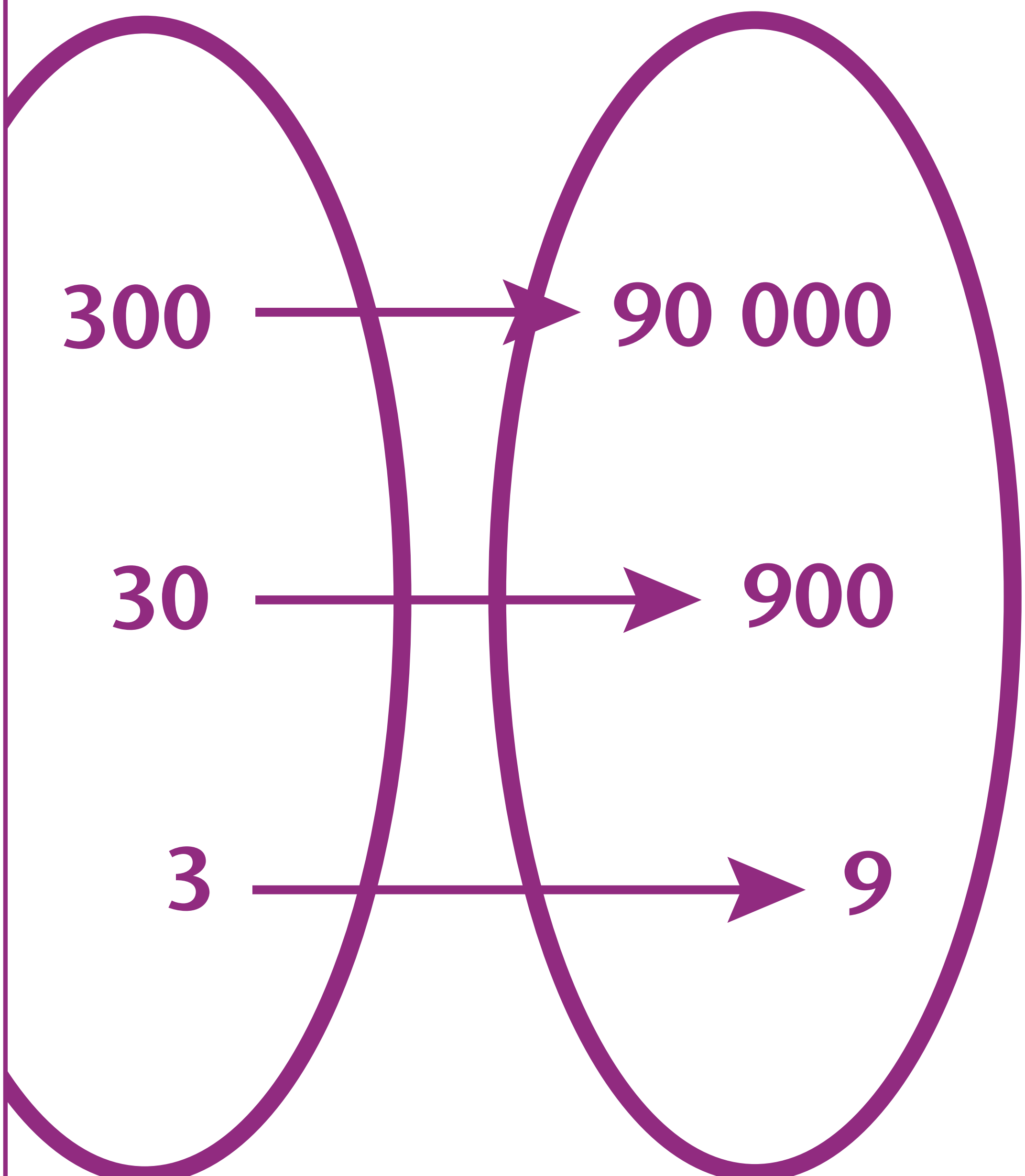
LITERACY FOUNDATIONS MATH



LEVEL 6
NUMBER SENSE

To the Student

This resource covers topics from the British Columbia Ministry of Education's Literacy Foundations Math Level 6. You may find this resource useful if you're a Literacy Foundations Math student, or a K–12 student in grades 7 – 9.

We have provided learning material, exercises, and answers for the exercises, which are located at the back of each set of related lessons. We hope you find it helpful.

Literacy Foundations Math Prescribed Learning Outcomes

Literacy Foundations Mathematics Level 6 follows two pathways: Math Foundations (MF) and Apprenticeship and Workplace (AW). The Prescribed Learning Outcomes (PLOs) are grouped into four areas: Number (A), Patterns and Relations (B), Shape and Space (C), and Statistics and Probability (D). For a complete list of the PLOs in Level 6, go to the BC Ministry of Education's website and search for Literacy Foundations Math curriculum.

PLOs Represented in This Resource

The PLOs represented in this Level 5 resource are as follows:

Number

MF: All topics, A1 – A6, with the exception of a portion of A3, roots of fractions, and a portion of A4, word problems with powers
AW: All topics, A1 – A9, with the exception of a portion of A3, roots of fractions, and A6, word problems with powers

Patterns and Relations

MF: B3 and B4, with the exception of a portion of B4, rational coefficients with variables in more than one term
AW: B3 and B4

Shape and Space

MF: C1 and C3, with the exception of a portion of C3, composite objects
AW: C1 – C3, C5, C6, with the exception of a portion of C5, composite objects

PLOs Not Represented in This Resource

The PLOs for which no material is included in this resource are as follows:

Number

MF: portions of A3, roots of fractions and a portion of A4, word problems with powers
AW: portions of A3, roots of fractions and a portion of A6, word problems with powers

Patterns and Relations

MF: B1, linear relationships; B2, patterns to linear equations; portions of B4, rational coefficients and variables in more than one term; B5, single-variable linear inequalities; B6 – B8, polynomials
AW: B1, linear relationships; B2, patterns to linear equations; and B5 – B8, polynomials

Shape and Space

MF: C2, polygons and polyhedra; and portions of C3, composite objects and line and rotation symmetry
AW: C4, polygons and polyhedra; and portions of C5, composite objects; and C7, line and rotation symmetry

Acknowledgements and Copyright

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Lesson 1

Square Roots

Learning Outcomes

By the end of this section you will be better able to:

- describe and calculate perfect squares
- find square roots of perfect squares
- estimate square roots of numbers that are not perfect squares

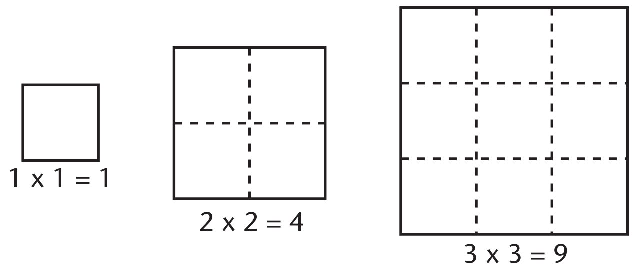
Perfect Squares

A perfect square is a number that is the area of a square whose sides are whole numbers.

That's a mouthful! Let's look at that one piece at a time.

Whole numbers are: 0, 1, 2, 3, . . .

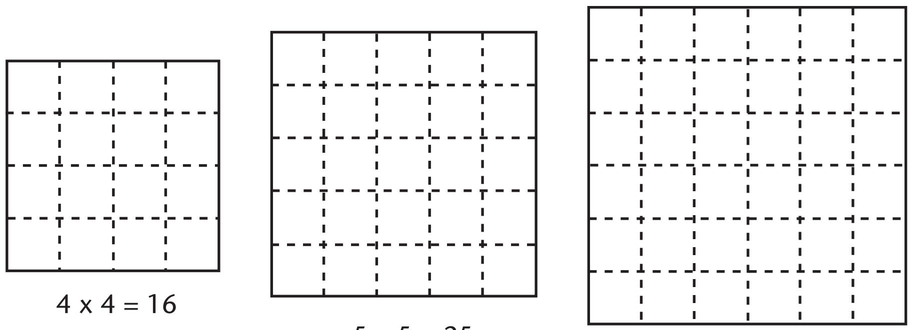
These are *squares whose sides are whole numbers*:



To find the area of a square, multiply the length by the width. In a square, the length and the width are the same. So, these numbers represent the area of a square whose sides are whole numbers.

In other words, **1**, **4**, and **9** are perfect squares.

What are the next three perfect squares? Here are pictures that represent each one.



4 x 4 = 16

5 x 5 = 25

6 x 6 = 36

Look at the definition again:

A perfect square is a number that is the area of a square whose sides are whole numbers.

The area of a square is the length of its side times itself.

So, we can find perfect squares by taking any whole number and multiplying it by itself.

$2 \times 2 = 4$	**4** is a perfect square.
$5 \times 5 = 25$	**25** is a perfect square.
$10 \times 10 = 100$	**100** is a perfect square.

Exercises 1.1

1. Multiply these numbers to find some more perfect squares.

   3 × 3 = 6 × 6 = 1 × 1 =

   9 × 9 = 4 × 4 =

2. Circle the perfect squares. Use the numbers you circled to complete the multiplication facts below.

121	22	100	144
55	16	6	56
25	1	71	12
63	64	49	8
9	81	4	36

   1 × 1 = 2 × 2 =

   3 × 3 = 4 × 4 =

   5 × 5 = 6 × 6 =

   7 × 7 = 8 × 8 =

   9 × 9 = 10 × 10 =

   11 × 11 = 12 × 12 =

Turn to the Answer Key at the end of the module to check your work.

You have learned that if you pick a whole number (say, "3") and multiply it by itself, the answer is called a perfect square.

For example: 3 multiplied by itself is 9.

9 is a perfect square.

$$3 \times 3 = 9$$

Another way to write this is: $3^2 = 9$

The little "2" that is written above the line means "multiply this number by itself." When you're reading an equation, say "squared."

That little number is called an exponent.

3^4 means $3 \times 3 \times 3 \times 3$
5^3 means $5 \times 5 \times 5$
2^7 means $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
You'll learn more about this in the lesson on Powers.

$$3^2 = 9$$

Read this as "three squared equals nine."

$$4^2 = 16$$

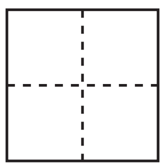
Read this as "four squared equals sixteen."

Exercises 1.2

Answer these questions without your calculator.

Remember: 3^2 means the same as 3×3.

1. $1^2 =$
2. $2^2 =$
3. $3^2 =$
4. $4^2 =$
5. $5^2 =$
6. $6^2 =$
7. $7^2 =$
8. $8^2 =$
9. $9^2 =$
10. $10^2 =$
11. This picture shows 2^2.



$2 \times 2 = 4$

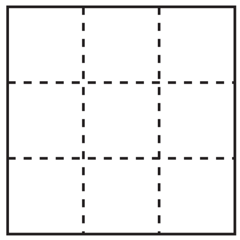
Draw a picture to show 5^2.

Turn to the Answer Key at the end of the module to check your work.

Square Roots of Perfect Squares

This equation and this diagram represent the same idea.

$$3^2 = 9$$



3 x 3 = 9

You already know that the answer when we multiply a whole number times itself is called a “perfect square.” The other number has a name too. That other number is the length of one side of the square. We could say that it is the **root** of the square. It is called the **square root**.

Exercises 1.3

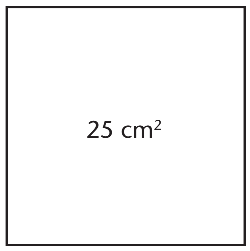
1. Draw a square with area 9. How long is the side of the square?

   What is the square root of 9?

2. Draw a square with area 4. How long is the side of the square?

   What is the square root of 4?

3. The area of this square is 25 cm^2. How long is each side?

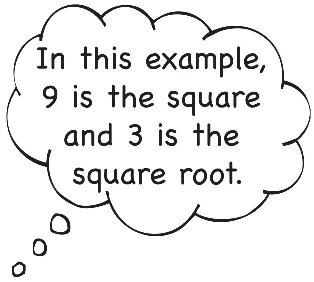


   What is the square root of 25?

Turn to the Answer Key at the end of the module to check your work.

The Square Root Symbol

There are many ways to describe the relationship between the square and the square root.



$9 = 3^2$	Nine is three squared.
$3^2 = 9$	Three squared is nine.
$\sqrt{9} = 3$	The square root of nine is three.
$3 = \sqrt{9}$	Three is the square root of nine.

This symbol, $\sqrt{}$, means “the square root of.” It is called a **square root symbol**.

Exercises 1.4

1. How many perfect squares are there between 1 and 100 (including 1 and 100)?

   List all the perfect squares between 1 and 100 (including 1 and 100).

   How can you be sure that your list is complete?

2. Draw a diagram to represent this equation:

   $\sqrt{25} = 5$

3. a. Four squared equals ________. $4^2 =$ ____

   b. Nine is ________ squared. $9 =$ ____^{2}

   c. ________ is the square root of four. ____ $= \sqrt{4}$

   d. One is the square root of ________. $1 = \sqrt{\_\_\_\_}$

   e. The square root of nine is ________. $\sqrt{9} =$ ____

   f. ________ is the square root of twenty-five. ____ $= \sqrt{25}$

4. a. $4^2 =$

   b. $6^2 =$

   c. $1^2 =$

   d. $8^2 =$

   e. $5^2 =$

5. a. $\sqrt{9} =$

   b. $\sqrt{81} =$

   c. $\sqrt{25} =$

   d. $\sqrt{49} =$

   e. $\sqrt{4} =$

6. a. $8^2 =$

   b. $16 = \_\_\_\_^2$

   c. $\sqrt{25} =$

   d. $7 = \sqrt{\_\_\_}$

   e. $100 = \_\_\_\_^2$

   f. $\sqrt{64} =$

Turn to the Answer Key at the end of the module to check your work.

Estimating Square Roots

Later on in this lesson, you will be using comparison symbols to compare two numbers.

Let's review how they are used.

= Equals

You use this symbol all the time. It means that the expression on the left is equal to the expression on the right.

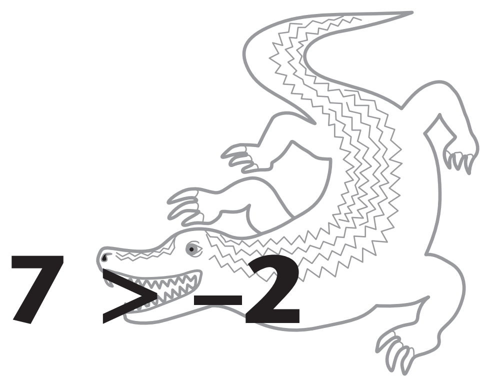
27 = 27 **2 + 3 = 5** **6 = (–2)(–3)** **–18 ÷ 3 = –6**

< Less Than and > Greater Than

You can use one of these symbols when the expression on the left is not equal to the expression on the right. The open side (the big side) of the symbol goes toward the expression that is bigger. The closed side (the small side) of the symbol goes toward the expression that is smaller.

2 < 3	**8 + 2 > 4**
2 is less than 3	**8 plus 2 is greater than 4**

Some people find it helpful to think of a greedy crocodile. Its mouth is always reaching for the bigger pile of fish!



Exercises 1.5

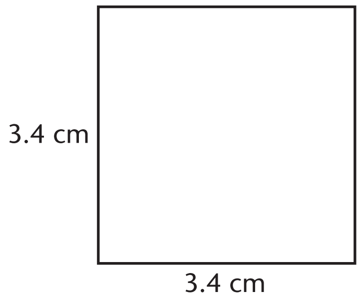
Complete each statement with the correct comparison symbol: =, <, or >.

1. $7 \ \square \ 3 + 4$
2. $-6 \ \square \ 5$
3. $2 \ \square \ 3$
4. $\sqrt{16} \ \square \ 4$
5. $8 \ \square \ 3^2$
6. $(-4)(-5) \ \square \ 12$
7. $3 \ \square \ \sqrt{25}$
8. $-6 + 5 \ \square \ 5 - 6$
9. $6^2 \ \square \ 36$
10. $5 \ \square \ \sqrt{36}$
11. $-24 \div 4 \ \square \ 3 - 10$
12. $2 \ \square \ 2^2$
13. $(-15)^2 \ \square \ (-15) \times (-15)$
14. $\sqrt{9} \ \square \ -2 + 5$

Turn to the Answer Key at the end of the module to check your work.

Not All Squares Are Perfect

You can take a line of any length you like and then use that line as the side of a square.



In fact—we can leave out the idea of a square shape and think about square numbers.

If you pick any number (say, 3.4) and multiply it by itself, the answer is called its square.

$3.4^2 = 3.4 \times 3.4 = 11.56$

3.4 squared is 11.56

11.56 is the square of 3.4

We could pick a negative number and multiply it by itself.

$(-3)^2 = (-3) \times (-3) = 9$

–3 squared is 9

9 is the square of –3

Every number can be squared.

$-17^2 = 17 \times 17 = 289$ $1.7^2 = 1.7 \times 1.7 = 2.89$

$(-17)^2 = (-17) \times (-17) = 289$ $(-1.7)^2 = (-1.7) \times (-1.7) = 2.89$

Exercises 1.6

Remember: When a whole number is squared, the answer is called a perfect square.

1. $12^2 =$
2. $(-1.2)^2 =$
3. $5.1^2 =$
4. $0.2^2 =$
5. $(-23)^2 =$
6. $35^2 =$
7. $3.5^2 =$
8. $(-0.4)^2 =$
9. $20^2 =$

Circle the answers that are perfect squares.

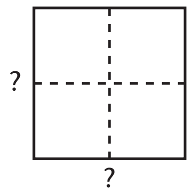
Turn to the Answer Key at the end of the module to check your work.

More Square Roots

We just learned that every number has a square. Does every number have a square root?

Let's look at a couple of squares that we already know a lot about. 4 and 9 are both perfect squares.

Think of a square with area 4.



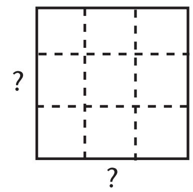
What is the root of this square?

In other words, how long is its side?

The length of the side is 2.

$$\sqrt{4} = 2$$

Think of a square with area 9.



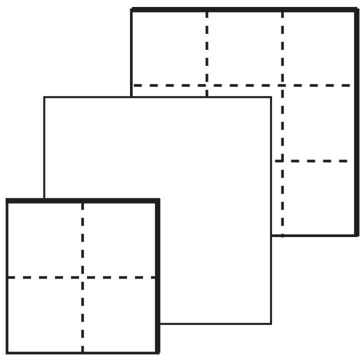
What is the root of this square?

In other words, how long is its side?

The length of the side is 3.

$$\sqrt{9} = 3$$

The expression $\sqrt{7}$ asks us to think about a square with area 7. What is the root of that square? How long is its side?



A square with area 7 is bigger than a square with area 4, and smaller than a square with area 9.

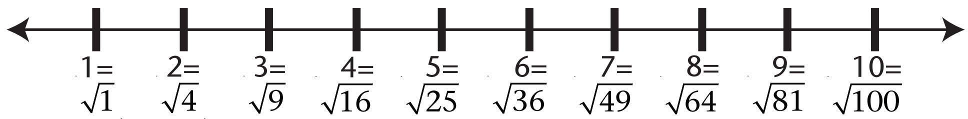
$$4 < 7 < 9$$

So, the side length of the 7 square will be longer than the side of the 4-square. It will be shorter than the side of the 9-square. $\sqrt{4} < \sqrt{7} < \sqrt{9}$

The square root of 7 is between 2 and 3. $2 < \sqrt{7} < 3$

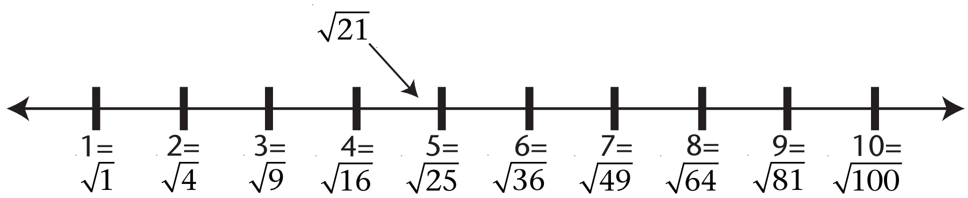
Make a number line that shows all integers from 1 to 10.

Now write $= \sqrt{\ }$ next to each number and fill in the numbers under the square root symbols.



This is a very useful tool for estimating square roots.

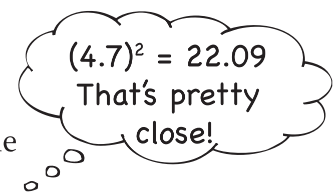
Where would $\sqrt{21}$ be on the number line?



21 is between 16 and 25	$16 < 21 < 25$
$\sqrt{21}$ is between $\sqrt{16}$ and $\sqrt{25}$	$\sqrt{16} < \sqrt{21} < \sqrt{25}$
$\sqrt{21}$ is between 4 and 5	$4 < \sqrt{21} < 5$

Can you estimate a value for $\sqrt{21}$? We know that $\sqrt{21}$ is between 4 and 5, so 4.5 would be a good guess.

Can we make a better guess than that? Is $\sqrt{21}$ closer to 4 or closer to 5? Look at the number line again. $\sqrt{21}$ is closer to 5. So 4.7 is a better guess.



Exercises 1.7

1. Make a list of the first 10 perfect squares.

   $1^2 =$ ________

   $2^2 =$ ________

   $3^2 =$ ________

   $4^2 =$ ________

   $5^2 =$ ________

   $6^2 =$ ________

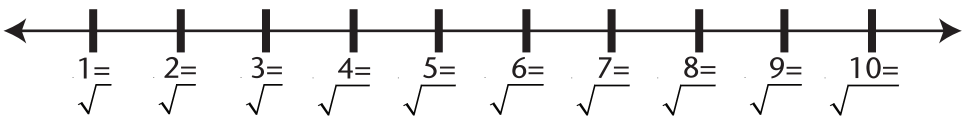
   $7^2 =$ ________

   $8^2 =$ ________

   $9^2 =$ ________

   $10^2 =$ ________

2. Complete this number line.



3. Use the number line you made to answer these questions. The first one is done for you.

   a. $2 < \sqrt{7} < 3$ $\sqrt{7}$ is between 2 and 3

   b. $4 < \sqrt{22} < \square$ $\sqrt{22}$ is between $\square$ and $\square$

   c. $\square < \sqrt{13} < \square$ $\sqrt{13}$ is between $\square$ and $\square$

d. $\square < \sqrt{61} < \square$ $\sqrt{61}$ is between $\square$ and $\square$

e. $\square < \sqrt{74} < \square$ $\sqrt{74}$ is between $\square$ and $\square$

f. $\square < \sqrt{42} < \square$ $\sqrt{42}$ is between $\square$ and $\square$

g. $\square < \sqrt{5} < \square$ $\sqrt{5}$ is between $\square$ and $\square$

h. $\square < \sqrt{57} < \square$ $\sqrt{57}$ is between $\square$ and $\square$

i. $\square < \sqrt{29} < \square$ $\sqrt{29}$ is between $\square$ and $\square$

j. $\square < \sqrt{3} < \square$ $\sqrt{3}$ is between $\square$ and $\square$

k. $\square < \sqrt{32} < \square$ $\sqrt{32}$ is between $\square$ and $\square$

l. $\square < \sqrt{95} < \square$ $\sqrt{95}$ is between $\square$ and $\square$

4. a. Is $\sqrt{7}$ closer to 2 or closer to 3?

b. Which of these numbers is the best estimate of $\sqrt{7}$?

2.2 2.5 2.7

Turn to the Answer Key at the end of the module to check your work.

Lesson 2

Fractions: Basic Skills

Learning Outcomes

By the end of this section you will be better able to:

- describe a quantity using fractions
- identify and calculate equivalent fractions
- reduce fractions to lowest terms

A fraction is a number that describes a piece of something. In this lesson, all of the fractions describe an amount less than one. $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{13}{16}$ are examples of fractions. The number above the bar is called the numerator of the fraction; the number below the bar is called the denominator.

Consider the square shown in Figure 1 below. This square is divided into 4 equal parts; 1 of these 4 parts is shaded. The fraction $\frac{1}{4}$, tells us what portion of the square is shaded. Similarly, the fraction $\frac{3}{4}$ tells us what portion is not shaded.

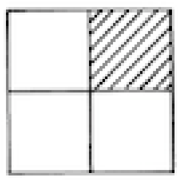


Figure 1

Now consider Figure 2. Here there are 12 small squares; 4 of them are shaded. We can say that $\frac{4}{12}$ of the rectangle is shaded. If, however, we consider the larger squares, we find that 1 of the larger squares is shaded, so we can also say that $\frac{1}{3}$ of the rectangle is shaded.

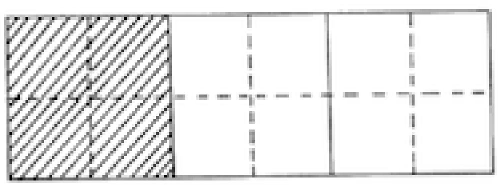


Figure 2

Self Test

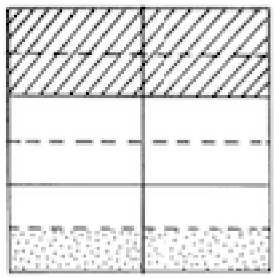
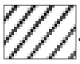
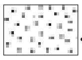


Figure 3

1. Give three fractions that tell what part of the square in Figure 3 is marked .

2. Give two fractions that tell what part of the square is marked .

Now consider the set of circles and triangles shown in Figure 4.

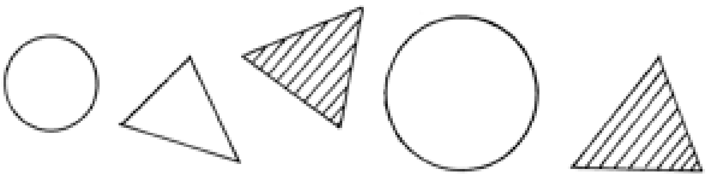


Figure 4

Of the 5 figures, 2 are circles; the fraction $\frac{2}{5}$ tells us what part of the set of figures is circles. Similarly, the fraction $\frac{3}{5}$ tells us what part of the set is triangles.

3. In Figure 4, what fractional part of the set of figures is shaded?

4. In Figure 4, what fraction of the set of triangles is shaded?

Answers

1. $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}$
2. $\frac{2}{12}, \frac{1}{6}$
3. $\frac{2}{5}$
4. $\frac{2}{3}$

Equivalent Fractions

Figure 5 illustrates pairs of equivalent fractions.

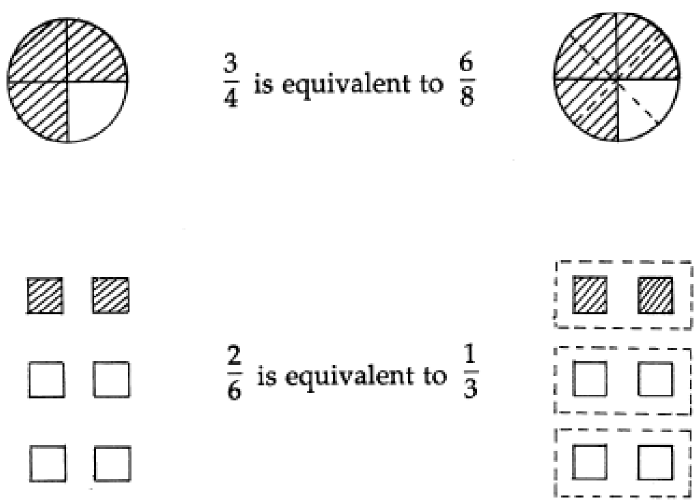
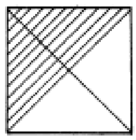


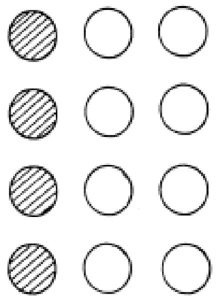
Figure 5

The fractions $\frac{3}{4}$ and $\frac{6}{8}$ represent equivalent portions of the circles; these fractions are called **equivalent fractions**. The fractions $\frac{2}{6}$ and $\frac{1}{3}$ are also equivalent; they represent equivalent parts of the sets of squares.

Self Test

Give the pair of equivalent fractions suggested in each case.

1. 

2. 

Answers

1. $\frac{2}{4}, \frac{1}{2}$

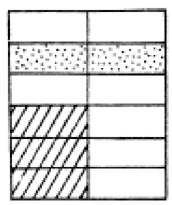
2. $\frac{4}{12}, \frac{1}{3}$ $\left(\text{or } \frac{8}{12}, \frac{2}{3}\right)$

Exercises 2.1

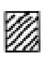
1. In each of (a) and (b) give two fractions that tell what part of the rectangle is marked as indicated.

(a)  ______________

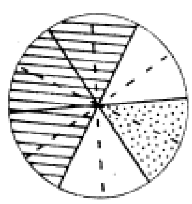
(b)  ______________



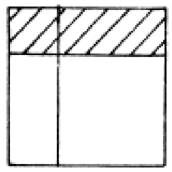
2. In each of (a) and (b), give two fractions that tell what part of the circle is marked as indicated.

(a)  ______________

(b)  ______________



3. Is $\frac{1}{2}$ of the square shaded?
   Explain your answer.



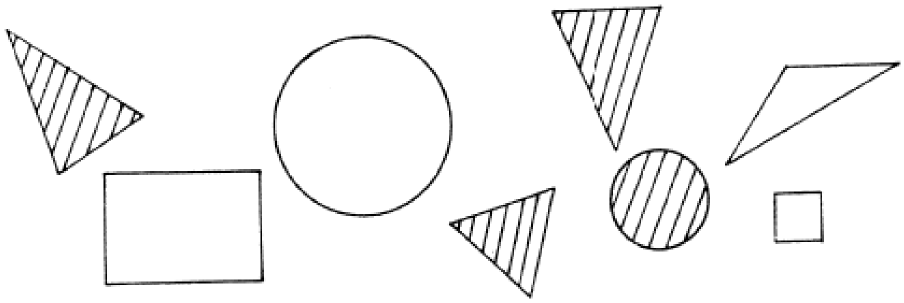
   ____

   ____

   ____

   ____

4. This question refers to the set of figures shown below. (Express your answer in lowest terms.)



   (a) What fraction of the set of figures is triangles?

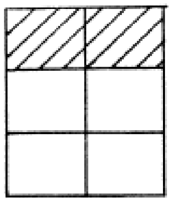
   (b) What fraction of the set of figures is circles?

   (c) What fraction of the triangles is shaded?

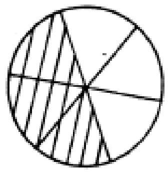
   (d) What fraction of the circles is shaded?

5. Give two equivalent fractions suggested by the shaded region.

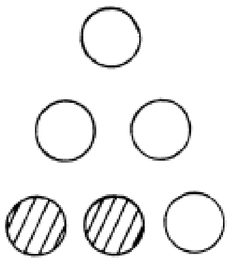
(a)



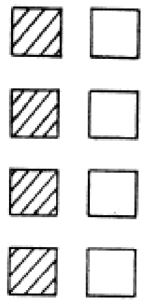
(b)



(c)



(d)



Turn to the Answer Key at the end of the module to check your work.

The Fraction "1"

We will now consider fractions in which the two numbers are the same, that is, in which the numerator is the same as the denominator. These fractions are rather special in that every one of them is equivalent to 1.

$\frac{2}{2}$ is equivalent to 1.

$\frac{3}{3}$ is equivalent to 1.

$\frac{4}{4}$ is equivalent to 1.

....

....

$\frac{29}{29}$ is equivalent to 1.

....

....

To see that this is, in fact, the case, consider the shaded portion of the rectangle in Figure 6(a).

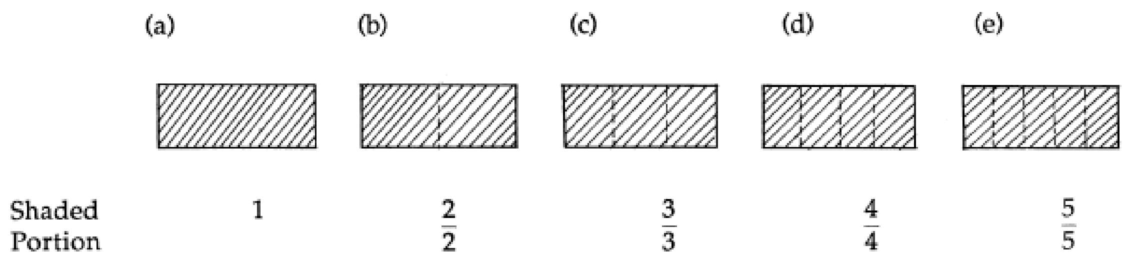


Figure 6

The entire rectangle is shaded, so we can therefore represent the shaded portion by the number 1. (There is 1rectangle, and all of it is shaded.) Suppose now that we divide this same rectangle into 2 equal regions, Figure 6 (b). Now the fraction $\frac{2}{2}$ represents the shaded portion. (There are 2 regions and both are shaded.) Hence, we see that the fraction $\frac{2}{2}$ is equivalent to 1. If we divide the rectangle into 3 regions, we find that the fraction $\frac{3}{3}$ is also equivalent to 1. Continuing in this way, we find that any fraction in which the numerator and denominator are the same is equal to 1.

More Equivalent Fractions

Consider the equivalent fractions illustrated in Figure 7.

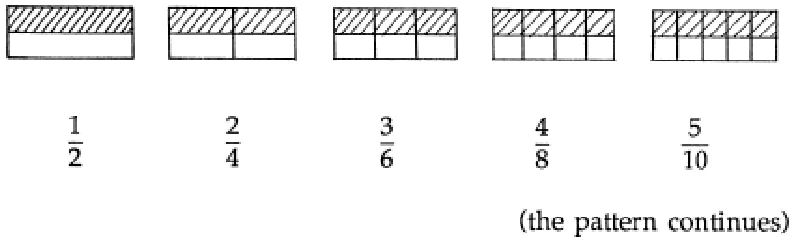
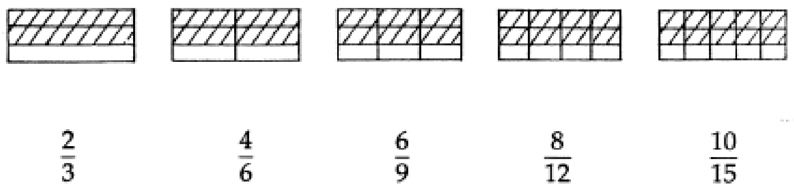


Figure 7

Each of the fractions $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ and so on, are equivalent to the fraction $\frac{1}{2}$. We can say that Figure 7 illustrates ***fractions equivalent*** to $\frac{1}{2}$.

Self Test

In Figure 8, what fraction are all the illustrations equivalent to?



Answer

$\frac{2}{3}$

Notice that each of the numerators has the numerator, 2, of $\frac{2}{3}$ as a factor and that each of the denominators has the denominator, 3, of $\frac{2}{3}$ as a factor; that is, 2 will divide into each of the numerators, and 3 will divide into each of the denominators.

$$\frac{2}{3} \quad \frac{4}{6} \quad \frac{6}{9} \quad \frac{8}{12} \quad \frac{10}{15}$$

$$\frac{1\times2}{1\times3} \quad \frac{2\times2}{2\times3} \quad \frac{3\times2}{3\times3} \quad \frac{4\times2}{4\times3} \quad \frac{5\times2}{5\times3}$$

Note the pattern. The first fraction has 1 as factors in numerator and denominator, the second has 2 as factors, the third, 3 as factors, and so on. Do you see that the sixth fraction (although not shown) will have 6 as factors and will be

$$\frac{6\times2}{6\times3}=\frac{12}{18}$$

and that the seventh fraction will have 7 as factors and will be

$$\frac{7\times2}{7\times3}=\frac{14}{21}?$$

Now consider the fractions equivalent to $\frac{1}{2}$.

$$\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \frac{4}{8} \quad \frac{5}{10}\ldots$$

$$\frac{1\times1}{1\times2} \quad \frac{2\times1}{2\times2} \quad \frac{3\times1}{3\times2} \quad \frac{4\times1}{4\times2} \quad \frac{5\times1}{5\times2}\ldots$$

The sixth fraction will be $\frac{6\times1}{6\times2}=\frac{6}{12}$ the ninth will be $\frac{9\times1}{9\times2}$ and so on.

Self Test

1. Write the 12th fraction in the series of fractions equivalent to $\frac{1}{2}$.

2. Write the 8th fraction in the series of fractions equivalent to $\frac{1}{3}$.

Answers

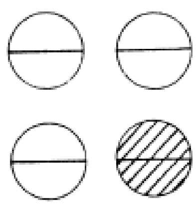
1. $\frac{12\times1}{12\times2}=\frac{12}{24}$
2. $\frac{8\times1}{8\times2}=\frac{8}{24}$

Exercises 2.2

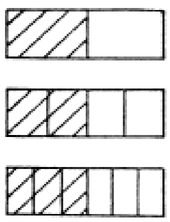
1. Each figure suggests two equivalent fractions. Give these fractions.

(a)

(b)



2. Give the three equivalent fractions that the figures suggests.



3. The fractions $\frac{3}{3}$, $\frac{17}{17}$, and $\frac{31}{31}$ are equivalent to ____________________.

4. Give the missing fractions, following the given pattern below.

$$\begin{array}{cccccc}
\frac{1\times4}{1\times5} & \frac{2\times4}{2\times5} & \underline{\qquad\qquad} & \frac{4\times4}{4\times5} & \underline{\qquad\qquad} & \frac{6\times4}{6\times5} \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\frac{4}{5} & \frac{8}{10} & \frac{12}{15} & \underline{\qquad} & \frac{20}{25} & \underline{\qquad}
\end{array}$$

5. Give the three indicated fractions for each group of equivalent fractions.

(a) $\frac{5}{6}$, $\frac{10}{12}$, $\frac{15}{18}$, ______ , ______ , ______ , …

(b) $\frac{3}{10}$, $\frac{6}{20}$, ______ , $\frac{12}{40}$, ______ , ______ , …

(c) $\frac{5}{8}$, $\frac{10}{16}$, $\frac{15}{24}$, ______ , ______ , $\frac{30}{48}$, ______ , …

(d) $\frac{1}{8}$, $\frac{2}{16}$, ______ , ______ , $\frac{5}{40}$, ______ , …

Turn to the Answer Key at the end of the module to check your work.

Lowest Terms

A fraction is in lowest terms if the numerator and the denominator have no common factor other than 1. For example, the fraction $\frac{8}{9}$. in lowest terms since 1 is the only common factor of 8 and 9.

$\frac{8\ \{1,2,4,8\}}{9\ \{1,3,9\}}$ No common factor other than 1.

If you have forgotten what a factor is, refer to Section 2.

The fraction $\frac{12}{15}$ is **not** in lowest terms since 1 and 3 are common factors of 12 and 15.

$\frac{12\ \{1,2,3,4,6,12\}}{15\ \{1,3,5,15\}}$ Common factors 1 and 3.

As further example, consider the fractions $\frac{16}{18}$ and $\frac{14}{15}$.

$\frac{16\ \{1,2,4,8,16\}}{18\ \{1,2,3,6,9,18\}}$ Common factors 1 and 2.
$\therefore$ not in lowest terms.

$\frac{14\ \{1,2,7,14\}}{15\ \{1,3,5,15\}}$ No common factors other than 1.
$\therefore$ in lowest terms.

Self Test

1. List the factors of 15.

____

2. List the factors of 20.

____

3. Is the fraction $\frac{15}{20}$ in lowest terms? Explain.

____

____

Answers

1. 1, 3, 5, 15
2. 1, 2, 4, 5, 10, 20
3. No, because common factors of 15 and 20 are 1 and 5.

We can build a series of equivalent fractions from a given lowest-terms fraction. For example, given the fraction $\frac{1}{3}$, we can determine a series of six equivalent fractions.

$$\frac{1\times1}{1\times3}, \quad \frac{2\times1}{2\times3}, \quad \frac{3\times1}{3\times3}, \quad \frac{4\times1}{4\times3}, \quad \frac{5\times1}{5\times3}, \quad \frac{6\times1}{6\times3}$$

$$= \quad \frac{1}{3}, \quad \frac{2}{6}, \quad \frac{3}{9}, \quad \frac{4}{12}, \quad \frac{5}{15}, \quad \frac{6}{18}$$

Note that we multiplied the numerator and the denominator of $\frac{1}{3}$ by 1, by 2, by 3, and so on. Similarly, given the fraction $\frac{1}{10}$, we can determine a series of four equivalent fractions.

$$\frac{1\times3}{1\times10}, \frac{2\times3}{2\times10}, \frac{3\times3}{3\times10}, \frac{4\times3}{4\times10} \longrightarrow \frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}$$

If we are given a series of equivalent fractions, we can determine the equivalent lowest-terms fraction. For example, in the series

$\frac{—}{}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots$ the lowest-term fraction is not given.

Given the series

$$\frac{—}{}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots$$
$$\rightarrow \frac{—}{}, \frac{2\times1}{2\times6}, \frac{3\times1}{3\times6}, \frac{4\times1}{4\times6}, \ldots$$

and we see that the lowest-terms fraction of this is $\frac{1}{6}$.

Similarly, since

$$\frac{—}{}, \frac{4}{18}, \frac{6}{27}, \frac{8}{36}, \ldots$$
$$\rightarrow \frac{—}{}, \frac{2\times2}{2\times9}, \frac{3\times2}{3\times9}, \frac{4\times2}{4\times9}, \ldots$$

we see that the lowest-terms fraction of this is $\frac{2}{9}$.

Self Test

1. From the fraction $\frac{1}{4}$ build a series of five equivalent fractions.

2. Find the lowest-terms fraction of this series. $—, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}$

Answers

1. $\frac{1\times 1}{1\times 4}, \frac{2\times 1}{2\times 4}, \frac{3\times 1}{3\times 4}, \frac{4\times 1}{4\times 4}, \frac{5\times 1}{5\times 4} \rightarrow \frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}$

2. $—, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \rightarrow —, \frac{2\times 2}{2\times 5}, \frac{3\times 2}{3\times 5}, \frac{4\times 2}{4\times 5}$

   The lowest-terms fraction is $\frac{2}{5}$.

Reducing Fractions to Lowest Terms

If we are given a fraction, we can find an equivalent lowest-terms fraction. This is called reducing the fraction to lowest terms.

We can reduce a fraction to lowest terms by ***dividing out*** common factors. For example, suppose that we wish to reduce $\frac{60}{80}$ to lowest terms. We can do this in two ways.

1. Divide out common factors until the fraction is in lowest terms.

$$\frac{60}{84}=\frac{60 \div 2}{84 \div 2}=\frac{30}{42}=\frac{30 \div 2}{42 \div 2}=\frac{15}{21}=\frac{15 \div 3}{21 \div 3}=\frac{5}{7}$$

OR

2. (a) Find the greatest common factor (GCF) of the numerator and the denominator.

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Factors of 84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

GCF of 60 and 84 is 12.

(b) Divide out the GCF.

$$\frac{60}{84}=\frac{60 \div 12}{84 \div 12}=\frac{5}{7}$$

If we know the GCF, all we need do is divide it out, as in 2 (b). If we do not know the GCF, the first method is perhaps the easier.

The following examples further illustrate the reducing of fractions to lowest terms.

$$\frac{20}{48}=\frac{20 \div 2}{48 \div 2}=\frac{10}{24}=\frac{10 \div 2}{24 \div 2}=\frac{5}{12}$$

$$\frac{9}{36}=\frac{9 \div 3}{36 \div 3}=\frac{3}{12}=\frac{3 \div 3}{12 \div 3}=\frac{1}{4}$$

Note that in each step, we must divide the numerator and the denominator by the same number.

Thus, $\frac{10}{24} = \frac{10 \div 2}{24} = \frac{5}{24}$ is incorrect, because we divided only the numerator by 2.

Also, $\frac{10}{24} = \frac{10 \div 2}{24 \div 3} = \frac{5}{8}$ is incorrect, because we divided the numerator and the denominator by different numbers.

Be very careful not to make these mistakes.

Exercises 2.3

1. (a) List the factors of 8.

   (b) List the factors of 12.

   (c) Is the fraction $\frac{8}{12}$ in lowest terms? Explain.

2. (a) List the factors of 15.

   (b) List the factors of 28.

   (c) Is the fraction $\frac{15}{28}$ in lowest terms? Explain.

3. Explain why each fraction is not in lowest terms.

   (a) $\frac{6}{10}$

   (b) $\frac{25}{30}$

4. Starting with lowest-term fraction, build a group of five equivalent fractions.

   (a) $\frac{1}{4}$

(b) $\frac{5}{8}$

5. Reduce each fraction to lowest terms.

(a) $\frac{9}{21}$

(b) $\frac{15}{70}$

(c) $\frac{14}{28}$

(d) $\frac{25}{75}$

(e) $\frac{24}{30}$

(f) $\frac{16}{30}$

(g) $\frac{15}{36}$

(h) $\frac{30}{48}$

(i) $\frac{6}{52}$

Turn to the Answer Key at the end of the module to check your work.

Review

1. A fraction is a number that describes a piece of something. In the fraction $\frac{2}{3}$ the 2 is the numerator and the 3 is the denominator. A fraction in which the numerator and the denominator are the same is equivalent to 1.

2. Equivalent fractions represent equal portions of a region or the same rational number.

3. A fraction may be changed to an equivalent fraction by multiplying its numerator and denominator by the same number.

4. A fraction is in lowest terms if its numerator and denominator have no common factor other than 1. A fraction may be reduced to lowest terms by dividing out common factors.

Lesson 3

Adding and Subtracting Fractions

Learning Outcomes

By the end of this section you will be better able to:

- add and subtract fractions with the same denominator
- add and subtract fractions with different denominators
- solve word problems using operations (+, –, ×, ÷) with fractions

Now we shall consider the addition and subtraction of rational numbers. First, however, let's briefly review some of the main points of the previous section in which introduced these numbers.

1. Equivalent fractions represent the same rational number. For example $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ each represent the same rational number.

$$\frac{4}{8} = \frac{1}{2} \qquad \frac{3}{6} = \frac{1}{2} \qquad \frac{2}{4} = \frac{1}{2}$$

2. A fraction may be reduced to lowest terms by dividing out common factors in the numerator and denominator. For example, $\frac{8}{10}$ may be reduced to lowest terms by dividing out the common factor 2.

$$\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

Write the following in lowest terms:

(a) $\frac{20}{35}$

(b) $\frac{28}{42}$

(c) $\frac{40}{50}$

Did you get the following answers?

(a) $\frac{20 \div 5}{35 \div 5} = \frac{4}{7}$

(b) $\frac{28 \div 14}{42 \div 14} = \frac{2}{3}$

(c) $\frac{40 \div 10}{50 \div 10} = \frac{4}{5}$

3. A fraction may be changed to an equivalent fraction by multiplying the numerator and denominator by the same number. For example, $\frac{2}{3}$ may be changed to the equivalent fraction $\frac{8}{12}$ by multiplying the numerator and denominator by 4.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Now it is time to learn the operations of addition and subtraction with fractions.

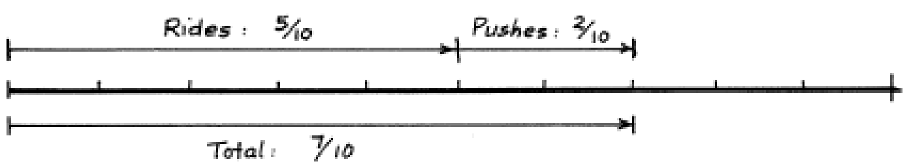
Addition of Fractions

Let us now see what we can discover about the addition of fractions. Consider the following.

Example 1

A boy rides a bicycle $\frac{5}{10}$ of a kilometre.

He then pushes the bicycle $\frac{2}{10}$ of a kilometre.

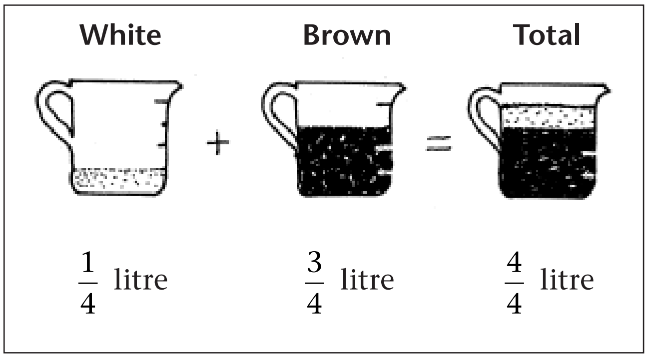


Total distance is $\frac{5}{10}$ km plus $\frac{2}{10}$ or $\frac{7}{10}$ km.

$$\frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

Example 2

A recipe calls for $\frac{1}{4}$ litre of white sugar and $\frac{3}{4}$ litre of brown sugar.



Total amount of sugar is $\frac{1}{4}$ litre plus $\frac{3}{4}$ litre or $\frac{4}{4}$ litre.

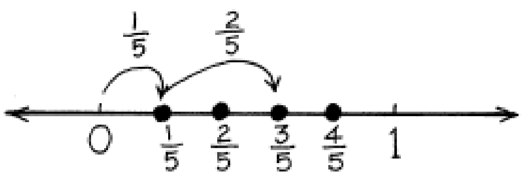
$$\frac{1}{4}+\frac{3}{4}=\frac{4}{4}$$

You will notice in each of the above examples, the denominators of the fractions are the same. We refer to this as like denominators.

Like Denominators

It is easy to add two or more fractions that have the same denominator. We simply add the numerators and keep the same denominator.

For example, let us add $\frac{1}{5}$ and $\frac{2}{5}$.



$$\frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}=\frac{3}{5}$$

Let's work through a few examples together.

Examples

Add the numerators.

(a) $\frac{2}{9}+\frac{5}{9}=\frac{2+5}{9}=\frac{7}{9}$

Keep the same denominator.

(b)
$$\begin{array}{r} \frac{3}{7} \\ +\ \frac{4}{7} \\ \hline \end{array}$$

$\frac{3+4}{7}=\frac{7}{7}=1$ $\left(\text{Reduce } \frac{7}{7} \text{ to } 1\right)$

(c) $\frac{5}{11}+\frac{4}{11}=\frac{5+4}{11}=\frac{9}{11}$

UnLike Denominators

This is when the denominators of at least two of the fractions are different.

Examples

(a) $\frac{1}{2}+\frac{3}{8}=?$

We need the least common multiple (LCM) of the two denominators, 2 and 8. This is called the least common denominator (LCD).

The multiples of 2 are 0, 2, 4, 6, 8, 10, ...

The multiples of 8 are 0, 8, 16, 24, 32, ...

The LCD of 2 and 8 is 8.

The least common denominator (LCD) of two numbers is the smallest nonzero common multiple. Zero is not an LCM.

From what we observed above, the common denominator is 8.We need a fraction that is equivalent to 12 that has a denominator of 8. $\frac{1}{2} = \frac{4}{8}$

$$\frac{1}{2}+\frac{3}{8}$$
$$=\frac{4}{8}+\frac{3}{8}$$
$$=\frac{7}{8}$$

(b) $\frac{3}{7}+\frac{2}{5}=?$

First, the multiples of 7 are 0, 7, 14, 21, 28, 35, ...

The multiples of 5 are 0, 5, 10, 15, 20, 25, 30, 35, ...

The LCD of 7 and 5 is 35.

$\frac{3}{7}=\frac{15}{35}$ and $\frac{2}{5}=\frac{14}{35}$

$$\frac{3}{7}+\frac{2}{5}=\frac{15}{35}+\frac{14}{35}=\frac{29}{35}$$

We need equivalent fractions that have a denominator of 35. If you want to review equivalent fractions, look at the previus lesson befor continuing.

Self Test

Add the following fractions. Show your work.

a. $\frac{1}{4} + \frac{2}{4}$

b. $\frac{1}{3} + \frac{1}{2}$

c. $\frac{3}{8} + \frac{1}{3}$

Answers

a. $\frac{1}{4} + \frac{2}{4}$

$= \frac{1+2}{4}$

$= \frac{3}{4}$

b. $\frac{1}{3} + \frac{1}{2}$

$= \frac{2}{6} + \frac{3}{6}$

$= \frac{5}{6}$

c. $\frac{3}{8} + \frac{1}{3}$

$= \frac{9}{24} + \frac{8}{24}$

$= \frac{17}{24}$

Subraction of Fractions

Like Denominators

This is very similar to the procedure used in addition.

Examples

Subtract the numerators.

(a) $\frac{7}{9} - \frac{3}{9} = \frac{7-3}{9} = \frac{4}{9}$

Keep the same denominator.

(b) $\frac{9}{10} - \frac{3}{10} = \frac{9-3}{10} = \frac{6}{10} = \frac{3}{5}$ $\left(\text{Reduce } \frac{6}{10} \text{ to } \frac{3}{5}\right)$

(c)
$$\begin{array}{r} \frac{7}{16} \\ -\ \frac{3}{16} \\ \hline \end{array}$$

$\frac{7-3}{16} = \frac{4}{16} = \frac{1}{4}$ $\left(\text{Reduce } \frac{4}{16} \text{ to } \frac{1}{4}\right)$

UnLike Denominators

Once again, it is necessary to find a common denominator first.

Examples

(a) $\frac{9}{12} - \frac{2}{3} = ?$

We need the LCM of the two denominators, 12 and 3.
This is called the least common denominator (LCD).

The multiples of 12 are 0, 12, 24, 36, 48, ...

The multiples of 3 are 0, 3, 6, 9, 12, 15, ...

The LCD of 12 and 3 is 12.

$$\frac{9}{12} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

(b) $\frac{4}{5} - \frac{1}{2} = ?$ The LCD of 5 and 2 is 10.

$$\frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$$

Self Test

Subtract the following fractions. Show your work.

(a) $\frac{5}{6} - \frac{1}{6}$

(b) $\frac{7}{12} - \frac{1}{4}$

(c) $\frac{11}{15} - \frac{2}{3}$

Answers

(a) $\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

(b) $\frac{7}{12} = \frac{7}{12}$, $\frac{1}{4} = \frac{3}{12}$; $\frac{7}{12} - \frac{3}{12} = \frac{4}{12} = \frac{1}{3}$

(c) $\frac{11}{15} = \frac{11}{15}$, $\frac{2}{3} = \frac{10}{15}$; $\frac{11}{15} - \frac{10}{15} = \frac{1}{15}$

Did you remember to reduce your fractions?

Exercises 3.1

1. Add. Reduce your answers to simplest terms.

(a) $\begin{array}{r} \frac{2}{7} \\ + \frac{4}{7} \\ \hline \end{array}$

(b) $\begin{array}{r} \frac{3}{10} \\ + \frac{2}{10} \\ \hline \end{array}$

(c) $\begin{array}{r} \frac{7}{9} \\ + \frac{2}{9} \\ \hline \end{array}$

(d) $\begin{array}{r} \frac{1}{6} \\ + \frac{3}{6} \\ \hline \end{array}$

(e) $\begin{array}{r} \frac{2}{3} \\ + \frac{1}{6} \\ \hline \end{array}$

(f) $\begin{array}{r} \frac{1}{4} \\ + \frac{3}{8} \\ \hline \end{array}$

(g) $\frac{1}{5}+\frac{3}{10}=$

(h) $\frac{1}{6}+\frac{1}{2}=$

(i) $\frac{2}{5}+\frac{1}{10}=$

2. Subtract. Reduce your answer to simplest terms.

(a) $\begin{array}{r} \frac{7}{9} \\ -\ \frac{2}{9} \\ \hline \end{array}$

(b) $\begin{array}{r} \frac{8}{10} \\ -\ \frac{3}{10} \\ \hline \end{array}$

(c) $\begin{array}{r} \frac{7}{12} \\ -\ \frac{3}{12} \\ \hline \end{array}$

(d) $\begin{array}{r} \frac{5}{6} \\ -\ \frac{5}{6} \\ \hline \end{array}$

(e) $\dfrac{5}{6} - \dfrac{3}{6}$

(f) $\dfrac{7}{9} - \dfrac{1}{3}$

(g) $\dfrac{8}{12} - \dfrac{1}{4}$

(h) $\dfrac{5}{10} - \dfrac{1}{2}$

(i) $\dfrac{9}{10} - \dfrac{2}{5}$

(j) $\dfrac{5}{8} - \dfrac{1}{4}$

Turn to the Answer Key at the end of the module to check your work.

Summary

1. To find the sum (or difference) of two fractions that have the same denominator, we find the sum (or difference) of the numerators and leave the denominators unchanged.

$$\frac{1}{7}+\frac{2}{7}=\frac{3}{7}$$

$$\frac{5}{9}-\frac{1}{9}=\frac{4}{9}$$

2. To find the sum (or difference) of two fractions that have different denominators, we must consider two equivalent fractions that have common denominators. The smallest of these common denominators is called the least common denominator of the two fractions.

$$\frac{1}{3}+\frac{1}{2}=\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$$

$$\frac{1}{3}-\frac{1}{4}=\frac{4}{12}-\frac{3}{12}=\frac{1}{12}$$

3. The least common denominator of two fractions is the LCM of the two denominators.

Solving Problems with Fractions

Word Problems and Fractions

Before we start using fractions in word problems, try these exercises to review adding and subtracting with fractions. Remember that you will need a common denominator for each of these questions.

Self Test

1. Add.

(a) $\frac{5}{9} + \frac{1}{9} =$

(b) $\frac{7}{8} + \frac{1}{3} =$

(c) $\frac{2}{3} + \frac{3}{4} =$

(d) $2\frac{1}{2} + 1\frac{3}{4} =$

(e) $11\frac{1}{6} + 9\frac{7}{18} =$

2. Subtract.

(a) $\frac{5}{6} - \frac{1}{6} =$

(b) $\frac{2}{3} - \frac{1}{4} =$

(c) $7 - \frac{1}{5} =$

Answers

1. (a) $\frac{6}{9} = \frac{2}{3}$ (b) $\frac{29}{24} = 1\frac{5}{24}$ (c) $\frac{17}{12} = 1\frac{5}{12}$

(d) $3\frac{5}{4} = 4\frac{1}{4}$ (e) $20\frac{10}{18} = 20\frac{5}{9}$

2. (a) $\frac{4}{6} = \frac{2}{3}$ (b) $\frac{5}{12}$ (c) $6\frac{4}{5}$

In this part of the lesson we'll consider some word problems involving the operations of subtraction and addition.

Work through Examples 1 and 2 and their solutions.

Example 1

A cake recipe asks for 2 eggs and the frosting recipe for the cake asks for 1egg.

What part of a dozen eggs is needed:

(a) For the cake alone?

(b) For the frosting alone?

(c) For the cake and the frosting?

Solution

(a) The cake requires 2 eggs or $\frac{2}{12} = \frac{1}{6}$ dozen.

(b) The frosting requires 1 egg or $\frac{1}{12}$ dozen.

(c) The cake and the frosting require $\frac{1}{4}$ dozen, since

2 eggs + 1 egg = 3 eggs or $\frac{3}{12} = \frac{1}{4}$ dozen,

or $\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ dozen.

Example 2

John had to measure out juices for a punch he was making. This is a list of the juices.

Apple juice	$\frac{1}{2}$ litre
Orange juice	$\frac{1}{3}$ litre
Pineapple juice	$\frac{3}{4}$ litre
Cherry juice	$\frac{1}{4}$ litre

If he had a 2-litre measure, how much space would be left below the 2-litre mark if he put the apple, orange, pineapple, and cherry juice in the measure at the same time?

Solution

$$\begin{array}{rl} & \frac{1}{2} = \frac{6}{12} \\ & \frac{1}{3} = \frac{4}{12} \\ & \frac{3}{4} = \frac{9}{12} \\ + & \frac{1}{4} = \frac{3}{12} \\ \hline & \frac{22}{12} = 1\frac{10}{12} = 1\frac{5}{6} \end{array}$$

$$\text{space} = 2 - 1\frac{5}{6} = \frac{1}{6} \text{ litre}$$

There would be $\frac{1}{6}$ litre of space left.

Notice that it is always necessary to write a sentence as a summary in each question.

Self Test

A frog jumped $2\frac{7}{8}$ m. It then jumped $1\frac{3}{4}$ m.

1. How far did it jump in the two jumps?

2. How much further was the first jump than the second?

Answers

1.
$$\begin{array}{rl} & 2\frac{7}{8} = 2\frac{7}{8} \\ + & 1\frac{3}{4} = 1\frac{6}{8} \\ \hline & \quad 3\frac{13}{8} = 3 + 1\frac{5}{8} = 4\frac{5}{8} \end{array}$$

The frog jumped $4\frac{5}{8}$ m in its two jumps.

2.
$$\begin{array}{rl} & 2\frac{7}{8} = 2\frac{7}{8} \\ - & 1\frac{3}{4} = 1\frac{6}{8} \\ \hline & \qquad 1\frac{1}{8} \end{array}$$

The first jump was $1\frac{1}{8}$ m longer than the second.

First read the problem carefully to decide what is required. Then write down the necessary information and find the sum or difference. Answer the question with a sentence.

Exercises 3.2

1. Add.

(a) $\frac{1}{4} + \frac{1}{4} =$

(b) $\frac{1}{5} + \frac{3}{10} =$

(c) $\frac{3}{8} + \frac{2}{5} =$

(d) $1\frac{3}{5} + 2\frac{1}{5} =$

(e) $2 + 3\frac{2}{5} =$

(f) $4\frac{5}{8} + 6\frac{1}{3} =$

2. Subtract.

(a) $\frac{4}{5} - \frac{1}{5} =$

(b) $\frac{11}{20} - \frac{7}{20} =$

(c) $\frac{7}{15} - \frac{1}{5} =$

(d) $\frac{13}{20} - \frac{5}{8} =$

(e) $2\frac{3}{4} - 1\frac{1}{3} =$

(f) $1\frac{3}{4} - \frac{7}{10} =$

3. Cassandra took $\frac{3}{4}$ h to walk to school. Joe rides to school on his bicycle in $\frac{1}{3}$ h.

   (a) Who takes longer to go to school?

   (b) How much longer?

4. On Saturday, it snowed for $4\frac{1}{2}$ h. On Sunday it snowed for $1\frac{3}{4}$ h. How many more hours did it snow on Saturday than on Sunday?

5. It took Paula $8\frac{1}{2}$ min to get up the ski slope. It took $1\frac{3}{4}$ min to ski down. How much longer did it take her to go up the slope?

6. In Allan's record collection, $\frac{1}{2}$ of the records are rock music and $\frac{1}{3}$ are jazz. What fraction of his collection is either rock or jazz?

Turn to the Answer Key at the end of the module to check your work.

Lesson 4

Multiplying and Dividing Fractions

Learning Outcomes

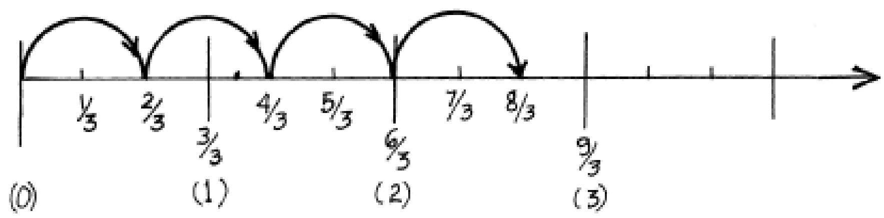
By the end of this section you will be better able to:

- multiply and divide with fractions
- solve word problems using operations (+, –, ×, ÷) with fractions

You may have already learned about multiplication and division with fractions in other math courses. We'll review those skills before we look at some word problems.

Diagrams can be used to help you understand the skill of multiplying fractions.

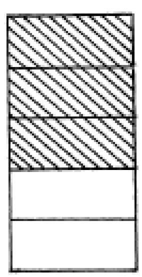
The diagram below shows why $4 \times \frac{2}{3} = \frac{8}{3}$



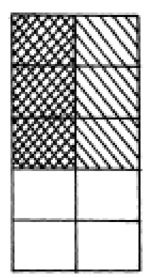
Another way of illustrating multiplying fractions is shown below.

The diagrams show why $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

Step 1: Show $\frac{3}{5}$



Step 2: Show $\frac{1}{2} \times \frac{3}{5}$



Drawing diagrams to find answers is very time-consuming. There is a very simple rule for multiplying fractions:

To multiply fractions, multiply the numerators and multiply the denominators.

Example 1

Find the product.

$$\frac{1}{5} \times \frac{3}{4}$$

Solution

$$\frac{1}{5}\times\frac{3}{4}=\frac{1\times3}{5\times4}=\frac{3}{20}$$

Example 2

Find the product.

$$\frac{3}{4} \times \frac{2}{9}$$

Solution

$$\frac{3}{4}\times\frac{2}{9}=\frac{3\times2}{4\times9}=\frac{6}{36}=\frac{1}{6}$$ (Express your answer in lowest terms.)

In some questions, we can simplify your work before we multiply. This method involves dividing out common factors to make our work easier. This is called **cancellation**.

Example 3

Consider $\frac{4}{15}\times\frac{10}{28}$.

Solution

$\frac{\cancel{4}^{1}}{15}\times\frac{10}{\cancel{28}_{7}}$ Divide out 4

$\frac{\cancel{4}^{1}}{\cancel{15}_{3}}\times\frac{\cancel{10}^{2}}{\cancel{28}_{7}}$ Divide out 5

$\frac{\cancel{4}^{1}}{\cancel{15}_{3}}\times\frac{\cancel{10}^{2}}{\cancel{28}_{7}}=\frac{2}{21}$ Multiply

This same procedure can be followed if one of the factors is a whole number.

Example 4

$\cancel{24}^{8} \times \frac{7}{\cancel{9}_{3}}$ Divide out 3

$\cancel{24}^{8} \times \frac{7}{\cancel{9}_{3}} = \frac{56}{3}$ Multiply

Self Test

Find the products. First divide out common factors.

1. $\frac{4}{9} \times \frac{3}{8}$

2. $\frac{5}{6} \times \frac{12}{5}$

Answers

1. $\frac{4}{9} \times \frac{3}{8} = \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} = \frac{1}{6}$

2. $\frac{5}{6} \times \frac{12}{5} = \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}} \times \frac{\overset{2}{\cancel{12}}}{\underset{1}{\cancel{5}}} = 2$

The skills we learn to multiply fractions extend to multiplying mixed numbers, as is shown in the next example.

First, write each mixed number as an improper fraction. Then continue with multiplication, just as you did in the previous questions.

Example 5

Find the product of $1\frac{3}{4} \times 2\frac{1}{2}$.

Solution

$$1\frac{3}{4} \times 2\frac{1}{2} = \frac{7}{4} \times \frac{5}{2} = \frac{7 \times 5}{4 \times 2} = \frac{35}{8} \text{ or } 4\frac{3}{8}$$

Self Test

Give the fraction for (a) and the mixed number for (b).

1. $1\frac{1}{2} \times 1\frac{2}{3} = \frac{3}{2} \times \frac{5}{3} =$ (a) $\qquad =$ (b)

2. $2\frac{1}{2} \times 1\frac{1}{4} = \frac{5}{2} \times \frac{5}{4} =$ (a) $\qquad =$ (b)

Answers

1. (a) $\frac{15}{6}$ (b) $2\frac{1}{2}$
2. (a) $\frac{25}{8}$ (b) $3\frac{1}{8}$

Exercises 4.1

1. Find the products. Give your answers in lowest terms.

   (a) $\frac{4}{7} \times \frac{1}{5}$

   (b) $9 \times \frac{1}{5}$

   (c) $\frac{1}{9} \times \frac{1}{10}$

   (d) $\frac{1}{3} \times 3$

   (e) $6 \times \frac{9}{10}$

   (f) $\frac{1}{4} \times \frac{5}{6}$

2. Find the products. First divide out common factors.

   (a) $\frac{5}{12} \times \frac{3}{10}$

   (b) $\frac{6}{5} \times \frac{15}{18}$

   (c) $\frac{3}{8} \times \frac{8}{9}$

   (d) $\frac{3}{4} \times \frac{20}{27}$

   (e) $\frac{7}{10} \times \frac{100}{21}$

   (f) $\frac{7}{8} \times \frac{12}{14}$

3. Find the products. First change each mixed number to an improper fraction. Give answers in lowest terms.

(a) $3\frac{1}{2} \times 1\frac{3}{4}$

(b) $3\frac{1}{5} \times 2$

(c) $\frac{3}{4} \times 2\frac{1}{2}$

(d) $1\frac{1}{8} \times 2\frac{1}{3}$

(e) $\frac{1}{6} \times 1\frac{2}{5}$

Turn to the Answer Key at the end of the module to check your work.

Dividing Fractions

You have probably used fact families—groups of related math facts—before.

For example:

$6 \div 3 = 2$ because $2 \times 3 = 6$

This discussion about fractions uses that same idea.

$$1 \div \frac{2}{5} = \frac{5}{2} \quad \text{because} \quad \frac{5}{2} \times \frac{2}{5} = 1$$

$$1 \div \frac{2}{3} = \frac{3}{2} \quad \text{because} \quad \frac{3}{2} \times \frac{2}{3} = 1$$

$$1 \div \frac{19}{21} = \frac{21}{19} \quad \text{because} \quad \frac{21}{19} \times \frac{19}{21} = 1$$

What do you notice about each of the problems? How are the quotients related to the fractions we divided by?

Note that in each problem we are dividing the number 1 by a fraction. Also notice that, in each case, the quotient is the reciprocal of the fraction we divided by.

1 ÷ a fraction = reciprocal of the fraction

Be certain to remember this fact. It is very easy to divide 1 by a fraction. The answer is the reciprocal of the fraction, and the reciprocal of a fraction is simply the fraction turned upside down.

Do you also remember that you can use this fact to help find other quotients? Review the following questions.

(a) $1 \div \frac{3}{4} = \frac{4}{3}$

$5 \div \frac{3}{4} = 5 \times \frac{4}{3} = \frac{20}{3}$

(b) $1 \div \frac{4}{9} = \frac{9}{4}$

$\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \times \frac{9}{4} = \frac{18}{12}$

Self Test

1. $1 \div \frac{3}{2} =$ ________ $\quad 6 \div \frac{3}{2} = 6 \times$ ________

2. $1 \div \frac{3}{4} =$ ________ $\quad \frac{7}{6} \div \frac{3}{4} = \frac{7}{6} \times$ ________

3. $1 \div \frac{5}{2} =$ ________ $\quad 9 \div \frac{5}{2} =$ ________ $\times \frac{2}{5}$

4. $1 \div \frac{5}{6} =$ ________ $\quad \frac{4}{9} \div \frac{5}{6} =$ ________ $\times \frac{6}{5}$

Answers

1. $\frac{2}{3}$ $\frac{2}{3}$

2. $\frac{4}{3}$ $\frac{4}{3}$

3. $\frac{2}{5}$ 9

4. $\frac{6}{5}$ $\frac{4}{9}$

To divide by a fraction, we multiply by the reciprocal of the fraction.

This rule also applies to division by a whole number, since a whole number such as 9 can be written as $\frac{9}{1}$.

$$\frac{4}{3} \div 9 = \frac{4}{3} \times \frac{1}{9} = \frac{4}{27}$$

$$\frac{3}{8} \div 2 = \frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$$

Self Test

Find the missing values.

1. $6 \div \frac{5}{9} = n \times \frac{9}{5}$ $n =$

2. $\frac{3}{5} \div \frac{1}{2} = n \times 2$ $n =$

3. $\frac{4}{5} \div 5 = \frac{4}{5} \times n$ $n =$

4. $\frac{7}{4} \div \frac{4}{5} = \frac{7}{4} \times n$ $n =$

Answers

1. 6
2. $\frac{3}{5}$
3. $\frac{1}{5}$
4. $\frac{5}{4}$

Exercises 4.2

1. Find the quotients. Use multiplication by reciprocals. Reduce your answers to simplest form.

(a) $\frac{3}{8} \div \frac{1}{2}$

(b) $\frac{1}{4} \div \frac{2}{3}$

(c) $\frac{5}{3} \div \frac{5}{4}$

(d) $\frac{4}{7} \div 8$

(e) $6 \div \frac{2}{3}$

(f) $\frac{4}{9} \div 6$

(g) $10 \div \frac{3}{2}$

(h) $\frac{6}{5} \div \frac{7}{2}$

Turn to the Answer Key at the end of the module to check your work.

Division and Mixed Numbers

We know that in order to divide by a fraction, we multiply by the reciprocal of the fraction. If we wish to divide by a mixed number, we first change the mixed number to an improper fraction. For example, suppose that we wish to find the quotient, $\frac{3}{8} \div 2\frac{3}{4}$.

$$2\frac{3}{4} = \frac{11}{4}$$

$$\therefore \quad \frac{3}{8} \div 2\frac{3}{4} = \frac{3}{8} \div \frac{11}{4}$$

$$= \frac{3}{8} \times \frac{4}{11} = \frac{12}{88} = \frac{3}{22}$$

Note that we reduced the answer to lowest terms. Previously we showed that in finding certain products, we can use a short cut called cancellation to make our work easier. In this case, we can divide by 4.

$$\frac{3}{\cancel{8}_2} \times \frac{\cancel{4}^1}{11} = \frac{3}{22}$$

If we wish to divide one mixed number by another mixed number, we change both mixed numbers to improper fractions. Suppose that we wish to divide $2\frac{1}{2}$ by $1\frac{3}{4}$.

$$2\frac{1}{2} = \frac{5}{2} \qquad 1\frac{3}{4} = \frac{7}{4}$$

$$2\frac{1}{2} \div 1\frac{3}{4} = \frac{5}{2} \div \frac{7}{4}$$

$$= \frac{5}{\cancel{2}_1} \times \frac{\cancel{4}^2}{7}$$ Multiply by reciprocal and simplify.

$$= \frac{10}{7} = 1\frac{3}{7}$$

As another example, consider the quotient $2\frac{1}{9} \div 1\frac{5}{6}$.

$$2\frac{1}{9} = \frac{19}{9} \qquad 1\frac{5}{6} = \frac{11}{6}$$

$$2\frac{1}{9} \div 1\frac{5}{6} = \frac{19}{9} \div \frac{11}{6}$$

$$= \frac{19}{\cancel{9}_3} \times \frac{\cancel{6}^2}{11}$$ Multiply by reciprocal and simplify

$$= \frac{38}{33}$$

$$= 1\frac{5}{33}$$ Reduced

Self Test

1. Change to improper fractions.

   (a) $2\frac{5}{8}$ (b) $1\frac{2}{7}$

2. Find the quotient using the improper fractions of these mixed numbers.

   $2\frac{5}{8} \div 1\frac{2}{7}$

Answers

1. (a) $\frac{21}{8}$ (b) $\frac{9}{7}$

2. $\frac{21}{8} \div \frac{9}{7} = \frac{\overset{7}{\cancel{21}}}{8} \times \frac{7}{\underset{3}{\cancel{9}}} = \frac{49}{24} = 2\frac{1}{24}$

Exercises 4.3

1. Find the quotients.

(a) $1\frac{1}{2} \div \frac{3}{4}$

(b) $3\frac{1}{4} \div \frac{5}{6}$

(c) $3\frac{1}{2} \div 2\frac{1}{4}$

(d) $5\frac{2}{3} \div 6\frac{3}{4}$

(e) $\frac{4}{5} \div 2\frac{2}{3}$

(f) $1\frac{3}{8} \div 4\frac{1}{3}$

(g) $2 \div 1\frac{1}{4}$

(j) $1\frac{1}{4} \div 1\frac{2}{3}$

(h) $3\frac{1}{3} \div 1\frac{3}{10}$

(i) $2\frac{2}{5} \div \frac{4}{15}$

Turn to the Answer Key at the end of the module to check your work.

Division and Word Problems

The following are a few examples of word problems that can be solved by using division of fractions.

Example 1

Write and solve an equation to find how many $\frac{1}{4}$ hour periods there are in 8 hours.

Divide the 8 hours into $\frac{1}{4}$ hour periods. If we let q be the number of periods,

$$q = 8 \div \frac{1}{4}.$$

$$q = 8 \div \frac{1}{4}$$

$$= 8 \times \frac{4}{1} = 32$$

There are 32 $\frac{1}{4}$ hour periods in 8 hours.

As a check, we note that 32 $\frac{1}{4}$ hour periods make up

$$32 \times \frac{1}{4} = \frac{32}{4} = 8 \text{ hours.}$$

Example 2

John walks at a rate of $\frac{7}{3}$ km/h. How long does it take him to walk 1 km?

In one hour John walks $\frac{7}{3}$ km. Divide the distance he walks into $\frac{7}{3}$ km sections; each section will take him 1 hour. Let t be the number of sections,

$$t = \text{distance} \div \frac{7}{3} = 1 \div \frac{7}{3}.$$

$$t = 1 \div \frac{7}{3}$$

$$= 1 \times \frac{3}{7} = \frac{3}{7}$$

It takes him $\frac{3}{7}$ of an hour to walk 1 km.

As a check, we note that John walks for $\frac{3}{7}$ of an hour at $\frac{7}{3}$ km/h or $\frac{3}{7} \times \frac{7}{3} = 1$ km

Example 3

How many pieces of paper, each $\frac{1}{8}$ as large as a square, does it take to cover $\frac{1}{4}$ of a square?

We think of dividing $\frac{1}{4}$ of a square into $\frac{1}{8}$'s.

$$\begin{aligned}\text{The number of pieces} &= \frac{1}{4} \div \frac{1}{8} \\ &= \frac{1}{4} \times \frac{8}{1} = 2\end{aligned}$$

As a check, we note that 2 pieces, each covering $\frac{1}{8}$ of a square, will cover $2 \times \frac{1}{8}$ or $\frac{1}{4}$ of a square.

Example 4

16 candy bars are divided equally among 5 people. How many does each person receive?

We think of dividing 16 bars into 5 equal groups.

Each receives $16 \div 5 = 16 \times \frac{1}{5} = 3\frac{1}{5}$ bars.

As a check, 5 people each receive $3\frac{1}{5}$ bars, for a total of $5 \times 3\frac{1}{5} = 16$ bars.

Carefully study these examples. Note that in each case we are dividing a given quantity into a number of groups. In Examples 1 to 3, we divided by the number representing the size of the groups. In Example 4 we divided by the number of groups. As shown in these examples, it is usually quite easy to check your answer by multiplication.

In some cases, we may find it helpful to make a rough sketch to illustrate the problem. Thus, in Example 1, we could draw a clock face and on it mark the 8 hours and the $\frac{1}{4}$ hour periods.

As with all word problems, it is important to read these problems carefully.

Estimation

We can estimate the answer to a problem such as 83 ÷ 19 by using approximate values. The ≐ symbol means "is approximately equal to".

$$83 \div 19 \doteq 80 \div 20 = 4$$

In a similar way, we can estimate the answer ≐ to a problem such as $8\frac{1}{4} \times 3\frac{3}{8}$ by considering the nearest whole numbers.

$$8\frac{1}{4} \times 3\frac{3}{8} \doteq 8 \times 3 = 24$$

Example

A length of wire is cut into four pieces. The original length of the wire was 19.7 m.

Estimate the length of each piece.

$$\text{Length} = 19.7 \div 4 \doteq 20 \div 4 = 5$$

Each piece is approximately 5 m long.

Exercises 4.4

Solve the problems by using division. Check your answers by multiplication.

1. A man walks at a speed of $\frac{7}{3}$ km/ h. How long does it take him to walk 12 km?

2. A rope $5\frac{1}{4}$ m long is cut into 7 pieces of equal length. How long is each of the pieces?

3. (a) A litre of nuts weighs $\frac{2}{5}$ kg. How many litres of nuts are in a pile weighing 10 kg?

(b) Each bag of nuts weighs $\frac{2}{5}$ kg. How many bags of nuts are in 1kg?

4. (a) How many $\frac{1}{2}$ hour periods are there in 5 hours?

(b) How many $\frac{1}{2}$ hour periods are there in $\frac{3}{4}$ hour?

Turn to the Answer Key at the end of the module to check your work.

Review

The main points are as follows.

1. To multiply fractions, we multiply the numerators and we multiply the denominators.

$$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

2. In finding the products, we can sometimes divide out common factors to make our work easier.

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{1}{2}$$

3. In multiplying mixed numbers, it is sometimes helpful to first change them to improper fractions.

$$1\frac{1}{3} \times 2\frac{1}{4} = \frac{4}{3} \times \frac{9}{4} = \frac{4 \times 9}{3 \times 4} = \frac{36}{12} = 3$$

4. Division problems involving fractions can be solved by solving the equivalent multiplication problems.

You find this quotient ↓ $n \times \frac{1}{2} = 1$

$1 \div \frac{1}{2} = n$ ↑ when you find this factor.

5. 1 ÷ (a fraction) = (reciprocal of the fraction)

$$1 \div \frac{3}{4} = \frac{4}{3}$$

6. To divide by a fraction, we multiply by the reciprocal of the fraction.

$$\frac{2}{3} \div \frac{1}{5} = \frac{2}{3} \times \frac{5}{1} = \frac{10}{3} = 3\frac{1}{3}$$

7. To divide one mixed number by another, we change both mixed numbers to improper fractions.

$$1\frac{1}{3} \div 2\frac{1}{5} = \frac{4}{3} \div \frac{11}{5} = \frac{4}{3} \times \frac{5}{11} = \frac{20}{33}$$

Lesson 5

Powers

Learning Outcomes

By the end of this section you will be better able to:

- understand powers with whole number bases and whole number exponents
- show that a any number to the power of zero is equal to one
- solve problems involving powers

Exponents and Powers

To save time and space, we can write a number such as 10 000 as a **power** of 10. Because 10 000 = 10 × 10 × 10 × 10, we write 10 000 = 10^4. We call 10^4 either 10 to the fourth power or the fourth power of 10

The raised 4 is called the **exponent**.

10^4

The 10 is called the **base**.

When we write 10 000 as 10^4, we say that it is in **exponential form**. We call it this because the numeral consists of a base and an exponent.

What will 100 be in exponential form?

Because 100 = 10 × 10, 100 = 10^2. We call 10^2 either *10 to the second power* or *10 squared*.

What will 1000 be in exponential form?

1000 = 10 × 10 × 10, so 1000 = 10^3. We call 10^3 either *10 to the third power* or *10 cubed*. Notice that the exponent tells us how many zeros are in the number we start with.

Thus 10^5 = 100 000 and 10^1 = 10.

Here is a chart for you to study. Cover parts of it to see if you can complete the chart in your mind.

Exponential Form	Factors	Decimal Form
10^1	$= 10$	$= 10$
10^2	$= 10 \times 10$	$= 100$
10^3	$= 10 \times 10 \times 10$	$= 1000$
10^4	$= 10 \times 10 \times 10 \times 10$	$= 10\ 000$
10^5	$= 10 \times 10 \times 10 \times 10 \times 10$	$= 100\ 000$

In a similar way we can also use exponents with bases other than 10. Study these examples.

Exponential Form	Factors	Product	What We Say
2^1	$= 2$	$= 2$	two to the first power
2^2	$= 2 \times 2$	$= 4$	two squared
2^3	$= 2 \times 2 \times 2$	$= 8$	two cubed
3^1	$= 3 \times 3$	$= 9$	three squared
3^5	$= 3 \times 3 \times 3 \times 3 \times 3$	$= 243$	three to the fifth power
300^2	$= 300 \times 300$	$= 90\ 000$	three hundred squared

Self Test

1. 10^5 =

2. 5^2 =

3. 3^3 =

4. 9^3 =

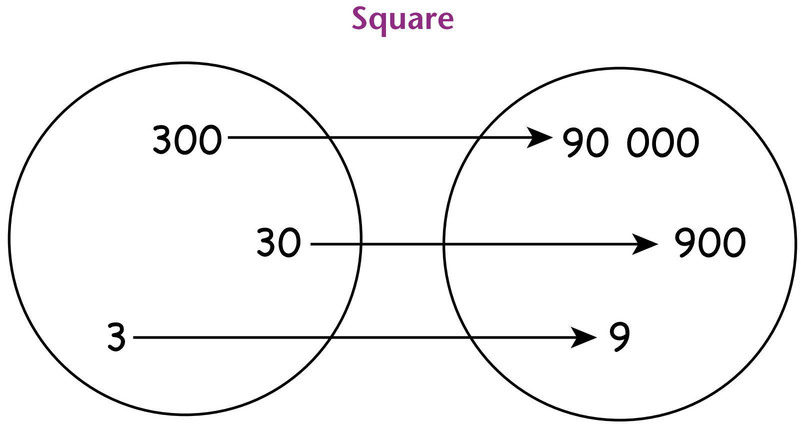
5. 6^3 =

6. 2^4 =

Answers

1. 100 000
2. 5 × 5 = 25
3. 3 × 3 × 3 = 27
4. 9 × 9 × 9 = 729
5. 6 × 6 × 6 = 216
6. 2 × 2 × 2 × 2 = 16

Here are some examples for you to study. See if you can spot some short-cuts for arriving at the answers below the diagram.



300 *squared* means $300^2 = 300 \times 300 = 90\,000$

When you square a number like 300 or 30, count the number of zeros in the number. The answer will have twice this many.

For example, $400^2 = 160\,000$

$5000^2 = 25\,000\,000$

$60^2 = 3600$

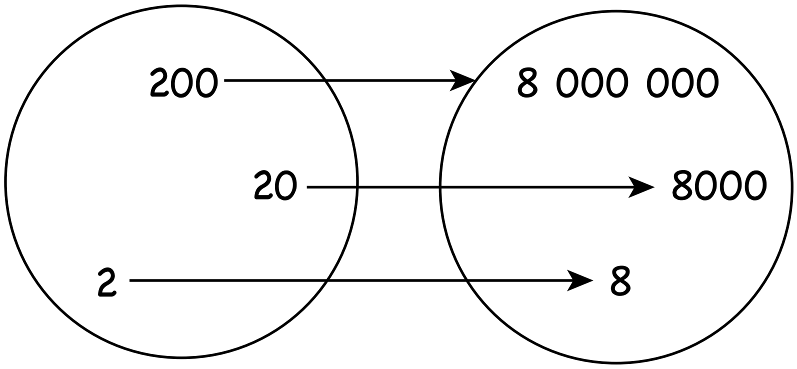
Question: How many zeros in the answer?

$\mathbf{50^2}$ $\mathbf{200^2}$

Did you get 2 and 4?

Now look at these examples. Find two short-cuts, similar to those on the previous page, for arriving at the answers below the diagram.

Cube



200 *cubed* means $200^3 = 200 \times 200 \times 200 = 8\ 000\ 000$

When you cube a number like 200 or 20, count the number of zeros in the number. The answer will have three times this many. For example,
$400^3 = 64\ 000\ 000$, $500^3 = 125\ 000\ 000$, and $3000^3 = 27\ 000\ 000\ 000$.

Try these:

a. $600^2 =$ ____________________ b. $20^3 =$ ____________________

c. $40^2 =$ ____________________ d. $500^2 =$ ____________________

The answers are:

a. 360 000 b. 8000
c. 1600 d. 250 000

Exercises 5.1

Evaluate.

1. $10^2 =$
2. 200^2
3. $10^3 =$
4. 500^3
5. $30^2 =$
6. 300^2
7. $2^5 =$
8. 6^3

Turn to the Answer Key at the end of the module to check your work.

Exponents and Variables

You will often meet mathematical sentences and phrases which contain variables with exponents. The following examples will show you how to read such phrases.

First of all, let's review some vocabulary:

Base: the number to be multiplied by itself.

Exponent: the small raised number telling how many times to multiply the base by itself.

Variable: a name for the letter or unknown in an expression.

exponent	exponent
2^3	x^2
..... base	.... variable (also the base)

Examples

(a) a^2 This is read *a squared*.
It means $a \times a$.

(b) x^3 This is read *x cubed*.
It means $x \bullet x \bullet x$.

(c) $(ab)^2$ This is read *ab times ab or ab all squared*.
It means $ab \times ab = (a \times b) \times (a \times b)$

You will often be asked to evaluate mathematical phrases which contain variables and exponents. Here are some examples for you to study.

Examples

(a) Evaluate a^3 for $a = 4$.

$$\begin{aligned} a^3 &= 4 \times 4 \times 4 \\ &= 64 \end{aligned}$$

(b) Evaluate $(3a)^2$ for $a = 2$.

$$\begin{aligned}(3a)^2 &= (3 \times 2)^2 \\ &= (6)^2 \\ &= 6 \times 6 \\ &= 36\end{aligned}$$

(c) Evaluate $(2c)^3$ for $c = 2$.

$$\begin{aligned}(2c)^3 &= (2 \bullet 2)^3 \\ &= (4)^3 \\ &= 4 \times 4 \times 4 \\ &= 64\end{aligned}$$

(d) Evaluate $(ab)^2$ for $a = 2, b = 3$.

$$\begin{aligned}(ab)^2 &= (2 \bullet 3)^2 \\ &= (6)^2 \\ &= 6 \times 6 \\ &= 36\end{aligned}$$

Self Test

Evaluate the following for r = 2, s = 3, t = 5.

(a) $3r =$

(b) $r^2 =$

(c) $4s^2 =$

(d) $(3t)^2 =$

(e) $(rt)^2 =$

(f) $(rst)^2 =$

Answers

(a) $3 \times 2 = 6$

(b) $2^2 = 2 \times 2 = 4$

(c) $4 \times 3^2 = 4 \times 9 = 36$

(d) $(3 \times 5)^2 = 15^2 = 225$

(e) $(2 \bullet 5)^2 = (10)^2 = 10 \times 10 = 100$

(f) $(2 \bullet 3 \bullet 5)^2 = (30)^2 = 30 \times 30 = 900$

Exercises 5.2

1. Evaluate.

   a. 2^3

   b. 1^5

   c. 10^3

   d. 6^3

   e. 10^6

2. Evaluate each of the following for $a = 3$, $b = 4$, $c = 10$.

(a) $3b =$

(b) $c^2 =$

(c) $a^3 =$

(d) $(2a)^2 =$

(e) $(3a)^2 =$

(f) $(4c)^2 =$

(g) $b^3 =$

(h) $(ac)^2 =$

(i) $(2a)^3 =$

(j) $(ab)^2 =$

Turn to the Answer Key at the end of the module to check your work.

Multiplying with Exponents

Multiplying with powers of the same number can be simplified by the use of exponents. Fill in the chart below. Then see if you can discover a quick way to find answers to questions like these, by making use of exponents.

$$10^2 \times 10^3 = \underline{\ 100\ } \times \underline{\ 1000\ } = \underline{\ 100\,000\ } = \underline{\ 10^5\ }$$

$$10^3 \times 10^1 = \underline{\qquad} \times \underline{\qquad} = \underline{\qquad} = \underline{\qquad}$$

$$10^4 \times 10^2 = \underline{\qquad} \times \underline{\qquad} = \underline{\qquad} = \underline{\qquad}$$

Check that you have these **answers**:

$$10^3 \times 10^1 = \underline{\ 1000\ } \times \underline{\ 10\ } \times \underline{\ 10\,000\ } = \underline{\ 10^4\ }$$

$$10^4 \times 10^2 = \underline{\ 10\,000\ } \times \underline{\ 100\ } \times \underline{\ 1\,000\,000\ } = \underline{\ 10^6\ }$$

Did you find a quick way? Here is the property:

> If *a* and *b* are whole numbers, and *n* is not equal to zero, then $n^a \times n^b = n^{a+b}$

Now look again at our three examples:

$$n^a \times n^b = 10^2 \times 10^3 = 10^{2+3} = 10^5$$

$$n^a \times n^b = 10^3 \times 10^1 = 10^{3+1} = 10^4$$

$$n^a \times n^b = 10^4 \times 10^2 = 10^{4+2} = 10^6$$

We can also use this method with bases other than 10. The only thing we must make sure of is that powers of the **same** base are multiplied.

$3^2 \times 5^3 = ?$

$\uparrow \quad \uparrow$

The bases are 3 and 5. We **cannot** use our property in this question.

So $3^2 \times 5^3 = 9 \times 125 = 1125$

$2^3 \times 2^4 = ?$

$\uparrow \quad \uparrow$

The bases are the same.

By the long method, $2^3 \times 2^4 = 8 \times 16 = 128 = 2^7$

By the property, $2^3 \times 2^4 = 2^{3+4} = 2^7$

When powers of the **same** base are multiplied, the product is also a power of that same base. The exponent of the product is equal to the **sum** of the exponents of the factors.

Self Test

Evaluate.

1. a. $10^5 \times 10^2 =$

   b. $3^4 \times 3^2 =$

   c. $n^4 \times n^2 =$

   d. $2^2 \times 2^3 =$

   e. $2^2 \times 3^2 =$

Answers

1. a. $10^{5+2} = 10^7 = 100\ 000\ 00$
   b. $3^{4+2} = 3^6 = 729$
   c. $n^{4+2} = n^6$
   d. $2^{2+3} = 2^5 = 32$
   e. $4 \times 9 = 36$ (bases are different)

Exercises 5.3

1. Express each product in exponential form.

   a. $2^2 \times 2^3 =$ ______________________________

   b. $10^1 \times 10^2 =$ ______________________________

   c. $2^4 \times 2^5 =$ ______________________________

   d. $6^2 \times 6^2 \times 6^2 =$ ______________________________

2. Evaluate each product. Show work.

   a. $3^2 \times 3^3 =$

   b. $5^1 \times 5^2 =$

   c. $2^2 \times 2^1 \times 2^2 =$

   d. $10^3 \times 10^1 =$

   e. $6^2 \times 2^2 =$

f. $3^2 \times 5^2 =$

g. $2^4 \times 2^2 =$

h. $3^2 \times 4^3 =$

Turn to the Answer Key at the end of the module to check your work.

Dividing with Exponents

Dividing with powers of the same number can also be simplified by the use of exponents. Fill in the chart below. Then see if you can discover a quick way to find answers to questions like these, by making use of exponents.

$$10^5 \div 10^3 = \frac{10^5}{10^3} = \frac{\cancel{10}^1 \times \cancel{10}^1 \times \cancel{10}^1 \times 10 \times 10}{\cancel{10}_1 \times \cancel{10}_1 \times \cancel{10}_1} = 10 \times 10 = 100 = 10^2$$

$$10^4 \div 10^1 = \_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_ = \_\_\_\_$$

$$10^3 \div 10^2 = \_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_ = \_\_\_\_$$

Check that you have these **answers**:

$$10^4 \div 10^1 = \frac{10^4}{10^1} = \frac{\cancel{10}^1 \times 10 \times 10 \times 10}{\cancel{10}_1} = 10 \times 10 \times 10 = 1000 = 10^3$$

$$10^3 \div 10^2 = \frac{10^3}{10^2} = \frac{\cancel{10}^1 \times \cancel{10}^1 \times 10}{\cancel{10}_1 \times \cancel{10}_1} = 10 = 10 = 10^1$$

Did you find a quick way? Here is the property:

> If *a* and *b* are whole numbers, and *a* is greater than or equal to *b*, and *n* is not equal to zero, then $n^a \times n^b = n^{a-b}$

Now look again at our three examples:

$$n^a \times n^b = 10^5 \div 10^3 = 10^{5-3} = 10^2$$

$$n^a \times n^b = 10^4 \div 10^1 = 10^{4-11} = 10^3$$

$$n^a \times n^b = 10^3 \div 10^2 = 10^{3-2} = 10^1$$

We can also use this method with bases other than 10. But once again, we must be sure that powers of the **same** base are divided.

$3^4 \div 2^3 = ?$
$\uparrow \quad \uparrow$

The bases are 3 and 2. We **cannot** use our property in this question.

So $\frac{3^4}{2^3} = \frac{81}{8} = 10\frac{1}{8}$

$3^5 \div 3^4 = ?$
$\uparrow \quad \uparrow$

The bases are the same.

$$3^5 \div 3^4 = \frac{3^5}{3^4} = \frac{\cancel{3}^1 \times \cancel{3}^1 \times \cancel{3}^1 \times \cancel{3}^1 \times 3}{\cancel{3}_1 \times \cancel{3}_1 \times \cancel{3}_1 \times \cancel{3}_1} = 3^1$$

or $3^5 \div 3^4 = 3^{5-4} = 3^1 = 3$

> When powers of the **same** base are divided, the quotient is also a power of that same base. The exponent of the quotient is equal to the **difference** found by subtracting the exponents of the divisor from the exponent of the dividend.

Try the following.

Self Test

Evaluate.

1. $10^5 \div 10^2 =$

2. $3^4 \div 3^2 =$

3. $8^7 \div 8^5 =$

4. $10^6 \div 10^4 =$

5. $9^5 \div 9^4 =$

Answers

1. $10^{5-2} = 10^3 = 1000$
2. $3^{4-2} = 3^2 = 9$
3. $8^{7-5} = 8^2 = 64$
4. $10^{6-4} = 10^2 = 100$
5. $9^{5-4} = 9^1 = 9$

One and Zero as Exponents

Note that $10 = 10^1$. It is a good idea to write in a little 1 when you see $10^5 \times 10 = ?$ or $3^4 \div 3 = ?$

Then $10^5 \times 10^1 = 10^{5+1} = 10^6$ and $3^4 \div 3^1 = 3^{4-1} = 3^3$

Zero as an Exponent

We define any number, except zero, raised to the zero power, as 1. For example, $5^0 = 1$, $0.2^0 = 1$, and $10^0 = 1$. There is a reason for this!

Consider this division question:

$$10^3 \div 10^3 = \frac{10^3}{10^3} = \frac{\cancel{10}^1 \times \cancel{10}^1 \times \cancel{10}^1}{\cancel{10}_1 \times \cancel{10}_1 \times \cancel{10}_1} = 1$$

If we use the property for dividing with exponents for questions in which the exponents are equal, our answer must be the same as when we do it the long way, as above.

$$10^3 \div 10^3 = 10^{3-3} = 10^0$$

It follows that 10^0 must equal 1, because the quotient for $10^3 \div 10^3$ can only have the value of 1, because 10^3 goes into 10^3 one time.

This reasoning will apply to any number, except zero. Zero to the zero power has no meaning.

For any number, *n*, except zero, $n^0 = 1$.

Self Test

Evaluate.

a. 8^0

b. $\frac{3^0}{11} =$

c. $10^2 \times 10 =$

d. $\frac{3^4 \times 3}{3^5} =$

Answers

a. 1

b. $\frac{1}{11}$ (the numerator only is to the 0 power)

c. $10^2 \times 10^1 = 10^3 = 1000$

d. $\frac{3^4 \times 3^1}{3^5} = \frac{3^5}{3^5} = 3^{5-5} = 3^0 = 1$

Exercises 5.4

1. Evaluate.

   a. $10^2 \times 10^5 =$

   b. $10^3 \times 10 =$

   c. $2^4 \times 2^2 =$

   d. $3^2 \times 3^0 =$

   e. $8^2 \times 8 =$

2. Express each quotient in exponential form, and then work out the answer.

   a. $9^5 \div 9^3 =$ ______________________________

   b. $10^5 \div 10^2 =$ ______________________________

   c. $5^4 \div 5^4 =$ ______________________________

   d. $10^4 \div 10 =$ ______________________________

   e. $8^{10} \div 8^8 =$ ______________________________

3. Solve for n. (What number must n equal to make the following true?)

a. $3^5 \times 3^4 = 3^n$ $n =$ ____________

b. $10^2 \times 10^n = 10^5$ $n =$ ____________

c. $2^n \times 2^7 = 2^7$ $n =$ ____________

d. $n^5 \times n^2 = 6^7$ $n =$ ____________

e. $n^n \times n^n = 1$ $n =$ ____________

f. $8^6 \div 8^2 = 8^n$ $n =$ ____________

g. $5^4 \div 5^n = 5$ $n =$ ____________

h. $7^n \div 7^4 = 7^3$ $n =$ ____________

i. $3^4 \div 3^n = 3^0$ $n =$ ____________

j. $5^4 \div 5 = n^3$ $n =$ ____________

Turn to the Answer Key at the end of the module to check your work.

Lesson 6
Order of Operations

Learning Outcomes

By the end of this section you will be better able to:

- use order of operations to evaluate expressions

Something that tells us what to do with a number or numbers is called an operation.

Addition, subtraction, multiplication, and division are the operations that you know about already. These are called the basic operations.

You have probably done questions about adding and subtracting more than two numbers in other math courses.

Previously, you learned about multiplying and dividing with more than two numbers.

What do you do with a question that involves many different operations?

BEDMAS: More Than Just a Weird Word

$$(-2)(3)^2 \div 6 + 9 - 14 \div (9 - 2)$$

BEDMAS is an acronym that helps you to remember the order of operations.

BEDMAS
First, work out everything that is in brackets.

$$\begin{aligned} &(-2)(3)^2 \div 6 + 9 - 14 \div \mathbf{(9 - 2)} \\ = &(-2)(3)^2 \div 6 + 9 - 14 \div \quad 7 \end{aligned}$$

B**E**DMAS

Next, simplify all of the exponents.

$$\begin{aligned} &(-2)(3)^2 \div 6 + 9 - 14 \div \mathbf{7} \\ &= (-2)(9) \div 6 + 9 - 14 \div 7 \end{aligned}$$

BE**DM**AS

Do all of the division and multiplication in the order they appear from left to right.

$$\begin{aligned} &= \mathbf{(-2)(9)} \div 6 + 9 - 14 \div 7 \\ &= \mathbf{-18 \div 6} + 9 - \mathbf{14 \div 7} \\ &= -3 + 9 - 2 \end{aligned}$$

BEDM**AS**

Finally, do the addition and subtraction.

$$\begin{aligned} &= \mathbf{-3 + 9} - 2 \\ &= \mathbf{6 - 2} \\ &= 4 \end{aligned}$$

Exercises 4.1

Solve the following.

1. $(-3)(7) =$
2. $4 \times 9 =$
3. $-13 \times 3 =$
4. $42 \div (-6) =$
5. $(8)(-1)(-4) =$
6. $12 + 6 =$
7. $15 \div 5 + 7 =$
8. $2 - 3 \times 4 =$
9. $3 + \frac{4}{2} =$
10. $-16 + (4)(3) =$
11. $-5 - (2)(-1)(-18) \div 4 =$
12. $6 \times 5 \div 3 =$
13. $18 \div 2 + 4 =$
14. $18 \div (2 + 4) =$

15. $6 \times 8 + 12 + 3 \times 9 =$

16. $3 + 11 \times 4 + 12 \div 3 =$

17. $7 - 3 \times 5 =$

18. $(7 - 3) \times 5 =$

19. $36 \div 9 + 2 + 1 \times 9 + 6 - 5 =$

20. $6 \times 7 \div 14 - 3 + 2 \times 4 =$

21. $5 - 1 + 2 - 4 \times 3 \div 6 =$

22. Solve these expressions.

Remember BEDMAS.
Brackets
Exponents
Division and **M**ultiplication (in order from left to right)
Addition and **S**ubtraction (in order from left to right)

a. $2^2 + 1 =$

b. $(–2)(3)^2 – 4 =$

c. $(–4 + 7) + 4^2 – 18 \div 3 =$

Turn to the Answer Key at the end of the module to check your work.

Answer Key

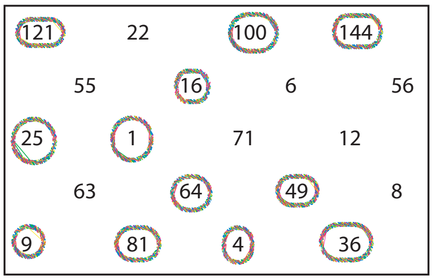
Lesson 1: Square Roots

Exercises 1.1

1. $3 \times 3 = 9$ $\quad$ $6 \times 6 = 36$ $\quad$ $1 \times 1 = 1$

   $9 \times 9 = 81$ $\quad$ $4 \times 4 = 16$

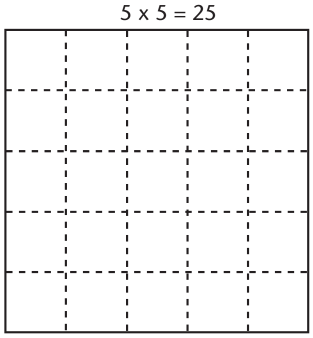
2.



$1 \times 1 = 1$	$2 \times 2 = 4$
$3 \times 3 = 9$	$4 \times 4 = 16$
$5 \times 5 = 25$	$6 \times 6 = 36$
$7 \times 7 = 49$	$8 \times 8 = 64$
$9 \times 9 = 81$	$10 \times 10 = 100$
$11 \times 11 = 121$	$12 \times 12 = 144$

Exercises 1.2

1. $1^2 = 1$
2. $2^2 = 4$
3. $3^2 = 9$
4. $4^2 = 16$
5. $5^2 = 25$
6. $6^2 = 36$
7. $7^2 = 49$
8. $8^2 = 64$
9. $9^2 = 81$
10. $10^2 = 100$
11. 5 x 5 = 25



Exercises 1.3

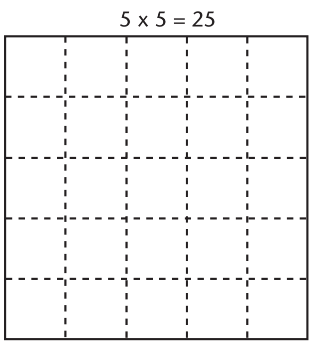
1. The length of the side of the square is 3.
   The square root of 9 is 3.
2. The length of the side of the square is 2.
   The square root of 4 is 2.
3. Each side is 5 cm long. 5 is the square root of 25.

Exercises 1.4

1. 10
   1, 4, 9, 16, 25, 36, 49, 64, 81, 100

   Answers may vary. Your answer might be something similar to this:

   I squared all of the integers in order, without skipping any. One squared is one, two squared is four, etc. When I got to 100, I stopped.

2. 

3. a. Four squared equals **sixteen**. $4^2 =$ **16**
   b. Nine is **three** squared. $9 =$ **3^2**
   c. **Two** is the square root of four. **2** $= \sqrt{4}$
   d. One is the square root of **one**. $1 = \sqrt{1}$
   e. The square root of nine is **three**. $\sqrt{9} =$ **3**
   f. **Five** is the square root of twenty-five. **5** $= \sqrt{25}$

4. a. 16
   b. 36
   c. 1
   d. 64
   e. 25

5. a. 3
   b. 9
   c. 5
   d. 7
   e. 2
6. a. 64
   b. 4
   c. 5
   d. 49
   e. 10
   f. 8

Exercises 1.5

1. $7 = 3 + 4$	2. $-6 < 5$
3. $2 < 3$	4. $\sqrt{16} = 4$
5. $8 < 3^2$	6. $(-4)(-5) > 12$
7. $3 < \sqrt{25}$	8. $-6 + 5 = 5 - 6$
9. $6^2 = 36$	10. $5 < \sqrt{36}$
11. $-24 \div 4 > 3 - 10$	12. $2 < 2^2$
13. $(-15)^2 = (-15) \times (-15)$	14. $\sqrt{9} = -2 + 5$

Exercises 1.6

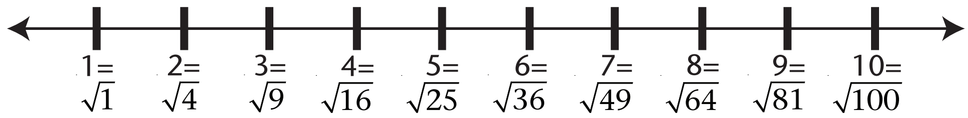
1. $12^2 = 12 \times 12 = \mathbf{144}$	2. $(-1.2)^2 = (-1.2) \times (-1.2) = \mathbf{1.44}$
3. $5.1^2 = 5.1 \times 5.1 = \mathbf{26.01}$	4. $0.2^2 = 0.2 \times 0.2 = \mathbf{0.04}$
5. $(-23)^2 = (-23) \times (-23) = \mathbf{529}$	6. $35^2 = 35 \times 35 = \mathbf{1225}$
7. $3.5^2 = = 3.5 \times 3.5 = \mathbf{12.25}$	8. $(-0.4)^2 = (-0.4) \times (-0.4) = \mathbf{0.16}$
9. $20^2 = 20 \times 20 = \mathbf{400}$	

The answers to questions 1, 5, 6, and 9 should be circled.

Exercises 1.7

1. $1^2 = 1$ $2^2 = 4$ $3^2 = 9$
   $4^2 = 16$ $5^2 = 25$ $6^2 = 36$
   $7^2 = 49$ $8^2 = 64$ $9^2 = 81$
   $10^2 = 100$

2.



3. a. 2 < $\sqrt{7}$ < 3 — $\sqrt{7}$ is between **2** and **3**
   b. 4 < $\sqrt{22}$ < **5** — $\sqrt{22}$ is between **4** and **5**
   c. **3** < $\sqrt{13}$ < **4** — $\sqrt{13}$ is between **3** and **4**
   d. **7** < $\sqrt{61}$ < **8** — $\sqrt{61}$ is between **7** and **8**
   e. **8** < $\sqrt{74}$ < **9** — $\sqrt{74}$ is between **8** and **9**
   f. **6** < $\sqrt{42}$ < **7** — $\sqrt{42}$ is between **6** and **7**
   g. **2** < $\sqrt{5}$ < **3** — $\sqrt{5}$ is between **2** and **3**
   h. **7** < $\sqrt{57}$ < **8** — $\sqrt{57}$ is between **7** and **8**
   i. **5** < $\sqrt{29}$ < **6** — $\sqrt{29}$ is between **5** and **6**
   j. **1** < $\sqrt{3}$ < **2** — $\sqrt{3}$ is between **1** and **2**
   k. **5** < $\sqrt{32}$ < **6** — $\sqrt{32}$ is between **5** and **6**
   l. **9** < $\sqrt{95}$ < **10** — $\sqrt{95}$ is between **9** and **10**

4. a. $\sqrt{7}$ is closer to 3
   b. 2.7 is the best estimate.

Lesson 2: Fractions: Basic Skills

Exercises 2.1

1. a. $\frac{3}{12}$ $\frac{1}{4}$
   b. $\frac{2}{12}$ $\frac{1}{6}$

2. a. $\frac{6}{12}$ $\frac{3}{6}$ $\frac{1}{2}$ (any two)

   b. $\frac{2}{12}$ $\frac{1}{6}$

3. No. The shaded part and the clear part are not the same size.

4. a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$ d. $\frac{1}{2}$

5. a. $\frac{1}{3}$ $\frac{2}{6}$

   b. $\frac{3}{6}$ $\frac{1}{2}$

   c. $\frac{2}{6}$ $\frac{1}{3}$

   d. $\frac{4}{8}$ $\frac{1}{2}$ or $\frac{2}{4}$ (any two)

Exercises 2.2

1. a. $\frac{3}{4}$ $\frac{6}{8}$

   b. $\frac{1}{4}$ $\frac{2}{8}$

2. $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$

3. one (1)

4. a. $\frac{3\times4}{3\times5}$ b. $\frac{5\times4}{5\times5}$ c. $\frac{16}{20}$ d. $\frac{24}{30}$

5. a. $\frac{20}{24},\frac{25}{30},\frac{30}{36}\cdots$

   b. $\frac{9}{30},\frac{15}{50},\frac{18}{60}\cdots$

   c. $\frac{20}{32},\frac{25}{40},\frac{35}{56}\cdots$

   d. $\frac{3}{24},\frac{4}{32},\frac{6}{48}\cdots$

Exercises 2.3

1. a. 1, 2, 4, 8
   b. 1, 2, 3, 4, 6, 12
   c. No, because 2 or 4 can divide into the top and bottom (numerator and denomintor)

2. a. 1, 3, 5, 15
   b. 1, 2, 4, 7, 14, 28
   c. Yes, because 1 is the only common factor.

3. a. Because 2 will divide into the numerator and the denominator.
   b. Because 5 will divide into the numerator and the denominator.

4. a. $\frac{1}{4}$ $\frac{2}{8}$ $\frac{3}{12}$ $\frac{4}{16}$ $\frac{5}{20}$
   b. $\frac{5}{8}$ $\frac{10}{16}$ $\frac{15}{24}$ $\frac{20}{32}$ $\frac{25}{40}$

5. a. $\frac{3}{7}$ (since 3 will divide into both)
   b. $\frac{3}{14}$ (since 5 will divide into both)
   c. $\frac{1}{2}$ (since 14 will divide into both)
   d. $\frac{2}{3}$ (since 25 will divide into both)
   e. $\frac{4}{5}$ (since 6 will divide into both)
   f. $\frac{8}{15}$ (since 2 will divide into both)
   g. $\frac{5}{12}$ (since 3 will divide into both)
   h. $\frac{5}{8}$ (since 6 will divide into both)
   i. $\frac{3}{26}$ (since 2 will divide into both)

Lesson 3: Adding and Subtracting Fractions

Exercises 3.1

1. a. $\frac{6}{7}$

   b. $\frac{5}{10}=\frac{1}{2}$

   c. $\frac{9}{9}=1$

   d. $\frac{4}{6}=\frac{2}{3}$

   e. $\frac{5}{6}$

   f. $\frac{5}{8}$

   g. $\frac{5}{10}=\frac{1}{2}$

   h. $\frac{4}{6}=\frac{2}{3}$

   i. $\frac{5}{10}=\frac{1}{2}$

2. a. $\frac{5}{9}$

   b. $\frac{5}{10}=\frac{1}{2}$

   c. $\frac{4}{12}=\frac{1}{3}$

   d. $\frac{0}{6}=0$

   e. $\frac{2}{6}=\frac{1}{3}$

(f) $$\begin{array}{rr} & \frac{7}{9} = \frac{7}{9} \\ - & \frac{1}{3} = \frac{3}{9} \\ \hline & \frac{4}{9} \end{array}$$

(g) $$\begin{array}{rr} & \frac{8}{12} = \frac{8}{12} \\ - & \frac{1}{4} = \frac{3}{12} \\ \hline & \frac{5}{12} \end{array}$$

(h) $$\begin{array}{rr} & \frac{5}{10} = \frac{5}{10} \\ - & \frac{1}{2} = \frac{5}{10} \\ \hline & \frac{0}{10} = 0 \end{array}$$

(i) $$\begin{array}{rr} & \frac{9}{10} = \frac{9}{10} \\ - & \frac{2}{5} = \frac{4}{10} \\ \hline & \frac{5}{10} = \frac{1}{2} \end{array}$$

(j) $$\begin{array}{rr} & \frac{5}{8} = \frac{5}{8} \\ - & \frac{1}{4} = \frac{2}{8} \\ \hline & \frac{3}{8} \end{array}$$

Exercises 3.2

1. (a) $\frac{2}{4} = \frac{1}{2}$

   (b) $\frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$

   (c) $\frac{15}{40} + \frac{16}{40} = \frac{31}{40}$

   (d) $3\frac{3}{5} + \frac{1}{5} = 3\frac{4}{5}$

   (e) $5\frac{2}{5}$

   (f) $$\begin{array}{rr} & 4\frac{5}{8} = 4\frac{15}{24} \\ + & 6\frac{1}{3} = 6\frac{8}{24} \\ \hline & 10\frac{23}{24} \end{array}$$

2. (a) $\frac{3}{5}$

   (b) $\frac{4}{20} = \frac{1}{5}$

   (c) $\frac{7}{15} - \frac{3}{15} = \frac{4}{15}$

   (d) $\frac{26}{40} - \frac{25}{40} = \frac{1}{40}$

   (e) $2\frac{9}{12} - 1\frac{4}{12} = 1\frac{5}{12}$

   (f) $1\frac{15}{20} - \frac{14}{20} = 1\frac{1}{20}$

3. (a) Cassandra

(b) $\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$ hour

4.
$$\begin{array}{rcccl} & 4\frac{1}{2} & = 4\frac{2}{4} & = 3\frac{6}{4} \\ - & 1\frac{3}{4} & = 1\frac{3}{4} & = 1\frac{3}{4} \\ \hline & & & 2\frac{3}{4} \end{array}$$

$\therefore$ It snowed $2\frac{3}{4}$ h longer on Saturday.

5.
$$\begin{array}{rcccl} & 8\frac{1}{2} & = 8\frac{2}{4} & = 7\frac{6}{4} \\ - & 1\frac{3}{4} & = 1\frac{3}{4} & = 1\frac{3}{4} \\ \hline & & & 6\frac{3}{4} \end{array}$$

$\therefore$ It took $6\frac{3}{4}$ min longer to get up the slope.

6. $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

$\therefore$ $\frac{5}{6}$ of his collection are rock or jazz.

Lesson 4: Multiplying and Dividing Fractions

Exercises 4.1

1. (a) $\frac{4 \times 1}{7 \times 5} = \frac{4}{35}$

   (b) $\frac{9}{1} \times \frac{1}{5} = \frac{9}{5} = 1\frac{4}{5}$

   (c) $\frac{1 \times 1}{9 \times 10} = \frac{1}{90}$

   (d) $\frac{1}{3} \times \frac{3}{1} = \frac{3}{3} = 1$

2. (a) $\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{12}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{10}}} = \frac{1}{8}$

   (b) $\frac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{3}{\cancel{18}}} = \frac{3}{3} = 1$

   (c) $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{1}{3}$

   (d) $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{5}{\cancel{20}}}{\underset{9}{\cancel{27}}} = \frac{5}{9}$

   (e) $\frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{10}}} \times \frac{\overset{10}{\cancel{100}}}{\underset{3}{\cancel{21}}} = \frac{10}{3} = 3\frac{1}{3}$

   (f) $\frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{\underset{2}{\cancel{14}}} = \frac{3}{4}$

3. (a) $\frac{7}{2} \times \frac{7}{4} = \frac{49}{8} = 6\frac{1}{8}$

   (b) $\frac{16}{5} \times \frac{2}{1} = \frac{32}{5} = 6\frac{2}{5}$

   (c) $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$

   (d) $\frac{\overset{3}{\cancel{9}}}{8} \times \frac{7}{\underset{1}{\cancel{3}}} = \frac{21}{8} = 2\frac{5}{8}$

   (e) $\frac{1}{6} \times \frac{7}{5} = \frac{7}{30}$

Exercises 4.2

1. (a) $\frac{3}{\underset{4}{\cancel{8}}} \times \frac{\overset{1}{\cancel{2}}}{1} = \frac{3}{4}$

(b) $\frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$

(c) $\frac{\overset{1}{\cancel{5}}}{3} \times \frac{4}{\underset{1}{\cancel{5}}} = \frac{4}{3}$ or $1\frac{1}{3}$

(d) $\frac{\overset{1}{\cancel{4}}}{7} \times \frac{1}{\underset{2}{\cancel{8}}} = \frac{1}{14}$

(e) $\frac{\overset{3}{\cancel{6}}}{1} \times \frac{3}{\underset{1}{\cancel{2}}} = 9$

(f) $\frac{\overset{2}{\cancel{4}}}{9} \times \frac{1}{\underset{3}{\cancel{6}}} = \frac{2}{27}$

(g) $\frac{10}{1} \times \frac{2}{3} = \frac{20}{3} = 6\frac{2}{3}$

(h) $\frac{6}{5} \times \frac{2}{7} = \frac{12}{35}$

Exercises 4.3

1. (a) $\frac{3}{2} \div \frac{3}{4} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}} = 2$

(b) $\frac{13}{4} \div \frac{5}{6} = \frac{13}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{5} = \frac{39}{10} = 3\frac{9}{10}$

(c) $\frac{7}{2} \div \frac{9}{4} = \frac{7}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{9} = \frac{14}{9} = 1\frac{5}{9}$

(d) $\frac{17}{3} \div \frac{27}{4} = \frac{17}{3} \times \frac{4}{27} = \frac{68}{81}$

(e) $\frac{4}{5} \div \frac{8}{3} = \frac{\overset{1}{\cancel{4}}}{5} \times \frac{3}{\underset{2}{\cancel{8}}} = \frac{3}{10}$

(f) $\frac{11}{8} \div \frac{13}{3} = \frac{11}{8} \times \frac{3}{13} = \frac{33}{104}$

(g) $\frac{2}{1} \div \frac{5}{4} = \frac{2}{1} \times \frac{4}{5} = \frac{8}{5} = 1\frac{3}{5}$

(h) $\frac{10}{3} \div \frac{13}{10} = \frac{10}{3} \times \frac{10}{13} = \frac{100}{39} = 2\frac{22}{39}$

(i) $\frac{12}{5} \div \frac{4}{15} = \frac{\overset{3}{\cancel{12}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{1}{\cancel{4}}} = \frac{9}{1} = 9$

(j) $\frac{5}{4} \div \frac{5}{3} = \frac{\overset{1}{\cancel{5}}}{4} \times \frac{3}{\underset{1}{\cancel{5}}} = \frac{3}{4}$

Exercises 4.4

1. In each hour he walks $\frac{7}{3}$ km.

   $\therefore$ We think of dividing 12 km into $\frac{7}{3}$ km sections.

   $$\begin{aligned}\text{Time} &= 12 \div \frac{7}{3}\\ &= \frac{12}{1} \times \frac{3}{7}\\ &= \frac{36}{7} = 5\frac{1}{7}\text{ h}\end{aligned}$$

   It would take him $5\frac{1}{7}$ h.

   Check: $\frac{7}{3} \times 5\frac{1}{7} = \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{12}{\cancel{36}}}{\underset{1}{\cancel{7}}} = 12$

2. We think of dividing $5\frac{1}{4}$ m rope into 7 pieces.

   $\therefore$ Length of each piece

   $$= 5\frac{1}{4} \div 7 = \frac{21}{4} \div 7 = \frac{\overset{3}{\cancel{21}}}{4} \times \frac{1}{\underset{1}{\cancel{7}}} = \frac{3}{4}\text{ m}$$

   Each piece is $\frac{3}{4}$ m.

   Check: $7 \times \frac{3}{4} = \frac{21}{4} = 5\frac{1}{4}$ m

3. (a) We think of dividing 10 kg of nuts into $\frac{2}{5}$ kg parts.

   $\therefore$ Number of litres $= 10 \div \frac{2}{5}$

   $$\begin{aligned}&= \overset{5}{\cancel{10}} \times \frac{5}{\underset{1}{\cancel{2}}}\\ &= 25\text{ L}\end{aligned}$$

   Check: $\overset{5}{\cancel{25}} \times \frac{2}{\underset{1}{\cancel{5}}} = 10$ kg

(b) We think of dividing 1 kg of nuts into $\frac{2}{5}$ kg parts.

$$\therefore \text{ Number of bags } = 1 \div \frac{2}{5}$$
$$= 1 \times \frac{5}{2}$$
$$= \frac{5}{2} = 2\frac{1}{2}$$

Check: $2\frac{1}{2} \times \frac{2}{5} = \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}} = 1$ kg

4. (a) We think of dividing the 5 hours into $\frac{1}{2}$ h periods.

$$\therefore \text{ Number of periods } = 5 \div \frac{1}{2}$$
$$= 5 \times \frac{2}{1}$$
$$= 10$$

Check: $10 \times \frac{1}{2} = \frac{10}{2} = 5$ h

(b) We think of dividing the $\frac{3}{4}$ h into $\frac{1}{2}$ h periods.

$$\therefore \text{ Number of periods } = \frac{3}{4} \div \frac{1}{2}$$
$$= \frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{1}$$
$$= \frac{3}{2} = 1\frac{1}{2}$$

Check: $1\frac{1}{2} \times \frac{1}{2} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$ h

Lesson 5: Powers

Exercises 5.1

1. 100
2. 40 000
3. 1000
4. 125 000 000
5. 900
6. 90 000
6. 32
8. 216

Exercises 5.2

1. a. 8 b. 1 c. 1000
   d. 216 e. 1 000 000

2. (a) $3 \bullet 4 = 12$
   (b) $10^2 = 10 \times 10 = 100$
   (c) $3^3 = 3 \times 3 \times 3 = 27$
   (d) $(2 \bullet 3)^2 = (6)^2 = 6 \times 6 = 36$
   (e) $(3 \bullet 3)^2 = (9)^2 = 9 \times 9 = 81$
   (f) $(4 \bullet 10)^2 = (40)^2 = 40 \times 40 = 1600$
   (g) $4^3 = 4 \times 4 \times 4 = 64$
   (h) $(3 \bullet 10)^2 = (30)^2 = 30 \times 30 = 900$
   (i) $(2 \bullet 3)^3 = (6)^3 = 6 \times 6 \times 6 = 216$
   (j) $(3 \bullet 4)^2 = (12)^2 = 12 \times 12 = 144$

Exercises 5.3

1. a. $2^{2+3} = 2^5$
   b. $10^{1+2} = 10^3$
   c. $2^{4+5} = 2^9$
   d. $6^{2+2+2} = 6^6$
2. a. $= 3^{2+3} = 3^5 = 243$
   b. $= 5^{1+2} = 5^3 = 125$
   c. $= 2^{2+1+2} = 2^5 = 32$
   d. $= 10^{3+1} = 10^4 = 10\ 000$
   e. $= 36 \times 4 = 144$ (bases are different)
   f. $= 9 \times 25 = 225$ (bases are different)
   g. $= 2^{4+2} = 2^6 = 64$
   h. $= 9 \times 64 = 576$ (bases are different)

Exercises 5.4

1. (a) $10^{2+5} = 10^7 = 10\,000\,000$ (b) $10^{3+1} = 10^4 = 10\,000$
   (c) $2^{4+2} = 2^6 = 64$ (d) $3^{2+0} = 3^2 = 9$
   (e) $8^{2+1} = 8^3 = 512$

2. (a) (i) $9^{5-3} = 9^2$ (ii) 81
   (b) (i) $10^{5-2} = 10^3$ (ii) 1000
   (c) (i) $5^{4-4} = 5^0$ (ii) 1
   (d) (i) $10^{4-1} = 10^3$ (ii) 1000
   (e) (i) $8^{10-8} = 8^2$ (ii) 64

3. (a) n = 9 (b) n = 3 (c) n = 0 (d) n = 6
   (e) n = 1 (f) n = 4 (g) n = 3 (h) n = 7
   (i) n = 4 (j) n = 5

Lesson 6: Order of Operations

Exercises 6.1

1. (–3)(7) = –21
2. 4 × 9 = 36
3. –13 × 3 = –39
4. 42 ÷ –6 = –7
5. (8)(–1)(–4) = 32
6. 12 + 6 = 18
7. 15 ÷ 5 + 7 = 10
8. 2 – 3 × 4 = –10
9. $3 + \frac{4}{2} = 5$
10. –16 + (4)(3) = –4
11. –5 – (2)(–1)(–18) ÷ 4 = –14
12. 6 × 5 ÷ 3 = 10
13. 18 ÷ 2 + 4 = 9 + 4 = 13
14. 18 ÷ (2 + 4) = 18 ÷ 6 = 3
15. 6 × 8 + 12 + 3 × 9 = 87
16. 3 + 11 × 4 + 12 ÷ 3 = 51
17. 7 – 3 × 5 = 7 – 15 = –8
18. (7 – 3) × 5 = 4 × 5 = 20
19. 36 ÷ 9 + 2 + 1 × 9 + 6 – 5 = 16
20. 6 × 7 ÷ 14 – 3 + 2 × 4 = 8
21. 5 – 1 + 2 – 4 × 3 ÷ 6 = 4

22. a. $2^2 + 1$
$= 4 + 1$
$= 5$

b. $(-2)(3)^2 - 4$
$= (-2)(9) - 4$
$= -18 - 4$
$= -22$

c. $(-4 + 7) + 4^2 - 18 \div 3$
$= 3 + 4^2 - 18 \div 3$
$= 3 + 16 - 18 \div 3$
$= 3 + 16 - 6$
$= 13$