x - 6) = 4$$
$$\underset{+12}{2x - 12} = \underset{+12}{4}$$
$$2x = 16$$
$$\frac{2x}{2} = \frac{16}{2}$$
$$x = 8$$

Exercises 2.6

Solve these equations. Check your work for each using either the Left Side—Right Side method or by putting a ? over the =.

1. $-3(x + 9) = x + 1$
2. $-1(6x + 6) = 2x + 18$
3. $3(x - 8) = 9$
4. $-4(x + 3) = x - 62$
5. $-2(3x - 1) = \mathrm{x} + 9$
6. $7(m - 5) = m + 1$

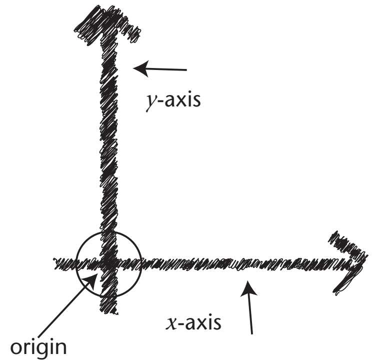
Turn to the Answer Key at the end of the module to check your work.

Answer Key

Lesson 1: Graphing

Exercises 1.1

1.

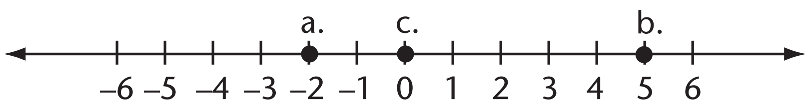


2. a. 3
   b. 4
   c. (3, 4)

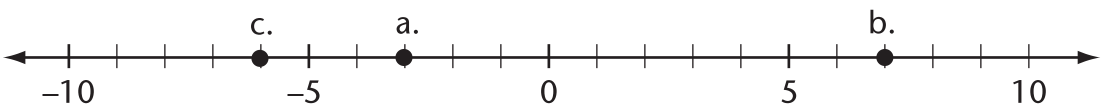
3. The Cartesian plane is an example of a **coordinate** system.

Exercises 1.2

1.



2.



Exercises 1.3

1. a. $y = x - 5$

$x = 0$
$y = x - 5$
$y = (0) - 5$
$y = -5$

$x = -3$
$y = x - 5$
$y = (-3) - 5$
$y = -8$

$x = 2$
$y = x - 5$
$y = (2) - 5$
$y = -3$

$x = 7$
$y = x - 5$
$y = (7) - 5$
$y = 2$

Ordered pairs: (–3, –8) (0, –5) (2, –3) (7, 2)

b. $y = -3x + 2$

$x = -5$
$y = -3x + 2$
$y = -3(-5) + 2$
$y = 15 + 2$
$y = 17$

$x = 0$
$y = -3x + 2$
$y = -3(0) + 2$
$y = 0 + 2$
$y = 2$

$x = 1$
$y = -3x + 2$
$y = -3(1) + 2$
$y = -3 + 2$
$y = -1$

$x = 3$
$y = -3x + 2$
$y = -3(3) + 2$
$y = -9 + 2$
$y = -7$

Ordered pairs: (–5, 17) (0, 2) (1, –1) (3, –7)

Exercises 1.4

Equation	Variables	Constant	Coefficient	Terms
$y = -2x + 3$	y, x	3	–2	$y, -2x, 3$
$a = \frac{1}{2}p + 8$	a, p	8	$\frac{1}{2}$	$a, \frac{1}{2}p, 8$
$c = 4q - 6$	c, q	–6	4	$c, 4q, -6$
$m = \frac{n}{3}$	m, n	0	$\frac{1}{3}$	$m, \frac{1}{3}n$ or $m, \frac{n}{3}$
$s = -4t - \frac{1}{2}$	s, t	$-\frac{1}{2}$	–4	$s, -4t, -\frac{1}{2}$

Exercises 1.5

1. a.

$$y = 2x + 8$$
$$11 = 2(2) + 8$$

Left Side (LS)	Right Side (RS)
11	**2(2) + 8**
11	**4 + 8**
11	**12**

Does the LS = RS? **No**

Does the point (2, 11) satisfy the equation $y = 2x + 8$? **No**

b. (15, 3)

$$y = \frac{1}{5}x$$

Left Side (LS)	Right Side (RS)
3	$\mathbf{\frac{15}{5}}$
3	**3**

Does the LS = RS? **Yes**

Does the point (15, 3) satisfy the equation $y = \frac{1}{5}x$? **Yes**

c. (–2, 8)

$$y = -3x + 2$$

Left Side (LS)	Right Side (RS)
8	**–3(–2) + 2**
8	**6 + 2**
8	**8**

Does the LS = RS? **Yes**

Does the point (2, 11) satisfy the equation $y = -3x + 2$? **Yes**

2.

$y = \frac{1}{2}x + 4$

$y = 3x - 2$

$\boxed{y = -x - 1}$

(–2, 3) (4, 10)

(4, 6) $\boxed{(0, -1)}$ (2, 4)

$\boxed{(-3, 2)}$

$\boxed{(2, -3)}$

(–4, 2)

(–3, –11)

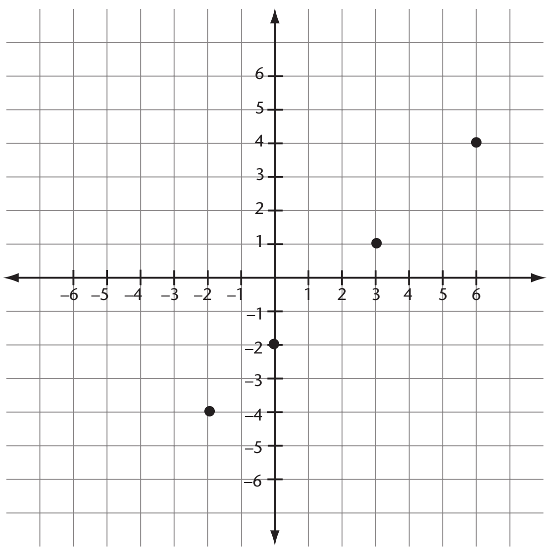
Exercises 1.6

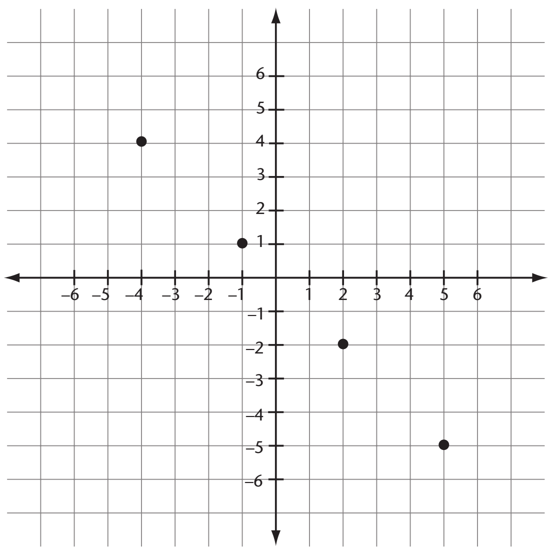
1. a. $y = x - 2$

x	y
–2	**–4**
0	**–2**
3	**1**
6	**4**

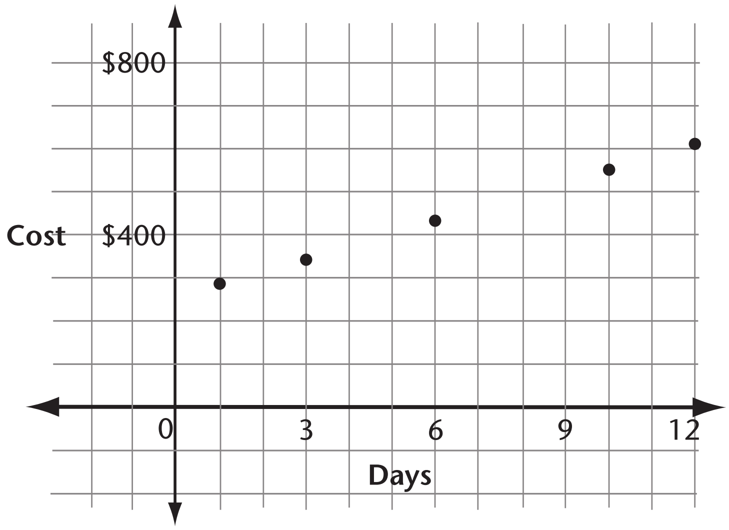
b. $y = -x$

x	y
–4	**4**
–1	**1**
2	**–2**
5	**–5**





2. **Example solutions:**



Days	1	2	3	4	5	6	10	11	12	13	14	15
Cost	280	310	340	370	400	430	550	580	610	640	670	700

Exercises 1.7

Answers will vary. Some examples of correct answers are:

1. Subtract 4 from each x-value to get the y-value. Each x is 4 more than y. $y = x - 4$.
2. It costs $5 to attend the dance. Each snack cost $1.
3. Multiply r by –3 to get t. $t = -3r$

Exercises 1.8

1. This graph touches the y–axis at –4

   Moving from left to right: For every one that x goes to the right, y goes up one

x	–2	0	2	4
y	–6	–4	–2	0

   Describe the relationship from the table of values: For every one that x increases, y increases by one. Or, y is 4 less than x.

2. This graph touches the y–axis at 0

   Moving from left to right: For every one that x goes to the right, y goes down by 4

x	–1	0	1	2
y	4	0	–4	–8

   Describe the relationship from the table of values: Whenever x goes up by 1, y goes down by 4. Or, y is –4 times x.

3. This graph touches the y–axis at 5

   Moving from left to right: For every one that x goes to the right, y goes up by 2

x	–2	–1	0	1
y	1	3	5	7

   Describe the relationship from the table of values: Whenever x goes up by 1, y goes up by 2. Or, y is 5 more than twice x.

Lesson 2: Algebra

Exercises 2.1

1.

Expression or Equation	What's happening to x?	What's the opposite?
$3x$	**multiplied by 3**	**divided by 3**
$x - 5$	**5 is subtracted**	**5 is added**
$\frac{x}{7}$	**divided by 7**	**multiplied by 7**
$-2x$	**multiplied by –2**	**divided by –2**
$14 + x$	**14 is added**	**14 is subtracted**
$-5x = 10$	**multiplied by –5**	**divided by –5**
$\frac{x}{-4} = 8$	**divided by –4**	**multiplied by –4**

2.

a. $\frac{3x}{3} = \frac{21}{3}$

$x = 7$

On the left,
the 3s simplify,
leaving just x.

b. $x + 16 = 8$

$-16 \quad -16$

$x = -8$

c. $x - 9 = 8$

$+9 \quad +9$

$x = 17$

3.

a. $\frac{x}{8} = 2$

$\frac{8x}{8} = 2(8)$

$x = 16$

b. $5x = -35$

$\frac{5x}{5} = \frac{-35}{5}$

$x = -7$

c. $x - 16 = -3$

$+16 \quad +16$

$x = 13$

Exercises 2.2

1.

a. $\frac{x}{5} - 8 = 2$

x is being <u>divided</u> by 5 and then <u>8</u> is subtracted.

$(+8) \quad (+8)$

To undo the "subtract 8," add 8 to the left side, and then add <u>8</u> to the <u>right</u> side to keep it <u>balanced.</u>

$\frac{x}{5} = 10$

$(5)\frac{x}{5} = 10(5)$

$x = 50$

To undo the "divided by 5," multiply the left side by <u>5</u> and then <u>multiply</u> the right side by <u>5</u> to keep it balanced.

b. $14 = -3x + 20$

$-20 \qquad -20$

x is multiplied by <u>–3</u> and then <u>20</u> is <u>added.</u>

Undo the "plus 20" by <u>subtracting</u> 20 from the <u>right</u> side, and then <u>subtracting</u> 20 from the <u>left</u> side to keep it balanced.

$-6 = -3x$

$\frac{-6}{(-3)} = \frac{-3x}{(-3)}$

Undo the "multiplied by –3" by <u>dividing</u> by –3 on the right side, and then <u>dividing</u> by <u>–3</u> on the left side to keep it <u>balanced.</u>

$2 = x$

$x = 2$

2.

a.

$-14 = 3x + 4$

$(-4) \qquad (-4)$

$\frac{-18}{(3)} = \frac{3x}{(3)}$

$-6 = x$

b.

$-14x + 1 = 43$

$(-1) \quad (-1)$

$\frac{-14x}{(-14)} = \frac{42}{(-14)}$

$x = -3$

c.

$\frac{y}{9} - 5 = -3$

$(+5) \quad (+5)$

$\frac{(9)y}{9} = 2(9)$

$y = 18$

d.

$13 = 3 + 5x$

$(-3) \quad (-3)$

$\frac{10}{-5} = \frac{-5x}{-5}$

$-2 = x$

Exercises 2.3

1. $6m + 4 = 28$ This question is wrong. ×
   $m = 3$

Check:

LS	RS
$6m + 4$	28
$6(3) + 4$	28
$18 + 4$	28
22	28

Correct solution:

$$6m + 4 = 28$$
$$-4 \quad -4$$
$$\frac{6m}{6} = \frac{24}{6}$$
$$m = 4$$

2. $-8 = -24 - 4x$ This question is correct. ✓
   $x = -4$

Check:

LS	RS
-8	$-24 - 4x$
-8	$-24 - 4(-4)$
-8	$-24 + 16$
-8	-8

3. $-5 - 9s = 13$ This question is correct. ✓
   $s = -2$

Check:

LS	RS
$-5 - 9s$	13
$-5 - 9(-2)$	13
$-5 + 18$	13
13	13

4. $7 = 4x - 13$ This question is wrong. ×
   $x = -5$

Check:

LS	RS
7	$4(-5) - 13$
7	$-20 - 13$
7	-33

Correct solution:

$$7 = 4x - 13$$
$$+13 \qquad +13$$
$$\frac{20}{4} = \frac{4x}{4}$$
$$5 = x$$

Exercises 2.4

1.

$$\begin{aligned}
2x + 3 + x &= 15\\
2x + x + 3 &= 15\\
3x + 3 &= 15\\
{\scriptstyle -3} \quad &\quad {\scriptstyle -3}\\
\frac{3x}{3} &= \frac{12}{3}\\
x &= 4
\end{aligned}$$

Check:

$$\begin{aligned}
2(4) + 3 + (4) &\stackrel{?}{=} 15\\
8 + 3 + 4 &\stackrel{?}{=} 15\\
15 &= 15 \checkmark
\end{aligned}$$

2.

$$\begin{aligned}
3x - 2 - x &= 6\\
3x + (-x) + (-2) &= 6\\
2x - 2 &= 6\\
{\scriptstyle +2} \quad &\quad {\scriptstyle +2}\\
\frac{2x}{2} &= \frac{8}{2}\\
x &= 4
\end{aligned}$$

Check:

$$\begin{aligned}
3(4) - 2 - (4) &\stackrel{?}{=} 6\\
12 - 2 - 4 &\stackrel{?}{=} 6\\
10 - 4 &\stackrel{?}{=} 6\\
6 &= 6 \checkmark
\end{aligned}$$

3.

$$\begin{aligned}
3x + 7 + 6x &= 61\\
3x + 6x + 7 &= 61\\
9x + 7 &= 61\\
{\scriptstyle -7} \quad &\quad {\scriptstyle -7}\\
\frac{9x}{9} &= \frac{54}{9}\\
x &= 6
\end{aligned}$$

Check :

$$\begin{aligned}
3(-6) + 7 + 6(-6) &\stackrel{?}{=} 61\\
18 + 7 + 36 &\stackrel{?}{=} 61\\
61 &= 61 \checkmark
\end{aligned}$$

4.

$$\begin{aligned}
-8a + 5 - 2a + 1 &= -44\\
-8a + (-2a) + 5 + 1 &= -44\\
-10a + 6 &= -44\\
{\scriptstyle -6} \quad &\quad {\scriptstyle -6}\\
\frac{-10a}{-10} &= \frac{-50}{-10}\\
a &= 5
\end{aligned}$$

Check :

$$\begin{aligned}
-8(-5) + 5 - 2(-6) + 1 &\stackrel{?}{=} -44\\
-40 + 5 - 10 + 1 &\stackrel{?}{=} -44\\
-44 &= -44 \checkmark
\end{aligned}$$

5.

$$\begin{aligned}
4x - 1 - x - 3 &= 5\\
4x + (-x) + (-1) + (-3) &= 5\\
3x + (-4) &= 5\\
{\scriptstyle +4} \quad &\quad {\scriptstyle +4}\\
\frac{3x}{3} &= \frac{9}{3}\\
x &= 3
\end{aligned}$$

Check :

$$\begin{aligned}
4(3) - 1 - 3 - 3 &\stackrel{?}{=} 5\\
12 - 1 - 3 - 3 &\stackrel{?}{=} 5\\
5 &= 5 \checkmark
\end{aligned}$$

6.

$$\begin{aligned}
-4g + 3 - 7g &= 25\\
-4g + (-7g) + 3 &= 25\\
-11g + 3 &= 25\\
{\scriptstyle -3} \quad &\quad {\scriptstyle -3}\\
\frac{-11g}{-11} &= \frac{22}{-11}\\
g &= -2
\end{aligned}$$

Check :

$$\begin{aligned}
-4(-2) + 3 - 7(-2) &\stackrel{?}{=} 25\\
8 + 3 - (-14) &\stackrel{?}{=} 25\\
11 + 14 &\stackrel{?}{=} 25\\
25 &= 25 \checkmark
\end{aligned}$$

Exercises 2.5

1. $6h + 19 = 4h + 29$

$$6h + 19 = 4h + 29$$
$$-4h \quad -4h$$
$$2h + 19 = 29$$
$$2h = 10$$
$$h = 5$$

Check:

$$6(5) + 19 \stackrel{?}{=} 4(5) + 29$$
$$30 + 19 \stackrel{?}{=} 20 + 29$$
$$49 = 49 \checkmark$$

2. $-5x + 20 = 8x - 6$

$$-5x + 20 = 8x - 6$$
$$-8x \quad -8x$$
$$-13x + 20 = -6$$
$$-13x = -26$$
$$x = 2$$

Check:

$$-5(2) + 20 \stackrel{?}{=} 8(2) - 6$$
$$-10 + 20 \stackrel{?}{=} 16 - 6$$
$$10 = 10 \checkmark$$

3. $-9x - 13 = 11x + 7$

$$-9x - 13 = 11x + 7$$
$$+9x \quad +9x$$
$$-13 = 20x + 7$$
$$-20 = 20x$$
$$-1 = x$$

Check:

$$-9(-1) - 13 \stackrel{?}{=} 11(-1) \cdot$$
$$9 - 13 \stackrel{?}{=} -11 + 7$$
$$-4 = -4 \checkmark$$

Exercises 2.6

1.
$$-3(x + 9) = x + 1$$
$$-3x + (-27) = x + 1$$
$$-4x + (-27) = 1$$
$$-4x = 28$$
$$x = -7$$

Check:

$$-3(-7 + 9) = -7 + 1$$
$$-3(2) = -6$$
$$-6 = -6 \checkmark$$

2.
$$-1(6x + 6) = 2x + 18$$
$$-6x + (-6) = 2x + 18$$
$$-8x + (-6) = 18$$
$$-8x = 24$$
$$x = -3$$

Check:

$$-1(6(-3) + 6) = 2(-3) + 18$$
$$-1(-18 + 6) = -6 + 18$$
$$-1(-12) = 12$$
$$12 = 12 \checkmark$$

3.
$$3(x - 8) = 9$$
$$3x - 24 = 9$$
$$3x = 33$$
$$x = 11$$

Check:

$$3(11 - 8) = 9$$
$$3(3) = 9$$
$$9 = 9 \checkmark$$

4.
$$-4(x + 3) = x - 62$$
$$-4x + (-12) = x - 62$$
$$-5x + (-12) = -62$$
$$-5x = -50$$
$$x = 10$$

Check:

$$-4(10 + 3) = 10 - 62$$
$$-4(13) = -52$$
$$-52 = -52 \checkmark$$

5.
$$-2(3x-1)=x+9$$
$$-6x+2=x+9$$
$$-7x+2=9$$
$$-7x=7$$
$$x=-1$$

Check:
$$-2\big(3(-1)-1\big)=(-1)+9$$
$$-2(-4)=8$$
$$8=8 \checkmark$$

6.
$$7(m-35)=m+1$$
$$7m-35=m+1$$
$$6m-35=1$$
$$6m=36$$
$$\frac{6m}{6}=\frac{36}{6}$$
$$m=6$$

Check:
$$7(6-5)=6+1$$
$$7(1)=7$$
$$7=7 \checkmark$$