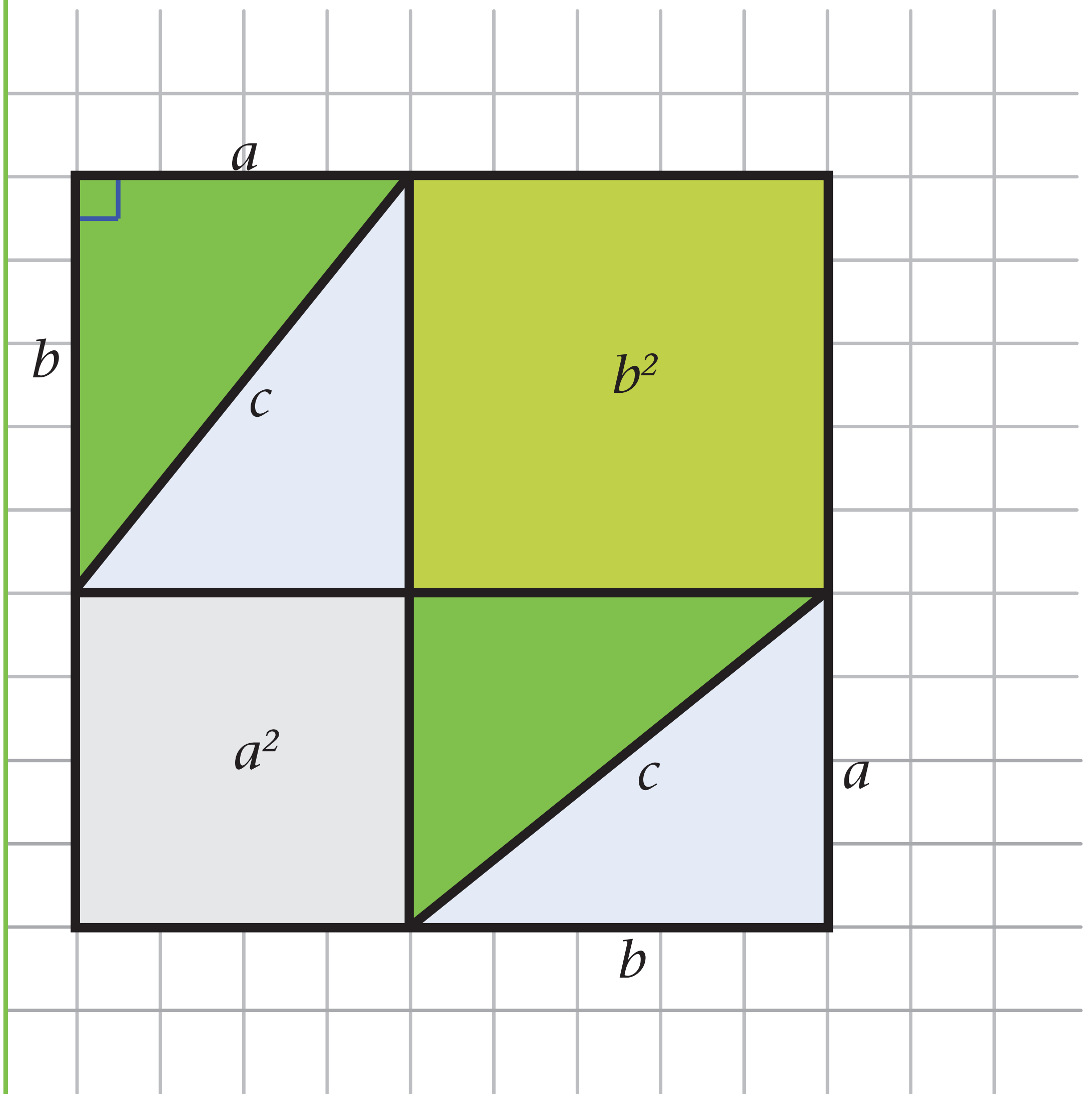
LITERACY FOUNDATIONS MATH



LEVEL 6
GEOMETRY

To the Student

This resource covers topics from the British Columbia Ministry of Education's Literacy Foundations Math Level 6. You may find this resource useful if you're a Literacy Foundations Math student, or a K-12 student in grades 7 – 9.

We have provided learning material, exercises, and answers for the exercises, which are located at the back of each set of related lessons. We hope you find it helpful.

Literacy Foundations Math Prescribed Learning Outcomes

Literacy Foundations Mathematics Level 6 follows two pathways: Math Foundations (MF) and Apprenticeship and Workplace (AW). The Prescribed Learning Outcomes (PLOs) are grouped into four areas: Number (A), Patterns and Relations (B), Shape and Space (C), and Statistics and Probability (D). For a complete list of the PLOs in Level 6, go to the BC Ministry of Education's website and search for Literacy Foundations Math curriculum.

PLOs Represented in This Resource

The PLOs represented in this Level 5 resource are as follows:

Number

MF: All topics, A1 – A6, with the exception of a portion of A3, roots of fractions, and a portion of A4, word problems with powers
AW: All topics, A1 – A9, with the exception of a portion of A3, roots of fractions, and A6, word problems with powers

Patterns and Relations

MF: B3 and B4, with the exception of a portion of B4, rational coefficients with variables in more than one term

AW: B3 and B4

Shape and Space

MF: C1 and C3, with the exception of a portion of C3, composite objects
AW: C1 – C3, C5, C6, with the exception of a portion of C5, composite objects

PLOs Not Represented in This Resource

The PLOs for which no material is included in this resource are as follows:

Number

MF: portions of A3, roots of fractions and a portion of A4, word problems with powers
AW: portions of A3, roots of fractions and a portion of A6, word problems with powers

Patterns and Relations

MF: B1, linear relationships; B2, patterns to linear equations; portions of B4, rational coefficients and variables in more than one term; B5, single-variable linear inequalities; B6 – B8, polynomials
AW: B1, linear relationships; B2, patterns to linear equations; and B5 – B8, polynomials

Shape and Space

MF: C2, polygons and polyhedra; and portions of C3, composite objects and line and rotation symmetry

AW: C4, polygons and polyhedra; and portions of C5, composite objects; and C7, line and rotation symmetry

Acknowledgements and Copyright

Project Manager: Christina Teskey
Writer: Angela Voll
Production Technician: Beverly Carstensen
Cover Design: Christine Ramkeesoon

New, October 2015

Table of Contents

Lesson 1

Pythagorean Theorem

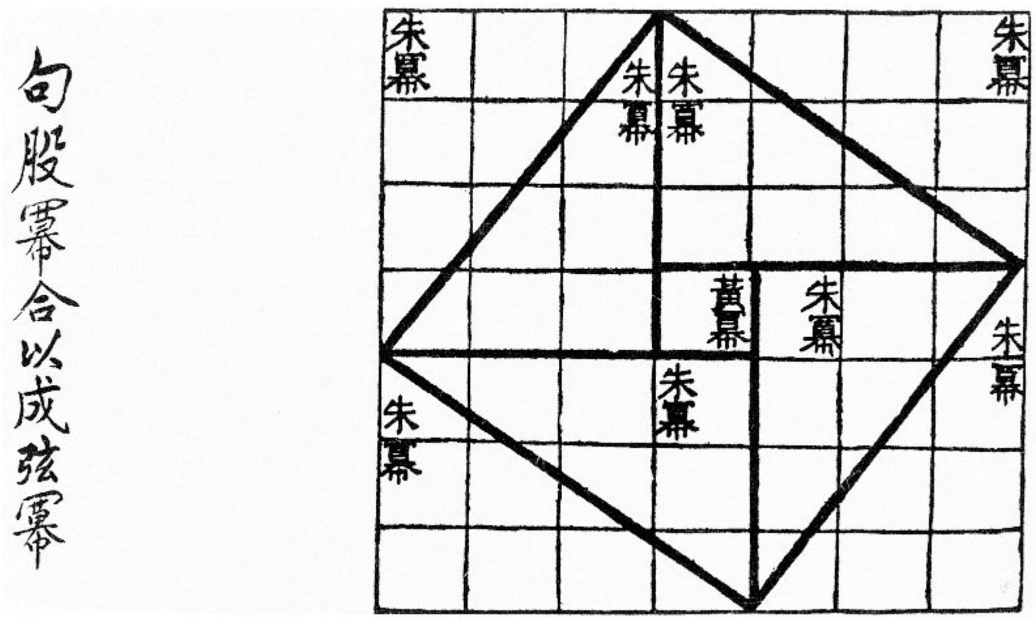
Learning Outcomes

By the end of this lesson you will be better able to:

- identify right angles and right triangles
- identify the legs and hypotenuse of a right triangle
- use the Pythagorean Theorem to determine if a given triangle is a right triangle
- understand and identify Pythagorean Triples
- use your calculator to find the square root of a number
- use the Pythagorean Theorem to find the length of a side of a right triangle

Right angles are very important. They're everywhere!

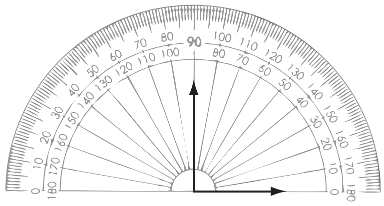
Many different societies in the ancient world had techniques to build and check for right angles—the Greeks, the Egyptians, the Babylonians, and the Chinese. It's possible that the Mayans and the Aztecs knew how to do calculations for right triangles, but we don't know enough about their mathematics to be sure.



Right Angles

An angle that measures 90° is called a **right angle**.

Why is a 90° angle so important? Why does it have a special name?



Look at these two fences.

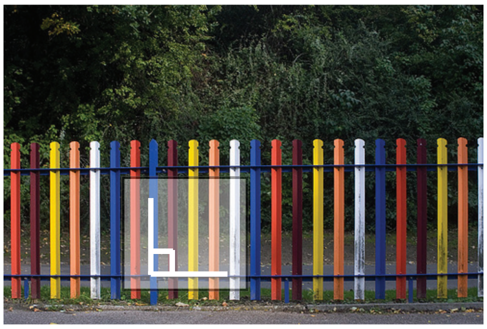
This fence is not going to last much longer.

This fence will be up for a long time. The posts are at right angles to the ground. The crossbeam forms a right angle with the posts.

A carpenter who is building a house needs to make sure that the walls form a right angle with the floor; otherwise the building will fall over.

The corners of this page are right angles.

This symbol is used to mark a right angle.

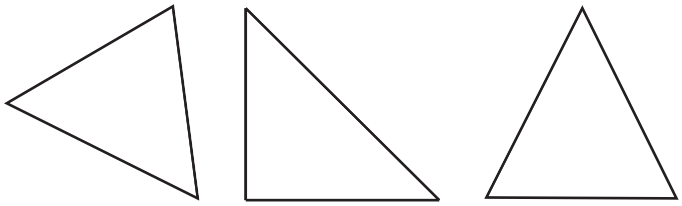


Exercises 1.1

Use the corner of a piece of paper to help you find the right angles in this picture. Mark the right angles that you find.

Right Triangles

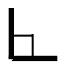
Do you know which triangle here is a right triangle? Why is it called a right triangle?



A **right triangle** is a triangle with one right angle.

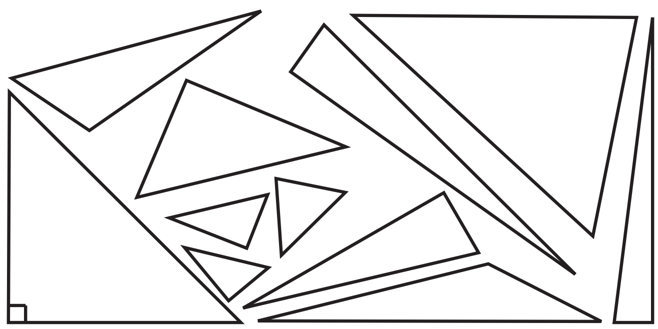
Exercises 1.2

Find all of the right triangles. Mark the right angles with this symbol:



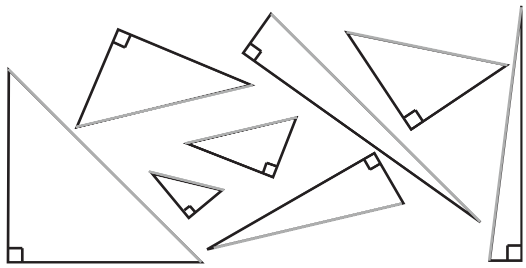
One has already been done.

Hint: You can use the corner of a piece of paper to help you find the right angles.

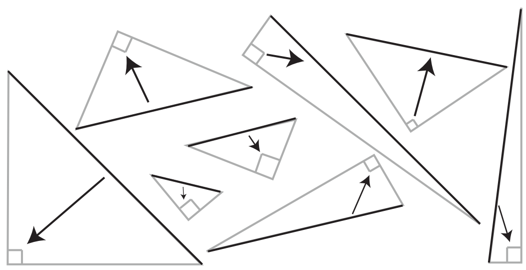


The Hypotenuse

The sides of a right triangle that form the right angle are called the legs.



There is one side of a right triangle that is not a leg. It's called the hypotenuse. It's always the longest side of a right triangle, and it's the side that is opposite the right angle.



Squares and Right Triangles

You will need:

- 2 sheets of graph paper from the Appendix
- coloured pencils
- scissors

In this activity you're going to build a proof of a famous mathematical theorem.

Step 1: Draw a right triangle. You need to decide how big it will be.

It doesn't matter how long the legs are. However, this activity will be easier to follow along with if the legs are of different sizes.

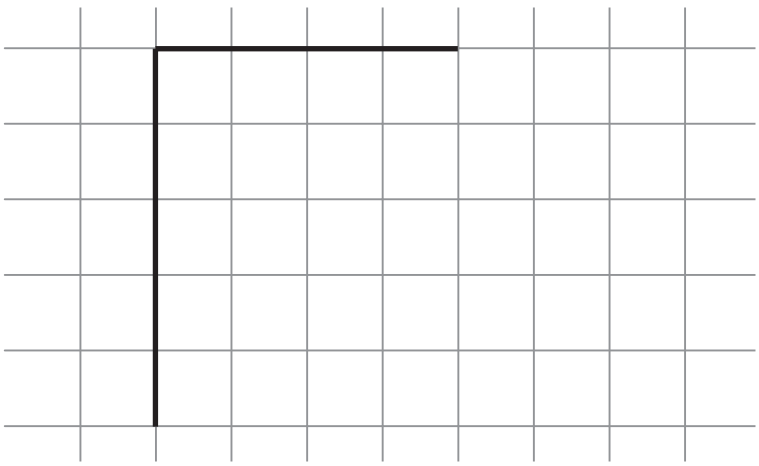
You will be repeating this triangle on your graph paper, so don't make your triangle too big. Choose a number between 2 and 6 for each leg of your triangle.

Fill in the blanks:

One leg of my triangle will be ___ units long.

The other leg of my triangle will be ___ units long.

Using the grid on your graph paper as a guide, draw the two legs of your right triangle in the upper left hand corner of the page.



In this example, one leg is 4 units long and the other is 5 units long.
Your triangle can be different as long as it has a right angle.

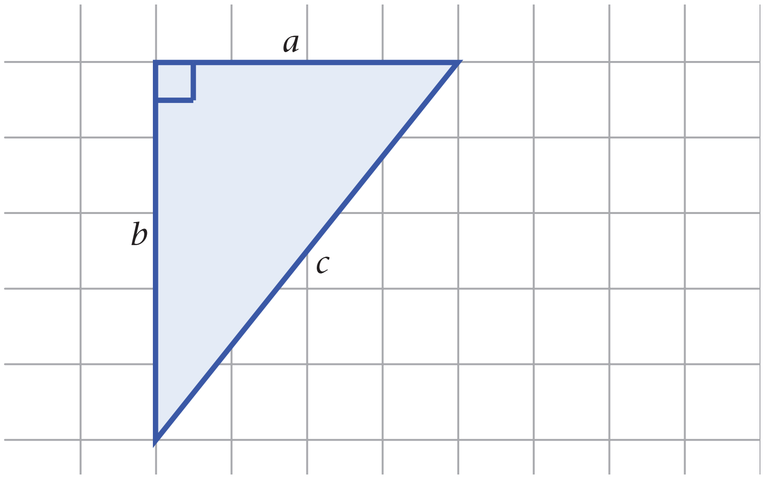
Step 2: Draw the hypotenuse of your triangle.

Colour the triangle blue.

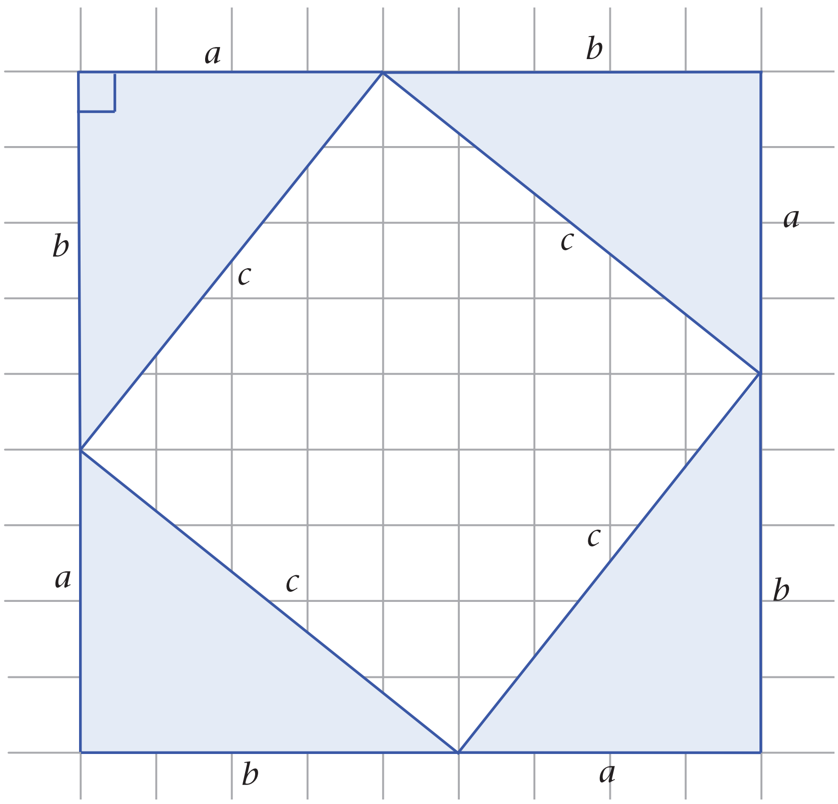
Label the short leg *a*.

Label the long leg *b*.

Label the hypotenuse *c*.



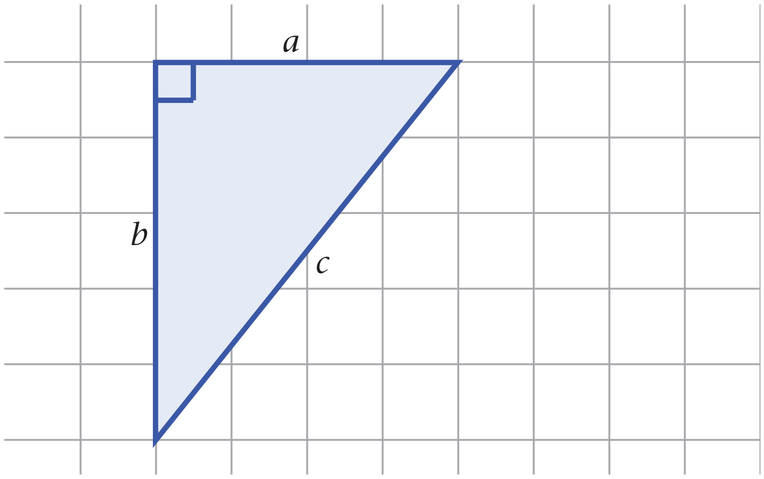
Step 3: Using the picture as a guide, draw three copies of your triangle. Colour them blue. Label the sides as you did in Step 2.



Examine the large square you have made. Can you see that the length of each side is $a + b$?

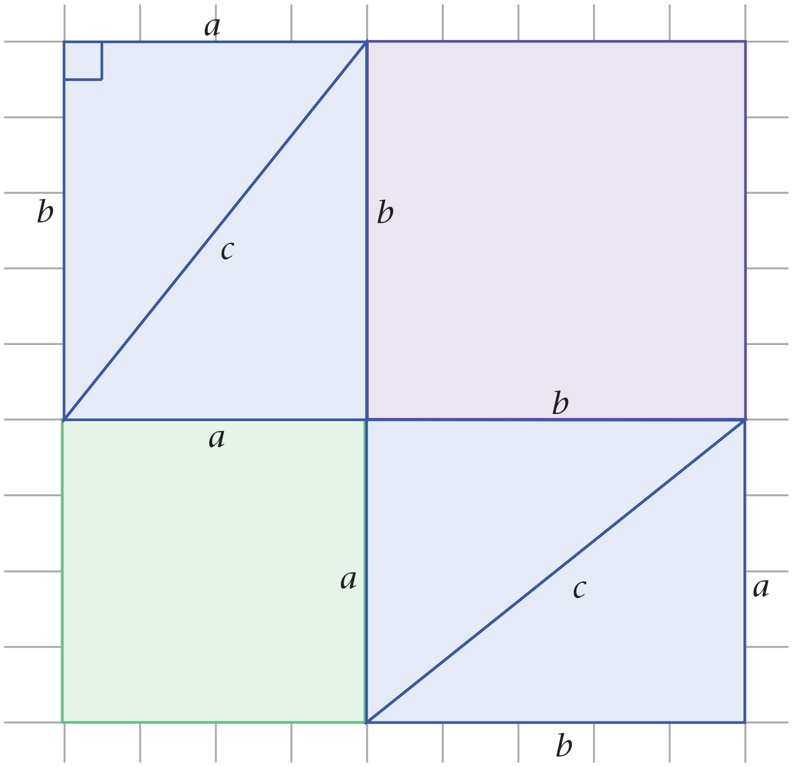
Look at the smaller white square inside. The length of each side of this square is c. The area of this square is c^2. Write "c^2" in the middle of the square.

Step 4: On another sheet of graph paper, repeat Steps 1 and 2 with the same size of triangle that you have been using so far.



Step 5: Using the picture as a guide, draw three copies of your triangle. Colour them blue. Label the sides.

Step 6: Using the picture as a guide, draw a square that encloses all of your triangles.

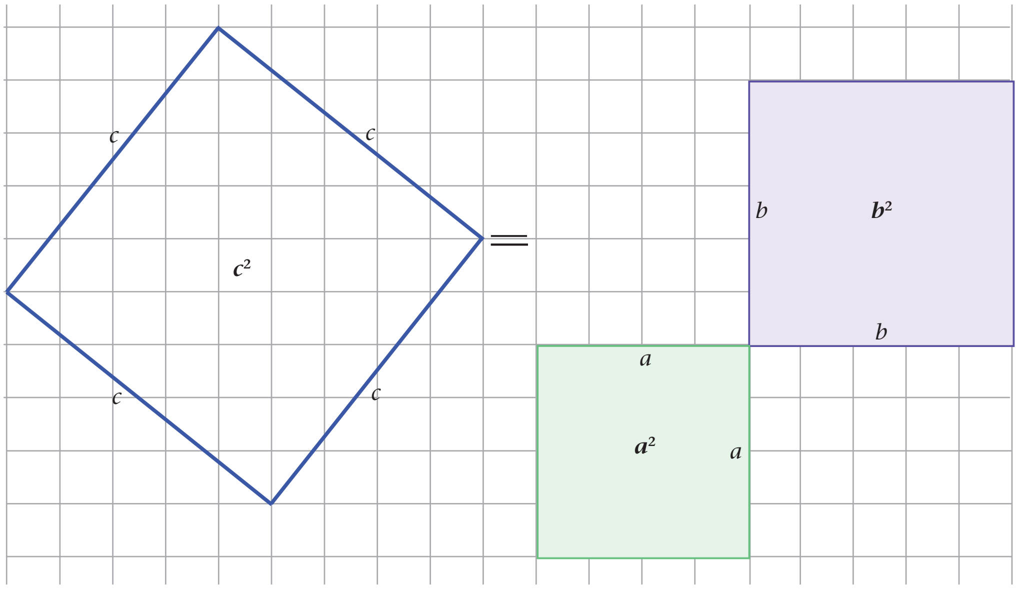


Now there are two squares inside your big square. Colour the smaller one green. Colour the other one purple.

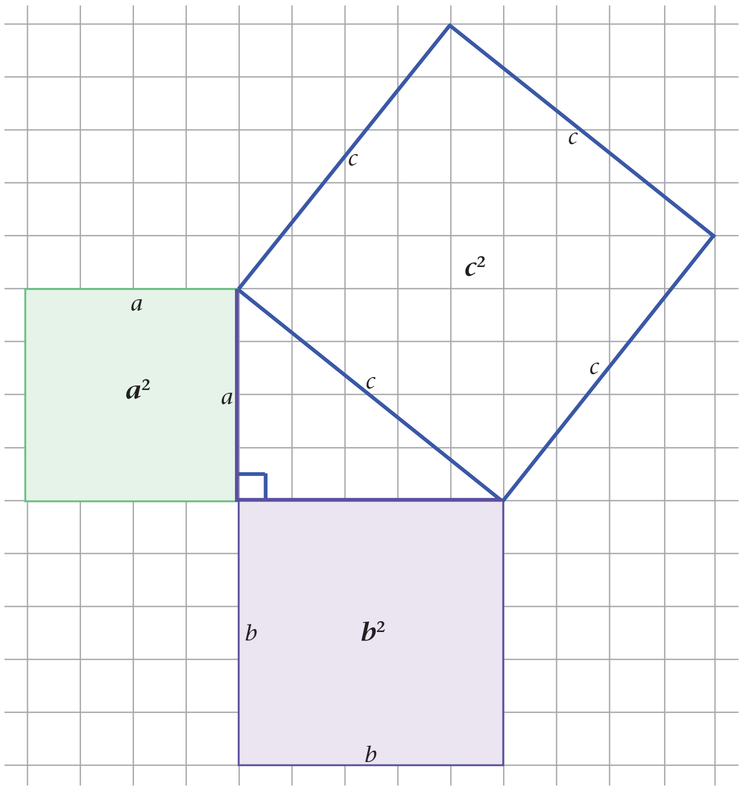
The length of each side of the green square is a. Its area is a^2. Write "a^2" in the middle of the green square.

The length of each side of the purple square is b. Its area is b^2. Write "b^2" in the middle of the purple square.

Examine the large square you have made. Can you see that the length of each side is $a + b$? It is exactly the same size as the first square you made!

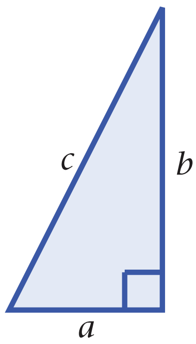


Step 7: Cut out one of your blue triangles. Cut out the squares. Arrange them as shown in the picture.



Try that activity again with right triangles of a different size. Maybe one leg is 3 squares long and the other is 7. Make four identical right triangles, and do the activity again.

You have just done a geometric proof of the Pythagorean Theorem!



The Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Even though Pythagoras was not the first to understand this property of right triangles, he was the first (we think) to express it in a general way that applies to all right triangles. That is why the theorem is named after him.

Exercises 1.3

1. State the Pythagorean Theorem.

2. Circle the whole numbers.

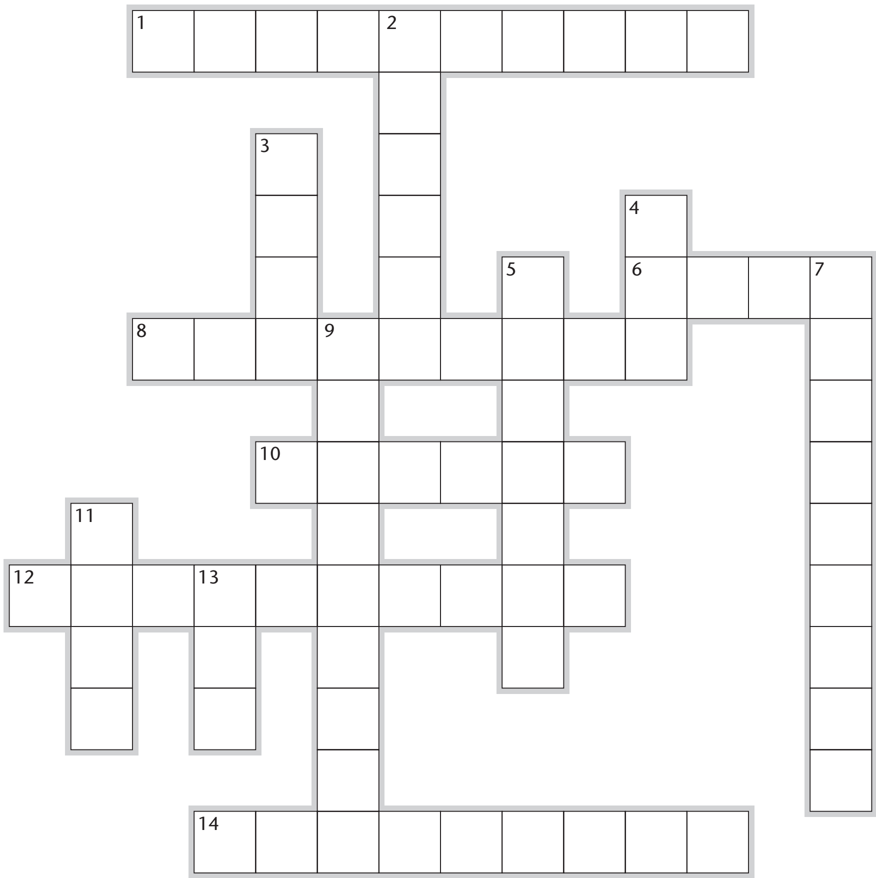
4 $\qquad \frac{1}{2}$ $\qquad 7.3$

5.1 $\qquad 6$ $\qquad \frac{1}{3}$

1.21 $\qquad 23$

3. Solve the clues in the crossword puzzle. Write out the answers in words.

   There are no spaces or dashes in crossword puzzle answers. If your answer is 41, write “FORTYONE” in the puzzle.



Across

1. 5^2
6. 3^2
8. 7^2
10. $9^2 + 3^2$
12. $8^2 – 1^2$
14. $8^2 + 1$

Down

2. $6^2 – 4^2$
3. 2^2
4. 1^2
5. 4^2
7. 9^2
9. 6^2
11. $3^2 – 2^2$
13. $3^2 + 1^2$

Turn to the Answer Key at the end of the module to check your work.

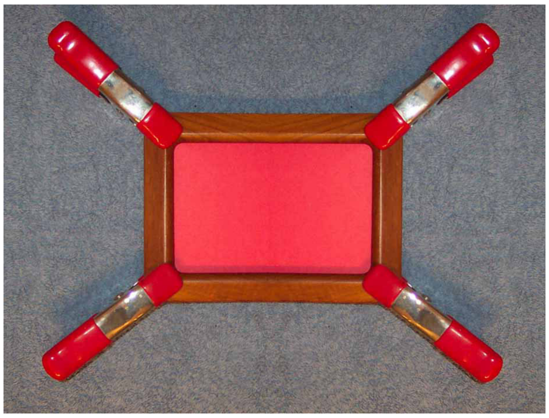
Pythagorean Triples

Using the Pythagorean Theorem

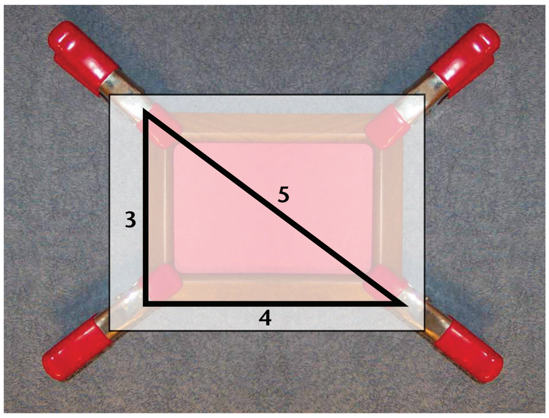
We can use the Pythagorean Theorem to check if an angle is a right angle.

Kent's Picture Frame

Kent is making picture frames. He has cut the wood for the frame and has it all clamped together. Before he glues it, he wants to make sure that the corners are right angles.



First, he measures the legs. They are 3 inches and 4 inches. Then he measures the diagonal, which is the hypotenuse of the right triangle. It is 5 inches.



We'll use the Pythagorean Theorem to find out if that corner is a right angle.

$$a^2 + b^2 = c^2$$

Fill in the lengths that Kent measured.

$$3^2 + 4^2 \stackrel{?}{=} 5^2$$

a and *b* represent the legs of the triangle
c represents the hypotenuse

We don't know yet if the left side of the equation equals the right side. Put a ? over the equals sign.

Calculate the square of each number.

$$9 + 16 \stackrel{?}{=} 25$$

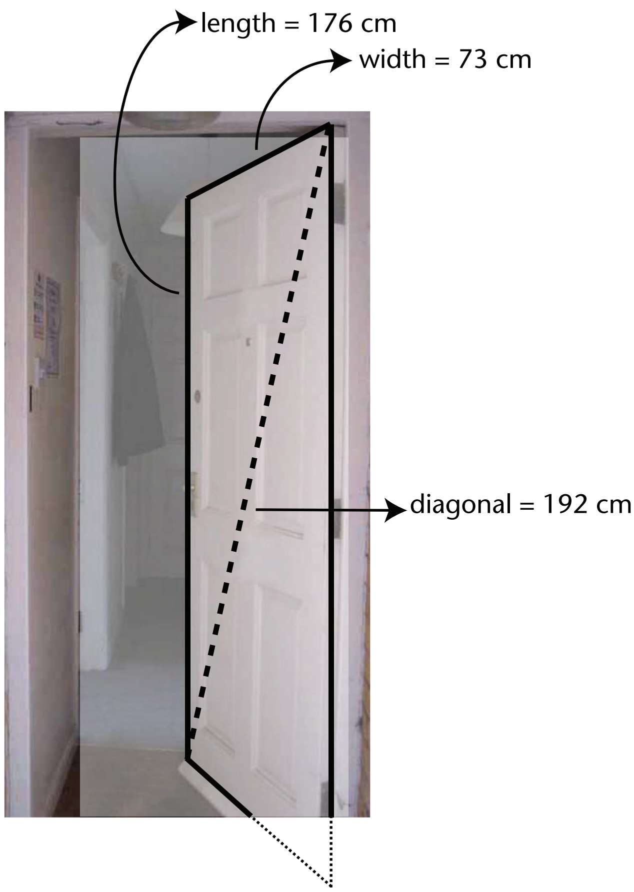
That's true! We don't need the question mark any more. Kent knows that corner of his picture frame is a right angle. He can glue it now.

$$25 = 25$$

Alexa's Door

Alexa is fixing a door. The corners weren't right angles and it didn't swing properly through the door frame.

She's been sanding the edges for a while now, and she thinks she's nearly done. She measures the lengths of the edges and the length of the diagonal, which is the hypotenuse of the right triangle.



Use the Pythagorean Theorem to check. $\boldsymbol{a^2 + b^2 = c^2}$

Fill in the lengths that Alexa measured.

c represents the hypotenuse $176^2 + 73^2 \stackrel{?}{=} 192^2$

We don't know yet if the left side of the equation equals the right side.

Put a ? over the equals sign.

Use your calculator.

Figure out the square of each number.

$30\,976 + 5329 \stackrel{?}{=} 36\,864$

That's not true! You don't need the question mark anymore and you can cross out the equals sign. Alexa knows that corner of her door is not a right angle. She still has to do a bit more sanding.

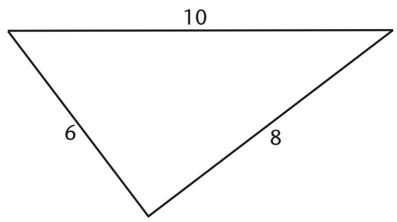
$36\,305 \neq 36\,864$

Exercises 1.4

Drawings can be deceiving! When there are no right angle markings, triangles may not always look like right triangles.

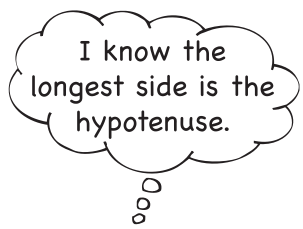
Use the Pythagorean Theorem to decide if these are right triangles or not.

1.



Use the Pythagorean Theorem to check. $a^2 + b^2 = c^2$

Fill in the lengths.
Put a ? over the equals sign.



$\_\_\_\_^2 + \_\_\_\_^2 \overset{?}{=} \_\_\_\_^2$

Figure out the square of each number. $\_\_\_\_ + \_\_\_\_ \overset{?}{=} \_\_\_\_$

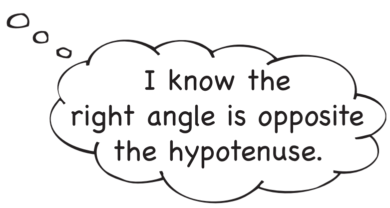
Is that true?

If it's not true, cross out the equals sign. $\_\_\_\_ = \_\_\_\_$

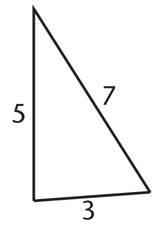
Is this triangle a right triangle or not?

If this is a right triangle, which angle is a right angle?

Mark the right angle.



2.



Use the Pythagorean Theorem to check.

Fill in the lengths.
Put a ? over the equals sign. $\underline{\qquad}^2 + \underline{\qquad}^2 \stackrel{?}{=} \underline{\qquad}^2$

Figure out the square of each number. $\underline{\qquad}^2 + \underline{\qquad}^2 \stackrel{?}{=} \underline{\qquad}^2$

Is that true?

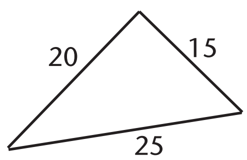
If it's not true, cross out the equals sign. $\underline{\qquad} = \underline{\qquad}$

Is this triangle a right triangle or not?

If this is a right triangle, which angle is a right angle?

Mark the right angle.

3.



Use the Pythagorean Theorem to check.

Is this triangle a right triangle or not?

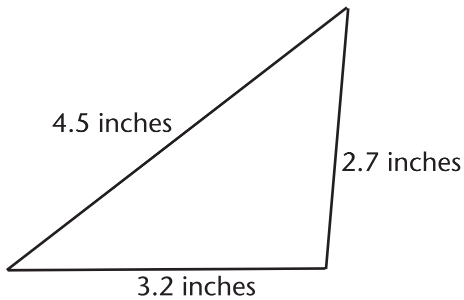
If this is a right triangle, which angle is a right angle?

Mark the right angle.

4. Draw a picture of a triangle with sides 13, 5, and 12 cm long. Is this a right triangle? Use the Pythagorean Theorem to check.

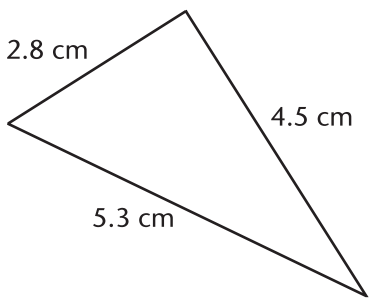
5. Draw a picture of a triangle with sides 11, 14, and 6 cm long. Is this a right triangle? Use the Pythagorean Theorem to check.

6. a. Is this triangle a right triangle? Use the Pythagorean Theorem to check.



b. Does this triangle have a hypotenuse? If not, why not? If so, how long is it?

7. a. Is this triangle a right triangle? Use the Pythagorean Theorem to check.

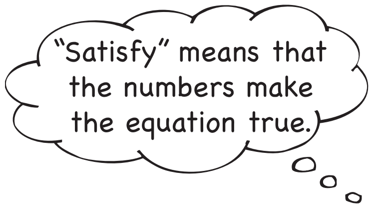


b. Does this triangle have a hypotenuse? If not, why not? If so, how long is it?

Turn to the Answer Key at the end of the module to check your work.

Pythagorean Triples

Look back at the example of Kent's picture frame.

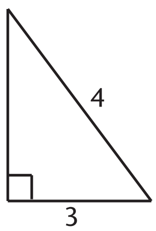


When three whole numbers **satisfy** the Pythagorean Theorem, these numbers are called a **Pythagorean Triple**.

Kent's picture frame measurements, 3, 4, and 5 inches form a Pythagorean Triple.

Alexa's door frame measurements do not satisfy the Pythagorean Theorem. These numbers are not a Pythagorean Triple.

Can you tell which number in the Pythagorean Triple 3, 4, 5 is the length of the hypotenuse? The hypotenuse is always the longest side. In this triple, 3 and 4 are the lengths of the legs. The length of the hypotenuse is 5.



Look at this triangle. Can we use the Pythagorean Triple 3, 4, 5 to solve this triangle? Could the missing measurement be 5?

No! If the missing measurement is 5, then one of the legs would be the longest side. The hypotenuse is ALWAYS the longest side.

Exercises 1.5

1. These are the measurements of the triangles you saw in Exercises 1.4. Put a check (✓) by the ones that are Pythagorean Triples.

   _________ 6, 8, 10

   _________ 3, 5, 7

   _________ 25, 15, 20

   _________ 13, 5, 12

   _________ 11, 14, 6

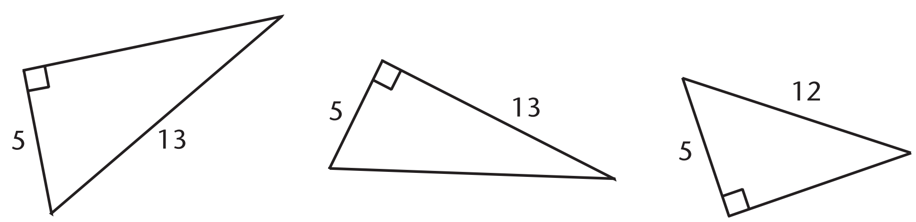
   _________ 2.7 inches, 3.2 inches, 4.5 inches

   _________ 2.8 cm, 4.5 cm, 5.3 cm

2. a. In the Pythagorean Triple 5, 12, 13, which number is the length of the hypotenuse?

   b. Which one of these triangles can be solved using the 5, 12, 13 Pythagorean Triple?

   Circle your answer. Write in the length of the missing side.



Turn to the Answer Key at the end of the module to check your work.

Using the Pythagorean Theorem

Algebra with Squares and Square Roots

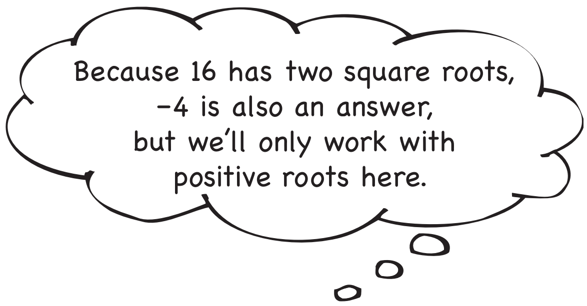
The variable in an equation is just a number that you don't know yet.

$$x^2 = 16$$

You read this equation by saying "x squared equals 16".

You can think of the equation like this: "I'm thinking of a number. If I square the number, the answer is 16. What is the number?"

If you can't think of the answer, start at the beginning with 1.



What is 1^2? ......not 16!

What is 2^2? ......not 16!

What is 3^2? ......not 16!

What is 4^2? 16! We found it! If $x^2 = 16$, then x must be 4.

It's not a bad method, but it only works with perfect squares. What would you do with this one?

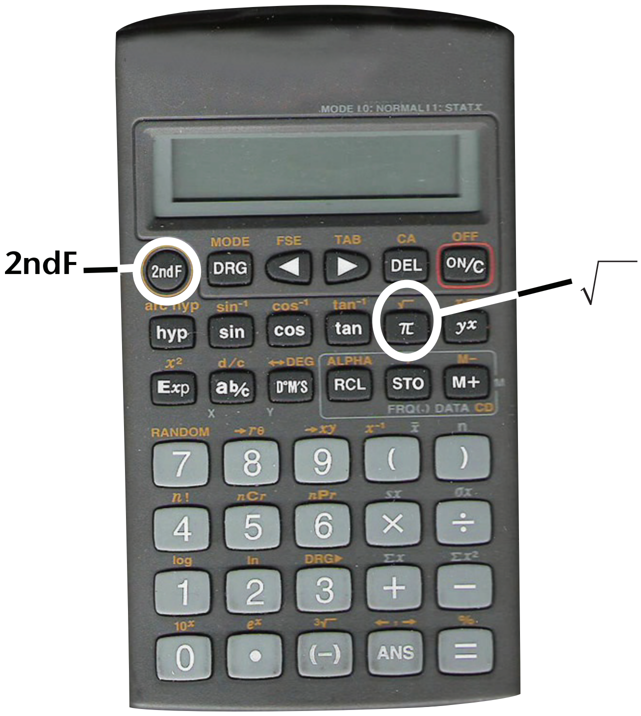
$$k^2 = 15$$

We need a way to find out what k equals. Is there something that we can do to k^2 that will leave us with k?

Take the square root of each side.

$$\sqrt{k^2} = \sqrt{15}$$

Here's an example of where you might find the square root button on a calculator.



On the calculator shown here, you have to press the 2nd function button first and then press the square root button.

Use your calculator to find $\sqrt{15}$. Round your answer to the nearest hundredth.

$$\sqrt{k^2} = \sqrt{15}$$

$$k \approx 3.87$$

Exercises 1.6

You will need:

- calculator

1. Find the value of these variables by taking the square root of both sides of the equation.

   Show all of your steps.

   You don't need your calculator to do any of these ones. Why not?

   a. $x^2 = 9$ b. $b^2 = 25$

   c. $k^2 = 100$ d. $a^2 = 49$

   e. $j^2 = 1$ f. $n^2 = 36$

Use your calculator for this question.

2. Find the value of these variables by taking the square root of both sides of the equation.

   Show all of your steps.

   Round your answers to the nearest hundredth.

   a. $x^2 = 10$

   b. $b^2 = 22$

   c. $k^2 = 107$

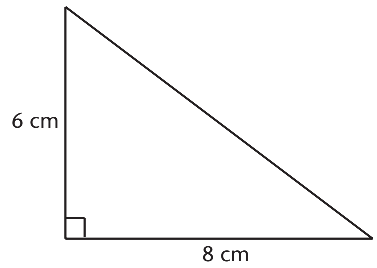
   d. $a^2 = 53$

   e. $j^2 = 8$

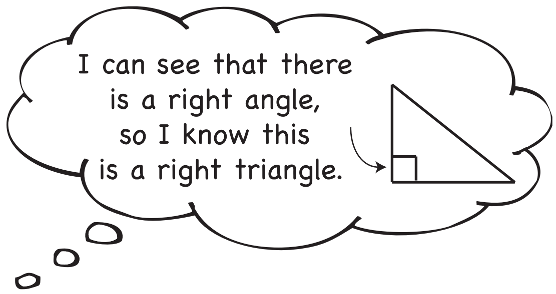
   f. $n^2 = 63$

Turn to the Answer Key at the end of the module to check your work.

Finding the Length of the Hypotenuse



How long is the hypotenuse of this triangle?



We know that this is a right triangle.

The Pythagorean Theorem tells us how the lengths of the legs of a right triangle are related to length of the hypotenuse. If we fill in the two sides that we know, we can figure out the third.

$$a^2 + b^2 = c^2$$

The lengths of the legs of the triangle are 6 and 8.

$$(6)^2 + (8)^2 = c^2$$

We don't know how long the hypotenuse is.

$$36 + 64 = c^2$$

Don't think about the variable. Figure out as much as you can.

$$100 = c^2$$

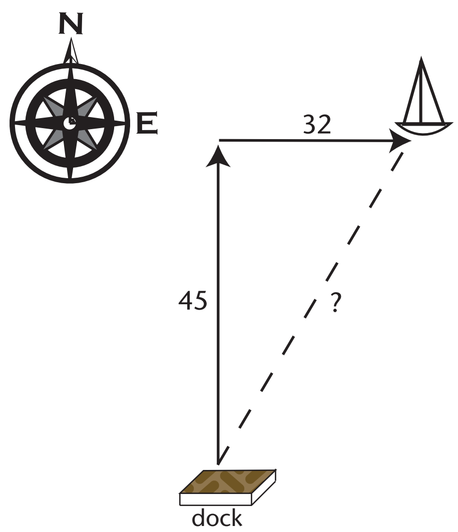
Take the square root of both sides.

$$\sqrt{100} = \sqrt{c^2}$$

The hypotenuse of the triangle is 10 cm long.

$$10 = c$$

The SS Minnow left the dock this afternoon. It sailed north for 45 nautical miles, then it sailed east for 32 nautical miles. How far away from the dock is the SS Minnow? Round your answer to the nearest nautical mile.



We know that this is a right triangle.

We can use the Pythagorean Theorem to describe the relationship of the lengths of the sides. $a^2 + b^2 = c^2$

The lengths of the legs of the triangle are 45 and 32. $(45)^2 + (32)^2 = c^2$

We don't know how long the hypotenuse is.

Don't think about the variable. Figure out as much as you can. $2025 + 1024 = c^2$

$3049 = c^2$

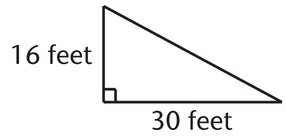
Take the square root of both sides. $\sqrt{3049} = \sqrt{c^2}$

$55.218 \approx c$

The SS Minnow is approximately 55 nautical miles away from the dock.

Exercises 1.7

1. Find the length of the hypotenuse of this right triangle.



2. A right triangle has legs that are 27 m and 53 m long.

   Draw a picture of this triangle.

   How long is the hypotenuse?

   Round your final answer to the nearest tenth of a metre.

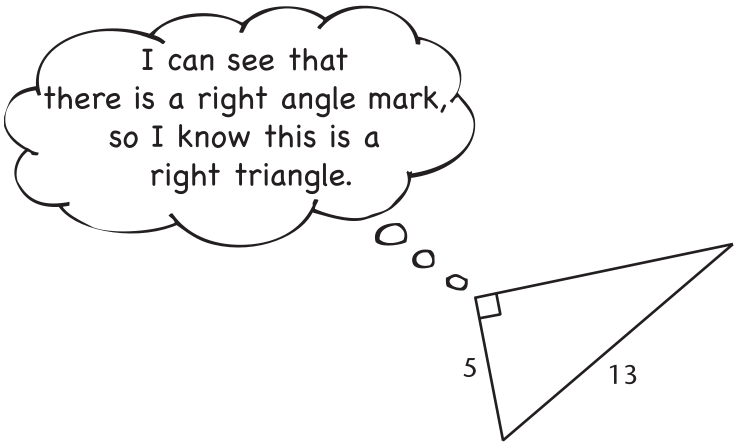
3.

If these corners are not right angles, the drawer will not slide in and out of the dresser properly.

Khira measured the width and the length of the drawer. How long should the hypotenuse be? Round your answer to the nearest tenth of a centimetre.

Turn to the Answer Key at the end of the module to check your work.

Finding the Length of a Leg of a Right Triangle



How long is the other leg of this triangle?

We know that this is a right triangle.

The Pythagorean Theorem tells us how the lengths of the three sides of a right triangle are related to each other. If we fill in the two sides that we know, we can figure out the third.

$$a^2 + b^2 = c^2$$

The length of one of the legs is 12. Substitute 12 for *a* or *b*. It doesn't matter which one. (Here we've substituted 12 for *b*.)

$$a^2 + (12)^2 = (13)^2$$

The length of the hypotenuse is 13. Substitute 13 for *c*. The hypotenuse is ALWAYS *c*.

Don't think about the variable. Figure out as much as you can.

$$a^2 + 144 = 169$$
$$a^2 + 144 - 144 = 169 - 144$$
$$a^2 = 169 - 144$$
$$a^2 = 25$$

Take the square root of both sides.

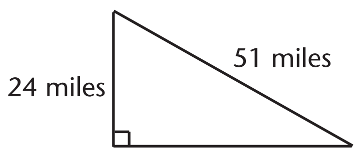
$$\sqrt{a^2} = \sqrt{25}$$

The length of the other leg is 5.

$$a = 5$$

Exercises 1.8

1. Find the length of the other leg.



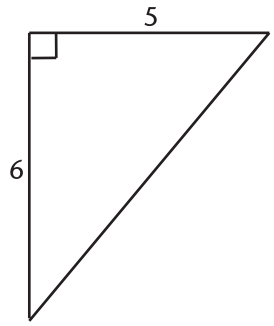
2. A right triangle has a hypotenuse that is 15 cm long. One of its legs is 7 cm long.

   Draw a picture of this triangle. Figure out the missing measurement.

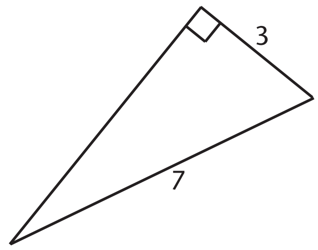
   Round your answer to the nearest tenth of a centimetre.

3. Find the length of the missing side for each of these triangles. All measurements are in centimeters. Round your answers to the nearest tenth.

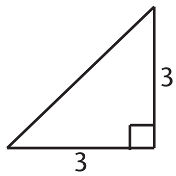
a.



b.

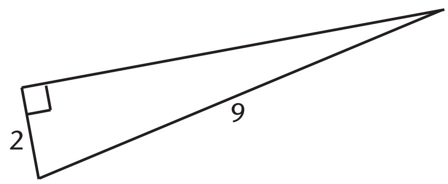


c.

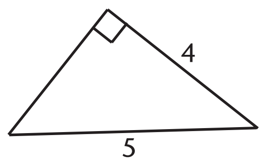


d.

e.

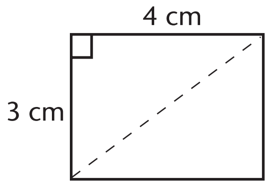


f.

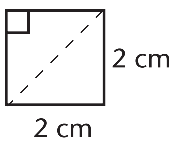


4. Find the length of each diagonal. Round your answers to the nearest hundredth.

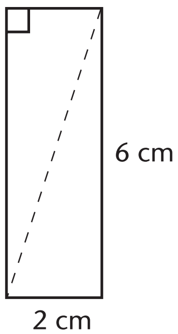
a.



b.



c.



Turn to the Answer Key at the end of the module to check your work.

Lesson 2

Nets and Views

Learning Outcomes

By the end of this lesson you will be better able to:

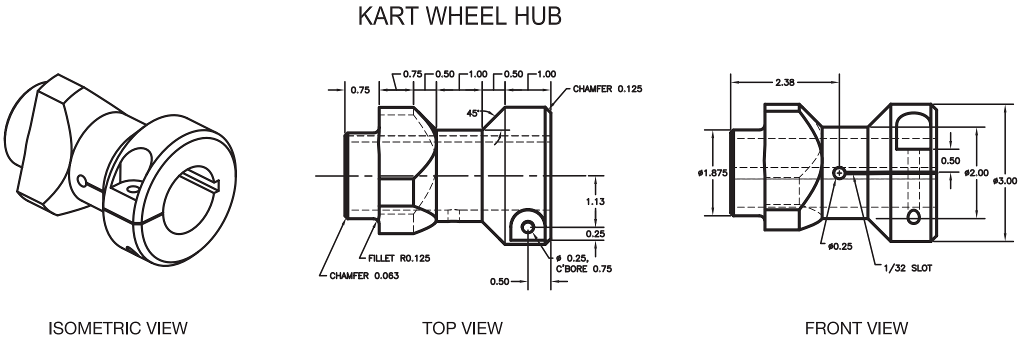
- identify right rectangular prisms, right triangular prisms, and cylinders
- draw nets of 3D objects
- draw and interpret top, front, and side views of simple 3D objects

An object that takes up space can be seen in many ways.

Our eyes see a go-kart in three dimensions.

However, developers such as engineers, designers, and fabricators will create the go-kart from a two-dimensional drawing with various views. Each of these views will look different, depending on what side of the go-kart they are looking at. All these different views help build the go-kart to the right size with the right components so it all works.

Similarly, developers create each component of a go-kart from drawings that detail it. For example, below are three views of a wheel hub for a go-kart.



Many professions, from architects to video game creators, use 2-D drawings to design and create various products.

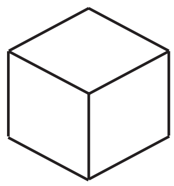
Nets

Nets of Rectangular Prisms

You will need:

- rectangular tissue box (empty)
- scissors
- tape
- templates (in Appendix)
- graph paper (in Appendix)
- isometric dot paper (in Appendix)
- Metric ruler

A cube is a 3-D shape with six equal sized squares as faces.



A die is an example of a cube.

Each of these figures below can be made into a cube… or can they?

Shade in each figure that you think can be folded into a cube.

Figure 1

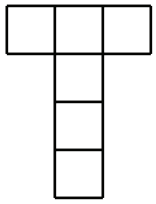


Figure 2

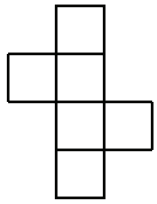


Figure 3

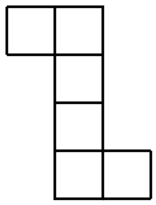
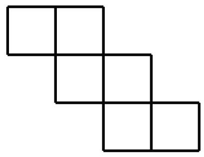


Figure 4



Now, go to the templates in the Appendix and cut out each figure. Fold along the lines to see if you can make a cube. Were you right?

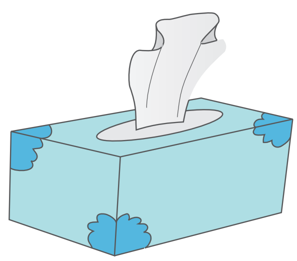
On graph paper, draw your own net made up of six connected squares that is different from Figures 1–4. Use a ruler. Cut out your drawing and fold it to see if your figure creates a cube or not.

The figure you drew and the figures you cut out are all called nets. A **net** is a 2-D figure that makes a 3-D object when it's folded.

Drawing a Net of a Rectangular Prism

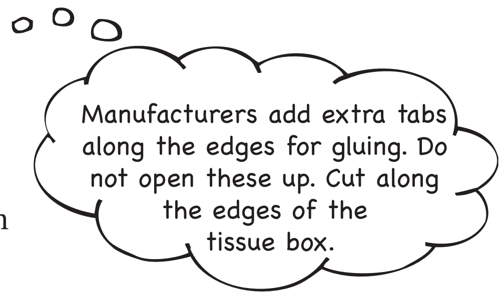
Here's another 3-D shape.

A tissue box is not a cube, but an example of a rectangular prism. One or more of its faces are rectangles that are all joined together at 90 degree angles.



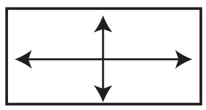
To draw a net of the tissue box, follow these instructions.

1. Take the tissue box you set aside for this lesson, and a pair of scissors.
2. Cut along each fold to flatten the tissue box.
3. Cut the flattened box into six rectangles.
4. Tape the pieces together to form the net of the tissue box.
5. Draw the net.

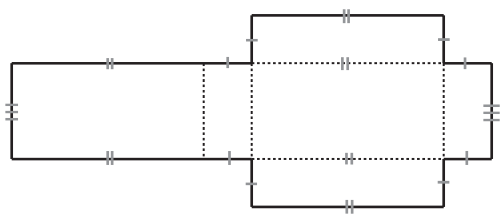


There are many edges of the rectangles above that end up taped together, and there are edges that don't. The edges that are taped together should be the same length as each other.

Also, you'll notice the opposite sides of each rectangle are the same length.



In the diagram below, the gray bars show you which edges have the same length. Taped edges are shown as dotted lines.

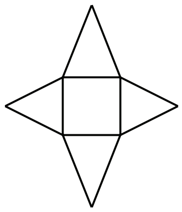


Look at your net again to see if each pair of taped edges is the same length.

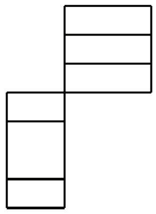
Exercises 2.1

1. Can you construct a 3-D object from each of the nets? If you think you can, shade the net in.

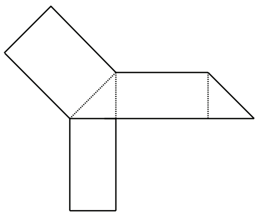
a.



b.



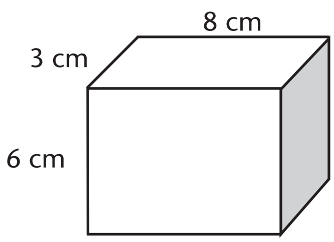
c.



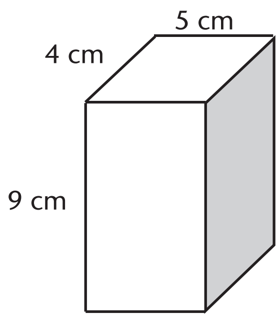
To check your answer, go to the templates in the Appendix. Cut out each net on this sheet and try to form a 3-D object to check your prediction(s).

2. Using a ruler and graph paper from the Appendix, draw a net for each of these 3-D objects.

a.



b.



3. Now cut out each net you drew in question 2. Construct your nets into shapes to confirm that they are correct.

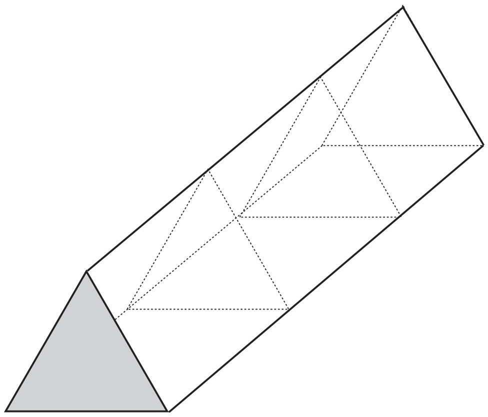
Turn to the Answer Key at the end of the module to check your work.

Nets of Triangular Prisms and Cylinders

The tissue box was an example of a rectangular **prism**. But what is a prism?

To understand the definition of a prism, we first need to understand the term **cross section**. When you slice bread, each slice has the same front and back view. When you have the same front and back view, you have a cross section.

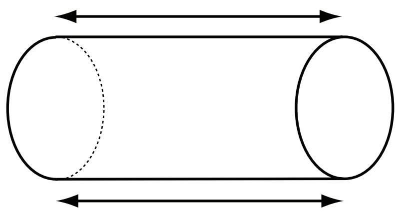
Prisms are 3-D shapes that have the same cross section along a length. In a triangular prism, all the cross sections are triangles.



In a rectangular prism, what shape are all the cross-sections?

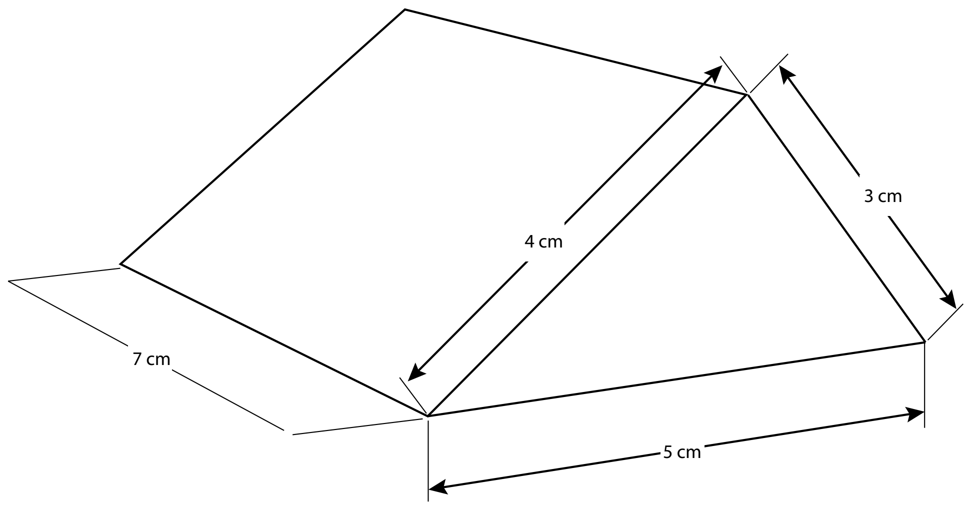
In a **rectangular prism**, the front and back views are parallel to each other and joined by rectangles.

In the case of a **cylinder**, the cross sections are all circles, and the front and back views are identical and parallel to each other.

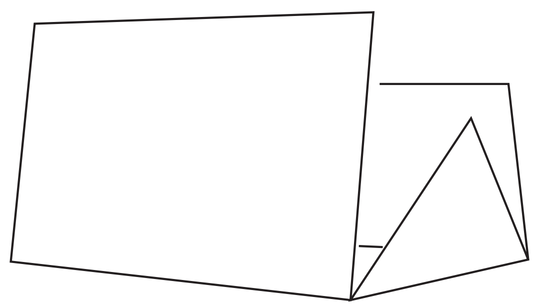


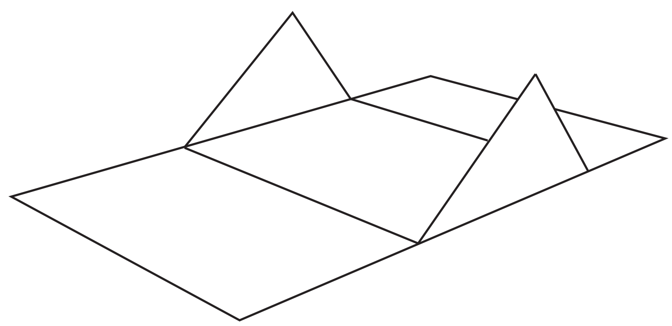
Drawing a Net of a Triangular Prism

Earlier in this lesson, you unfolded a tissue box. The flat piece of cardboard that you had when you were done was the net of the rectangular prism you started with.



When drawing the net of any object, imagine unfolding it.





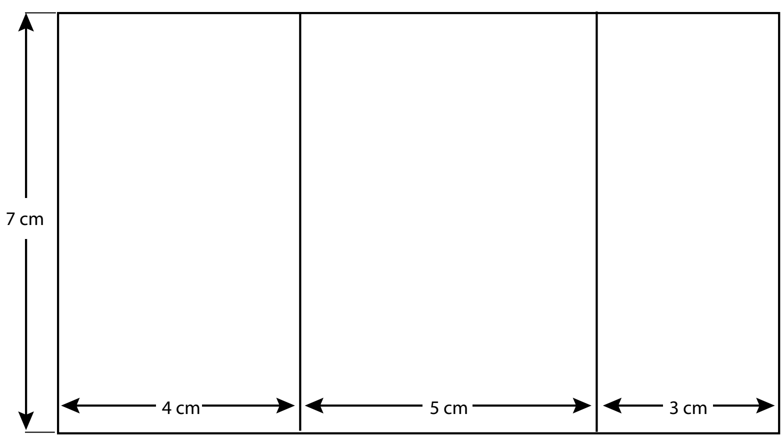
There are many ways to unfold an object to create its net. Your method might be different than the one in this example. No matter what method you use, thinking about this 'unfolding' will help you to plan the drawing of your net.

Use a ruler and graph paper to help you to draw nets with straight lines and right angles. If you know how to do geometric constructions with a compass and straightedge, your nets will be very accurate.

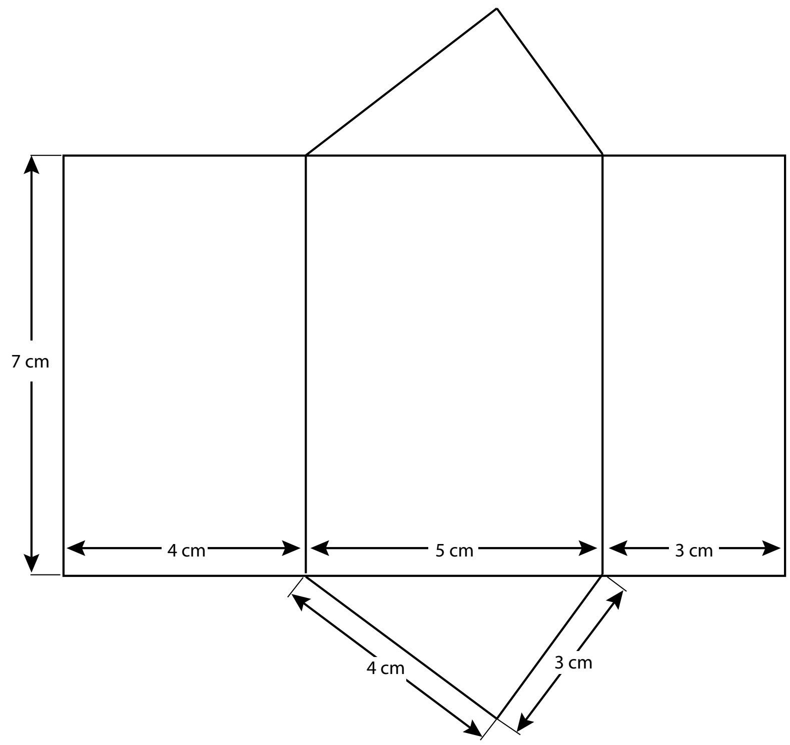
Step 1: To draw a net of this triangular prism, start with the rectangular faces. In the centre of your page, draw a rectangle that is 7 cm long and 5 cm wide.

Draw a second rectangle that is 7 cm long and 4 cm wide on the left side of your first rectangle.

Draw a third rectangle that is 7 cm long and 3 cm wide on the right side of your first rectangle.

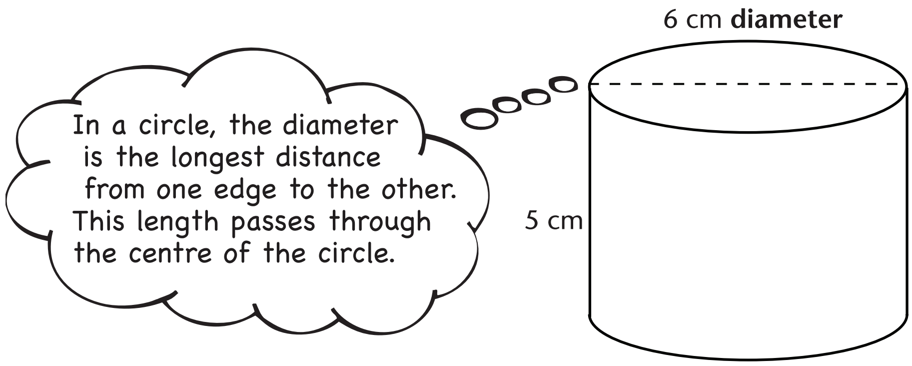


Step 2: Draw a triangle on either side of the middle rectangle.



Cut out your net and fold it into a triangular prism. Is there anything that you would do differently next time?

Drawing a Net for a Cylinder

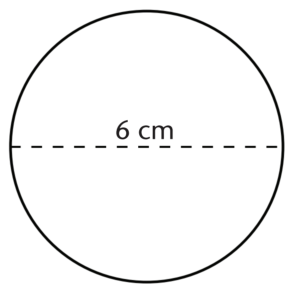


The tricky part in creating the net of a cylinder is figuring out how long to make the rectangle.

Use a ruler and graph paper to draw this net.

Step 1: Draw the top view.

The **top view** of a cylinder is a circle with a **diameter** of 6 cm.



Step 2: Draw the rectangle.

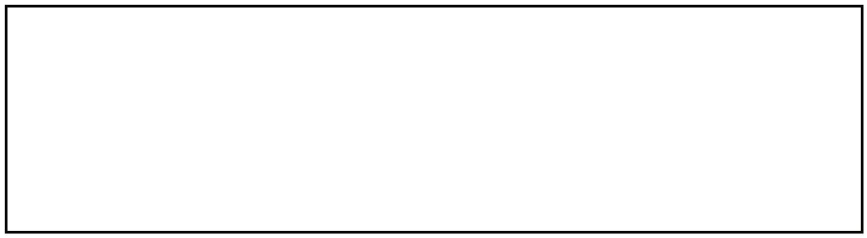
The height of the rectangle will be 5 cm. But what will the length be?

The length of the rectangle = length around the outside edge of the circle = πd

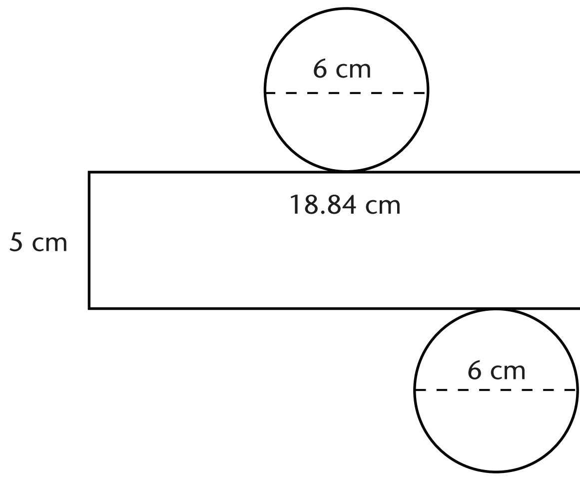
The diameter of the circle is 6 cm.

3.14×6 cm = 18.84 cm

Then the length around the outside edge of the circle is 18.84 cm.



Step 3: Draw the **bottom view** of the cylinder, which will be identical to the top view.



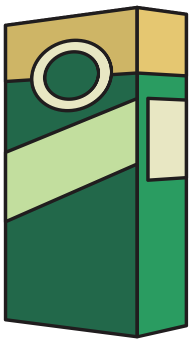
Cut out your net, and tape the edges together. Does it build into a cylinder?

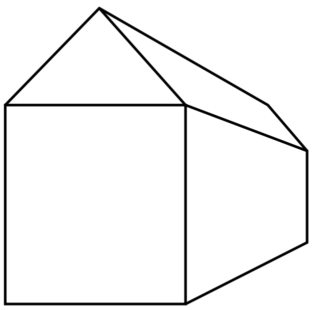
Exercises 2.2

1. Match the shape to the name. Draw lines. Each name can be used more than once.

______ 

______ 

______ 

______ 

a. rectangular prism

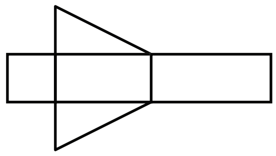
b. triangular prism

c. cylinder

d. rectangular prism and triangular prism

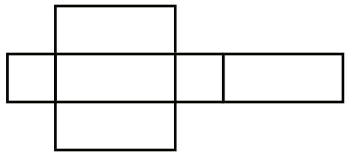
2. Which type of prism does each net form?

a.



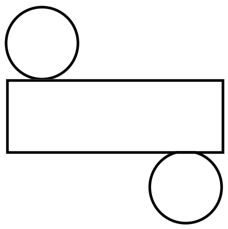
____

b.



____

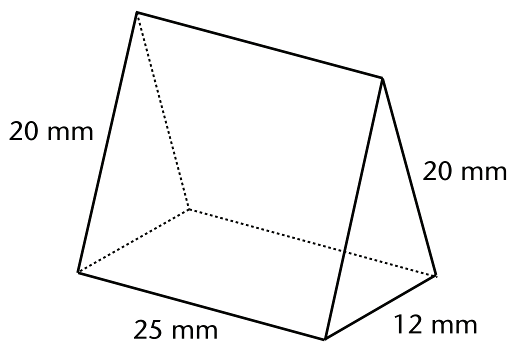
c.



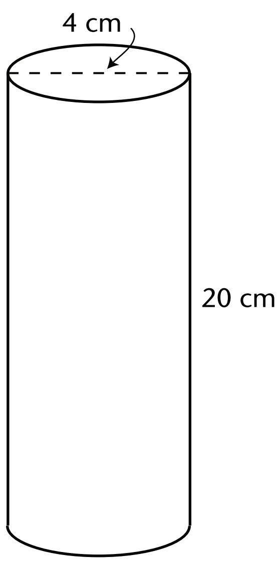
____

3. Use a ruler and graph paper found in the Appendix to sketch the net for each of the following. Label each edge with its length.

a.



b.



Turn to the Answer Key at the end of the module to check your work.

Views

Top, Front, and Side Views

You will need:

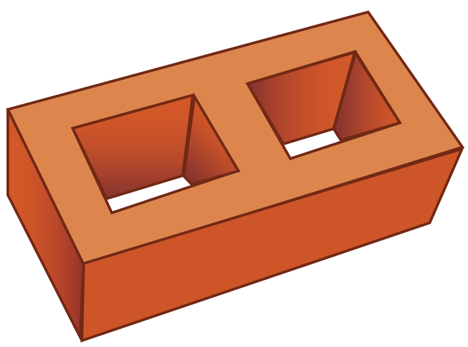
- graph paper in Appendix

Imagine someone gave you some tools, a pile of wood and some electrical wire and asked you to build a garage. Where would you start? You'd need to know what the garage was for to know how big it should be. You'd also need to know where the doors go, where lights and light switches need to be located, and many, many other details. How much easier would it be if you also had a set of drawings showing all the details about what the finished garage should look like?

For whatever we want to build we first need a plan. This is true whether we are building large complex things like a building or a car, right down to the small components within them.

The plan for anything we build is known as a working drawing. A working drawing contains 2-D views of the object. At a minimum, there are views of the the top, front, and side. These views contain measurements and all the other details needed to build the object. A 3-D view, often an isometric drawing, can also be part of a working drawing to help us visualize what the object will look like.

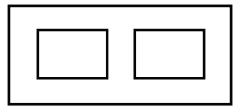
Let's work through the process backwards, starting with the finished product, a clay brick.



Use graph paper for the following sketches:

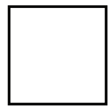
Sketch the **top view**.

Standing over top and looking down, what do you see?



Sketch the **side view**.

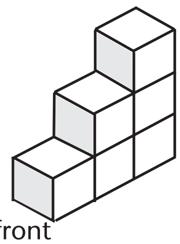
Standing to the side, either left or right what do you see?



Sketch the **front view**.

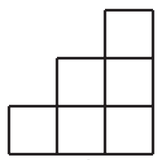
Standing in front, what do you see?

Here is a slightly more challenging 3-D shape.



front

Sketch the **side view**.

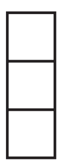


side

Sketch the **front view**.

We know from the 3-D drawing that the cubes step back like stairs.

But since we are drawing in 2-D, there is no depth. So we still draw these faces of the cubes all in one column, one on top of the other.

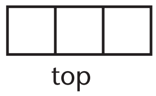


front

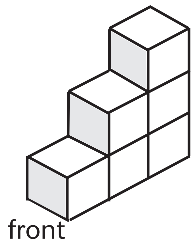
Sketch the **top view**.

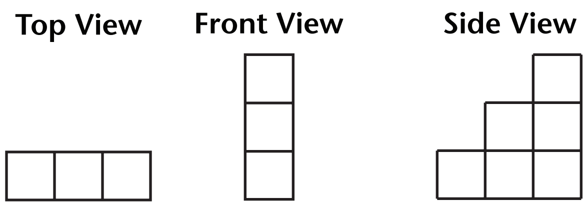
Looking from the top, one of the faces of the cubes starts closer to you and then as you move along, the next cube face is further away and the next is even further.

But, since we are drawing in 2-D, there is no depth. So we draw the faces of other cubes all in one row, one beside the other.



Here is the tower of cubes again with all of the views:

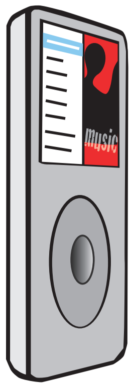




Exercises 2.3

1. Draw the top, front, and side view of each object on graph paper.

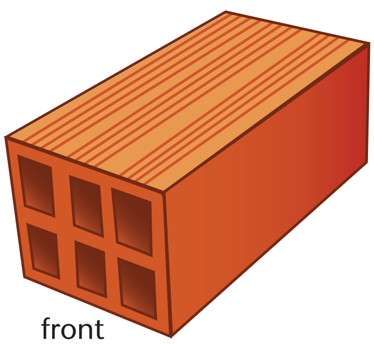
a.



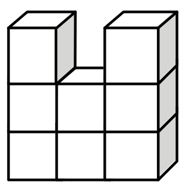
b.

2. Draw the top, front, and side views of each of these 3-D objects on graph paper.

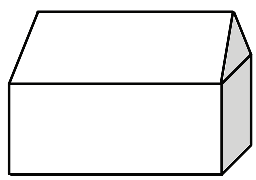
a.



b.



c.



Turn to the Answer Key at the end of the module to check your work.

Lesson 3

Surface Area

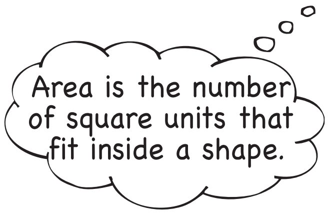
Learning Outcomes

By the end of this lesson you will be better able to:

- find the surface area of prisms and cylinders
- find the surface area of composite 3D objects

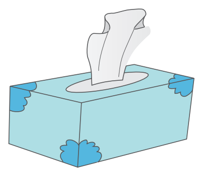
Surface Area of Prisms

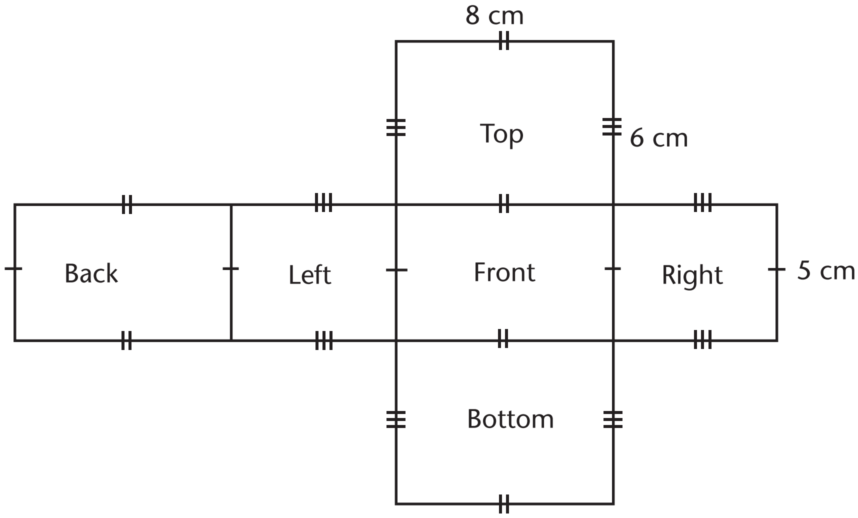
Every 3-D object is made up of 2-D faces. Each face has an **area**. All the faces together form a net.



Surface area is the area of all the faces in the net added together. The units of surface area are units squared, for example cm^2 or m^2.

A tissue box is made up of cardboard. Manufacturers figure out the surface area in order to figure out how much cardboard to buy.





The little ticks on the edges of this net tell you that many of the sides have the same length. On the far right of the net is a side with one tick and a length of 5cm. All of the other sides with one tick have a length of 5cm.

The net of the tissue box helps you calculate the surface area.

Fill in the missing numbers. The first one is done for you.

Front Area: 8×5

Back Area: ________

Left Side Area: ________

Right Side Area: ________

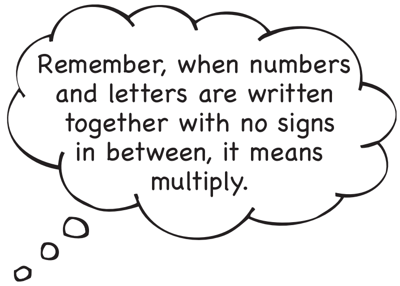
Top Area: ________

Bottom Area: ________

Notice that:

- The **front and back views** are the same size. The areas were found by multiplying 8×5 or $l \times h$. The area of both views together is $2lh$.
- The **left and right views** are the same size. The areas were found by multiplying 6×5 or $w \times h$. The area of both views together is $2wh$.
- The **top and bottom views** are the same size. The areas were found by multiplying 8×6 or $l \times w$. The area of both views together is $2lw$.

So a formula for finding the surface area of a rectangular prism is:



$$SA = 2lw + 2wh + 2lh$$

To use this formula, plug in 8 for *l*, 6 for w, and 5 for h.

$$SA = 2\ l\ w + 2\ w\ h + 2\ l\ h$$
$$SA = 2(8)(6) + 2(6)(5) + 2(8)(5)$$

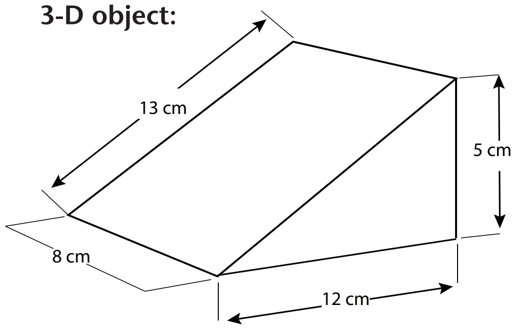
Enter these numbers in a scientific calculator exactly as shown here:

$$2 \times 8 \times 6 + 2 \times 6 \times 5 + 2 \times 8 \times 5 =$$
$$SA = 236\ \text{cm}^2$$

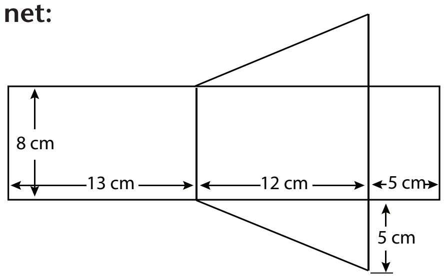
Surface Area of a Triangular Prism

The surface area of a triangular prism is found in the same way. Draw the net, find the area of each face, and then add them all together.

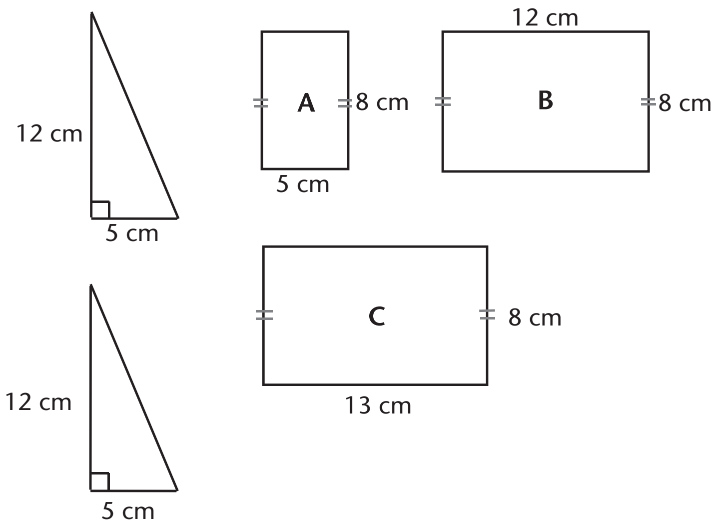
3-D object:



Its net:

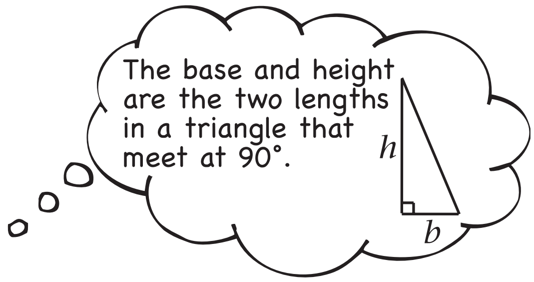


All the different faces:



This net has two identical faces that are triangles and three different sized rectangles.

The area of one triangle is:



$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(5)(12)$$

$$A = 30 \text{ cm}^2$$

So the area of both triangles is:

$$30 + 30 = 60 \text{ cm}^2$$

The area of the 3 different rectangles in the net are:

Rectangle A	**Rectangle B**	**Rectangle C**
A = lw	A = lw	A = lw
A = (5)(8)	A = (12)(8)	A = (13)(8)
A = 40 cm^2	A = 96 cm^2	A = 104 cm^2

The area of all three rectangles is:

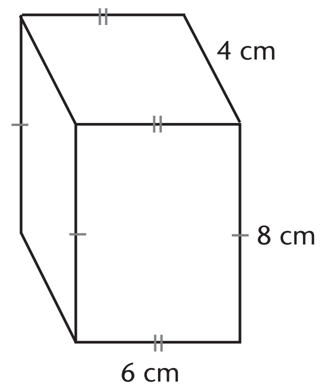
$$40 + 96 + 104 = 240 \text{ cm}^2$$

The surface area of the entire triangular prism is:

$$60 \text{ cm}^2 + 240 \text{ cm}^2 = 300 \text{ cm}^2$$

Exercises 3.1

1. For this rectangular prism:



a. Draw the net or the various views for the prism. Label the lengths of all the sides. Use the graph paper in the Appendix.

b. Find the area of the entire net.

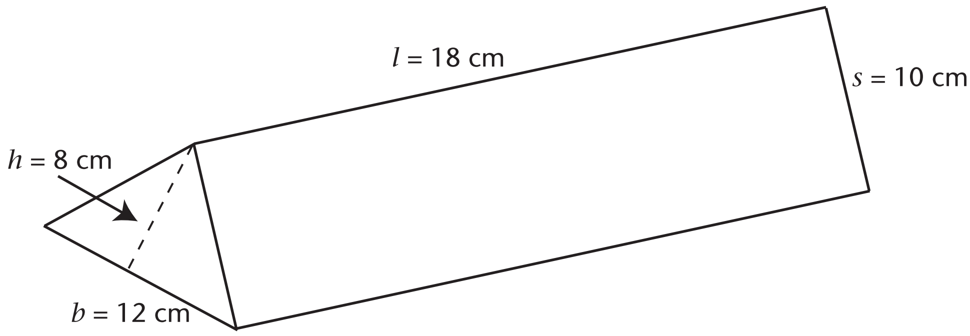
2. For the prism in question 1:

a. Identify the length, the width, and the height of the prism.

l = ______ w = ______ h = ______

b. Using the SA formula for a rectangular prism, find the surface area. Show your work.

3. For this triangular prism:



a. Draw the net using the graph paper in the Appendix.

b. Find the surface area.

Turn to the Answer Key at the end of the module to check your work.

Surface Area of a Cylinder

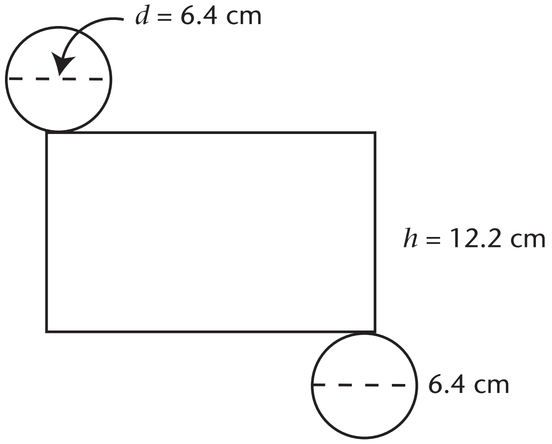
You will need:

- graph paper

Pop cans come in the form of a cylinder. They typically have a height of 12.2 cm and a diameter of 6.4 cm.

How much aluminum would cover this pop can?

To answer this question, we need to look at the net.



The net of a cylinder includes two circles and a rectangle.

The formula for the area of a circle is:

$$A = \pi \times r \times r \text{ or } A = \pi r^2$$

The length of the diameter is 6.4 cm.

> The radius goes from the centre of the circle to the edge. The radius is half as long as the diameter of the circle.

To use this formula you will plug in 3.2 cm for "*r*". Why? Because the radius is half the diameter (6.4 ÷ 2 = 3.2).

The area of each circle is:

> Use the π button on your calculator, not 3.14

$$A = \pi r^2$$
$$A = \pi \times r \times r$$
$$A = \pi \times 3.2 \times 3.2$$
$$A = 32.16990877\ldots..$$

You can round this to one decimal place, to get 32.2 cm^2.

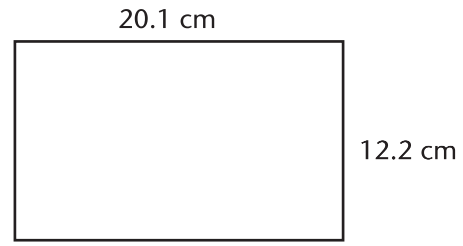
The area of both circles is 32.2 + 32.2 = 64.4 cm^2

Let's work on the area of the rectangle next.

> The **circumference of a circle** is the distance around the circle.

The length of the rectangle is equal to the circumference of the circle.

Length of the rectangle
= circumference of the circle
= $\pi d = \pi \times (6.4 \text{ cm})$
= 20.1 cm



So the rectangle has dimensions 20.1 cm by 12.2 cm.

The area of the rectangle is:

$$A = lw$$
$$A = (20.1)(12.2)$$
$$A = 245.22 \text{ cm}^2$$

You can round this to one decimal place, to get 245.2 cm^2.

The surface area of the entire pop can is:

$$64.4 \text{ cm}^2 + 245.2 \text{ cm}^2 = 309.6 \text{ cm}^2.$$

Just over 300 cm^2 of aluminum is needed to make a pop can.

In your calculations, notice that both of the circles are the same size, and the areas were found by multiplying $\pi \times r \times r$ which is the same as πr^2. The area of both circles together is $2\pi r^2$.

The length of the rectangle is equal to πd and the width of the rectangle is h. The area of the rectangle is length (πd) times width (h) so the area is πdh.

So a formula for finding the surface area of a cylinder is:

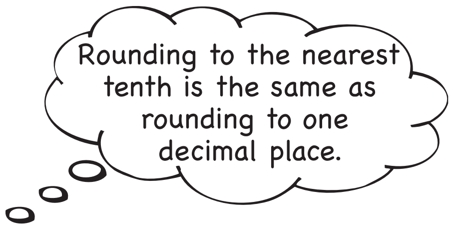
$$SA = 2\pi r^2 + \pi dh$$

To use this formula, plug in the 3.2 for *r*, 6.4 for *d*, and 12.2 for *h*.

$$SA = 2\pi \quad r \qquad r \quad + \pi \quad d \qquad h$$
$$SA = 2\pi(3.2)(3.2) + \pi(6.4)(12.2)$$

Put these into a scientific calculator exactly as they appear.

SA = 309.6353719…



Round this to the nearest tenth.

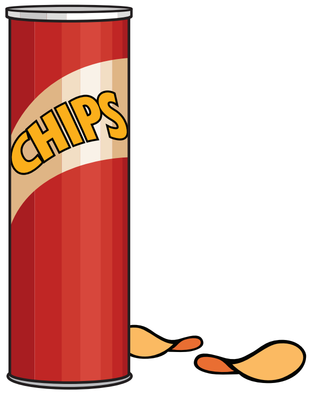
SA = 309.6 cm^2

Exercises 3.2

1. a. Using a ruler and graph paper, draw a cylinder with a diameter of 5 cm and a height of 7 cm.

   b. Using a ruler and graph paper, draw the net of this cylinder. Draw the length of the rectangle to the nearest millimetre.

2. A potato chip can is in the shape of a cylinder. The top and bottom are made of plastic. The rest is made of cardboard. The diameter of the can is 5 cm and the height is 7 cm.



   a. Find the total area covered in plastic. Round to one decimal place.

   b. How many square centimetres of cardboard (to the nearest tenth) does the can use?

   c. Use the formula for the surface area of a cylinder to find the total surface area of the can of chips. Round to the nearest tenth.

3. The formula for the SA of a cylinder is SA = $2\pi r^2 + \pi dh$, but another way of writing this formula is: SA = $2\pi r^2 + 2\pi rh$.

   a. What is the difference between these two formulas for the surface area of a cylinder?

   ____

   ____

   b. Explain why these two formulas will give you the same result.

   ____

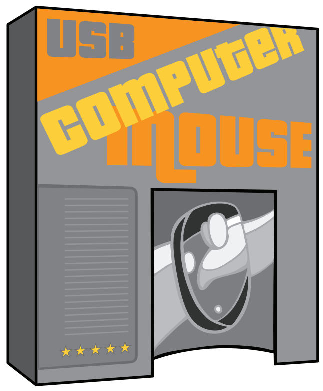
   ____

Turn to the Answer Key at the end of the module to check your work.

More About Area in 2D and 3D

Areas of Irregular Shapes

Packaging often looks like a prism or cylinder, and it may have cutouts.

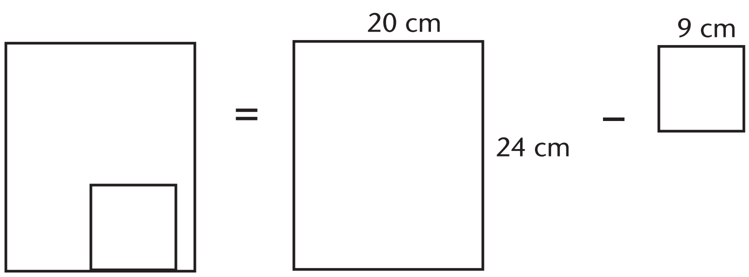


In the computer mouse package above, the front view has a square cut out of it. The square measures 9 cm by 9 cm. The front view has outer dimensions of 20 cm by 24 cm.

What is the area of the front face?

Here is a drawing of the front view with an explanation about how to find its area.

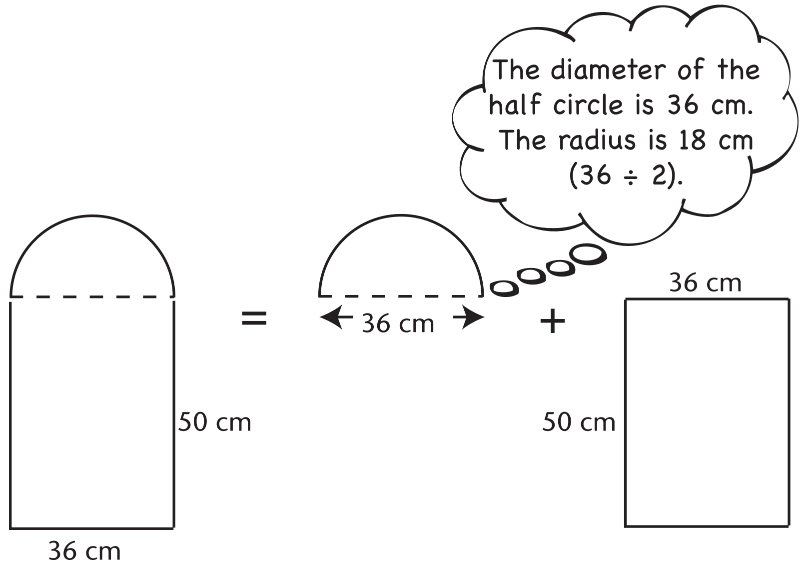
Front View



Area of the Irregular Shape = (Area of Rectangle) minus (Area of Square)

$$\begin{aligned} &= lw &&- \quad s^2 \\ &= 24 \times 20 &&- \quad 9^2 \\ &= 480 &&- \quad 81 \\ &= 399 \text{ cm}^2 \end{aligned}$$

Mirrors often come in shapes that are not simple rectangles. A mirror could appear as a rectangle with a half circle on top.



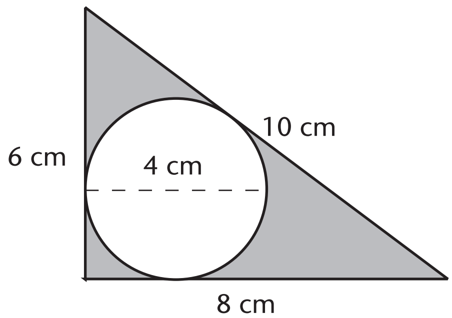
Area of the Irregular Shape = (Area of Rectangle) plus (Area of Half Circle)

$$
\begin{aligned}
&= lw && + && \pi r^2 \div 2 \\
&= 36 \times 50 && + && (\pi \times 18^2) \div 2 \\
&= 1800 && + && 508.9 \\
&= 2308.9\ \text{cm}^2
\end{aligned}
$$

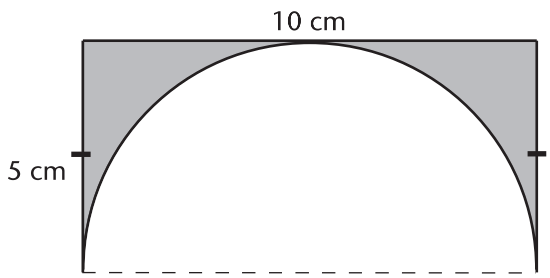
Exercises 3.3

1. Find the area of shaded region. Write answers to the nearest tenth.

a.

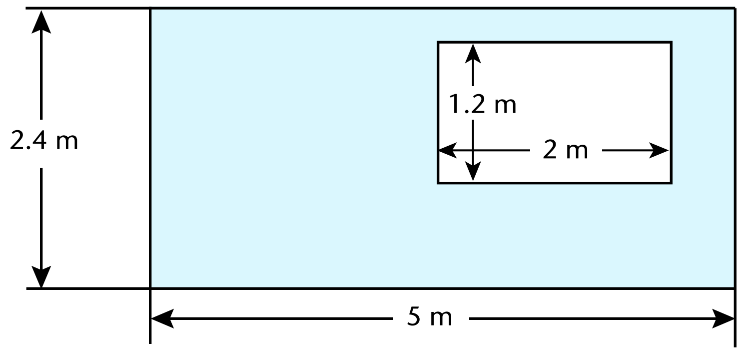


b.



2. You really want to paint your room a new colour, but you are only allowed to paint one wall. You choose the wall that has a window in the corner of it.

   a. Calculate the area that you need to paint.



   b. If a small can of paint will paint 8 m^2, will you have enough paint?

   c. If you need to do two coats of paint, how many small cans will you need?

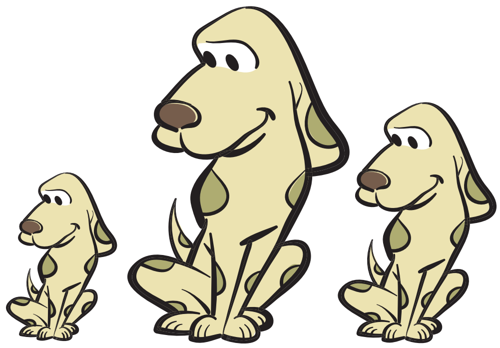
   d. A large can of paint will paint 50 m^2 and costs $22. A small can of paint will paint 8 m^2 and costs $10. Would you buy a large can of paint, or several small cans? Explain your answer.

Turn to the Answer Key at the end of the module to check your work.

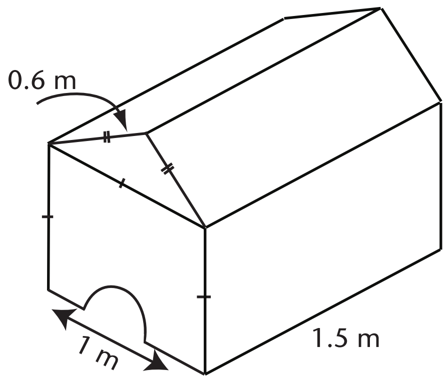
Area of Composite Figures

The size of a building or home will affect the cost to make it. In packaging, this is also the case, but the cost is not as great. Building materials cost much more than the paper and plastics needed to package a product.

To build a doghouse, the size of the dog will affect the size of the building.

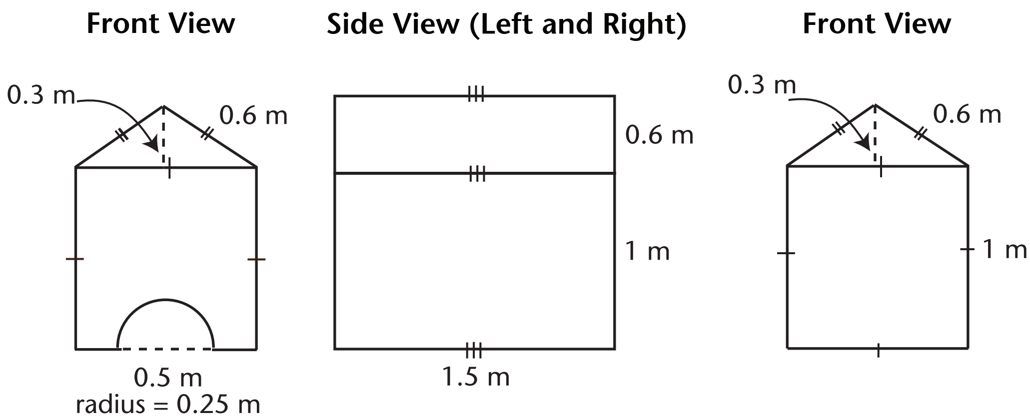


The doghouse you decide to build has a half circle opening at the front only. The half circle has a diameter of 0.5 m. There is no bottom on the doghouse.



The height of the triangle on the roof is 0.3 m.

The cost to build the doghouse depends on the amount of wood you need. The wood needed depends on the surface area of the doghouse. Let's find the surface area to two decimal places.



To find the area of the front:

Area of front = Area of triangle + Area of square − Area of half circle

$$\begin{aligned} &= \frac{1}{2}bh && + s^2 && - \left(\pi r^2\right) \div 2 \\ &= \frac{1}{2}(1)(0.3) && + 1^2 && - \left(\pi \times 0.25^2\right) \div 2 \\ &= 1.05182523\ldots \end{aligned}$$

Round this to two decimal places, so the area of the front = 1.05 m^2.

To find the area of the sides:

Area of the side = area of roof rectangle + area of bottom rectangle

$$\begin{aligned} &= lw && + lw \\ &= (1.5)(0.6) && + (1.5)(1.0) \\ &= 2.4 \end{aligned}$$

Since there are two identical sides: Area of the sides = 2 × 2.4 = 4.8 m^2.

To find the area of the back:

Area of back = Area of triangle + Area of square

$$= \frac{1}{2}bh \qquad + s^2$$

$$= \frac{1}{2}(1)(0.3) \qquad + 1^2$$

$$= 1.15$$

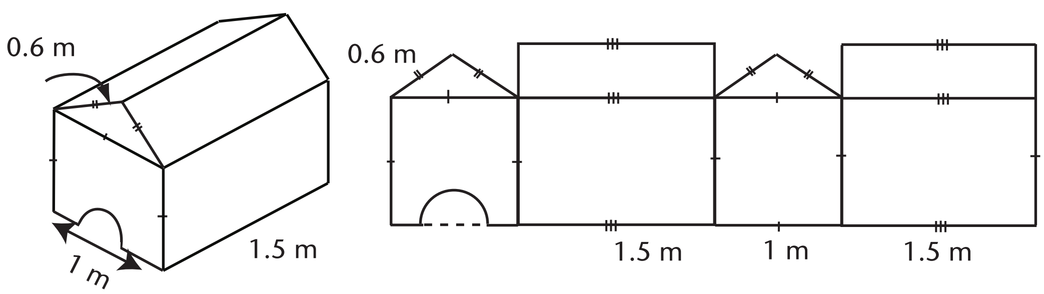
In total, the surface area of the doghouse is:

$$\text{SA} = \text{area of front} + \text{area of sides} + \text{area of back}$$
$$= 1.05\ \text{m}^2 + 4.8\ \text{m}^2 + 1.15\ \text{m}^2 = 7\ \text{m}^2$$

So you need to purchase approximately seven square metres of wood to build this doghouse. If the wood costs $4.89 per square metre, then the cost of wood would be:

$$\text{cost of wood} = 7\ \text{m}^2 \times \$4.89/\text{m}^2 = \$34.23$$

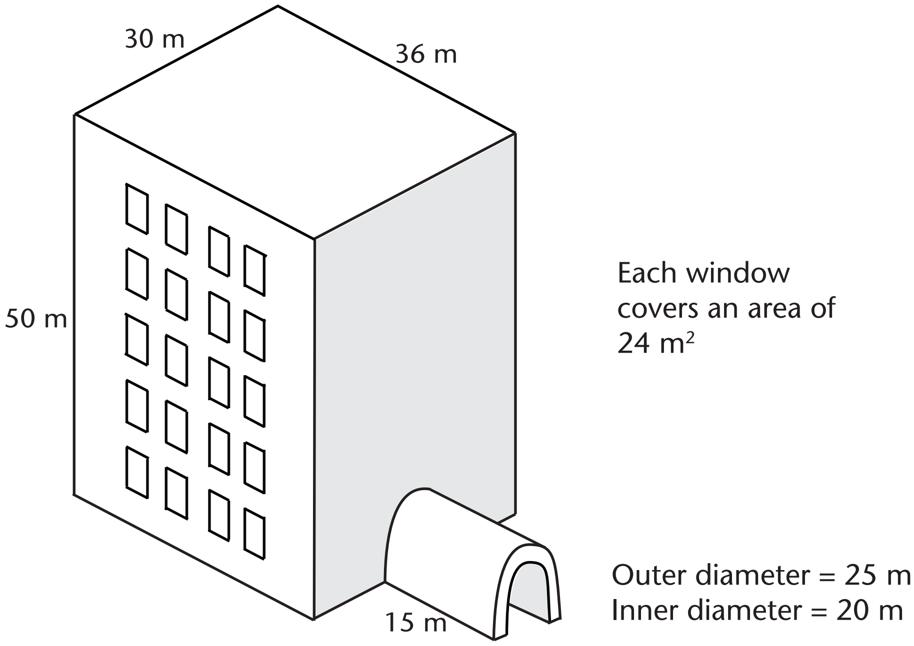
You can also use a net to find the surface area of the doghouse.



Draw the net, find the area of each face, and add all the areas together.

Exercises 3.4

1. A cement building is to be painted. The sides and back of the building have no windows. The roof will not be painted, but the rest of the building and the parking garage entrance will be.



   a. Draw the front view, back view, left view, and right view of this building. Include any lengths that you know in your diagram.

   Front View **Back View**

Left Side View **Right Side View**

b. For each view, find the area that will be painted. Do not include the parking garage in these calculations. Round to two decimal places.

Front view (without parking garage)

Back view

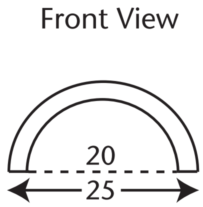
Left side view

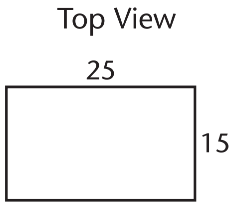
Right side view

c. What is the total area of the building that must be painted, without including the parking garage?

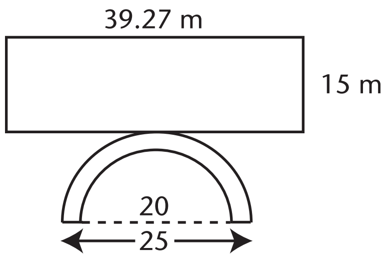
2. The two painters decide to figure out the area of the parking garage before painting.

Painter One uses views to find the surface area:





Painter Two draws the net:



a. Using the numbers in their drawings, Painter One ends up with the incorrect surface area. Painter Two ends up with the correct answer. Explain why Painter One arrives at the wrong answer.

b. Using Painter Two's diagram, find the area that needs to be painted (to two decimal places).

Turn to the Answer Key at the end of the module to check your work.

Lesson 4

Scale Diagrams

Learning Outcomes

By the end of this lesson you will be better able to:

- draw and interpret scale diagrams

Ratios and Scale Drawings

Example 1

A scale diagram of a bug is shown at the right with a scale of 20 cm (length on diagram) represents 1cm (length on actual object).

What is the actual length of the bug to 1 decimal place?

Solution

Remember all the skills you learned using ratios and proportions. The length of the bug in the diagram is 3.4 cm (measure with your ruler).

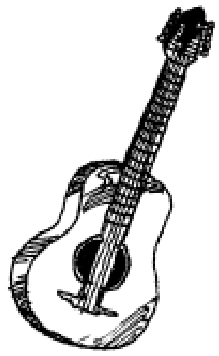
$$20 \text{ cm} \quad \leftrightarrow \quad 1 \text{ cm}$$

$$1 \text{ cm} \quad \leftrightarrow \quad \frac{1}{20} \text{ cm}$$

Therefore 3.4 cm $\leftrightarrow 3.4 \times \frac{1}{20} = 0.17$ cm.

The length of the bug is 0.2 cm (to 1decimal place).

Example 2



A scale diagram of a guitar is shown at the right. The real guitar is 100 cm long. The length of the scale diagram is 5 cm.

Then 5 cm ↔ 100 cm.

Calculate the width of the widest part of the guitar.

Solution

From the diagram, the width of the widest part of the guitar is 1.9 cm (measure with your ruler).

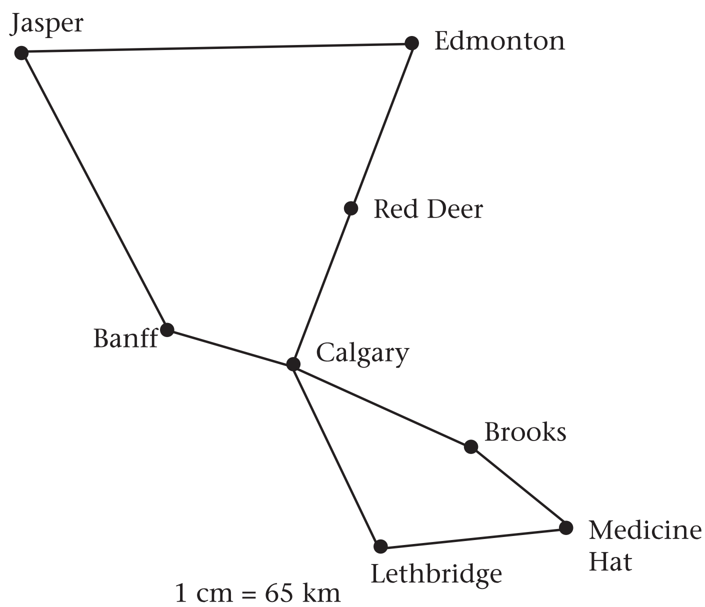
1 cm	↔	20 cm
1.9 cm	↔	1.9×20 cm = 38 cm

Thus the widest part of the guitar is 38 cm.

Sometimes we see scale diagrams in map scales. Once again, our use of ratio and proportion is necessary.

Example 3

Cindy lives in Lethbridge, Alberta. On Saturday, her family went to the Stampede in Calgary. On a map, the distance between the towns is 2.6 cm. The scale on the map is 1cm:65 km. Find the actual distance between Lethbridge and Calgary.



Solution

$$\frac{1}{65} = \frac{2.6}{n} \begin{array}{l} \leftarrow \text{ Map distance} \\ \leftarrow \text{ Actual Distance} \end{array}$$

$$1n = 2.6 \times 65$$

$$1n = 169$$

$$n = 169$$

The actual distance between Lethbridge and Calgary is approximately 169 km.

Exercises 4.1

1. An insect is drawn to the scale 20 cm represents 1 cm. What is the length of the actual insect of each of the following?

   a. 10 cm

   b. 1 cm

   c. 0.5 cm

2. The scale on a diagram is 1 cm represents 4 m. Calculate the actual length represented by each of the following measures on the scale diagram.

   a. 2 cm

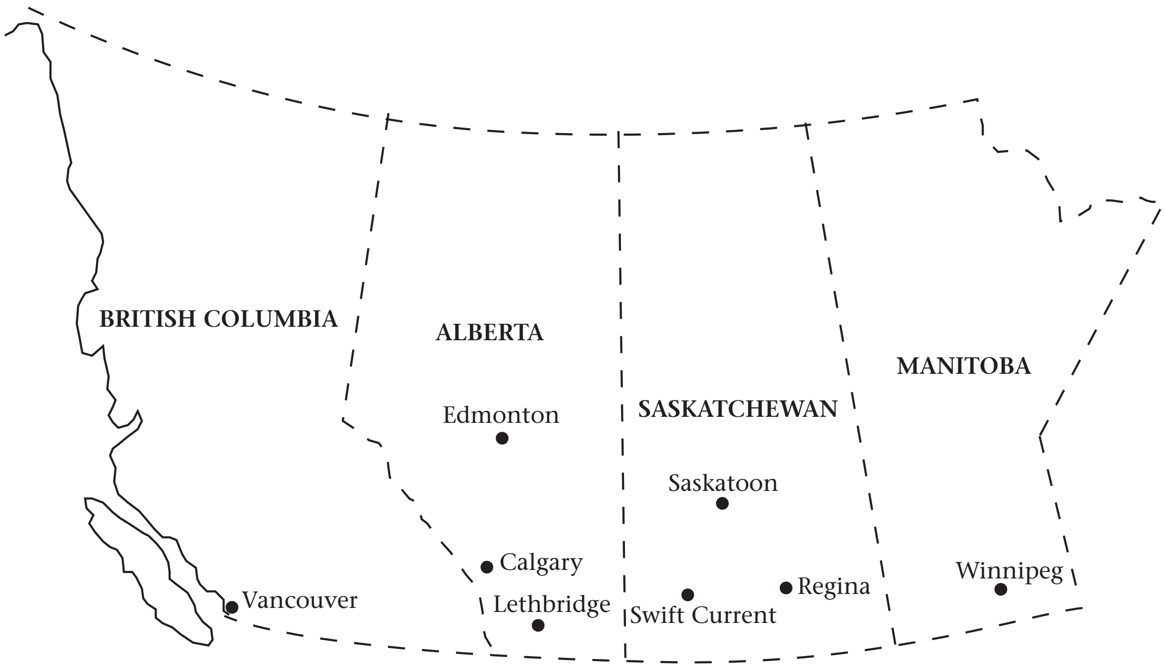
   b. 8 cm

   c. 12 cm

   d. 12.5 cm

   e. 1.45 cm

3. Jacque MacKenzie measured the distance between Regina and Winnipeg and found it to be 3 cm. What is the actual air distance? (Show your work.)

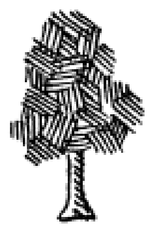


Scale: 1 cm = 160 km

4. The drawing of a screw is 4 times as large as the real screw. What scale is correct?

1:4 *or* 4:1? ____________________

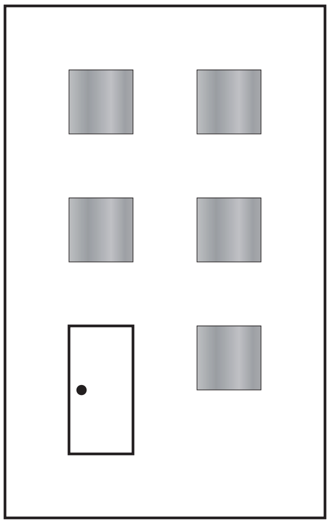
5. The drawing of the tree is $\frac{1}{100}$ as large as the real tree. What scale is correct?



1 cm = 1 m *or* 1 m = 1 cm? ____________________

6. Measure the height of the drawing of the building in centimetres. What is the real building's height?

   1 cm = 2 m



Turn to the Answer Key at the end of the module to check your work.

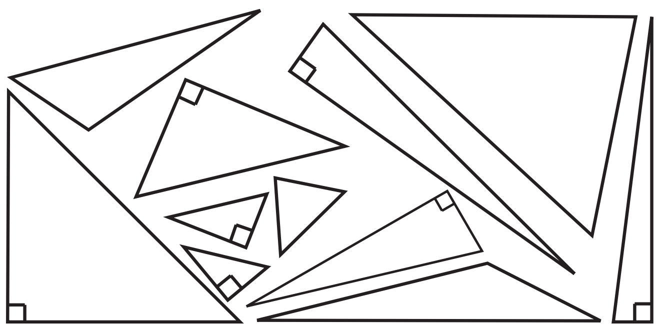
Answer Key

Lesson 1: The Pythagorean Theorem

Exercises 1.1

There are many more right angles in this picture than just the ones shown here. How many did you find?

Exercises 1.2

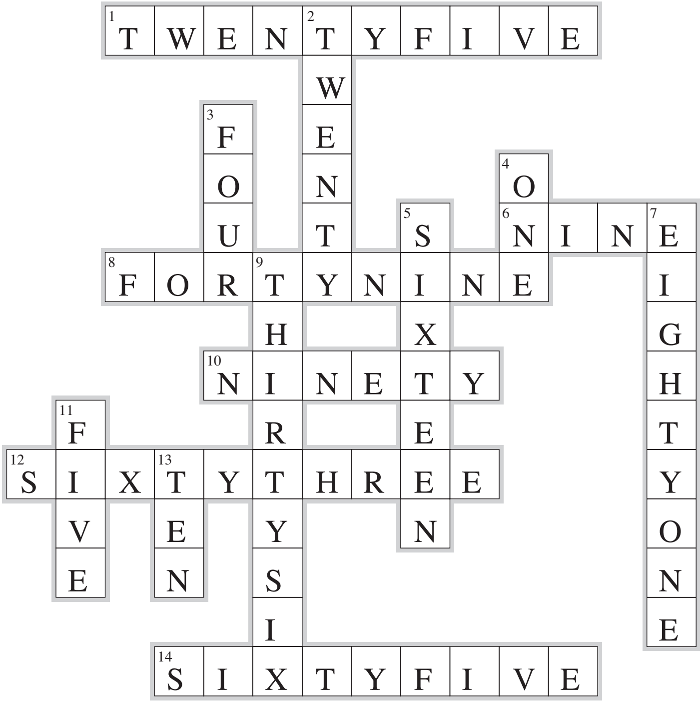


Exercises 1.3

1. $a^2 + b^2 = c^2$

2. 4 $\frac{1}{2}$ 7.3 5.1 6 $\frac{1}{3}$ 1.21 23

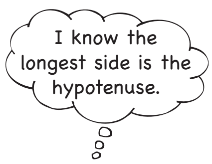
3.



Exercises 1.4

1.

Use the Pythagorean Theorem to check. $a^2 + b^2 = c^2$



Fill in the lengths.
Put a ? over the equals sign.

$$6^2 + 8^2 \stackrel{?}{=} 10^2$$

Figure out the square of each number. $36 + 64 \stackrel{?}{=} 100$

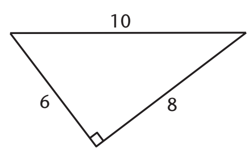
Is that true?

If it's not true, cross out the equals sign. $100 = 100$

Is this triangle a right triangle or not? Yes, this is a right triangle.

If this is a right triangle, which angle is a right angle?

Mark the right angle.



2.

Use the Pythagorean Theorem to check. $a^2 + b^2 = c^2$

Fill in the lengths.
Put a ? over the equals sign. $5^2 + 3^2 \stackrel{?}{=} 7^2$

Figure out the square of each number. $25 + 9 \stackrel{?}{=} 49$

Is that true?

If it's not true, cross out the equals sign. $34 \neq 49$

Is this triangle a right triangle or not? No, this is not a right triangle.

3.

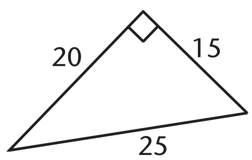
Use the Pythagorean Theorem to check.

$$a^2 + b^2 = c^2$$
$$20^2 + 15^2 \stackrel{?}{=} 25^2$$
$$400 + 225 \stackrel{?}{=} 625$$
$$625 = 625$$

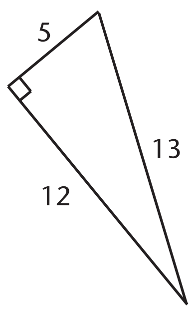
Is this triangle a right triangle or not? Yes, this is a right triangle.

If this is a right triangle, which angle is a right angle?

Mark the right angle.



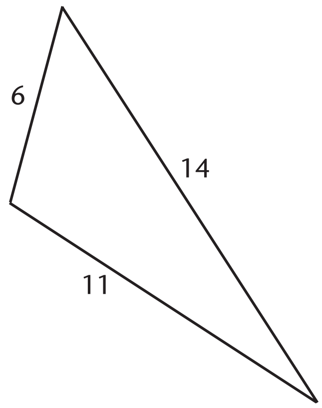
4.



$$a^2 + b^2 = c^2$$
$$12^2 + 5^2 \stackrel{?}{=} 13^2$$
$$144 + 25 \stackrel{?}{=} 169$$
$$169 = 169$$

Is this triangle a right triangle or not? Yes, this is a right triangle.

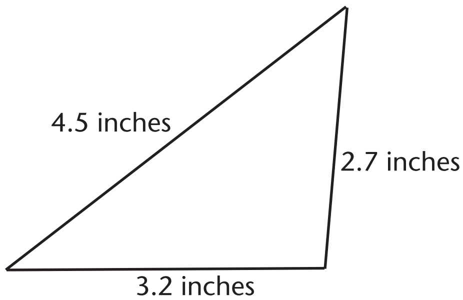
5.



$$a^2 + b^2 = c^2$$
$$6^2 + 11^2 \stackrel{?}{=} 14^2$$
$$36 + 121 \stackrel{?}{=} 196$$
$$157 \neq 196$$

Is this triangle a right triangle or not? No, this is not a right triangle.

6. a.

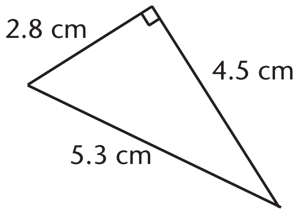


$$a^2 + b^2 = c^2$$
$$2.7^2 + 3.2^2 \stackrel{?}{=} 4.5^2$$
$$7.29 + 10.24 \stackrel{?}{=} 20.25$$
$$17.53 \neq 20.25$$

No, this is not a right triangle.

b. A hypotenuse is the longest side of a right triangle. This is not a right triangle, so this triangle does not have a hypotenuse.

7. a.



$$a^2 + b^2 = c^2$$
$$2.8^2 + 4.5^2 \stackrel{?}{=} 5.3^2$$
$$7.84 + 20.25 \stackrel{?}{=} 28.09$$
$$28.09 = 28.09$$

Yes, this is a right triangle.

b. This is a right triangle, so this triangle does have a hypotenuse. A hypotenuse is the longest side of a right triangle.

The hypotenuse is 5.3 cm long.

Exercises 1.5

1. The triangles in questions 1, 2, 4, and 7 are right triangles, but the lengths of the sides in question 7 are not whole numbers.

   The triangles in questions 1, 2, and 4 have side lengths that form Pythagorean Triples.

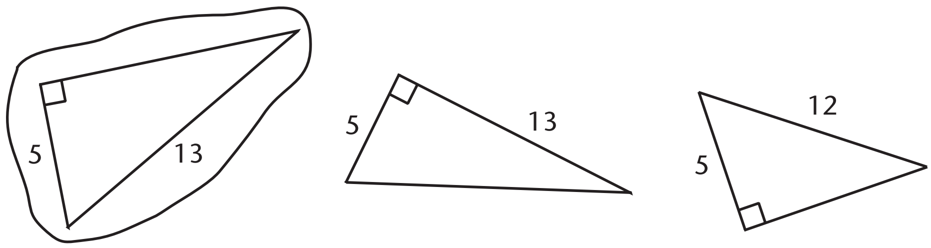
   The Pythagorean Triple in question 1 is 6, 8, 10.

   The Pythagorean Triple in question 2 is 15, 20, 25.

   The Pythagorean Triple in question 4 is 5, 12, 13.

2. a. The hypotenuse is the longest side. The length of the hypotenuse is 13.

   b.



Exercises 1.6

1. Answers will vary. A good answer will mention one or more of these points:
   All of the numbers in these questions are perfect squares. The square root is a whole number.

a. $x^2 = 9$
$\sqrt{x^2} = \sqrt{9}$
$x = 3$

b. $b^2 = 25$
$\sqrt{b^2} = \sqrt{25}$
$b = 5$

c. $k^2 = 100$
$\sqrt{k^2} = \sqrt{100}$
$k = 10$

d. $a^2 = 49$
$\sqrt{a^2} = \sqrt{49}$
$a = 7$

e. $j^2 = 1$
$\sqrt{j^2} = \sqrt{1}$
$j = 1$

f. $n^2 = 36$
$\sqrt{n^2} = \sqrt{36}$
$n = 6$

2. a. $x^2 = 10$
$\sqrt{x^2} = \sqrt{10}$
$x = 3.16$

b. $b^2 = 22$
$\sqrt{b^2} = \sqrt{22}$
$b = 4.69$

c. $k^2 = 107$
$\sqrt{k^2} = \sqrt{107}$
$k = 10.34$

d. $a^2 = 53$
$\sqrt{a^2} = \sqrt{53}$
$a = 7.28$

e. $j^2 = 8$
$\sqrt{j^2} = \sqrt{8}$
$j = 2.83$

f. $n^2 = 63$
$\sqrt{n^2} = \sqrt{63}$
$n = 7.94$

Exercises 1.7

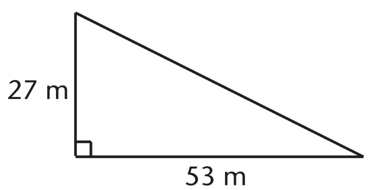
1. $a^2 + b^2 = c^2$

I know that the legs are 16 and 30. In the Pythagorean Theorem, *a* and *b* are the legs of the triangle.

$16^2 + 30^2 = c^2$
$256^2 + 900^2 = c^2$
$1156 = c^2$
$\sqrt{1156} = \sqrt{c^2}$
$34 = c$

The length of the hypotenuse is 34 feet.

2.



I know that the legs are 27 and 53. In the Pythagorean Theorem, *a* and *b* are the legs of the triangle.

The hypotenuse is 59.5 m long.

$a^2 + b^2 = c^2$
$27^2 + 53^2 = c^2$
$729^2 + 2809^2 = c^2$
$3538 = c^2$
$\sqrt{3538} = \sqrt{c^2}$
$59.481 = c$

3.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 43^2 + 34^2 &= c^2 \\ 1849^2 + 1156^2 &= c^2 \\ 3005 &= c^2 \\ \sqrt{3005} &= \sqrt{c^2} \\ 54.8 &= c \end{aligned}$$

The hypotenuse should be 54.8 cm long.

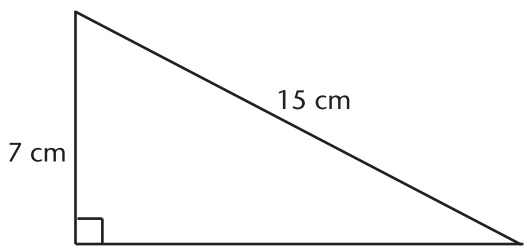
Exercises 1.8

1.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 24^2 + b^2 &= 51^2 \\ 576 + b^2 &= 2601 \\ b^2 &= 2601 - 576 \\ b^2 &= 2025 \\ \sqrt{b^2} &= \sqrt{2025} \\ b &= 45 \end{aligned}$$

The other leg is 45 miles long.

2.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 7^2 &= 15^2 \\ a^2 + 49 &= 225 \\ a^2 &= 176 \\ \sqrt{a^2} &= \sqrt{176} \\ a &= 13.3 \end{aligned}$$

The other leg is 13.3 cm long.

3. a. The missing side is the hypotenuse. The length of the missing side is 7.8 cm.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + 6^2 &= c^2 \\ 25 + 36 &= c^2 \\ 61 &= c^2 \\ \sqrt{61} &= \sqrt{c^2} \\ 7.8 &= c \end{aligned}$$

The length of the missing side is 7.8 cm.

b. The missing side is a leg.

$$a^2 + b^2 = c^2$$
$$3^2 + b^2 = 7^2$$
$$9 + b^2 = 49$$
$$b^2 = 49 - 9$$
$$b^2 = 40$$
$$\sqrt{b^2} = \sqrt{40}$$
$$b = 6.3$$

The length of the missing side is 6.3 cm.

c. The missing side is the hypotenuse.

$$a^2 + b^2 = c^2$$
$$3^2 + 3^2 = c^2$$
$$9 + 9 = c^2$$
$$18 = c^2$$
$$\sqrt{18} = \sqrt{c^2}$$
$$4.2 = c$$

The length of the missing side is 4.2 cm.

d. The missing side is the hypotenuse.

$$a^2 + b^2 = c^2$$
$$2^2 + 2^2 = c^2$$
$$4 + 4 = c^2$$
$$8 = c^2$$
$$\sqrt{8} = \sqrt{c^2}$$
$$2.8 = c$$

The length of the missing side is 2.8 cm.

e. The missing side is a leg.

$$a^2 + b^2 = c^2$$
$$4 + b^2 = 81$$
$$b^2 = 81 - 4$$
$$b^2 = 77$$
$$\sqrt{b^2} = \sqrt{77}$$
$$b = 8.8$$

The length of the missing side is 8.8 cm.

f. The missing side is a leg.

$$a^2 + b^2 = c^2$$
$$4^2 + b^2 = 5^2$$
$$16 + b^2 = 25$$
$$b^2 = 25 - 16$$
$$b^2 = 9$$
$$\sqrt{b^2} = \sqrt{9}$$
$$b = 3$$

Did you notice that this triangle could be solved with a Pythagorean Triple?
The length of the missing side is 3 cm.

4. a.

$$a^2 + b^2 = c^2$$
$$4^2 + 3^2 = c^2$$
$$16 + 9 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = \sqrt{c^2}$$
$$5 = c$$

Did you see the Pythagorean Triple?
The length of the diagonal is 5 cm.

b.

$$a^2 + b^2 = c^2$$
$$2^2 + 2^2 = c^2$$
$$4 + 4 = c^2$$
$$8 = c^2$$
$$\sqrt{8} = \sqrt{c^2}$$
$$2.83 = c$$

The length of the diagonal is 2.83 cm.

c.

$$a^2 + b^2 = c^2$$
$$2^2 + 6^2 = c^2$$
$$4 + 36 = c^2$$
$$40 = c^2$$
$$\sqrt{40} = \sqrt{c^2}$$
$$6.32 = c$$

The length of the diagonal is 6.32 cm.

Lesson 2: Nets and Views

Exercises 2.1

1. a. no b. no c. yes
2. Student is able to answer this when doing reconstructions, answers not required.
3. Student is able to answer this when doing reconstructions, answers not required.

Exercises 2.2

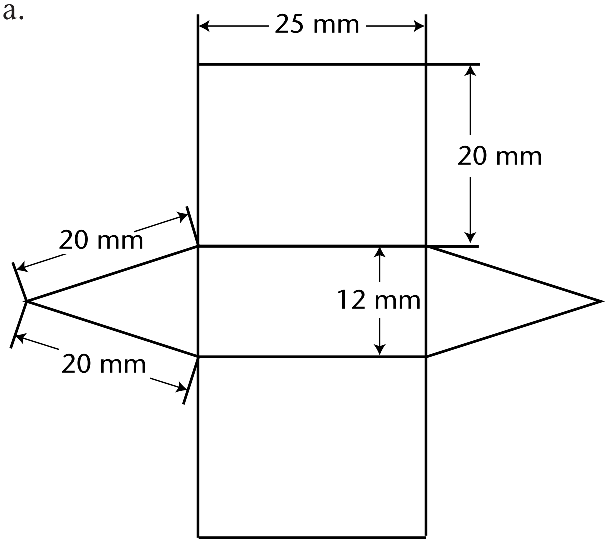
1. b

   c

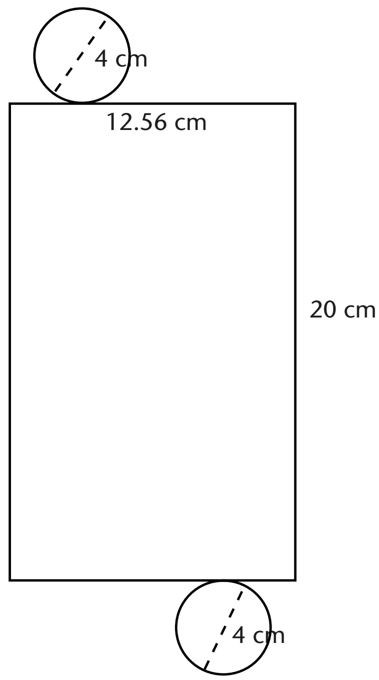
   a

   d

4. a. triangular prism b. rectangular prism c. cylinder
5. a.

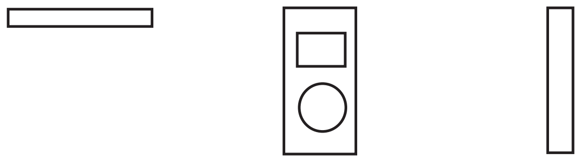


b.

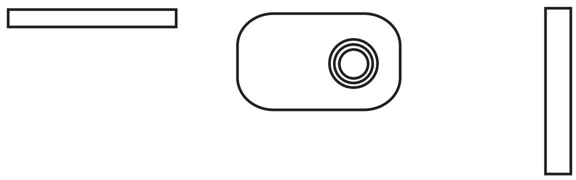


Exercises 2.3

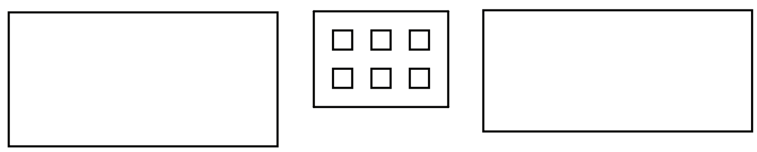
1. a. Top View Front View Side View



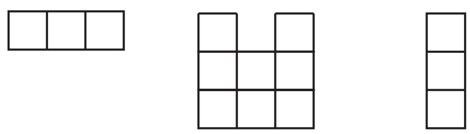
b. Top View Front View Side View



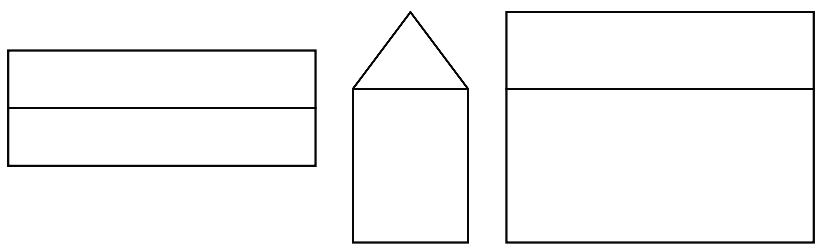
2. a. Top View Front View Side View



b. Top View Front View Side View



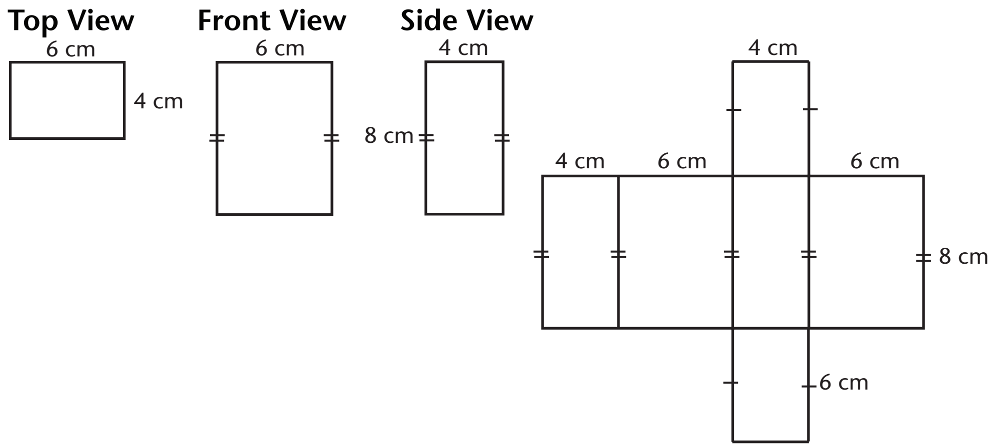
c. Top View Front View Side View



Lesson 3: Surface Area

Exercises 3.1

1. a. You can draw the shapes all connected or separately, but your drawing should contain:



   b. Surface area = 208 cm^2

   area of 2 larger rectangles = 2 × (6 × 8) = 96 cm^2

   area of 2 medium sized rectangles = 2 × (4 × 8) = 64 cm^2

   area of 2 smallest rectangles = 2 × (4 × 6) = 48 cm^2

2. For the prism in question 1:

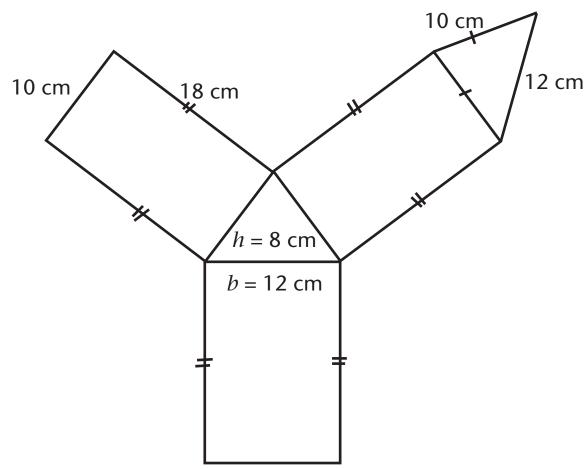
   a. l = 6 cm w = 4 cm h = 8 cm (length is typically longer than width)

   b. SA = $2lw + 2wh + 2lh$

   SA = 2(6)(4) + 2(4)(8) + 2(6)(8)

   SA = 208 cm^2

3. a. Answers will vary. This one is a possibility:



b. Surface area = 672 cm^2

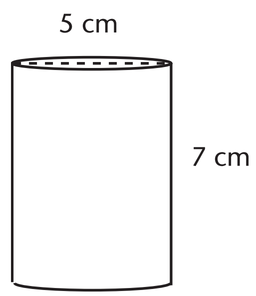
area of 2 triangles = $2 \times (\frac{1}{2} \times 12 \times 8) = 96$ cm^2

area of largest rectangle = $12 \times 18 = 216$ cm^2

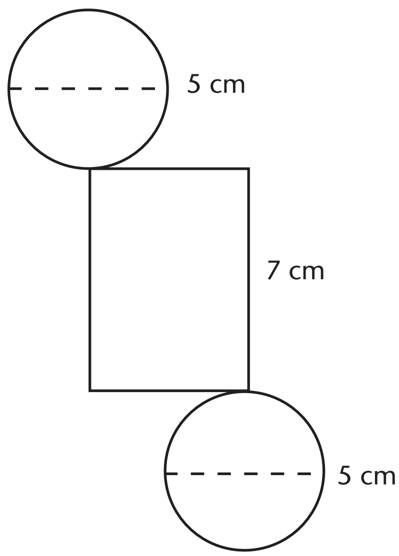
area of 2 other rectangles = $2 \times (10 \times 18) = 360$ cm^2

Exercises 3.2

1. a.



b.



2. a. The area of the 2 circles

$= 2 \times (\pi \times 2.5 \times 2.5)$

$= 39.3$ cm^2

b. Area of the rectangle in the net
$= \text{circumference of the circle} \times \text{height}$
$= (\pi \times 5) \times 7$
$= 110 \text{ cm}^2$

c. $SA = 2\pi r^2 + \pi dh$
$= 2\pi(2.5)^2 + \pi(5)(7)$
$= 149.3 \text{ cm}^2$

3. a. They are the same except for the last part. One formula has πdh, the other formula has $2\pi rh$

b. Since the diameter of a circle is 2 times the radius ($d = 2r$), then the formulas πdh and $2\pi rh$ are the same.

Exercises 3.3

1. a. area of the irregular shape

= (area of the triangle) minus (area of the circle)

$= \frac{1}{2}bh \quad - \quad \pi r^2$

$= \frac{1}{2}(8)(6) \quad - \quad \pi(2)^2$

$= 24 \quad - \quad 12.6$

$= 11.4 \text{ cm}^2$

b. area of the irregular shape
= area of the rectangle minus area of the half circle
$= lw \quad - \quad (\pi r^2) \div 2$
$= (10)(5) \quad - \quad \pi(5)^2 \div 2$
$= 50 \quad - \quad 39.3$
$= 10.7 \text{ cm}^2$

2. a. One way to do this is by subtracting the window area from the wall area.

Area to be painted =
(area of the wall) – (area of the window)
$= lw - lw$
$= (5)(2.4) - (2)(1.2)$
$= 12 - 2.4$
$= 9.6 \text{ m}^2$

b. no

c. two coats of paint = 9.6 + 9.6 = 19.2 m^2.

Two cans of paint covers 8 + 8 = 16 m^2, not enough.

Three cans of paint covers 8 + 8 + 8 = 24 m^2, more than enough.

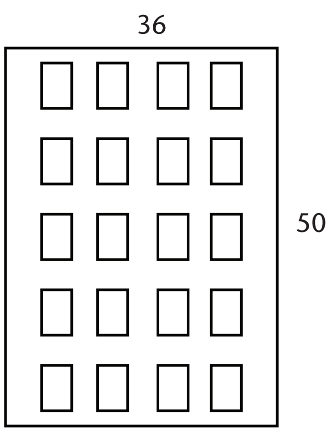
You need three small cans of paint.

d. Answers will vary. If you buy three small cans, it costs $30 and you have 3.2 m^2 of leftover paint. If you buy one large can, it costs $22, and you would have 30.8 m^2 of leftover paint. If the amount of savings is most important, you would buy one large can. If you are concerned about the paint waste, you may choose to buy the 3 small cans.

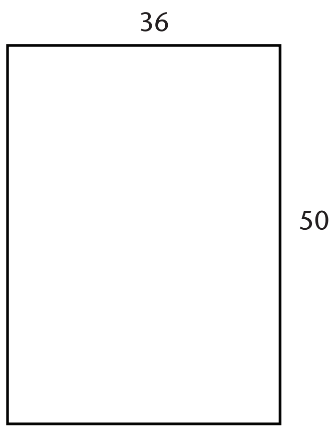
Exercises 3.4

1. a.

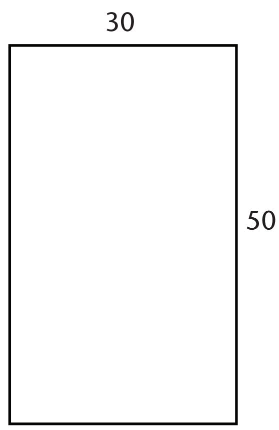
Front View



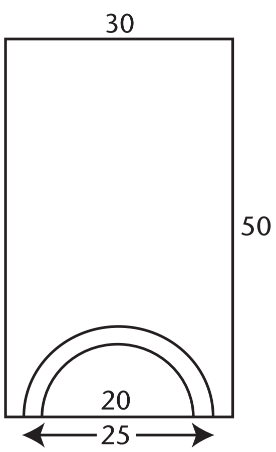
Back View



Left Side View



Right Side View



b. **Front View** = area of front wall – area of all 20 windows

$= 36 \text{ m} \times 50 \text{ m} - 20 \times 24 \text{ m}^2$
$= 1800 \text{ m}^2 - 480 \text{ m}^2$
$= 1320 \text{ m}^2$

Back View = $36 \times 50 = 1800 \text{ m}^2$

Left Side View = $30 \times 50 = 1500 \text{ m}^2$

Right Side View = same as left side view – area of parking garage entrance

$= 30 \times 50 - (\pi \times 12.5^2) \div 2$
$= 1254.56 \text{ m}^2$

c. $1320 + 1800 + 1500 + 1254.56 = 5874.56 \text{ m}^2$

2. a. Answers will vary. The top view was labelled 25 by 15. Since the view is of a curved surface, the actual dimensions are 39.27 by 15. These dimensions give a larger area, which is why Painter One was wrong.

   b. Area that needs to be painted = 677.41 m^2

   Rectangle = $39.3 \times 15 = 589.05 \text{ m}^2$

   Area of larger half circle – area of smaller half circle

   $= (\pi \times 12.5^2) \div 2 - (\pi \times 10^2) \div 2$

   $= 88.36 \text{ m}^2$

Lesson 4: Scale Diagrams

Exercises 4.1

1. a. 0.5 cm
   b. 0.5 mm (or 0.05 cm)
   c. 0.25 mm (or 0.025 cm)

2. a. 8 m
   b. 32 m
   c. 48 m
   d. 50 m
   e. 5.8 m

3. $$\frac{1 \text{ cm}}{160 \text{ cm}} = \frac{3 \text{ cm}}{x}$$
$$1.x = 3.160$$
$$x = 480$$

4. 1:4

5. 1 cm = 1 m
6. The scale is 1 cm = 2 m

   1 cm = 200 cm

   Thus 4.5 cm = 4.5 × 200

   = 900 cm

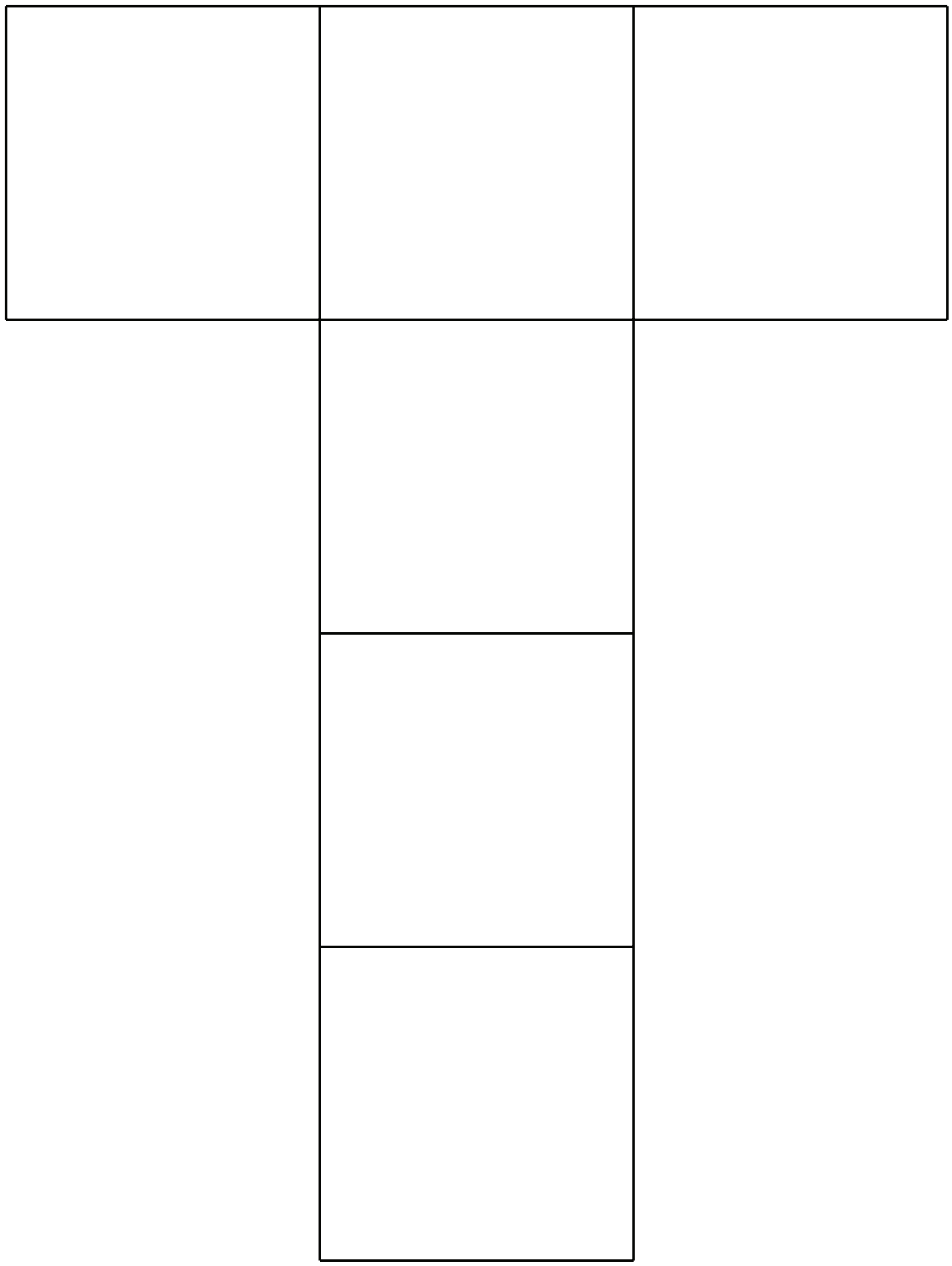
   or 9 m

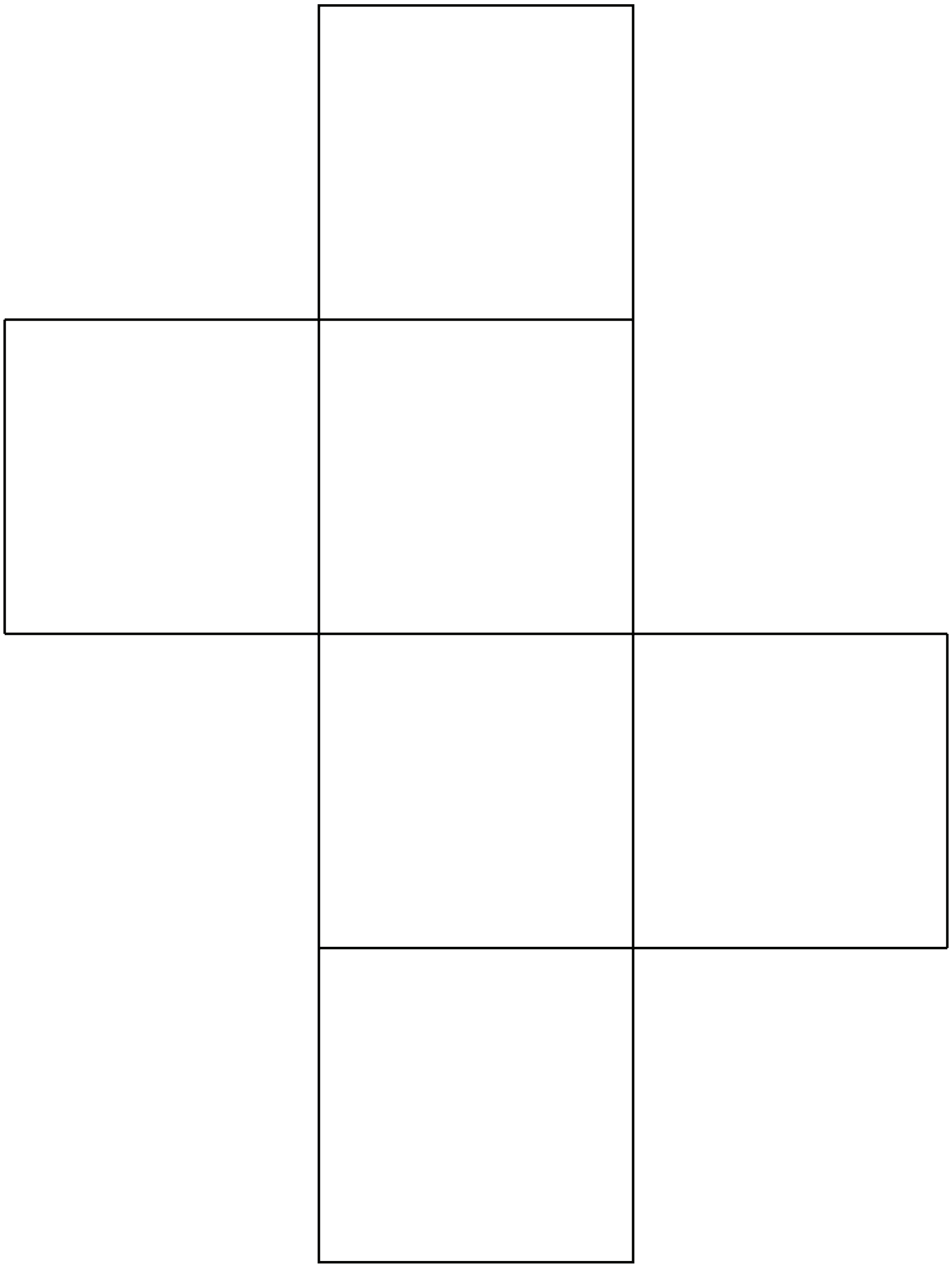
   The building is 9 m high.

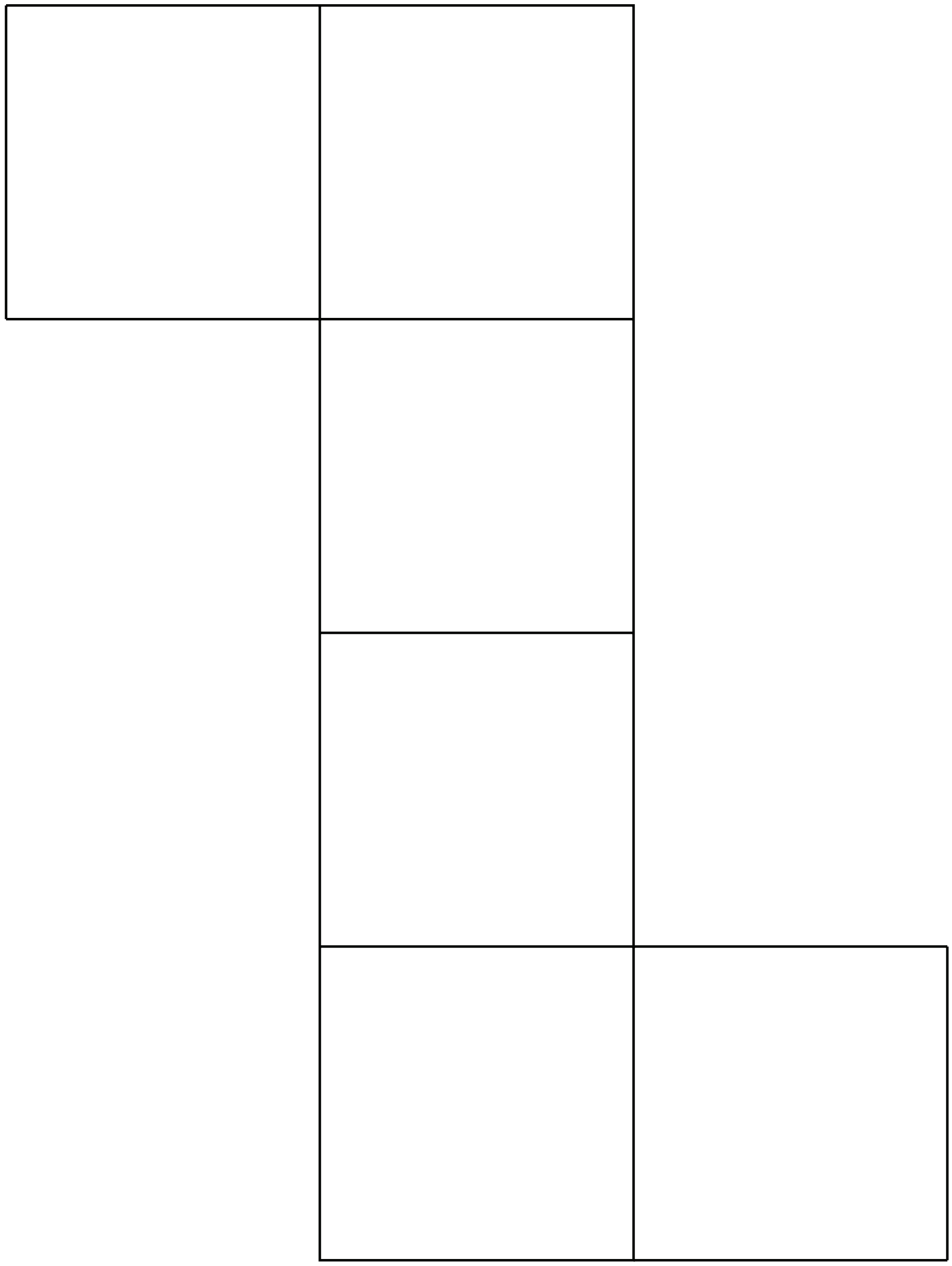
Templates

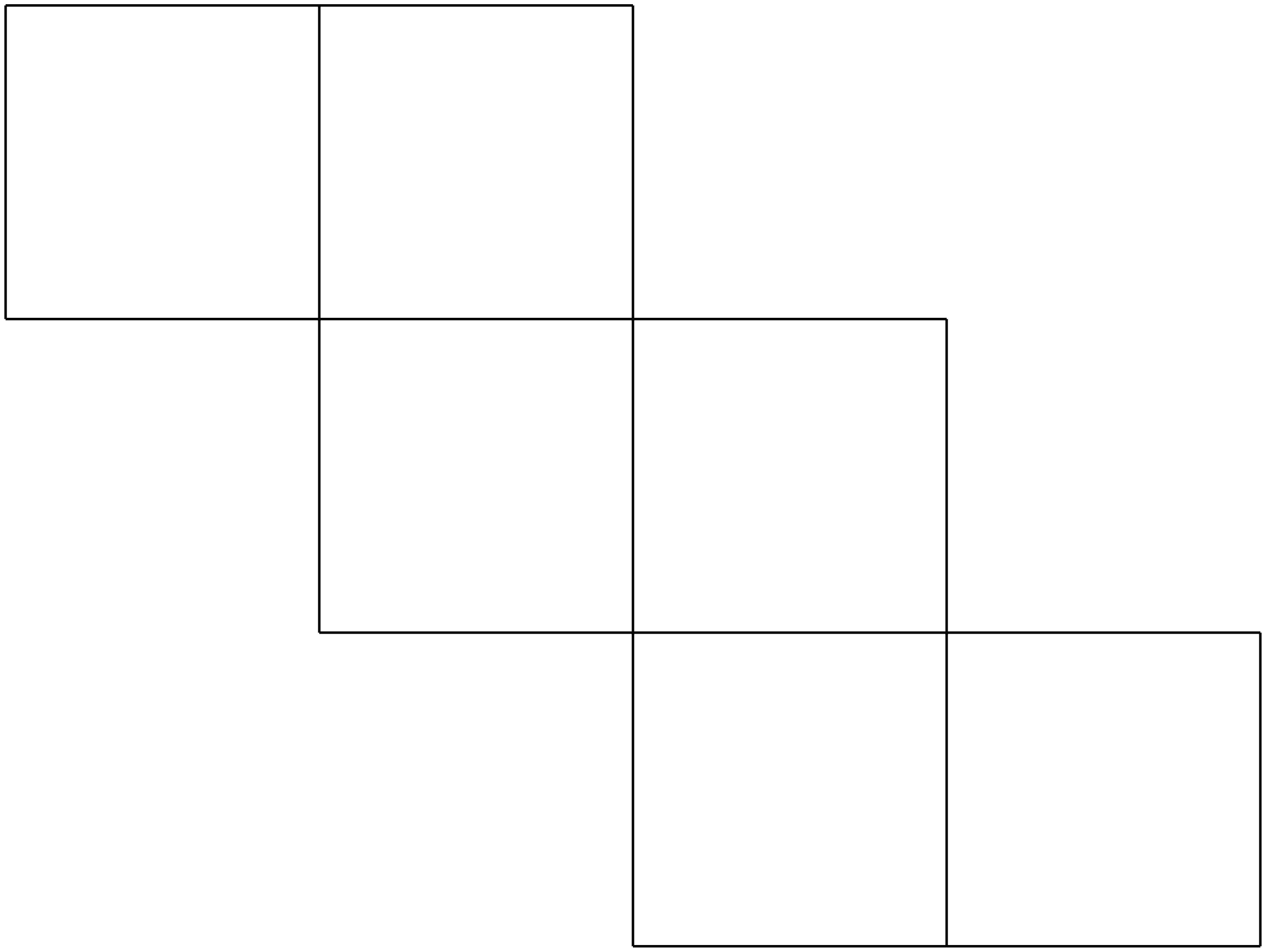
Lesson 2: Nets and Views

Nets of Rectangular Prisms



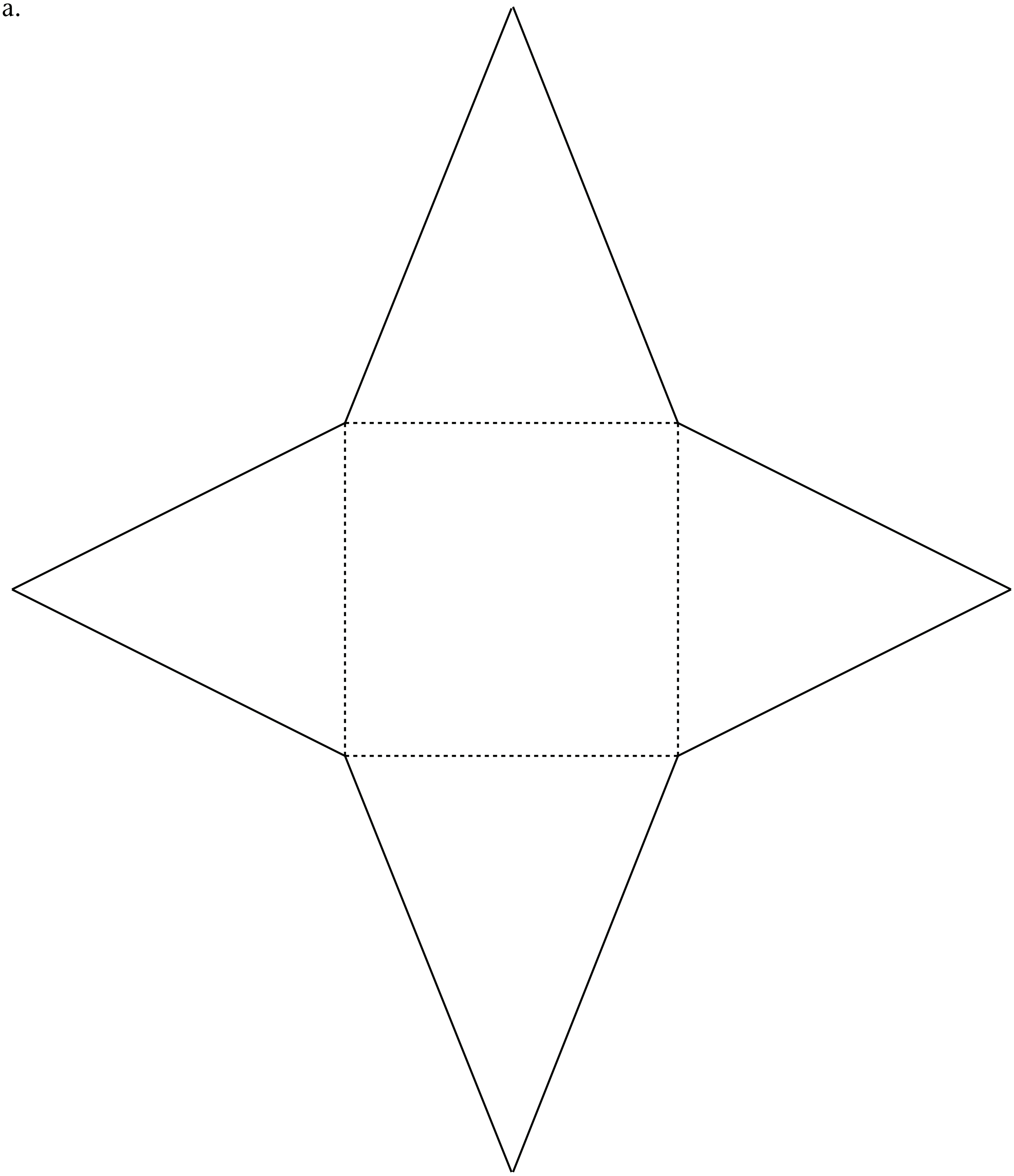




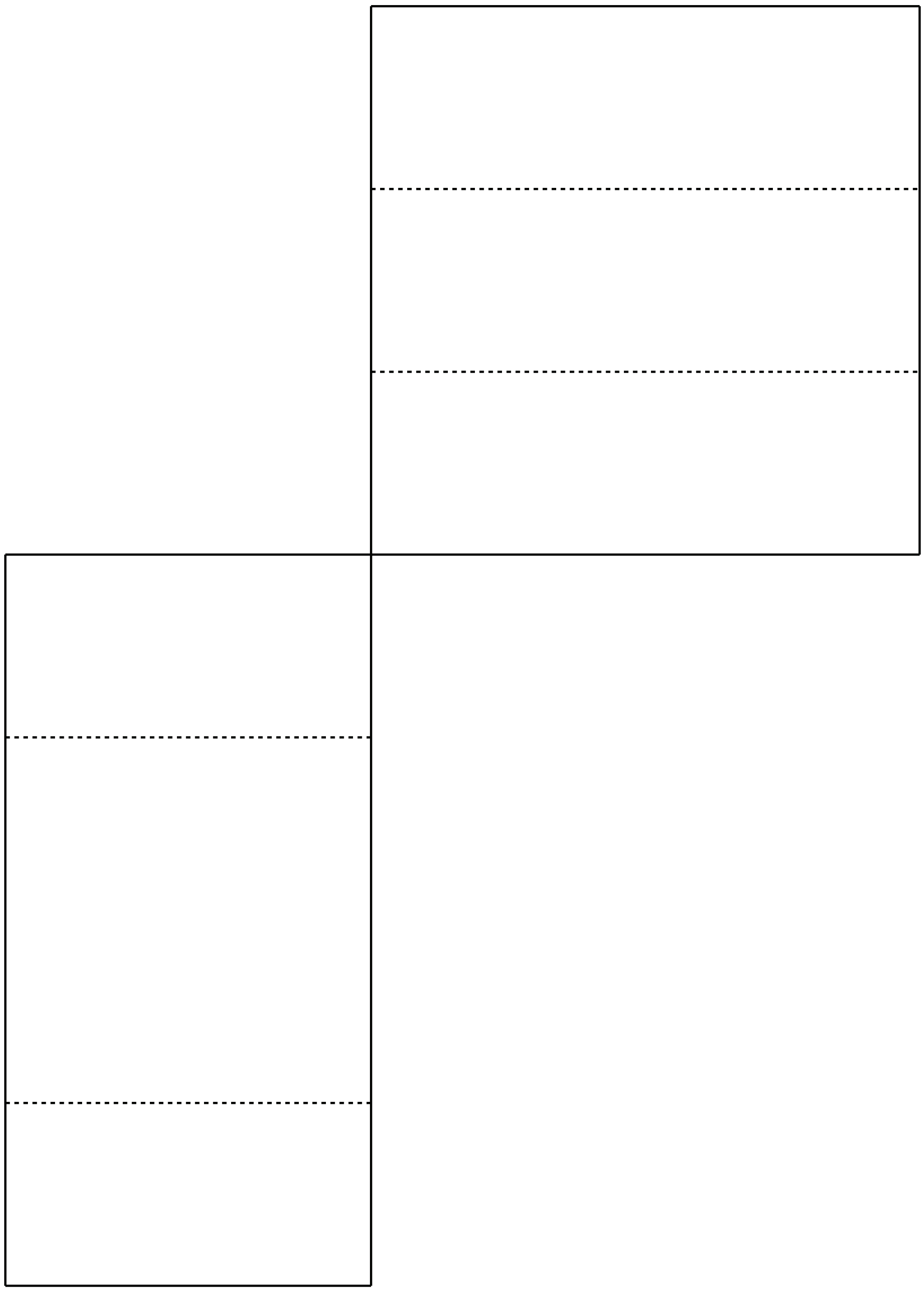


Exercises 2.1

a.



b.



c.