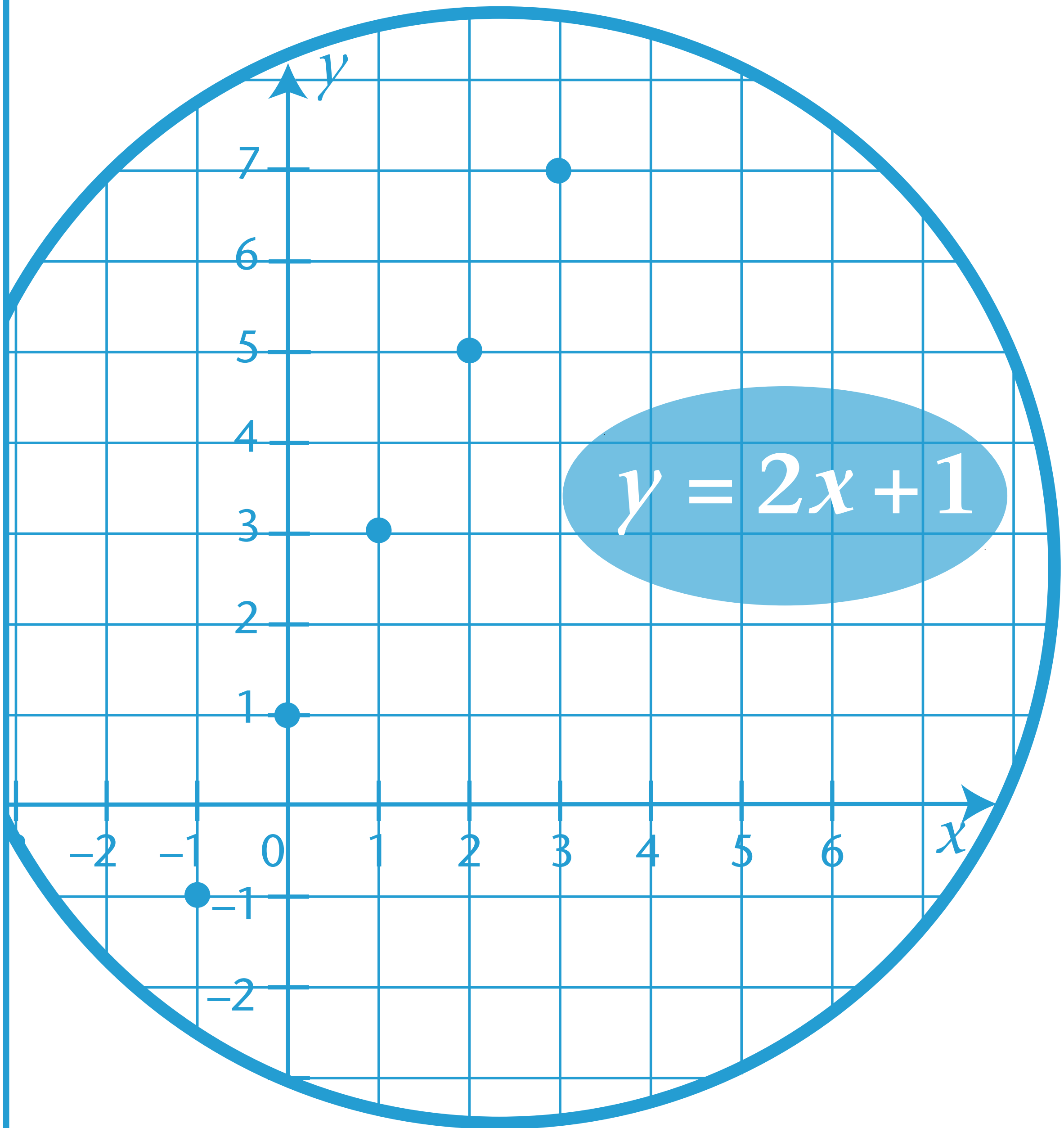
LITERACY FOUNDATIONS MATH



LEVEL 5

PATTERNS AND RELATIONS

To the Student

This resource covers topics from the British Columbia Ministry of Education's Literacy Foundations Math Level 5. You may find this resource useful if you're a Literacy Foundations Math student, or a K-12 student in grades 7 – 9.

We have provided learning material, exercises, and answers for the exercises, which are located at the back of each set of related lessons. We hope you find it helpful.

Literacy Foundations Math Prescribed Learning Outcomes

The Literacy Foundations Math Prescribed Learning Outcomes (PLOs) are grouped into four areas: Number (A), Patterns and Relations (B), Shape and Space (C), and Statistics and Probability (D). For a complete list of the PLOs in Level 5, search for Literacy Foundations Math curriculum on the BC Ministry of Education's website.

PLOs Represented in This Resource

The PLOs represented in this Level 5 resource are as follows:

Number
All topics, A1 – A12

Patterns and Relations
All topics, B1 – B6

Shape and Space
All topics, C1 – C3

Statistics and Probability
D2

PLOs Not Represented in This Resource

The PLOs for which no material is included in this resource are as follows:

Statistics and Probability
There is no material for D1, line graphs from data sets.

Acknowledgements and Copyright

Project Manager: Christina Teskey
Writer: Angela Voll
Production Technician: Beverly Carstensen
Cover Design: Christine Ramkeesoon

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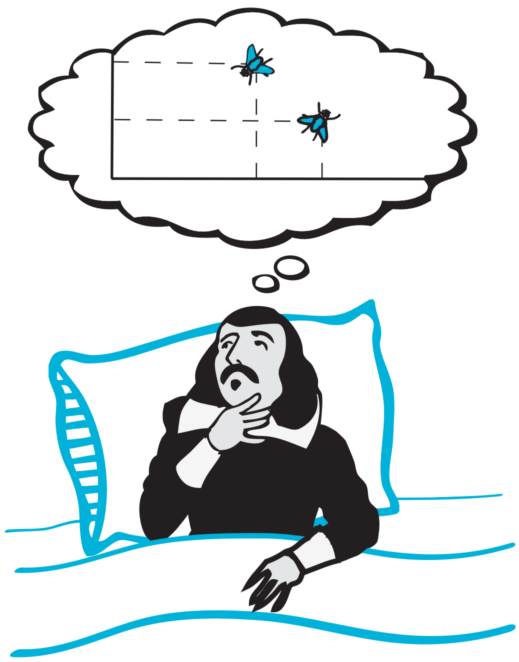
Lesson 1
Cartesian Plane

Learning Outcomes

By the end of this section you will be better able to:

- identify the features of the Cartesian Plane.
- use ordered pairs to describe the location of points in the Cartesian plane.
- plot points in the Cartesian plane

René Descartes was home sick in bed in the early 1600s. He watched a fly crawl around on the ceiling. René noticed that he could describe the fly's position no matter where it was by giving its distance from the corner of the room in two directions.



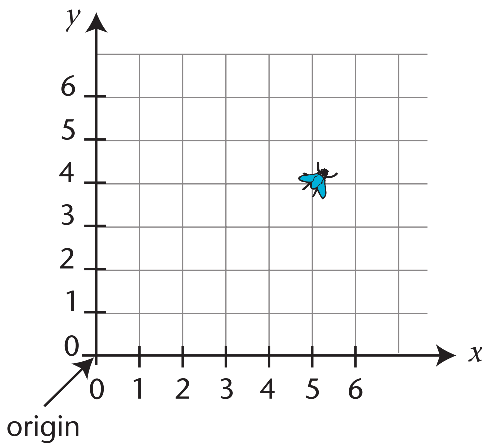
There are many situations where we need to clearly describe the location of an object. Video game designers, architects, and your GPS system all use René Descartes' bug-finding idea to precisely describe information about location.

A flat surface is called a plane. We call René's bug-finder the **Cartesian plane**.

The corner of the room is the **origin**. That just means the place where we start. All of our descriptions of distances will be measured from this spot.

The horizontal direction is called ***x***. The horizontal number line is called the ***x*-axis**.

The vertical direction is called ***y***. The vertical number line is called the ***y*-axis**.



The fly is called a **point**.

To describe the location of the fly, we ALWAYS give the distance in the ***x*** direction first. This fly is located 5 units to the right of the origin and 4 units above the origin. The fly is at (5,4).

The first number describes the distance in the *x* direction. This number is called the ***x*-coordinate**. The ***x***-coordinate of the location of the fly is 5.

The second number describes the distance in the *y* direction. This number is called the ***y*-coordinate**. The *y*-coordinate of the location of the fly is 4.

When we write the two coordinates together, they are ALWAYS in round brackets. The two numbers are separated by a comma. The **coordinates** of the location of the fly are (5,4).

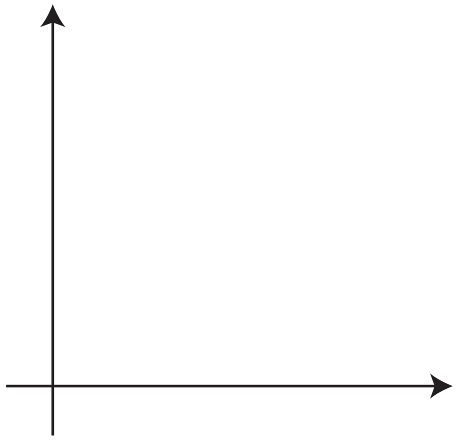
Sometimes we call coordinates a coordinate pair or an **ordered pair**. The Cartesian plane is just one example of a **coordinate system**.

Exercises 1.1

1. Label the origin.

   Label the ***x***-axis.

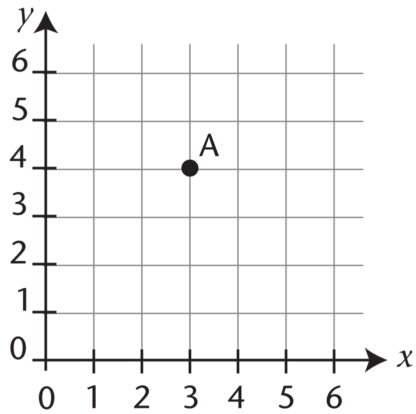
   Label the ***y***-axis.



2 a. What is the *x*-coordinate of point A?

  b. What is the *y*-coordinate of point A?

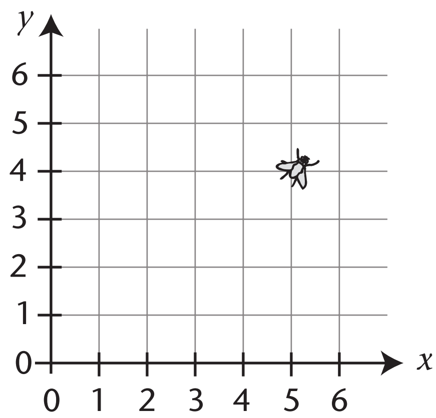
  c. What are the coordinates of point A?



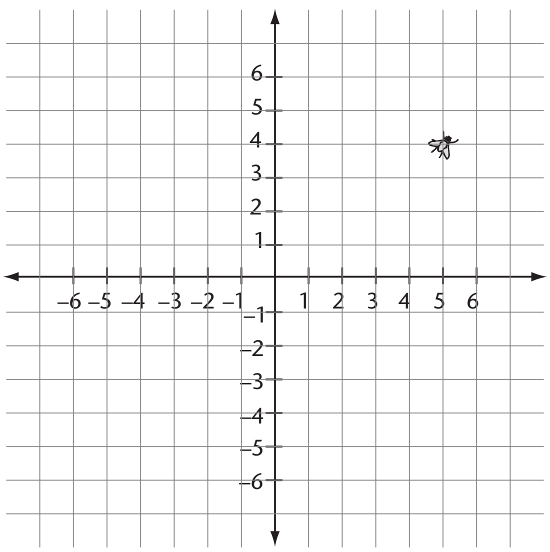
3. The Cartesian plane is an example of a ____________________ system.

Turn to the Answer Key at the end of the module to check your work.

Until now, we have been using only half of a number line for each axis: *x* and y.



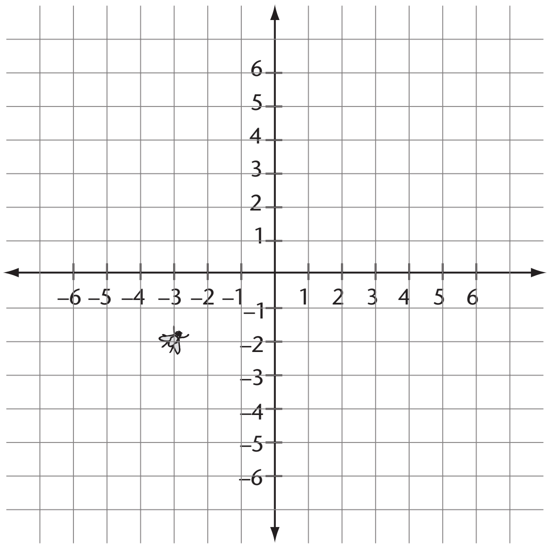
It's time to stretch out. Let's use a complete number line for each axis.



Now our fly isn't stuck walking around on René's bedroom ceiling anymore. It can go as far as it likes in any direction.

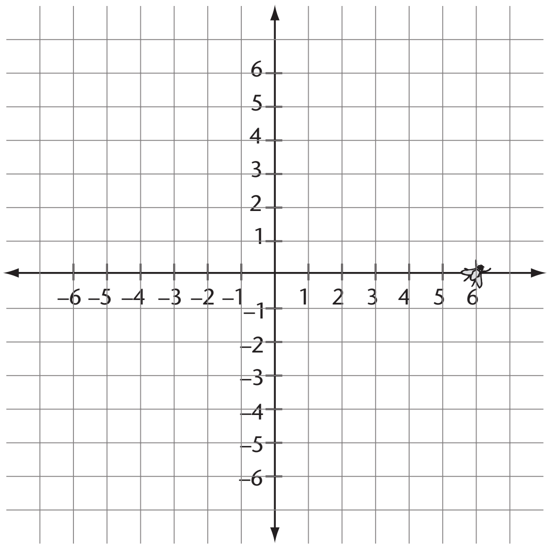
The positive numbers on the *x*-axis describe distances to the right of the origin, just like you have already seen. If the *x*-coordinate is 2, we know that the point is 2 units to the right of the origin. When we want to describe distances to the left of the origin, we use the negative numbers on the *x*-axis. If the *x*-coordinate is –2, we know that the point is 2 units to the left of the origin.

The y-axis works in a similar way. The *positive* numbers on the y-axis describe distances *above* the origin. If the y-coordinate is 5, we know that the point is 5 units above the origin. When we want to describe distances *below* the origin, we use the *negative* numbers on the y-axis. If the y-coordinate is –5 we know that the point is 5 units below the origin.



The fly moved. Can you describe its new location?

It is 3 units to the LEFT of the origin. The x-coordinate is now –3. It is 2 units BELOW the origin. The y-coordinate is –2. The fly is at coordinates (–3,–2).

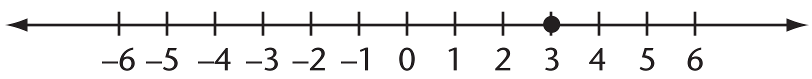


When the fly has landed on an axis, we still have to describe its position. This fly is 6 units to the right of the origin. The x-coordinate is 6. However, the fly is neither above nor below the origin. The y-coordinate is 0. The fly is at coordinates (6, 0).

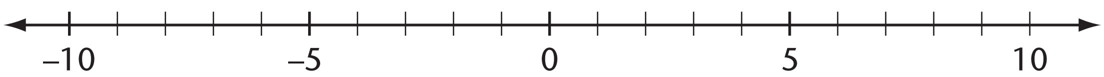
Exercises 1.2

René's bug-finder is like two number lines stuck together.

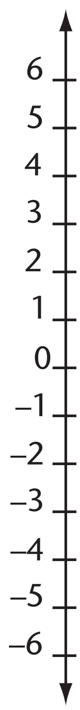
1. There's already a point on 3. Put a point at each of the following locations.
   a. –2
   b. 5
   c. 0



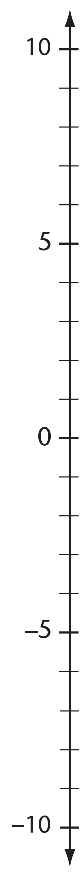
2. Put a point at:
   a. –3
   b. 7
   c. –6



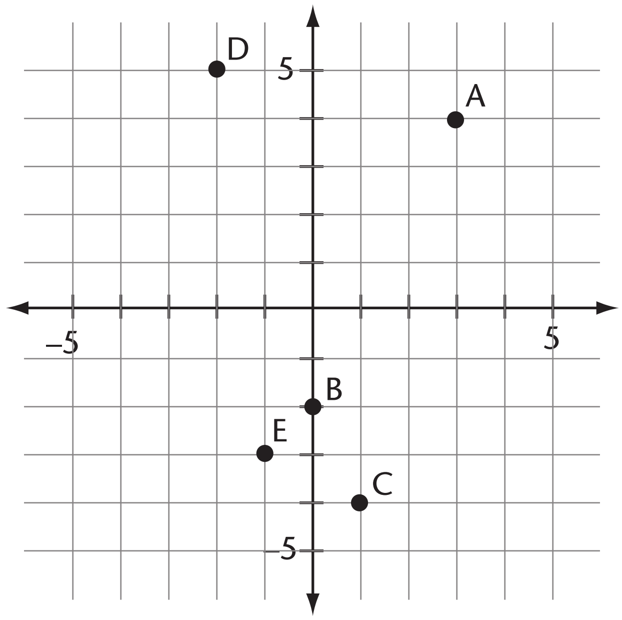
3. Put a point at:
   a. –3
   b. 4
   c. 1



4. Put a point at:
   a. –7
   b. 9
   c. –4



5. Give the coordinates of each point.



A (,)

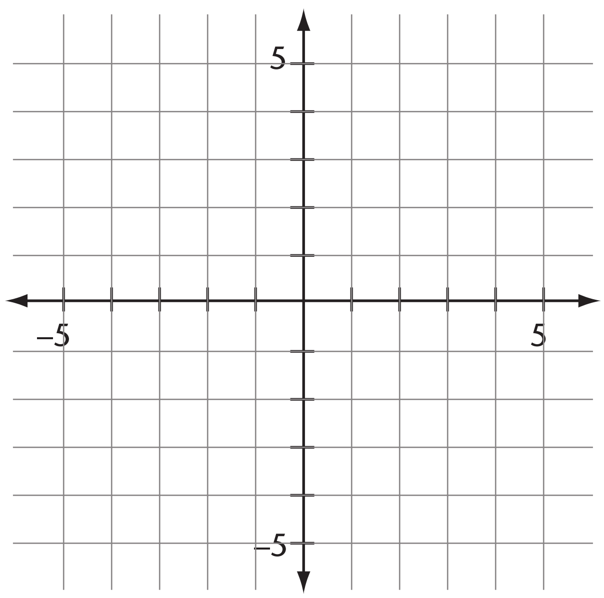
B (,)

C (,)

D (,)

E (,)

6. Plot each point on the Cartesian plane.



F (–3, 1)

G (–1, 4)

H (–2 , –4)

I (0, –2)

J (2, 1)

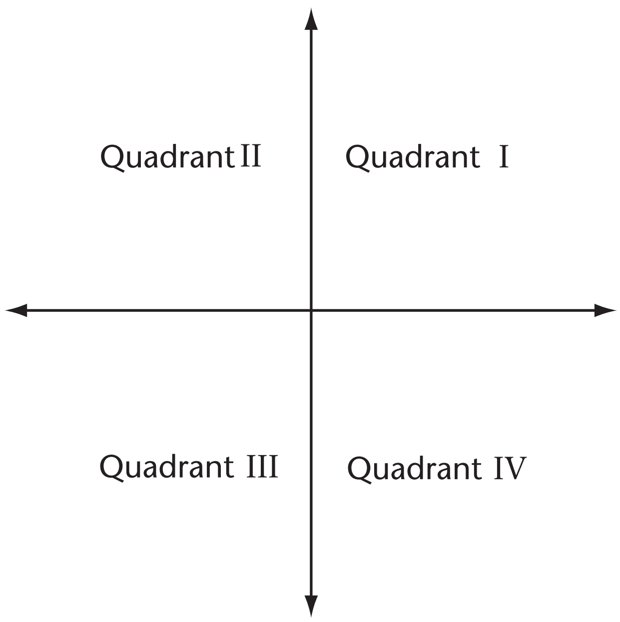
Turn to the Answer Key at the end of the module to check your work.

A quad is a dirt-bike with four wheels. Quadruplets are four babies born at the same time. A building divided into four apartments is a quadruplex.

Have you ever heard of a quadrille? It is a dance for couples. How many? You guessed it—four.

The quadriceps muscle in your thigh is actually four different muscles that work together to extend your knee.

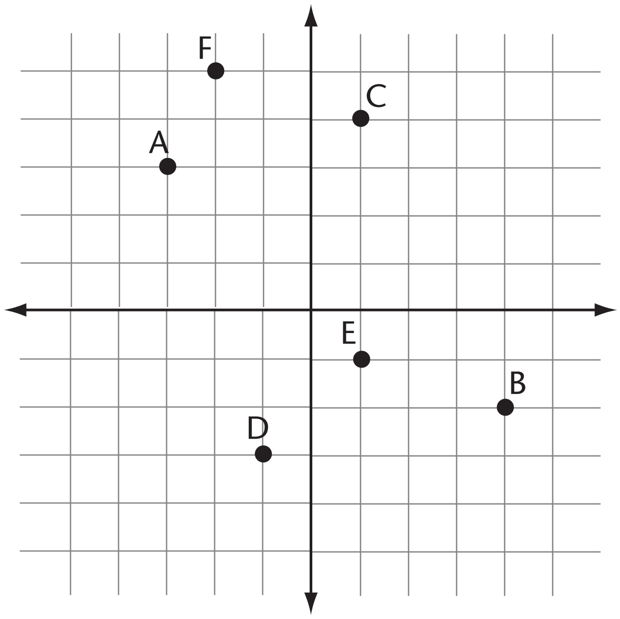
We have one more new word to learn. The *x*-axis and the *y*-axis split the Cartesian plane into four sections. Each section is called a **quadrant**.



Notice that the quadrants are numbered with Roman numerals: I, II, III, IV. Fancy!

Exercises 1.3

1. Give the quadrant number of each point. The first one has been done for you.



Point	Quadrant
A	II
B	_____
C	_____
D	_____
E	_____
F	_____

2. What are the coordinates of the points in Quadrant IV?

3. Fill in each blank with "positive" or "negative."

   The *x*-coordinate of all the points in Quadrant IV is ______________________

   The y-coordinate of all the points in Quadrant IV is ______________________

Turn to the Answer Key at the end of the module to check your work.

Lesson 2

Expressions

Learning Outcomes

By the end of this section you will be better able to:

- describe relationships between numbers using the symbols of algebra
- evaluate an expression given the value of the variable

Write an Expression

We've looked at some of the ways you can use words to describe patterns of numbers.

- *four more cats*
- *four years older*
- *four kilograms heavier*
- *four centimetres taller*

Each phrase uses different words, but the pattern of the numbers in each of those situations is the same.

Now we're going to describe patterns with symbols—the symbols of math. This will let us concentrate on the pattern of the numbers without the other details of the situation.

Exercises 2.1

In this lesson, you will be thinking about how numbers are related to each other. You already know a lot about how numbers are related.

1. What number does each statement describe?

   a. a number that is 3 more than 9

   b. a number that is 5 less than 11

   c. a number that is half of 8

   d. This number increased by 6 is 2.

2. Darian walks 17 blocks to get to school. Teagan walks 11 blocks. Who walks farther? How much farther?

Turn to the Answer Key at the end of the module to check your work.

Explore

Let's look at a very simple relationship between numbers.

2	5
7	10
–2	1

The number on the right is
3 more than
the number on the left.

That sentence does a great job of describing the relationship between the pairs of numbers. For each row, that sentence is true. The specific numbers change, but that sentence still describes the relationship.

Can we do the same job with numbers and symbols?

"3 more than"—That sounds like adding 3. Let's try it. The number on the right is:

$$2 + 3$$

That works, but only for the first row. What's going on? The sentence was accurate for every row, but our numbers and symbols aren't. Isn't math supposed to be *better* at describing patterns of numbers?

Read that sentence again. *The number on the right is 3 more than the number on the left*. It doesn't say, "The number on the right is 3 more than 2." We need a symbol that will do the same job as the phrase *the number on the left.*

That's what a **variable** is for. A variable is a symbol in an expression. It is usually a letter. A variable stands for a number that might change.

Number starts with "n", so we'll use the letter *n* for our variable. Our variable *n* will be doing the same job as the phrase, *the number on the left.*

We're ready to try again. Describe the relationship using numbers and symbols. The number on the right is:

$$n + 3$$

We did it! We have expressed the idea, *3 more than the number on the left*, using only symbols.

Look at the examples below. In each case the numbers and symbols express the same idea as the descriptions that use words.

$n + 3$	three more than a number
$3n$	a number multiplied by three
$n - 3$	three less than a number
$\frac{n}{3}$	a number divided by three

When you understand these examples, try the practice activity.

Exercise 2.2

Match each description in words to the numbers and symbols that express the same idea.

$n - 6$	twice a number
$n + 3$	a number times five
$2n$	five more than a number
$n + 5$	a number decreased by six
$\frac{n}{4}$	four less than a number
$4 - n$	a number divided by four
$5n$	half of a number
$n - 4$	three more than a number
$\frac{n}{2}$	four minus a number

Turn to the Answer Key at the end of the module to check your work.

Explore

The groups of numbers and symbols that we have been looking at have a name—they are called **expressions**.

Expressions have parts that are added together. Each part that is added is called a **term**. The following expression has two terms. Each term in the expression is underlined.

$$\underline{2x} + \underline{1}$$

The first term is "2*x*". You already know about variables. The variable here is *x*. The number in front of the *x* is called the **coefficient** of *x*. This term means "2 times *x*," even though the × is missing.

The second term is "1". There is no variable here. Nothing in this term can ever change, so we call this a **constant** term.

Try another example.

$$\underline{3m} + \underline{5n} - \underline{7}$$

How many terms are in this expression? three terms

What are the variables in this expression? m and n

What are the coefficients? 3 and 5

Let's take a close look at that last term before we answer the next question. Remember, terms are pieces of an expression that are *added* together. Remember also that subtracting means the same as adding the negative of a number.

What is the constant? −7

Let's look at one more expression before you do some practice with these new words.

$$\underline{2p} + \underline{q} - 6$$

How many terms are in this expression? three terms

What are the variables in this expression? p and q

What are the coefficients? The coefficient of the first term is 2. The second also has a coefficient. The variable *q* by itself means the same at "1 times *q*". The coefficient of the second term is 1.

What is the constant? −6

Exercises 2.3

1. For each expression, underline each term. Then fill in the chart. The first one has been done for you.

	How many terms?	What are the variables?	List any coefficients.	Is there a constant term? What is it?
$m + 4$	2	m	1	4
$2x - 9$				
$-3s$				
$5 + p$				
$x + 3y + 7$				
$b + 2$				
$4t$				

2. Write an expression with two terms. One term has a variable, w, and a coefficient, 7. The other term is a constant, 5.

Turn to the Answer Key at the end of the module to check your work.

Explore

Now you're going to write some of your own expressions.

Example 1

Tosh is 4 years older than his brother, Evan.

How old is Tosh when his brother is 4?

How old is Tosh when his brother is 15?

We can figure out Tosh's age by adding four to his brother's age. We need an expression that means "add four to his brother's age". You can choose any letter you like for the variable. We'll use e as a variable meaning his brother's age.

Tosh's age is: $e + 4$

We could also have written $4 + e$. The order of the terms doesn't matter.

Example 2

Think about Evan and Tosh again. Evan's age is 4 years **less** than Tosh's age.

How old is Evan when Tosh is 13?

How old is Evan when Tosh is 35?

We can figure out Evan's age by subtracting four from Tosh's age.

Use t as a variable meaning Tosh's age. Write an expression for Evan's age.

Evan's age is: $t - 4$

Can we change the order of terms here? We must be very careful and remember that $t - 4$ actually means $t + (-4)$.

$t - 4$ and $4 - t$ do NOT have the same meaning!

Example 3

Milos likes to drive fast! Last summer when he was in Germany, he rented a sports car and drove on the Autobahn. His speed was 125 km per hour.

How far did he drive in one hour?

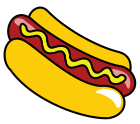
How far did he drive in four hours?

We figure out the distance by multiplying Milos' speed by the number of hours he was driving.

Use *t* as a variable meaning time measured in hours. Write an expression for the distance Milos travels at this speed.

$$125t$$

Another Example!



Hotdogs at the park cost $2.50 each.

How would you figure out the cost of two hotdogs?

How would you figure out the cost of seven hotdogs?

We figure out the cost of the hotdogs by multiplying the price of one hotdog by the number of hotdogs we are buying.

Use the variable *h* to describe the number of hotdogs we're buying. Write an expression for the cost of the hotdogs.

$$2.50h$$

Drinks at the park cost $1.25 each. Use the variable *d* to describe the number of drinks we're buying. Write an expression for the cost of the drinks.

$$1.25d$$

Now we can combine these expressions and write one expression to describe the total cost of the hotdogs and the drinks.

$$2.50h + 1.25d$$

Expressions like these tell the computer at the grocery checkout how to calculate your total.

Yet Another Example!

A paintball birthday party costs $25 plus $6 for every player.

How much does a party with seven players cost?

How much does a party with twelve players cost?

You can calculate the cost of the party in two steps:

1. multiply $6 by the number of players
2. then add $25

Use *p* as a variable meaning the number of players. Write an expression for the cost of the party.

$$6p + 25$$

Try this!

You have $100 to spend on the party.

Use the expression $6p + 25$ to figure out how many people you can invite.

Exercises 2.4

1. Augustin is 6 years younger than Maggie.

    a. Choose a variable to represent Maggie's age. Write an expression for Augustin's age.

    b. Choose a variable to represent Augustin's age. Write an expression for Maggie's age.

2. Quyen noticed that the number of bags of popcorn she sells is usually about half of the number of people in the theatre. Write an expression for the bags of popcorn she can expect to sell.

3. Write an expression for each phrase.

    a. four less than a number

    b. a number increased by three

    c. twice a number

    d. one more than twice a number

    e. two fewer than a number

    f. a number divided by five

    g. three less than twice a number

4. Write a phrase to describe each expression.

   a. $3n$

   b. $5 + n$

   c. $n - 2$

   d. $\frac{n}{3}$

Turn to the Answer Key at the end of the module to check your work.

Evaluate an Expression

Introduction

We started out looking at patterns and describing those patterns with words. Then we thought about the words we used to describe the patterns. What words do we use for increasing patterns? What words do we use for decreasing patterns?

Next, we learned how variables, coefficients, and constants combine to make terms. Terms combine to make expressions.

Now we are going to use expressions—those ideas of patterns—to discover more information about the patterns and predict future results.

Exercises 2.5

We'll start with a review of what you've learned so far.

1. $2x + 5$

   a. How many terms does this expression have?

   b. What is the variable in this expression?

   c. What is the coefficient of *x*?

   d. Is there a constant? If yes, what is it?

2. Let j = Jim's age now

   Write an expression that means:

   a. Jim's age in 3 years

   b. 3 times Jim's age

   c. one more than 3 times Jim's age

3. Complete these questions without a calculator. Remember: (6)(2) means 6×2.

   a. $(6)(2)$
   b. $7 - 1$
   c. $6 - 10$
   d. $8 + 3$
   e. $15 \div 3$
   f. $(4)(8)$
   g. $-3 + 9$
   h. $5(7)$
   i. $16 \div 2$
   j. $-2 + 6$

k. $21 \div 7$

l. $5 + 9$

m. $\frac{1}{2}(10)$

n. $12 \div 4$

o. $4 - 9$

p. $(9)(3)$

q. $-1 + 3$

r. $\frac{1}{3}(12)$

s. $20(6)$

t. $-5 - 6$

Turn to the Answer Key at the end of the module to check your work.

Explore

When the idea of a pattern is written as an expression, we can use it to find out anything we like about the pattern.

One way to learn more about the pattern is to find the value of the expression for a specific value of the variable. Finding the value of an expression once we know the value of the variable is called **evaluating** the expression.

Evaluate the expression **4*t* + 3** when $t = 2$.	$4t + 3$
First, replace the *t* with brackets.	$4(\ \ ) + 3$
Fill-in the value for *t*. In this question, $t = 2$.	$4(2) + 3$
Now we work it out. If you don't know what to do first, underline the terms.	$\underline{4(2)} + \underline{3}$
The first term tells us to multiply. Do that.	$= 8 + 3$
Now the first term is 8. There is nothing to do there.	
The second term is 3—nothing to do there either.	
Each term has been evaluated. Now we can combine the terms.	$= 11$
Your finished work should look like this:	$4(2) + 3$ $= 8 + 3$ $= 11$

Try this one!

What is the value of $2p + 1$ when $p = -3$?

First, substitute with brackets.	$2(-3) + 1$
Second, evaluate each term.	$= -6 + 1$
Finally, combine terms.	$= -5$

We'll do one more together. This time there are two variables. What is the value of $4m + 5n - 7$ when $m = 6$ and $n = 2$?

First, substitute with brackets.	$4(6) + 5(2) - 7$
Second, evaluate each term.	$= 24 + 10 - 7$
Finally, combine terms.	$= 34 - 7$ $= 27$

Evaluating expressions isn't just part of math homework. Dr. John Current evaluates expressions all the time.

Doctors often need to know the surface area of a person's body. This helps them figure out how much fluid the body needs and the dosages of certain medications. You know how to calculate the area of squares, triangles, and many other shapes. However, the surface of your body is a much more complicated shape. The calculations are also complicated. Unfortunately, the calculations are often inaccurate for babies and very young children.

Dr. John Current realized that there was a simple relationship between the surface area of a baby's skin and the baby's weight. The surface area of a baby is an expression that uses a variable, *w*, for the baby's weight (in grams). The body surface area (in cm^2) is:

$$1321 + 0.3433w$$

When Dr. Current needs to know the BSA (body surface area) for a baby that weighs 6000 g, he evaluates his formula.

First, he substitutes with brackets.	$1321 + 0.3433(6000)$
Second, he evaluates each term.	$= 1321 + 2059.8$
Finally, he combines terms.	$= 3380.8$

Exercises 2.6

Work neatly in the space provided or on a separate sheet of paper.

Remember the steps:

First, substitute with brackets.
Second, evaluate each term.
Finally, combine terms.

1. What is the value of each expression when $b = 4$?

   a. $b - 7$

   d. $3b - 4$

   b. $3 + 2b$

   e. $6b$

   c. $6 + b$

   f. $9 - 5b$

2. Evaluate the expression $4 + 2n$ for each value of n.

   a. $n = 3$

b. $n = 5$

c. $n = 0$

d. $n = 2.3$

e. $n = \frac{3}{4}$

3. Find the value of each expression for $f = 3$, $g = -2$, and $h = 1$.

a. $f + 2h$

b. $g + h + 4$

c. $2f - 3h$

d. $-8f + 4h - 3$

e. $g + 5h + 6.8$

4. Evaluate $3p - 2q$.

a. $p = 3.4$, $q = 4$

b. $p = 7$, $q = 5.6$

c. $p = 1.4$, $q = 2.1$

5. This expression represents the cost of hotdogs and drinks at the park: $2.50h + 1.25d$.

   The variable h represents the number of hotdogs. The variable d represents the number of drinks. What is the total cost of five hotdogs and three drinks? Remember to show your steps.

Turn to the Answer Key at the end of the module to check your work.

Lesson 3

Describing Relationships

Learning Outcomes

By the end of this section you will be better able to:

- represent and describe patterns and relationships using graphs and a table of values.

Start With the Table of Values

Linear relationships might sometimes be given by a table of values.

x	y
2	1
4	2
6	3
8	4

If we look at the values, we can discover the relationship between the two variables.

Look at the first row in the table of values (2, 1). Two possibilities for the relationships are:

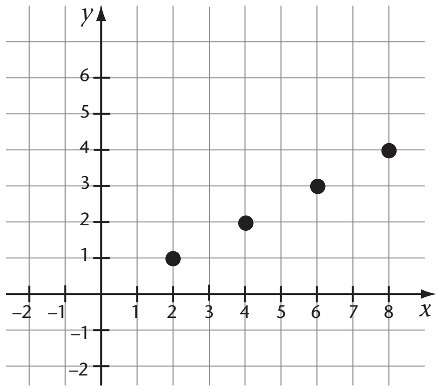
1. Take away 1 from x to get y (2 – 1 = 1) OR
2. Divide x by 2 to get y (2 ÷ 2 = 1).

We won't know which one it is until we test another point.

Look at the point (8, 4). We can confirm that you divide x by 2 to get y, not subtract (8 – 1 is not 4). So we can say that x is twice as big as y, or the other way around, that y is one–half the value of x.

Be sure to check it against the other points that you have listed in order to confirm your work.

Here's the graph:



x	y
2	1
4	2
6	3
8	4

The following table shows Corey's costs when he went to the summer fun fair.

Number of Rides	0	1	2	5	8	10
Cost	\$8	\$10	\$12	\$18	\$24	\$28

Looking at the table of values, you can see the following:

When he went on one ride, it cost him \$2 more than going on no rides (\$10 – \$8 = \$2).

When he went on two rides, it cost him \$2 more than going on one ride (\$12 – \$10 = \$2).

It looks as though each ride cost him \$2.

You can also see from the table that it cost Corey \$8 even when he didn't go on any rides. It probably cost him \$8 just to get into the fairgrounds.

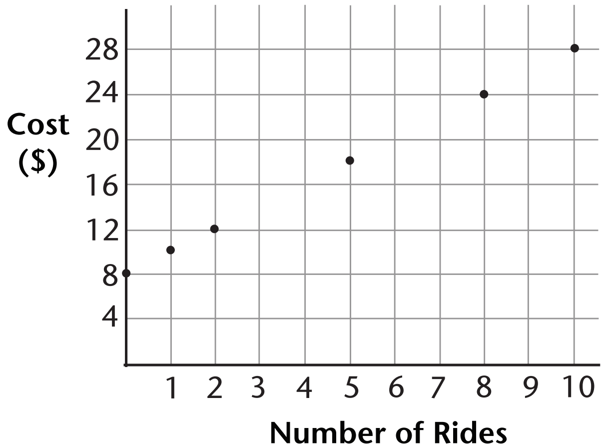
You can test this out by calculating what it would cost Corey to go on 10 rides, and then compare your answer to the one given in the table of values.

Our guess: admission to the fairgrounds is \$8, and each ride is \$2

For 10 rides: cost = admission + amount for 10 rides
cost = \$8 + 10 (\$2)
cost = \$8 + \$20
cost = \$28

This matches the value given in the table of values, so our description of the cost is probably accurate. To be sure, you could check it against more points.

You can also check it against the graph.



To describe this graph, we would say that it starts at $8 on the y–axis. Then for every one it goes to the right, it goes up two.

Exercises 3.1

Look at the following tables of values. Describe the relationships between the variables.

1.

x	y
2	–2
8	4
10	6
12	8

Relationship: ______________________________

2.

Number of Snacks Consumed at Dance	0	1	3	5
Cost to Attend Dance	$5	$6	$8	$10

Relationship: ______________________________

3.

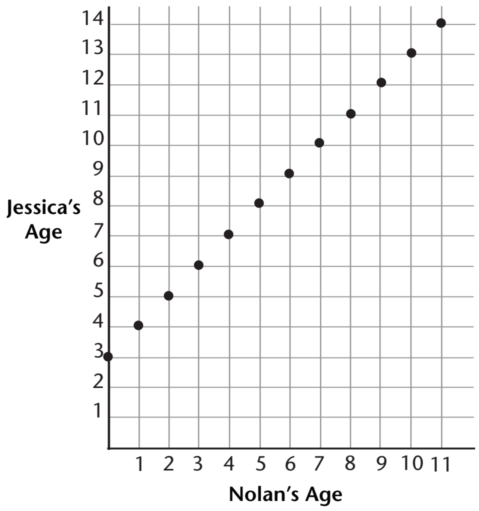
r	t
–3	9
–1	3
0	0
4	–12

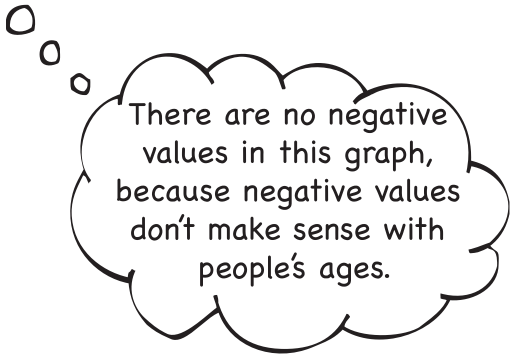
Relationship: ______________________________

Turn to the Answer Key at the end of the module to check your work.

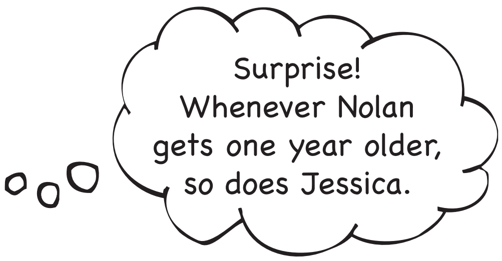
Start With the Graph

Here's a graph of the relationship between Jessica's age and the age of her little brother, Nolan.





This graph starts at three on the *y*–axis. Following the points from left to right, you can see that every time you move over one space, you also move up one space.



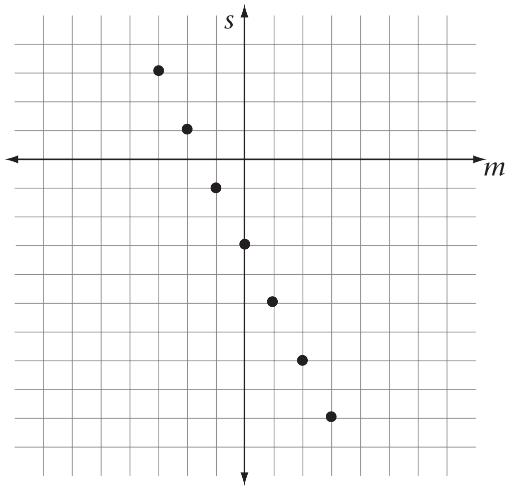
When you're given only the graph, it's a good idea to make a table of values containing at least three ordered pairs. These points can give you a better idea how these two numbersare related.

Nolan's Age	0	2	11
Jessica's Age	3	5	14

This means that when Nolan was 0 (just born), Jessica was three years old. Then when Nolan was two, Jessica was five. And now Nolan is 11, and Jessica is 14.

You may have already noticed that Jessica is three years older than her brother Nolan. To put it into more algebraic terms, Jessica's age is Nolan's age plus three years.

Here is another graph. We don't know what m and s represent, but it is clear that there is a linear relationship between them.



This graph touches the vertical axis at –3. Looking at the points from left to right, you can see that for every one space the graph goes to the right, it goes down two spaces.

Find some ordered pairs from the graph and list them in a table of values:

m	0	1	2	3
s	–3	–5	–7	–9

You can see from the table of values that for every one that m increases, s decreases by two.

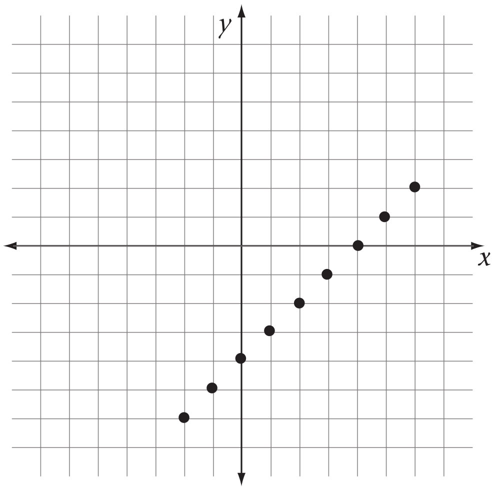
Exercises 3.2

Examine the graphs and describe each one by explaining:

- at which point it touches the y–axis
- how the position of the points change as you move from left to right

Then list four ordered pairs in the table of values, and describe the relationship between the two variables.

1.



This graph touches the y–axis at ______________.

Moving from left to right: ____

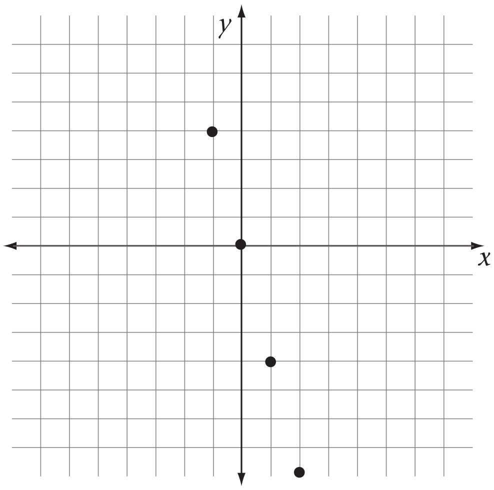
____

x	–2	0	2	4
y				

Describe the relationship from the table of values: __________________

____

2.



This graph touches the y–axis at ____________.

Moving from left to right: ________________________________

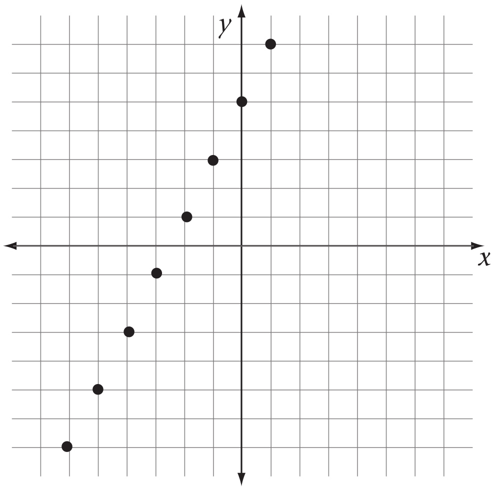
____

x	–1	0	1	2
y				

Describe the relationship from the table of values: _______________

____

3.



This graph touches the y–axis at ______________.

Moving from left to right: ____

____

x	–2	–1	0	1
y				

Describe the relationship from the table of values: __________________

____

Turn to the Answer Key at the end of the module to check your work.

Lesson 4

Algebra, Part 1: The Language of Algebra

Learning Outcomes

By the end of this section you will be better able to:

- Use the operations of addition, subtraction, multiplication and division to keep an algebraic equation balanced.
- Identify terms, variables, coefficients, and constants in an algebraic expression.
- Express ideas about pattern using English words and algebraic expressions.

Algebraic equations are a unique language made up of numbers, letters and symbols. (Algebra is also a universal language, meaning that it is used by people in many different countries.) Algebraic equations are useful because they help us to interpret problems and organize the information into number sentences. They provide scientists, engineers, chemists, economists, business persons, bankers and many others with a valuable tool for organizing ideas and solving problems.

Learning how to use algebra will help you in your other courses, like Science and Social Studies, and in your future career. The skills you gain by learning algebra will help you:

- organize your ideas
- show how you arrive at an answer, and
- prove that your answer is correct.

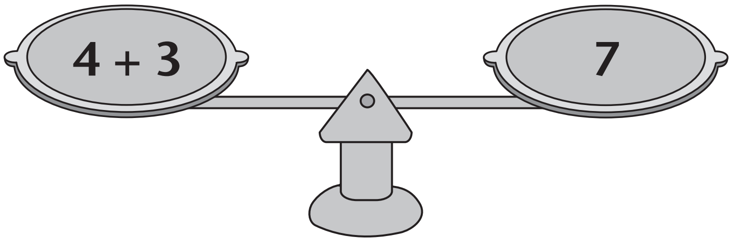
So even if algebra looks like an awkward way to solve a simple problem that you could more easily do in your head, it is a skill worth learning and building on!

Usually, we think of an equal sign as leading the way to an answer. You write a question then an equal sign, followed by an answer. For example, 4 + 3 = 7 is thought of as "if I add four plus three, the answer will be equal to 7."

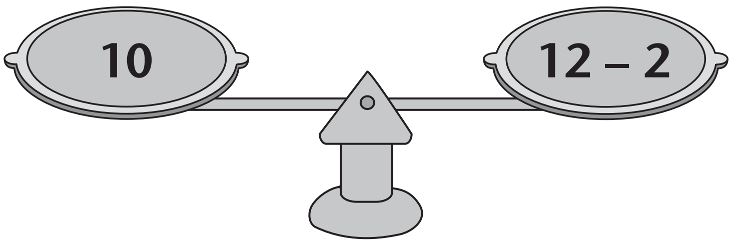
In this lesson, you need to start thinking of the equal sign (=) as something that says one side of the equal sign "is the same as" the other side of the equal sign. The equal sign is the balancing point between the two sides, just like the balancing post on a see-saw.

Examples:

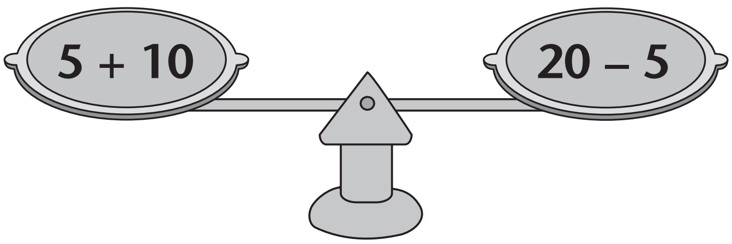
a. 4 + 3 = 7
(4 + 3 is the same as 7)



b. 10 = 12 – 2
(10 is the same as 12 – 2)



c. 5 + 10 = 20 – 5
(5 + 10 is the same as 20 – 5)



What is an Equation?

An equation is made up of two expressions separated by an equal sign. Both sides of the equal sign are equal to each other.

An **expression** is a phrase made up of a single number or variable, or a combination of numbers and variables combined with operations of addition, subtraction, multiplication,
and division.

EXAMPLES OF EXPRESSIONS	EXAMPLES OF EQUATIONS
$2x + 2$	4 + 3 = 7
$x - 3$	10 = 12 – 2
$5 - x$	5 + 10 = 20 – 5

Is 8 + 9 = 20 – 3 an equation?

Yes, it's an equation, both sides are equal: 8 + 9 = 17 and 20 – 3 = 17

Is 6 + 3 – 9 an equation?

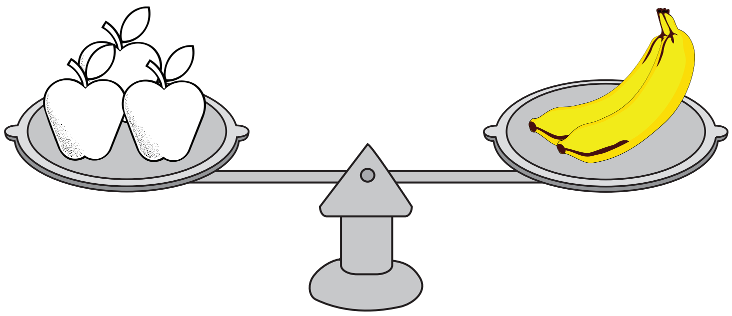
No, 6 + 3 – 9 is not an equation because there is no equal sign.

Is 10 = 6 + 3 an equation?

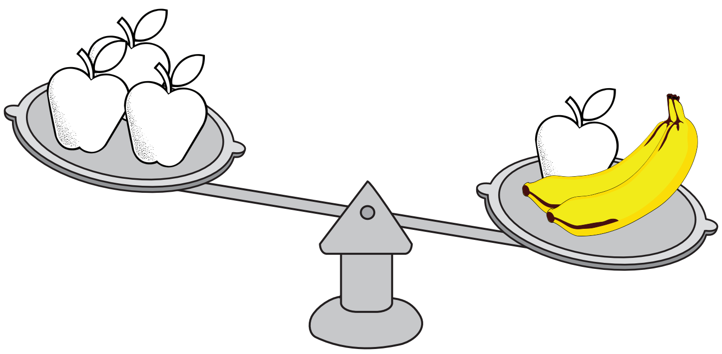
There's an equal sign, but it's not an equation, because both sides are not equal to each other: 6 + 3 is equal to 9, and not 10.

Keep the Scale Balanced

Imagine that you have a scale, as shown below with three apples on one side and two bananas on the other. In the centre of the scale is the fulcrum—the balancing point. The fulcrum is like an equal sign in an equation; both sides of the equation must remain balanced at all times.

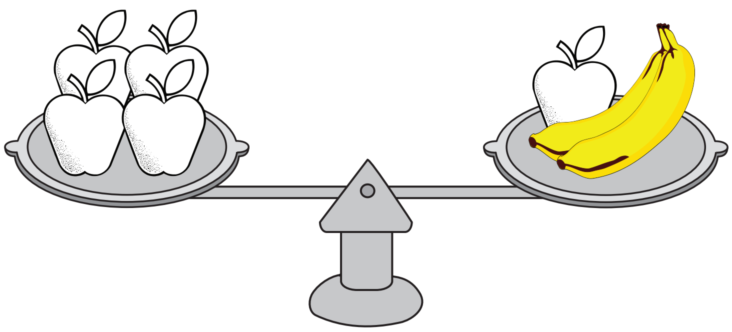


What happens if you add an apple to one side of the scale?



The scale tilts down on the side with the extra apple.

What happens if you add an apple to the other side?

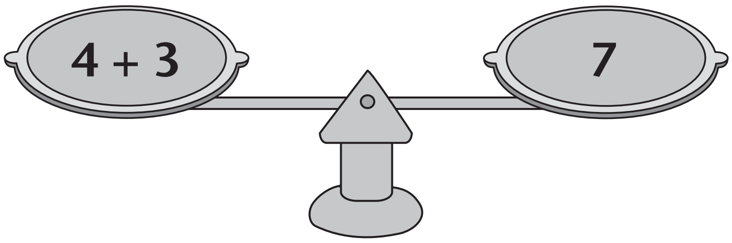


The scale becomes balanced again, and you have preserved the equality of the scale.

It's the same for an equation. We preserve equality by performing the same operation on both sides of the equation.

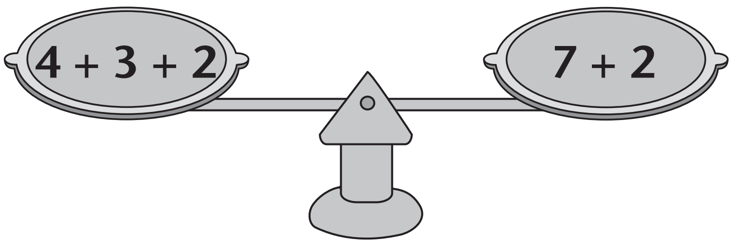
Examples:

1. $4 + 3 = 7$

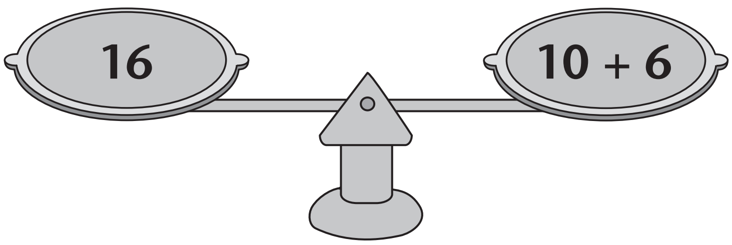


If you add 2 to one side, you must add two to the other side:

$4 + 3 + 2 = 7 + 2$

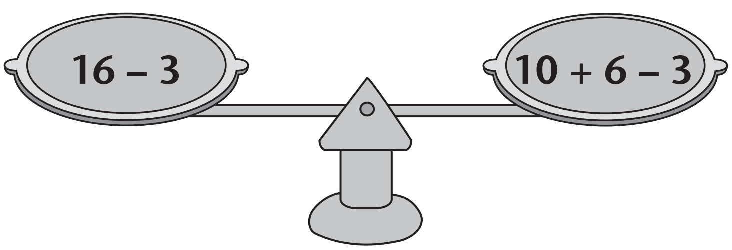


2. $16 = 10 + 6$



If you subtract 3 from one side, you must subtract three from the other side.

$16 – 3 = 10 + 6 – 3$

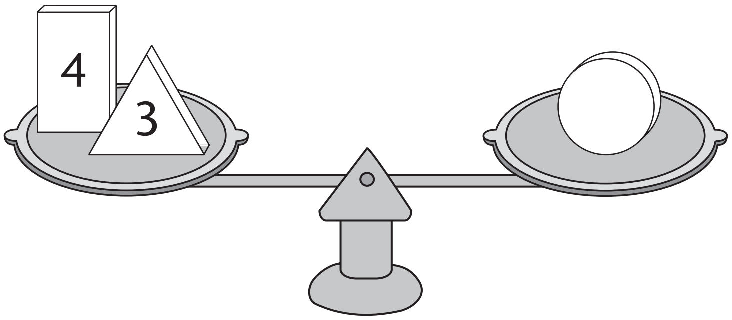


Exercises 4.1

Balance the Scale

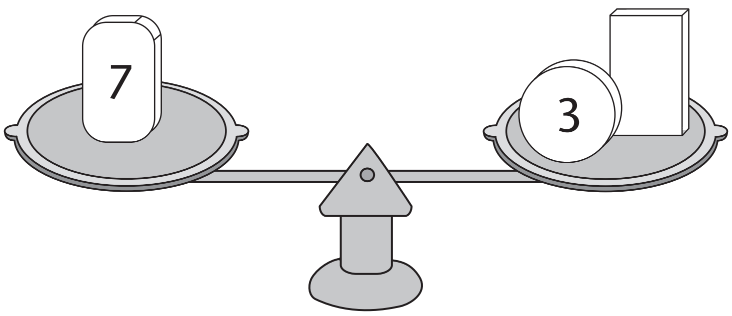
Look at each scale, and write down an equation that describes the balanced scale.

Example

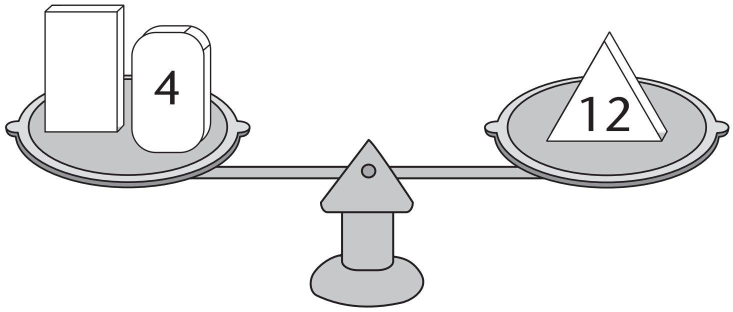


The equation is: 4 + 3 = 7

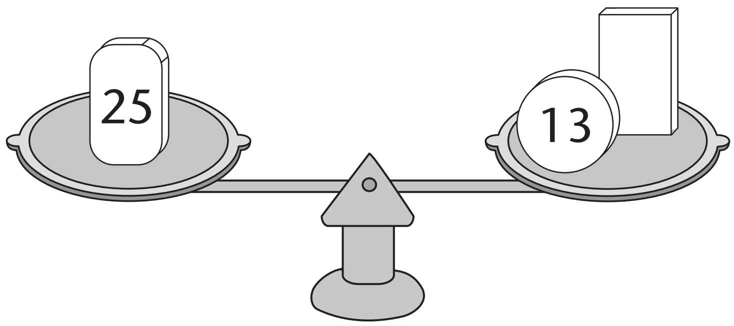
1. The equation is:



2. The equation is:

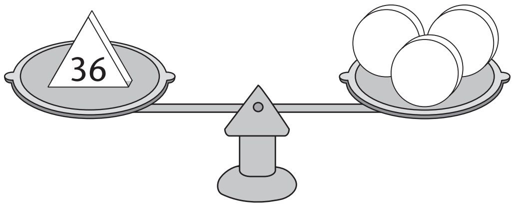


3. The equation is.



Turn to the Answer Key at the end of the module to check your work.

This example is similar to those you've just seen, but this time there are three missing values.



How can you find the value of each block? Try to figure it out on your own first, and then go through the solution shown below.

Since each block is the same size, they must weigh the same on the scale. This means that each block is the same value.

Method 1:

You could add up three of the same numbers to find three that add to 36.

10 + 10 + 10 = 30 (close)
11 + 11 + 11 = 33 (closer)
12 + 12 + 12 = 36 (correct)
So, the value of each block is 12.
The equation is: 36 = 12 + 12 + 12

Method 2:

You could divide the number 36 by 3 to the value of each block.

36 ÷ 3 = 12

Again, the value of each block is 12.

The equation is: 36 ÷ 3 = 12

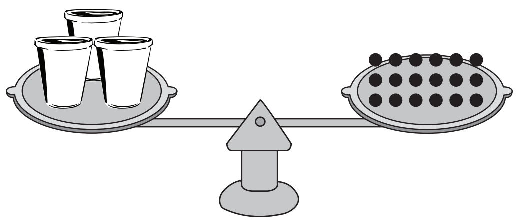
Or you can write: 36 = 3 × 12

Preserving Equality

You can add, subtract, multiply, and divide on both sides of the equation. You must remember that what you do to one side of the equal sign, you must do to the other. It's important that you preserve the equality, or it's no longer an equation. Each side of the equal sign must equal the other.

What you do to one side of the equation, you MUST do to the other side.

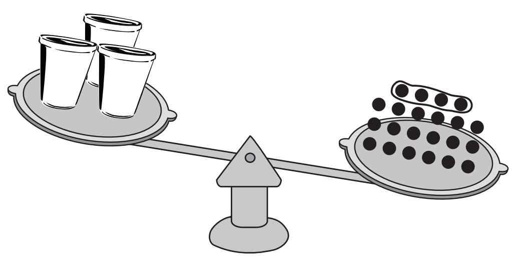
A. Addition



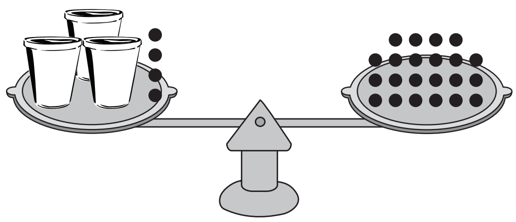
This scale is balanced. It doesn't matter how many counters are in each cup. All we need to know is that the scale is balanced.

3 cups = 18 counters
OR
$3x = 18$

What happens if we add four counters to one side of the balance?



To preserve equality, we have to add four counters to the other side of the balance as well.



$$3x + 4 = 18 + 4$$
$$3x + 4 = 22$$

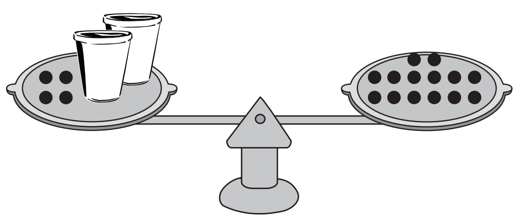
If we want to add 4 to one side, we have to add 4 to the other side.

Fill in the blanks:

What you do to _____ _______ of the equation, you MUST do to the _____ _______.

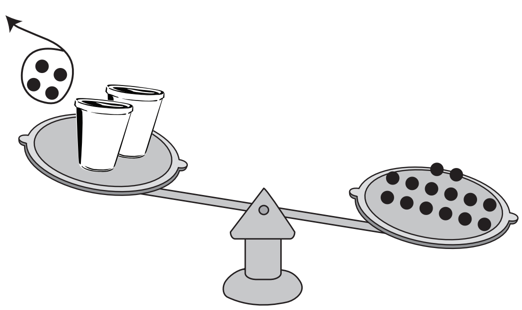
B. Subtraction

This scale is also balanced.

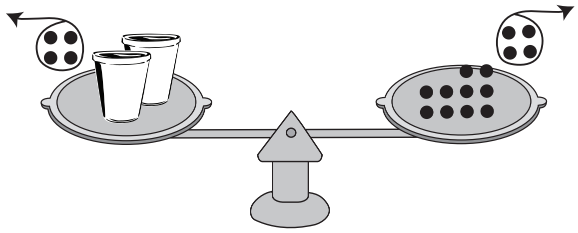


2 cups and 4 counters = 14 counters
$$2x + 4 = 14$$

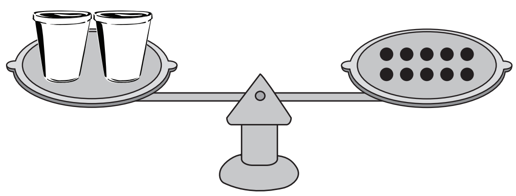
If we remove four counters from the left side of the balance, how do we preserve equality? What do we have to do?



We have to subtract four counters from the other side, too.



$$2x + 4 - 4 = 14 - 4$$
$$2x = 10$$

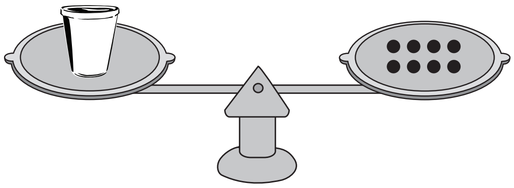


If we want to subtract four from one side, we have to subtract four from the other side, too.

What you do _____ _____ _______ _____ _____ _________,   **you MUST do _____ _____ _____ _______.**

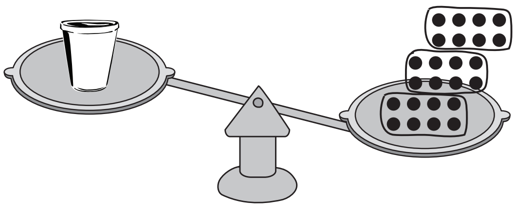
C. Multiplication

This scale is balanced.

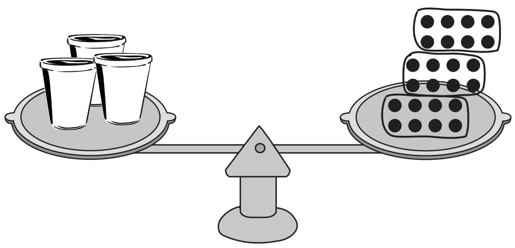


$x = 8$

If we multiply by three on one side of the equation,



we have to multiply by three on the other side, too.

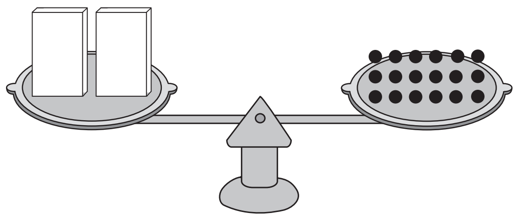


$3(x) = 3(8)$

3(8) means 3 x 8

$3x = 24$

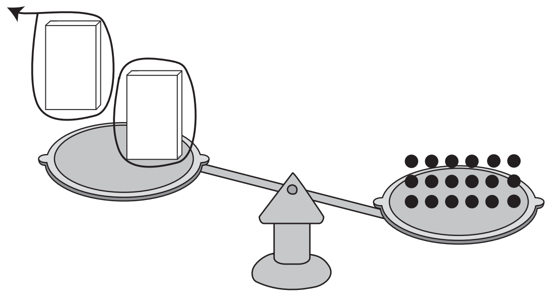
D. Division



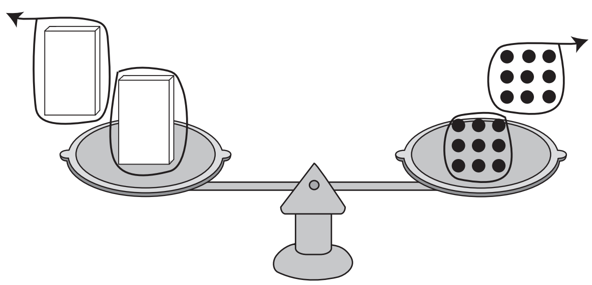
This scale is balanced.

2 blocks = 18 counters
$2x = 18$

If we divide by two on one side of the equation,



we have to divide by two on the other side, too.



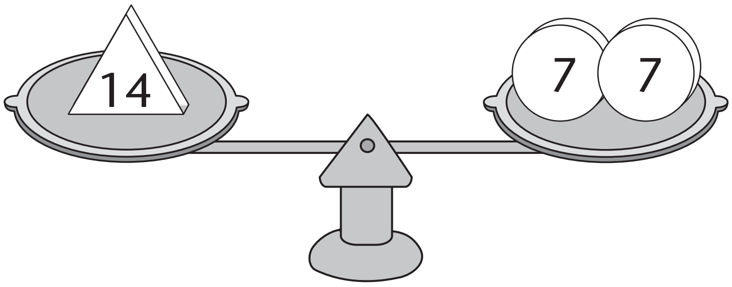
$$\frac{2x}{2} = \frac{18}{2}$$
$$x = 9$$

What you do _____ _____ _______ _____ _____ _________, **you MUST do _____ _____ _____ _______.**

Exercises 4.2

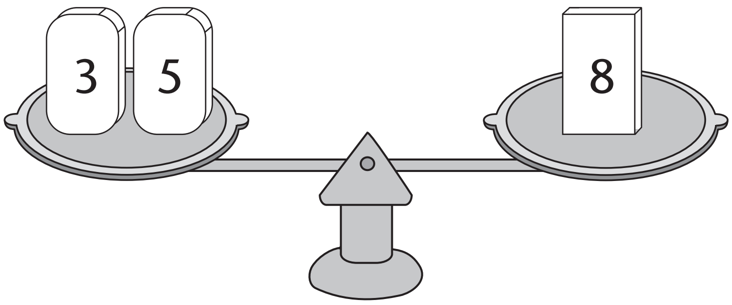
1. Determine if the following balances are equal or not equal.

a.



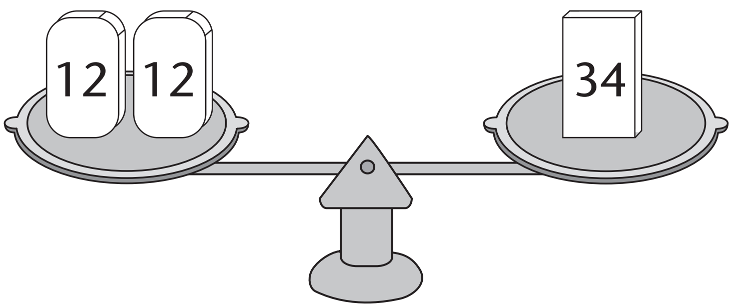
Equal Not Equal

b.



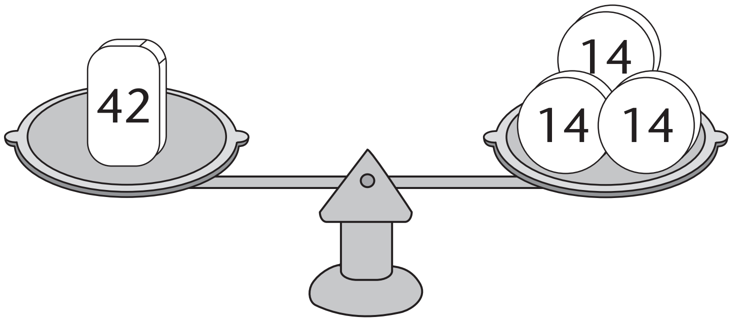
Equal Not Equal

c.



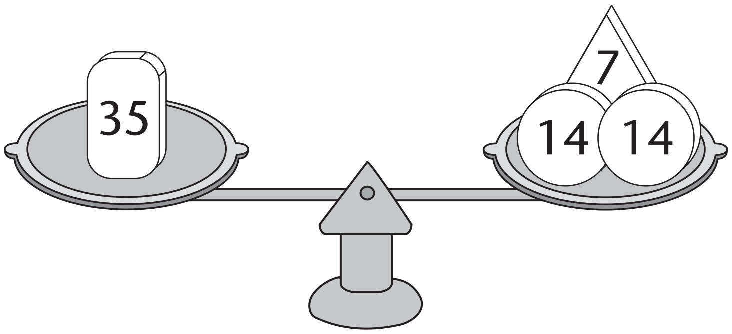
Equal Not Equal

d.



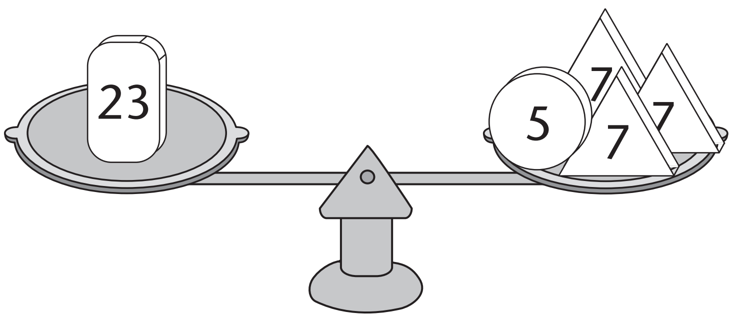
Equal Not Equal

e.



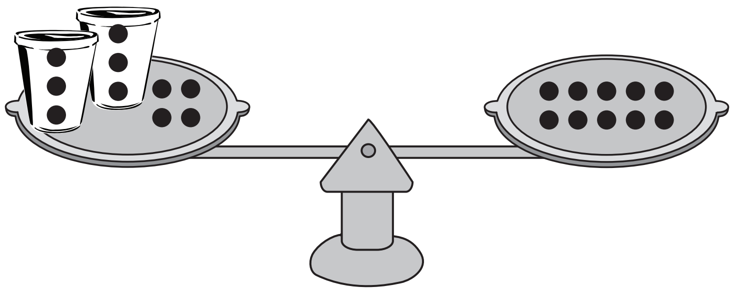
Equal Not Equal

f.



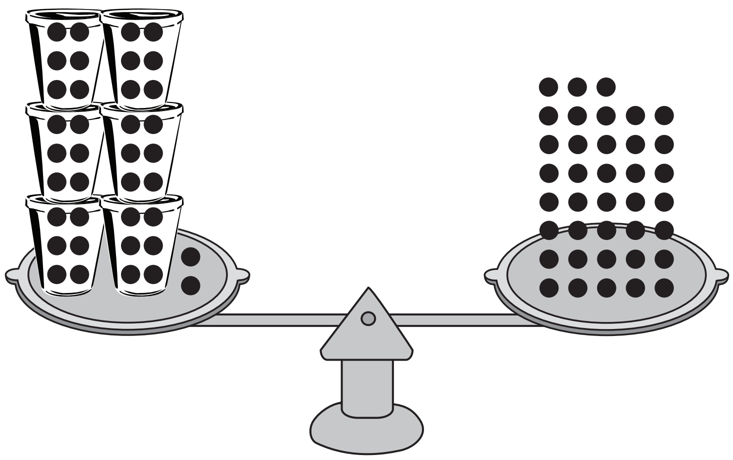
Equal Not Equal

g.



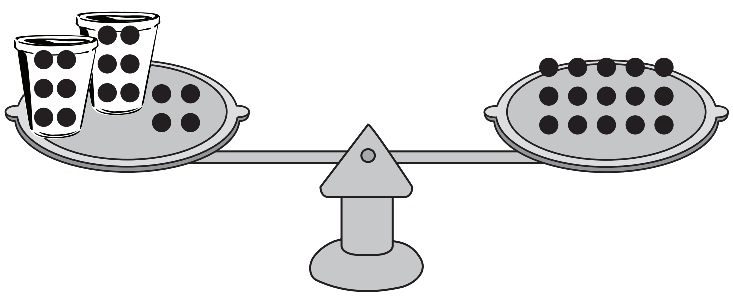
Equal Not Equal

h.



Equal Not Equal

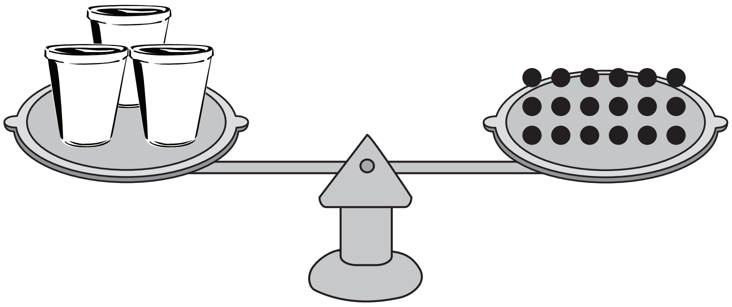
i.



Equal Not Equal

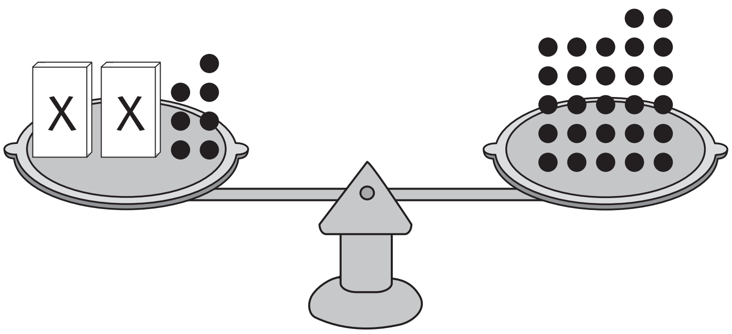
2. Solve the following problems:

a.



How many counters are in each cup?

b.

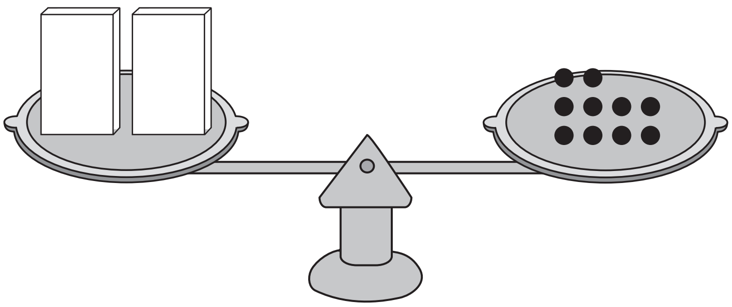


The 7 counters are removed from the right hand side of the scale. How many counters need to be removed from the left hand side?

What is the value of each block (assume that each block has the same value)?

c. 4 × 10 counters = 10 counters + __________ counters

d.



3. Write 5 different equations that are equal to 12.
   e.g., 10 + 2 = 12 or 12 × 1 = 12

Turn to the Answer Key at the end of the module to check your work.

Review: Terms, Variable, and Coefficient

Terms are the "words" that make up an expression or equation. They can be a number, a letter, or a product of a number and letter. Terms are separated by addition or subtraction signs.

Examples

$10 + 3x = 16$ The terms are 10, $3x$, 16

$2ab + 3ab = 5ab$ The terms are $2ab$, $3ab$, $5ab$

Remember:

- $a \times b$ is written as ab
- ab is the product of a and b: $a \times b$
- ba is the product of b and a: $b \times a$
- $ab = ba$, but $57 \neq 75$

Variable

1. A **variable** in an expression represents an "unknown."

   ### Example

   $2n + 1$ "n" can change in value

2. A variable in an equation is an "unknown" quantity of something that can be solved.

   ### Example

   $$2n + 1 = 5$$
   $$2(2) + 1 = 5$$
   $$n = 2$$

Coefficient

A **coefficient** is the number value in front of a variable.

Example

$5a$ coefficient = 5, variable = a

If there is no coefficient in front of a variable, the coefficient is understood to be 1.

Example

a coefficient = 1, variable = a

So $1a$ and a mean the same thing.

Constant Term

A **constant term** is a term that is not multiplied or divided by a variable. It remains a constant value.

Example

$3x + 2$ 2 is a constant term

Similarities and Differences Between Equations and Expressions

Similarities

EXPRESSIONS can include:	EQUATIONS can include:
• a single number • a single variable, • a combination of numbers and variables • single operations or a combination of operations: +, –, ×, or ÷	• a single number • a single variable, • a combination of numbers and variables • single operations or a combination of operations: +, –, ×, or ÷ Equations are made up of two expressions separated by an equal sign
Example $3x + 1$ $16 – 3$	Example $3x + 1 = 16 – 3$

Differences

EXPRESSIONS:	EQUATIONS:
• an expression is a mathematical phrase • you can only simplify an expression • you can't solve an expression • there is no equal sign in an expression	• an equation is an algebraic sentence • you can solve an equation • an equation includes an equal sign • one side of an equation is balanced with the other side
Example five less than a number $x - 5$ "x" can be many different numbers	Example five less than a number is 10 $x - 5 = 10$ $x = 15$

Exercises 4.3

1. Fill in the blank with either the word "equation" or "expression."

    a. 3 cups + 4 counters, is an ___________________.

    b. 2 cups + 3 counters = 3 rows of 5 counters each on a balance is an ___________________.

    c. $5x + 3$, is an ___________________.

    d. $(2 + 3) - 1$, is an ___________________.

    e. $(2 + 3) - 1 = 4$, is an ___________________.

    f. $18 = 6 + 7 + 5$, is an ___________________.

    g. $15 = 45 - 5x$, is an ___________________.

    h. $45 - 5x$, is an ___________________.

Turn to the Answer Key at the end of the module to check your work.

Translating Words to Symbols

Let's review words and symbols that are used to describe mathematical operations. For example, plus is written as a "+" sign in an expression, and minus is written as a "–" sign. It is important to be able to translate word sentences into algebraic sentences.

Exercises 4.4

1. Fill in as many of the expressions as you can in the right hand column using "*n*" as the number:

Table with Operational Symbols

WORD	OPERATION	SYMBOL	EXAMPLE	LET *n* = NUMBER
a. plus	add	+	a number plus five	$n + 5$
b. minus	subtract	–	a number minus three	
c. more than	add	+	six more than a number	
d. less than	subtract	–	two less than a number	
e. increased by	add	+	a number increased by four	
f. decreased by	subtract	–	a number decreased by one	
g. the sum of	add	+	the sum of a number and seven	

h. the difference between	subtract	–	the difference between a number and one	
i. the product of	multiply	×	the product of a number and four	
j. the quotient of	divide	÷	the quotient of a number and five	
k. twice	multiply by 2	× 2	twice a number	
l. triple	multiply by 3	× 3	triple the sum of a number and four	
m. half	multiply by ½ or divide by 2	× ½ or ÷ 2	half a number	
n. of	multiply	×	one-third of a number	

Turn to the Answer Key at the end of the module to check your work.

Translating Words to Equations

The following words and phrases translate to an equal sign in an equation:

- is
- equals
- is the same as
- the result is

Example

A number increased by four is seven.
Let n = a number
Equation: $n + 4 = 7$

Before you translate words into algebraic equations, you will find it helpful to highlight key words. You'll want to highlight the words for terms, operations, and the answer.

Examples

The product of four and a number is twenty.

The **product** of <u>four</u> and <u>a number</u> is **twenty**.

Equation: $4 \times n = 20$, or $4n = 20$

The sum of twice a number and ten is sixteen.

The **sum** of <u>twice a number</u> and <u>ten</u> is **sixteen**.

Equation: $2n + 10 = 16$

Translating Equations to Words

Let's translate algebraic equations back into words. Review the *Table with Operational Symbols* if you are having difficulty.

Example

$x + 7 = 10$
Sentence: A number plus seven equals ten.

Exercises 4.5

1. Translate these equations into words, letting "x" be a number.

   a. $4 + x = 6$
   Sentence:

   b. $x - 3 = 9$
   Sentence:

   c. $2 + x = 7$
   Sentence:

   d. $3x = 12$
   Sentence:

   e. $5x + 1 = 37$
   Sentence:

2. Match the sentence with the proper equation on the right.

_____	The sum of six and a number is eleven.	a.	$4(2x) = 16$
_____	Seven less than a number is five.	b.	$6 + x = 11$
_____	The product of five and a number is thirty-five.	c.	$n - 7 = 5$
_____	The sum of a number and 2 is eight.	d.	$\frac{1}{2}h = 12$
_____	One half a number is twelve.	e.	$2(x + 3) = 18$
_____	Four increased by a number is sixteen.	f.	$4 + x = 16$
_____	The product of four and twice a number is sixteen.	g.	$w + 2 = 8$
_____	Twice the sum of a number and three is eighteen.	h.	$5x = 35$

3. Match the equation with the correct sentence on the right.

_____	$x - 5 = 3$	a.	A number added to seven equals twelve.
_____	$7 + x = 12$	b.	A number decreased by five is three.
_____	$x \div 4 = 2$	c.	A number times 3 equals 0.
_____	$3x = 0$	d.	Three times the difference of a number and ten is 15.
_____	$8x = 6 + 2$	e.	Five multiplied by the product of 2 and a number equals 40.
_____	$5(2x) = 40$	f.	The product of 8 and a number is the same as six plus two.
_____	$3(x - 10) = 15$	g.	A number divided by 4 is 2.

Turn to the Answer Key at the end of the module to check your work.

Lesson 5

Algebra, Part 2: Solving Equations

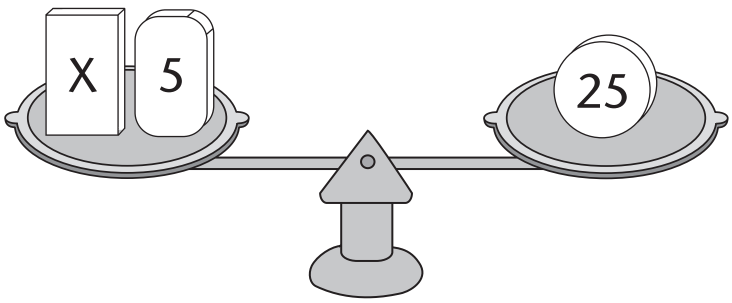
Learning Outcomes

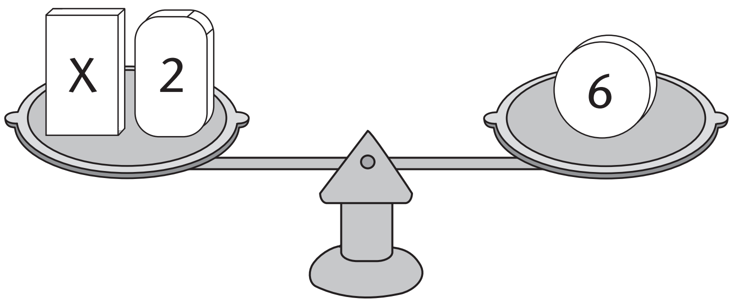
By the end of this section you will be better able to:

- Solve one-step and two-step algebraic equations.
- Use substitution to check your solution to an algebraic equation.
- Use algebra to solve word problems.

Balancing the Scales

Let's look at some scale balances and the equations that represent what we see.





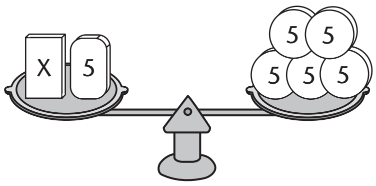
Can you think of a way to figure out what x represents in the above balance scales?

You must always keep the scales balanced. In other words:

What you do to one side of the equation, you MUST do to the other side.

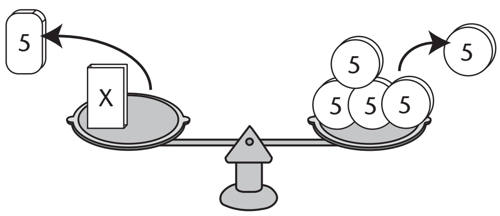
We need to **isolate** the variable so that we can determine what it equals (e.g., x = 25). The phrase <u>isolate the variable</u> means to get the variable by itself.

Let's look at an example.



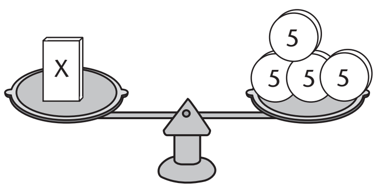
Write an equation that represents the balanced scale

$x + 5 = 25$



To get x by itself, remove 5. In the equation, remove 5 means subtract 5. Remember: keep the scale balanced!

$x + 5 = 25$
(–5) (–5)



What are we left with? Write an equation that represents the balanced scale.

$x = 20$

Notice that the operation in our original equation was *addition*. To isolate the variable, we had to take away, or *subtract*. Addition and subtraction are *inverse* operations.

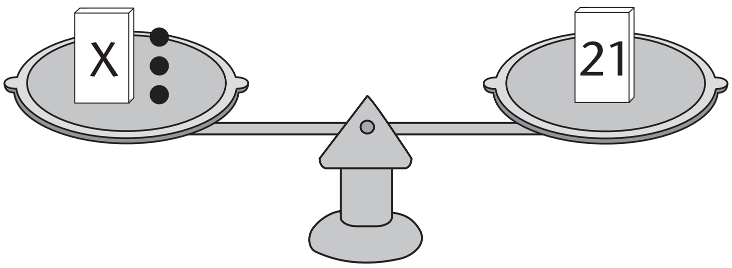
Addition undoes subtraction.
Subtraction undoes addition.

Now it's your turn to practise balancing the scales.

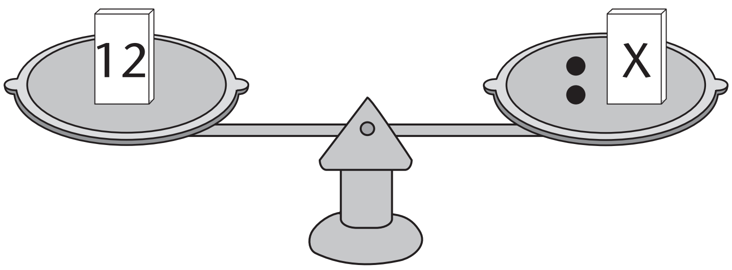
Exercises 5.1

1. Write an equation to represent the following scale balances:

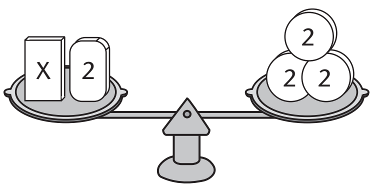
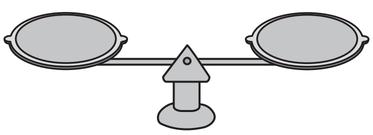
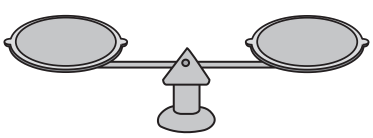
a.



b.



2. Follow the example on the previous page to solve this equation for each step.

Scales	Equation
	_____ + _____ = _____
	_____ + _____ = _____ ( ) ( )
	_____ = _____

Turn to the Answer Key at the end of the module to check your work.

Solving One-Step Equations

In this lesson, you will learn about solving one-step equations that are made up of one term added to or subtracted from another term. It is very similar to how you learned to balance the scales. Just remember:

- What you do to one side of the equation, you must do to the other side of the equation.
- The inverse of addition is subtraction, and the inverse of subtraction is addition.
- Our goal is to isolate the variable.

Here are some examples of one-step equations involving addition and subtraction:

$x + 3 = 6$

$16 + x = 21$

$x + 7 = 15$

$x - 9 = 1$

$x - 4 = 3$

Solving One-Step Equations (Addition)

We don't always need to draw a scale to help us solve an equation. Let's look at some examples to explore how to solve one-step equations. These examples are all equations with addition.

Example A

What is the value of x in the equation below?

$x + 2 = 6$

Maybe you already have a pretty good idea about the value of x in this equation. That's great.

It is important to learn algebra using simple equations. Then you will be able to apply the steps of algebra when the answers are not so clear.

Follow these steps to solve the equation:

Step 1: Write down the equation. $x + 2 = 6$

Step 2:	You want to have the unknown variable isolated on one side of the equation. Use the inverse operation.	(-2) (-2)
Step 3:	Write down your answer. Remember: You want to end up with x by itself (isolated).	$x = 4$

Example B

What is the value of x in this equation? $x + 2 = 9$

Step 1:	Write down the equation.	$x + 2 = 9$
Step 2:	Perform the inverse operation to isolate x.	(-2) (-2)
Step 3:	Write down your answer.	$x = 7$

Why do you think this is called a one-step equation?

Even though it looks like you're following three steps to solve the equation, there is one key step. The step that gives us our answer is the second one: performing the inverse operation.

In Example B, the equation is called a "one-step equation" because it requires the "one-step" of subtracting 2 from both sides to isolate x.

Example C

Try filling in the blanks to solve for x.

Solve:

$$2 + c = 7$$
$$(-\_\_) \qquad (-\_\_)$$
$$c = \_\_$$

Check your answer below:

$$2 + c = 7$$
$$(-2) \qquad (-2)$$
$$c = 5$$

Solving One-Step Equations (Subtraction)

These examples will show you how to solve for the unknown variable when subtraction is used in the equation. Remember: the inverse of subtraction is addition.

Example D

What is the value of x?

$$x - 9 = 7$$

Follow these steps to solve the equation:

Step 1:	Write down the equation.	$x - 9 = 7$
Step 2:	You want to have the unknown variable isolated on one side of the equation. Use the inverse operation.	$(+9)\ (+9)$
Step 3:	Write down your answer.	$x = 16$

Remember: You want to end up with *x* by itself (isolated).

Example E

Solve:

$$c - 8 = 20$$
$$(+\_\_)\ (+\_\_)$$
$$c = \_\_$$

Check your answer below:

$$c - 8 = 20$$
$$(+8)\ (+8)$$
$$c = 28$$

Exercises 5.2

Fill in the blanks in the equations below.

1. $c - 3 = 7$

   (+___) (+___)

   $c =$ ____

2. $x + 3 = 11$

   (___) (___)

   $x =$ ____

3. $p - 3 = 10$

   (___) (___)

   $p =$ ____

4. $m + 14 = 21$

   (___) (___)

   $x =$ ____

Turn to the Answer Key at the end of the module to check your work.

Checking your Solutions

"Solving an equation" means the same thing as "finding the solution that makes the equation true." To check your solution, you need to determine if both sides of the equation are balanced. You can do this by substituting your answer for the variable. If both sides of the equation are equal, the equation is "balanced."

Helpful Hint: Here's a catchy way to remember the steps when you are checking your solution—"Solve, Substitute, SMILE". The smile comes when you find out your answer is indeed correct!

- You *solve* the equation so that x is isolated on one side of the equation.
- You *substitute* the value of x into the original equation.
- Then you can *smile* if both sides of the equal sign are the same!

Let's look at an example:

You have a number of blueberries in a bowl. You add four strawberries, to equal 19 pieces of fruit in the bowl. How many blueberries do you have?

Solve:

Let x = number of blueberries in one bowl.

Equation: $x + 4 = 19$

You need to take off 4 pieces of fruit from both sides of the scale. Remember, what you do to one side of the scale you must do to the other to keep the scale balanced.

The equation is now:

$$x + 4 = 19$$

$$(-4) \quad (-4)$$

(Here we are applying the inverse of addition (subtraction) to solve our equation).

$$x = 15$$

If our calculations were correct, you should have 15 blueberries in your bowl.

Check your answer:

Substitute:

Check to see if $x = 15$

Equation:	$x + 4 = 19$
Substitute: 15 for the x	$15 + 4 = 19$
Are both sides balanced?	$19 = 19$

Both sides of the equation are balanced, so our solution is correct.

Smile! ☺

Looks like our calculations were correct. You have 15 blueberries in your bowl.

But what if we had made a mistake? How can checking our solution help us to find our mistakes? Let's see what would have happened if we had solved and got $x = 14$ as our answer:

Check to see if $x = 14$

Equation:	$x + 4 = 19$
Substitute 14 for the x:	$14 + 4 = 19$
Are both sides balanced?	$18 = 19$ oops! ☹

Both sides of the equation are not balanced, so my solution is incorrect. No smile.

When your check doesn't end in a smile, you know something's wrong. If you end up with an unbalanced statement like 18 = 19, you know you've made a mistake along the way. You should always go back and try solving the equation again.

Try some exercises to make sure you've got the hang of it.

Solve, Substitute, and Smile!

Exercises 5.3

1. Evaluate if $x = 4$

   a. $x + 6$

   b. $9 - x$

   c. $32 - x$

2. Check to see if $x = 3$ for these equations:

   a. $4 + x = 7$

   b. $21 - x = 19$

   c. $x - 49 = 3$

2. Solve the following equations showing the steps of your work. Check your answer.

   a. $3 + x = 8$

   b. $q - 11 = 18$

c. $x - 10 = 2$

d. $3 + x = 19$

e. $w - 25 = 50$

f. $a - 8 = 8$

g. $x + 15 = 27$

h. $x - 47 = 62$

Turn to the Answer Key at the end of the module to check your work.

Solving Word Problems

You now have many of the skills needed to solve word problems involving one-step addition and subtraction equations. Let's practice some together!

A number increased by 2 is equal to six. What is the number?

Example

Create a variable (e.g. "x") to represent an unknown number.

Let x = unknown number

Write out your equation.

$$x + 2 = 6$$

Remember: Solve, Substitute, Smile!

Solve: Isolate the unknown variable on one side of the equation by using the inverse operation.

In this equation, 2 is added to the variable. To solve, you will subtract 2 on both sides of the equation.)

$$\begin{aligned} x + 2 &= 6 \\ (-2) \quad & (-2) \\ x &= 4 \end{aligned}$$

Check your solution:

Substitute 4 for x in the original equation:

$$\begin{aligned} x + 2 &= 6 \\ (4) + 2 &= 6 \\ 6 &= 6 \end{aligned}$$

Yes, the solution is correct! Smile! ☺

Don't forget to answer the problem.

The number is 4.

Let's try another example:

Zachary has some hockey cards. He gives three of them to a friend and has 11 cards left. How many hockey cards did Zachary have to start with before he gave 3 cards away?

Example

Create a variable (e.g. "x") to represent an unknown number. Let x = unknown number of hockey cards

Write out your equation. $x - 3 = 11$

Solve: Isolate the unknown variable on one side of the equation, using the inverse operation of subtraction—addition.

Because 3 is subtracted from the variable, you must add 3 to bothsides of the equation to maintain balance.

$$x - 3 = 11$$

$$(+\ 3) \qquad (+\ 3)$$

$$x = 14$$

Check your solution.

Substitute 14 for x in the original equation:

$$x - 3 = 11$$

$$(14) - 3 = 11$$

$$11 = 11$$

Yes, the answer is correct. Smile! ☺

Make sure to answer the problem.

Zack had 14 hockey cards to start with before he gave 3 cards away.

Exercises 5.4

1. Express the following statements as equations and solve. Check your answers.
   Let x = unknown number

   a. 6 less than a number is 8.

   b. A number increased by 3 equals 12.

   c. Four plus a number is 25.

   d. A number increased by 11 gives 19.

2. Alex has some hockey cards and gives 7 cards away. He now has 31 hockey cards left. How many cards did he start with?

Turn to the Answer Key at the end of the module to check your work.

Solving One-Step Equations Involving Multiplication or Division (× or ÷)

Here's an algebra puzzle for you:

- Pick a number.
- Add 4 to the number.
- Then multiply by 2.
- Subtract 6.
- Divide it by 2.
- Now subtract 1 to get your answer.

What did you get? Your answer should be the same number that you started with. This will work for any number you choose. Try different numbers and see what happens.

To find out if the puzzle works, you can substitute numbers for the variable and see if the result is always the same. Checking your solutions of equation problems works the same way. Remember—Solve, Substitute, Smile!

To solve this puzzle, you have to use all four operations: +, –, ×, ÷. In the last lesson, you learned how to keep a scale or one-step equation balanced by adding or subtracting. You can also keep a scale or one-step equation balanced by multiplying or dividing.

In this lesson, you will be learning how to solve, substitute, and smile using one-step equations that involve multiplication or division
(× or ÷).

It will help you to review some vocabulary as well as multiplication and division before you begin this part of the lesson.

Remember: A term can be made up of a number and variable, multiplied or divided by each other. For example, $\frac{x}{3}$ is a term. $3x$ is also a term.

Multiplication

Sometimes we write multiplication using brackets.

For example:

3(6) is a term that means the same as 3×6.

3(6) and 3×6 both equal 18

Note: If you are multiplying more than 2 numbers together, proceed in steps by multiplying two numbers at a time.

For example: (4)(2)(15) is the same as $4 \times 2 \times 15$.

Here there are 3 numbers to multiply, so start by multiplying the first two numbers. Then multiply that answer by the third number.

(4) (2) (15)

= (8)(15)

= 120

Division

When we divide, we're splitting our starting number into equal groups, and counting the number of groups we can make. The answer in a division question is called the "quotient."

Example

Marc has 12 cookies to divide among 4 people. How many cookies will each person get?

Answer: Each person gets three cookies.

We already looked at one-step equations with addition or subtraction, but one-step equations can involve multiplication or division operations.

For example, $2x = 6$ and $x \div 2 = 5$ are both one-step equations.

You'll notice that on one side of each equation there is a single term: an unknown number written as a variable that is either multiplied or divided by a number. On the other side of each equation is a single, constant term.

To solve these kinds of one step equations, we undo the operation.

What do you think it means to "undo" the operation?

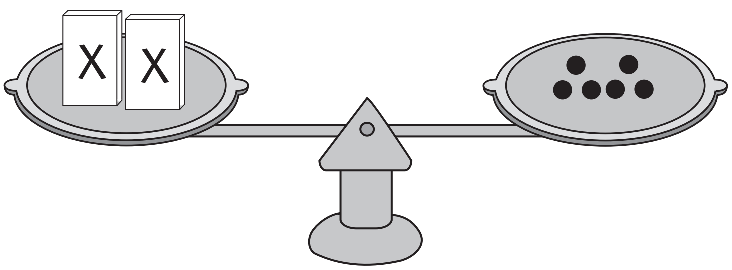
We undo multiplicaton by dividing.

We undo division by multiplying.

Solving One-Step Equations (Multiplication)

Let's work through some one-step equations involving multiplication. You will need to use division (the inverse operation of multiplication) to solve these problems.

Example A



How can you figure out the value of each block on the scale?

Let x = one block.

There are 2 blocks, so you write $2x$ as the total number of blocks.
x = the unknown number of counters represented by one block.

Now, write an equation to represent the balance using $2x$ for the blocks. There are six counters on the right hand side of the scale, so our equation should look like this:

$$2x = 6$$

Okay, let's solve for x.

Remember:

What you do to one side of the equation, you MUST do to the other side.

We are multiplying by 2. We can undo that by dividing by 2.

Your equation should look like this:

$$2x = 6$$

$$(\div 2) \quad (\div 2)$$

$$1x = 3$$

The value of one block is 3. So $x = 3$.

Note: sometimes when we divide both sides of the equation, we write it in a different way. Look at the two ways of showing your work: they both mean the same thing.

$$2x = 6$$

$$(\div 2) \quad (\div 2)$$

$$x = 3$$

OR

$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

Steps to follow when solving one-term equations involving multiplication:

Step 1: Choose a variable for the quantity that you are trying to find.

Step 2: Write down the equation that represents the problem.

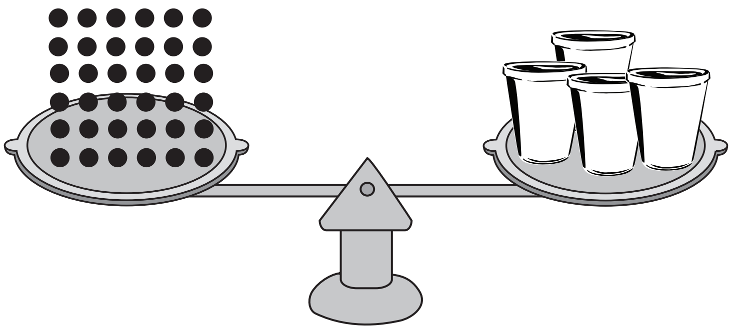
Step 3: Perform the inverse operation.

Remember:

What you do to one side of the equation, you MUST do to the other side.

Step 4: Write down your answer.

Example B



Solve:

Step 1: Choose a variable.
Let x = unknown quantity of counters in a cup.

Step 2: Write down the equation that represents the problem. $36 = 4x$

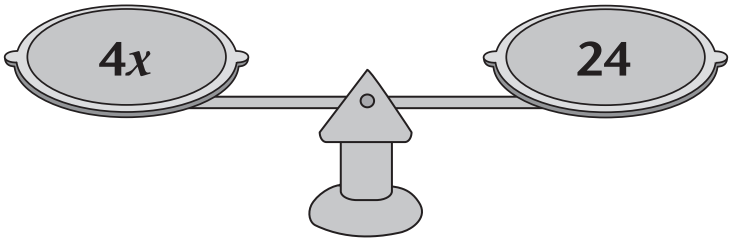
Step 3: Solve. We can undo multiplying with dividing. $(\div 4) \quad (\div 4)$

Divide both sides by 4. $9 = 1x$

Step 4: Write down your answer: $x = 9$

1 cup has 9 counters inside it.

Example C



Write an equation for the balance scale:

Using the steps above, solve for x:

Check to see that your answer looks like the one below. When you are ready, try the practice questions.

Answer:

$$4x = 24$$

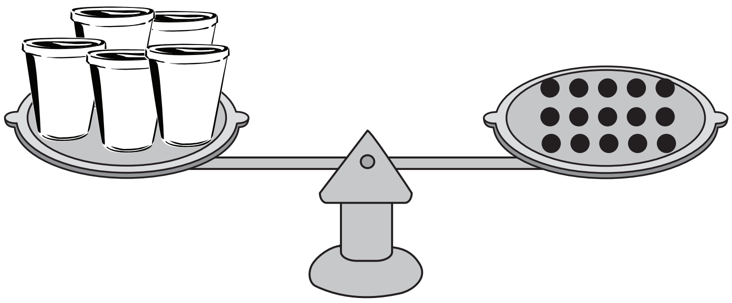
$$(\div 4) \quad (\div 4)$$

$$x = 6$$

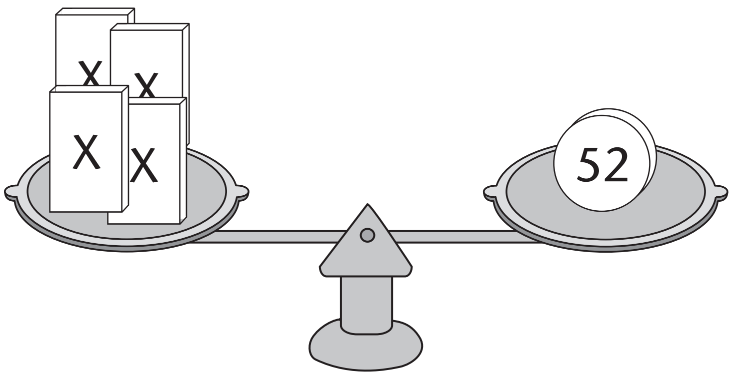
Exercises 5.5

1. Solve the following one-step equations involving multiplication:

   a. If each cup has the same number of counters, how many counters are in each cup?



   b. What is the value of one block?



2. Fill in the blanks.

   a.
$$3x = 24$$
$$3x = 24$$
$$(\div \_\_) \quad (\div \_\_)$$
$$x = \_\_\_\_$$

   b.
$$2x = 14$$
$$2x = 14$$
$$(\div 2) \quad (\div \_\_)$$
$$x = \_\_\_\_$$

c.
$$4y = 160$$
$$\frac{4y}{(\div 4)} = \frac{160}{(\div \_\_)}$$
$$y = \_\_\_\_$$

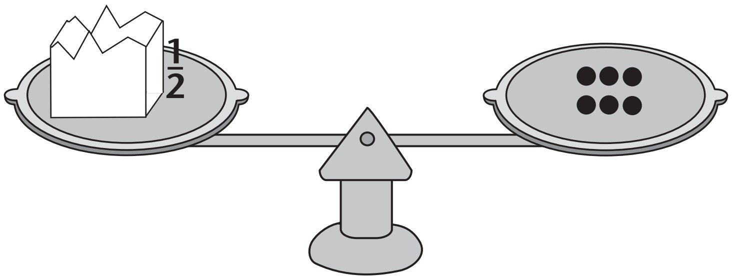
d.
$$5x = 50$$
$$\frac{5x}{(\div \_\_)} = \frac{50}{(\div 5)}$$
$$x = \_\_\_\_$$

Turn to the Answer Key at the end of the module to check your work.

Solving Equations (Division)

To solve one-term equations involving division, you use the inverse operation—multiplication.

Example D



If half a box balances equally with 6 widgets (half a box holds 6 widgets), how many will fit in a full box?

To solve this problem, let x represent the box.

$$\frac{1}{2}x = 6$$

Multiply both sides by 2.

$$2\left(\frac{1}{2}x\right) = 2(6)$$
$$\frac{2}{2}x = 12$$
$$x = 12$$

Twelve widgets fill a box.

Steps to follow when solving one-step equations involving division:

Step 1: Choose a variable for the quantity that you are trying to find.

Step 2: Write down the equation that represents the problem.

Step 3: Perform the inverse operation. The inverse of division is multiplication.

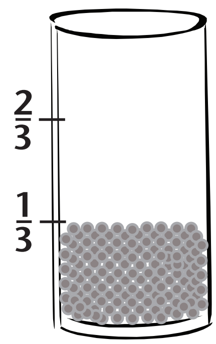
Remember:

What you do to one side of the equation, you MUST do to the other side.

Step 4: Write down your answer. Remember, the coefficient of a single variable x is 1, but you don't write the 1.
For example, for $1x$, you just write x.

Example E

Kayla has collected 121 nickels, which is one-third of the nickels needed to fill her collection jar. How many nickels will she have when the jar is full?

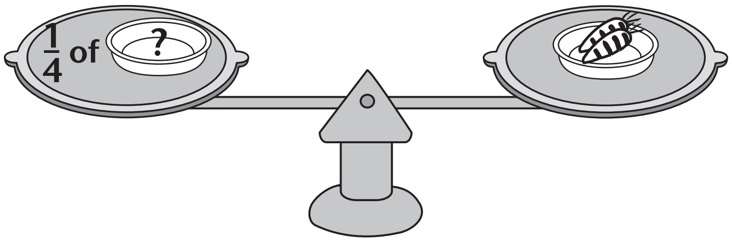


Step 1:	Let x = quantity that you are trying to solve.	Let x = total number of nickels.
Step 2:	Write down the equation that represents the problem.	$\frac{1}{3}x = 121$
Step 3:	Multiply both sides by 3. The inverse of dividing by 3 is multiplying by 3.	$(\times 3) \quad (\times 3)$
Step 4:	Write down your answer.	$x = 363$
	The collection jar can hold 363 nickels.	

Example F

Sam is eating some carrots out of a dish. He sees that he has only $\frac{1}{4}$ of the carrots left in the bowl. If there are 2 carrots left, how many

carrots were in the bowl when he started?



Write down an equation to represent the scale:

Let x = total carrots.

Equation:

$x \div 4 = 2$

$x \div 4 = 2$

$(\times 4) \quad (\times 4)$

$x = 8$

or

$$\frac{1}{4}x = 2$$

$$4\left(\frac{1}{4}x\right) = 2(4)$$

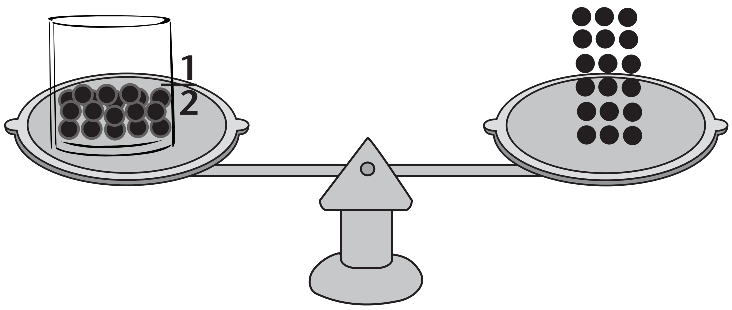
$$x = 8$$

There were 8 carrots in the bowl.

Exercises 5.6

1. Solve the following one-term equations involving division:

a.



How many counters fill one whole cup?

b. $\frac{1}{4}$ piece of paper = 42 circles of paper

How many paper circles can you make from 1 piece of paper?

2. Fill in the blanks.

a.
$$\frac{n}{3} = 12$$
$$(3)\frac{n}{3} = 12(3)$$
$$n = \_\_\_$$

b.
$$\frac{n}{4} = 72$$
$$(4)\frac{n}{4} = 72(4)$$
$$n = \_\_\_$$

c.
$$\frac{n}{5} = 35$$
$$(5)\frac{n}{5} = 35(\_\_\_)$$
$$n = \_\_\_$$

d.
$$\frac{n}{8} = 56$$
$$(8)\frac{n}{8} = 56(\_\_\_)$$
$$n = \_\_\_$$

e.
$$\frac{n}{6} = 36$$
$$(\_\_\_)\frac{n}{6} = 36(\_\_\_)$$
$$n = \_\_\_$$

3. Solve for the variable.

   a. $5r = 15$

   b. $7q = 42$

   c. $x \div 3 = 15$

   d. $y \div 2 = 14$

   e. $3a = 0$

Turn to the Answer Key at the end of the module to check your work.

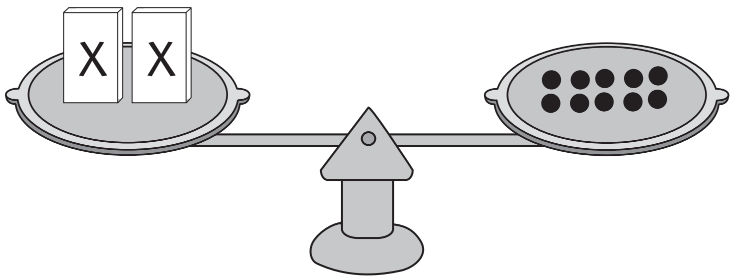
Check your Answers: Solve, Substitute, Smile!

In the last lesson, you learned how to solve, substitute, and smile when solving one-step equations (+ and –). You can use "Solve, Substitute, Smile" when you are solving one-step equations (× and ÷) too!

Remember to follow the steps from the last lesson when you check your solution: "Solve, Substitute, SMILE." The smile comes when you find out that your answer is correct.

- You *solve* the equation so that *x* is isolated on one side of the equation.
- You *substitute* the value of *x* into the original equation.
- Then you can *smile* if both sides of the equal sign are the same!

Example G



$$2x = 10$$
$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

Check your solution:

Sometimes, you can check your answer by inspection.

Use real objects to verify your results.

Using two cups and ten marbles, divide the marbles evenly between the cups. How many marbles do you have in each cup? You should have 5 marbles in each cup.

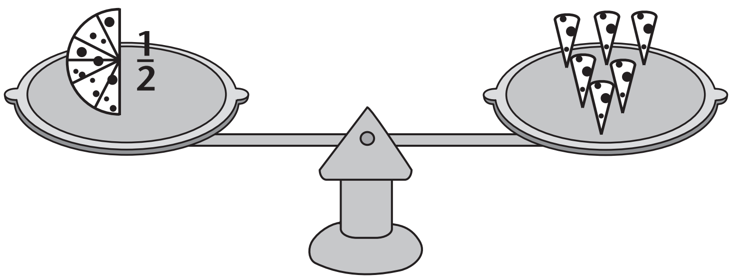
Otherwise, **substitute** the answer for the variable in the original equation.

$$\begin{aligned} 2x &= 10 \\ 2(5) &= 10 \\ 10 &= 10 \end{aligned}$$

Remember to answer the question.

The value of 1 block is equal to 5 counters.

Example H



How many pieces of pizza are there in a whole pizza?

Let x = 1 pizza.

$$\begin{aligned} \frac{1}{2}x &= 6 \\ (2)\frac{1}{2}x &= 6(2) \\ \frac{2x}{2} &= 12 \\ x &= 12 \end{aligned}$$

Check your solution:

Sometimes, you can check your answer by inspection:

Use real objects to verify your results.

Line up 6 blocks.

Line up another row of 6 below this row.

Count up how many blocks you have in total.

You should have 12.

Or, **substitute** the answer for the variable in the original equation.

$$\begin{aligned} \frac{1}{2}x &= 6 \\ \frac{1}{2}(12) &= 6 \\ 6 &= 6 \end{aligned}$$

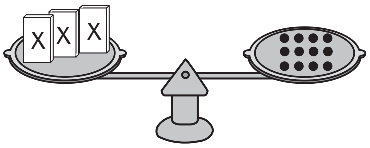
Answer the question:

There are 12 pieces in one whole pizza.

Exercises 5.7

1. Evaluate if $x = 4$
   a. $5x$
   b. $20 \div x$
   c. $x \div 4$
   d. $5x - 2x$
2. Check to see if $x = 3$ for these equations:
   a. $6x = 18$
   b. $21x = 64$
   c. $x \div 3 = 1$
   d. $54 \div x = 18$

3. How many counters are in each block?



a. Write down an equation to represent the scale.

b. Solve the equation.

c. Check your answer.

4. Fill in the blanks in the following equations and check your solution.

a.
$$4a = 32$$
$$\frac{4a}{4} = \frac{32}{4}$$
$$a = \_\_\_$$

b.
$$\frac{1}{3}a = 21$$
$$(3)\frac{1}{3}a = 21(3)$$
$$a = \_\_\_$$

c.
$$8x = 32$$
$$\frac{8x}{8} = \frac{32}{8}$$
$$a = \_\_\_$$

d.
$$\frac{1}{2}b = 6$$
$$(\_\_\_)\frac{1}{2}b = 6(\_\_\_)$$
$$a = \_\_\_$$

5. Evaluate if d = 4

a. $3d$

b. $8 \div d$

c. $21d$

d. $d \div 2$

Turn to the Answer Key at the end of the module to check your work.

Solving Two-Step Equations

Imagine you are weighing some apples and oranges at a grocery store. You place one apple on a scale. Forgetting to look at the scale to see how much one apple weighs, you add four more apples. Now, you add 8 oranges and the scale weighs 3 kg. You're curious to see how much just one apple weighs. What can you do to figure out how much just one apple weighs?

Well, you can remove the oranges that you added to the scale. Then you can divide the weight of five apples by five. Or, you could then remove four of the apples to leave one apple on the scale. You could then read the weight of just one apple on the scale.

In this lesson, you will learn how to solve two-step equations that involve more than one operation. It's just like adding and removing objects from a balance scale. Good luck!

You learned about terms and coefficients in previous lessons.

This expression has two terms:

$3x + 9$

The terms are $3x$ and 9.

The coefficient of x is 3.

Get ready for this lesson about solving two-step equations with these questions about terms and coefficients.

Exercises 5.8

1. Underline the term with the variable in it.

   Example: $\underline{3x} + 9$

   a. $2x + 7$
   b. $b - 3$
   c. $4 + 6q$
   d. $37 + 86t$
   e. $\frac{1}{2}y - 1$
   f. $7 + 2x$
   g. $-13 + 9q$

2. Here are those same expressions again. This time, give the coefficient of each variable.

   Example: $3x + 9$ Coefficient is $\underline{\ 3\ }$

   a. $2x + 7$
   b. $b - 3$
   c. $4 + 6q$
   d. $37 + 86t$
   e. $\frac{1}{2}y - 1$
   f. $7 + 2x$
   g. $-13 + 9q$

Turn to the Answer Key at the end of the module to check your work.

You have learned how to solve one-step equations involving addition, subtraction, multiplication and division operations.

You are now going to learn how to solve two-step equations involving both multiplication or division *and* addition or subtraction.

Some examples of two-step equations are shown below:

$2x + 7 = 9$

$3x - 2 = 7$

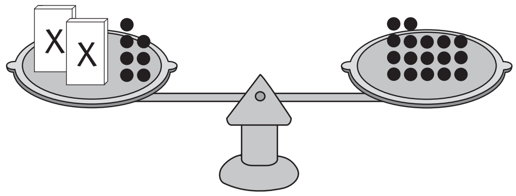
$\frac{1}{2}x + 3 = 8$

Why do you think these equations are called two-step equations?

There are two operations in each of these equations, so you solve these equations using two steps.

Let's find out how.

Solving Two-Step Equations Using a Balance Scale

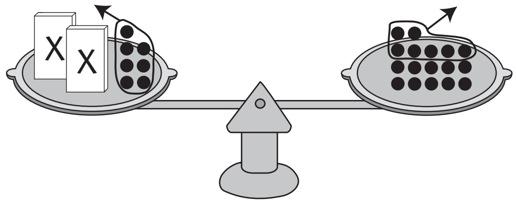


What do you think is the first step in figuring out the value of each block?

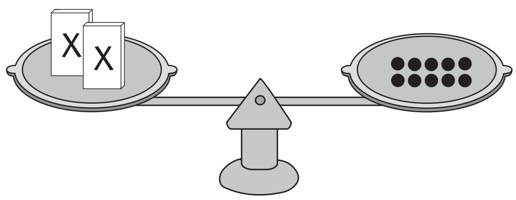
You want to isolate a single block on one side of the scale, so get rid of the counters on the left side first.

Step 1: Remove the 7 counters from the left side of the balance.

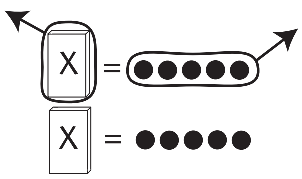
What you do to one side, you must do to the other. So, remove 7 counters from the right side.



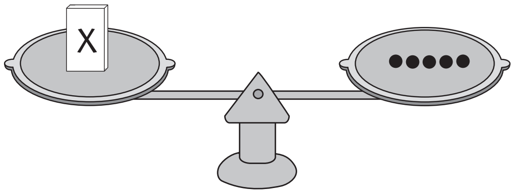
The balance now looks like this:



Step 2: Think about how many counters each block represents.



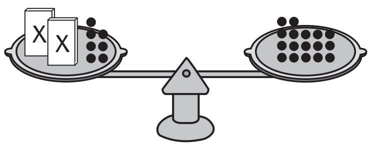
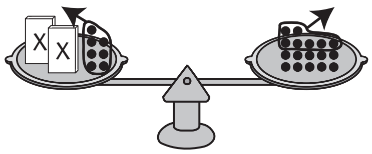
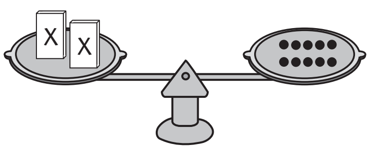
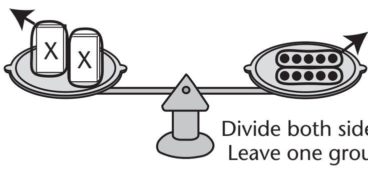
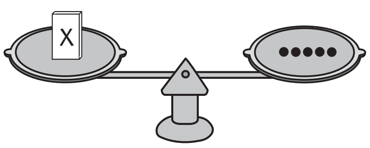
Since 2 blocks balance 10 counters, each block must represent 5 counters. Remember that you have to keep the scale balanced, but you want to isolate one block.



Step 3: Answer the question:

The value of 1 block is equal to 5 counters.

Let's see if we can write equations to go with the scale pictures in this example.

	$2x + 7 = 17$
	$2x + 7 = 17$ $(-7) \quad (-7)$
	$2x = 10$
 Divide both sides into 2 groups. Leave one group on each side.	$\frac{2x}{2} = \frac{10}{2}$
	$x = 5$

Exercises 5.9

1. Fill in the blanks using these words:

subtract	substituting
division	isolate
inverse	solution
divide	other
equation	quotient
product	

a. An algebraic _________ is a complete sentence that contains an equal sign.

b. To check if your answer is correct, you verify your ___________.

c. Addition and subtraction are ____________ operations.

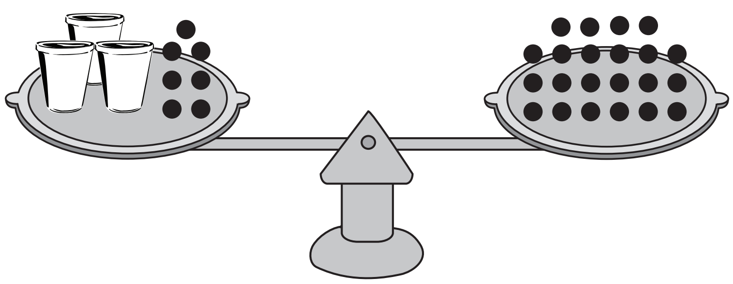
d. Multiplication and ____________ are inverse operations.

e. It is possible to verify your solution by ______________ the answer for the variable in the original equation.

f. What you do to one side of the equation, you must do to the ____________ side of the equation.

g. To solve an equation, you ______________ the variable on one side of the equation.

2. How many counters are in each cup?



Turn to the Answer Key at the end of the module to check your work.

Solving Two-Step Equations

The examples we looked at so far illustrated the process behind solving two-step equations. Linking pictures to the algebra is a useful strategy to approach equation-solving. Let's look a little closer at the algebra now.

Let's solve

$$3x + 4 = 10$$

First, isolate the term with the variable in it.

You've already practised finding this term.

$$3x + 4 = 10$$

In this equation, "3*x*" is the term with the variable in it. We need to isolate that term. We need to get that term by itself.

How can we get rid of "+ 4" ?

We need to subtract 4. Remember: What we do to one side of the equation, we must do to the other side.

$$\begin{aligned} 3x + 4 &= 10 \\ (-4) \quad & \quad (-4) \\ 3x &= 6 \end{aligned}$$

Ta da! The term with the variable is by itself.

Second, isolate the variable.

Do this by getting rid of the coefficient. You've already practised finding coefficients.

$$3x = 6$$

The coefficient of *x* is "3". It is telling us to multiply by 3.

How do we get rid of "multiply by 3"?

We need to divide by 3. What we do to one side, we must do to the other side.

$$\frac{3x}{3} = \frac{6}{3}$$
$$x = 2$$

Have we solved the equation? We need to check our answer to be sure.

Check by substitution:

$$3x + 4 = 10$$
$$3(2) + 4 = 10$$
$$6 + 4 = 10$$
$$10 = 10 \quad ☺$$

Try another example.

$$5x - 4 = 16$$

First, isolate the term with the variable in it.

The term with the variable is "5*x*". We need to get rid of "– 4."

$$5x - 4 = 16$$
$$(+4) \qquad (+4)$$
$$5x = 20$$

Second, isolate the variable.

The variable is "*x*". We need to get rid of the coefficient "5."

$$\frac{5x}{5} = \frac{20}{5}$$
$$x = 4$$

Now we check our answer.

$$5x - 4 = 16$$
$$5(4) - 4 = 16$$
$$20 - 4 = 16$$
$$16 = 16 \quad ☺$$

We'll do one more together.

$$\frac{x}{2} - 5 = 8$$

First, isolate the term with the variable in it.

The term with the variable is "$\frac{x}{2}$." We need to get rid of "– 5."

$$\frac{x}{2} - 5 = 8$$
$$(+5) \quad (+5)$$
$$\frac{x}{2} = 13$$

Second, isolate the variable.

In the equation, we are dividing *x* by 2. We get rid of that by doing the inverse. We need to multiply by 2.

$$(2)\frac{x}{2} = 13(2)$$
$$x = 26$$

Check the answer:

$$\frac{x}{2} - 5 = 8$$
$$\frac{26}{2} - 5 = 8$$
$$13 - 5 = 8$$

You're ready to do some questions on your own. If you need help, look back at these examples.

Exercises 5.10

Fill in the blanks to solve the following equations:

1. $2x - 5 = 7$

Solve:

Step 1: $2x - 5 = 7$

$+\_\_\_\_ \quad + \_\_\_\_\_$

$2x = \_\_\_\_\_\_$

Step 2: $2x = 12$

$\div\_\_\_ \quad \div\_\_\_$

$x = \_\_\_\_\_$

Check your answer.

$2x - 5 = 7$

$2(\_\_\_\_) - 5 = 7$

$\_\_\_ - 5 = 7$

$\_\_\_ = 7$

2. $4y + 6 = 18$

Solve:

Step 1:

$4y + 6 = 18$

$(- \_\_\_) \ (- \_\_\_)$

$4y = \_\_\_\_\_\_$

Step 2:

$4y = 12$

$\div\_\_\_ \quad \div\_\_\_$

$y = \_\_\_\_\_$

Check your answer.

3. $\frac{y}{4} - 2 = 14$

Solve: $\frac{y}{4} - 2 = 14$

$(+\_\_\_)(+\_\_\_)$

$\frac{y}{4} = \_\_\_$

$(\times 4)(\times 4)$

$y = \_\_\_$

Check your answer.

Substitute:

4. $\frac{x}{3} + 6 = 10$

Solve: $\frac{x}{3} + 6 = 10$

Check your answer.

Step 1: $\frac{x}{3} + 6 = 10$

$(-\_\_\_)(-\_\_\_)$

$\frac{x}{3} = \_\_\_$

Step 2: $\frac{x}{3} = \_\_\_$

$(3)\frac{x}{3} = \_\_\_(3)$

$x = \_\_\_$

Substitute.

Turn to the Answer Key at the end of the module to check your work.

Solving a Word Problem

You already have all the skills you need to solve word problems.

- Translate an English sentence into an algebraic equation.
- Solve an algebraic equation.

Now you need to put all of those skills together.

Example 1

The product of an unknown number and two, plus seven, is fifteen.

What is the number?

		Example
Step 1:	Highlight or underline information that is needed to solve the problem.	product of a number and two plus seven is fifteen
Step 2:	Let the unknown number be represented by a variable such as n	Let n = unknown variable.
Step 3:	Translate the word problem into an algebraic equation.	product of a number and two is written as $(2n)$ plus seven is written as $(+ 7)$ is fifteen is written as $(=15)$ The equation is: $2n + 7 = 15$.
Step 4:	Solve by isolating the n on one side of the equation.	$2n + 7 = 15$ $(-7) \quad (-7)$ $2n = 8$ $(\div 2) \quad (\div 2)$ $n = 4$

Step 5: Check your answer.	$2x + 7 = 15$ $2(4) + 7 = 15$ $8 + 7 = 15$ $15 = 15$ Smile!
Step 6: Answer the problem.	The unknown number is 15.

Example 2

Scott collects sea shells and rocks. He has 37 rocks in total. The number of rocks is five more than four times the number of sea shells. How many sea shells does Scott have in his collection?

		Example
Step 1:	Highlight or underline information that is needed to solve the problem.	37 rocks in total number of rocks (is) (5 more) than (4 times the number of sea shells) I want to find out how many sea shells are in Scott's collection.
Step 2:	Let the unknown number be represented by a variable such as n	Let x = the number of sea shells.
Step 3:	Translate the word problem into an algebraic equation.	I know: "Is" means "equal to." "Five more" means "+ 5." "Four times the number of seashells" means "4 × the number of seashells." $4x + 5 = 37$

Step 4:	Solve by isolating the *x* on one side of the equation.	$4x + 5 = 37$ $(-5) \quad (-5)$ $4x = 32$ $(\div 4) \quad (\div 4)$ $x = 8$
Step 5:	Check your answer.	$4x + 5 = 37$ $4(8) + 5 = 15$ $32 + 5 = 37$ $37 = 37$ Smile!
Step 6:	Answer the problem.	Scott has 8 sea shells in his collection.

Exercises 5.11

1. Solve the following word problems using these steps:

   Step 1: Write a let statement to name the variable that represents the unknown number.

   Step 2: Translate the word problem into an algebraic equation.

   Step 3: Solve for the variable by isolating a single variable on one side. Show your work.

   Step 4: Verify your solution.

   a. The sum of twice a number and eight is thirty. What is the number?

   b. The product of 5 and a number less 4 is 11. What is the number?

   c. A number divided by three plus 6 equals 10.

2. Solve for the variable and verify the solution.

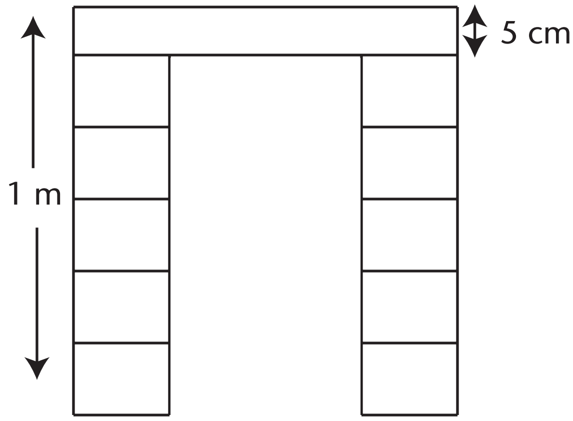
   a. $2x + 4 = 10$

   b. $3x - 5 = 4$

   c. $\frac{1}{2}x + 4 = 10$

   d. $\frac{1}{3}x - 2 = 4$

3. A platform 5 cm thick is supported by concrete blocks of equal thickness as shown. The height is 1m from the floor to the top of the platform. How thick is each concrete block?



4. Here's a challenge question for you. Try your best; if you get stuck. look at the solution in the answer key.

   Stephanie has a collection of leaves and pine cones. She has 46 pine cones in total. The number of pine cones is 5 less than 3 times the number of leaves. How many leaves does Stephanie have in her collection?

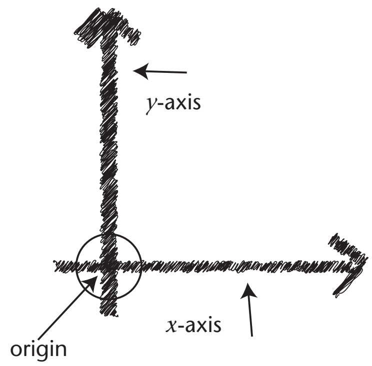
Turn to the Answer Key at the end of the module to check your work.

Answer Key

Lesson 1: Cartesian Plane

Exercises 1.1

1.

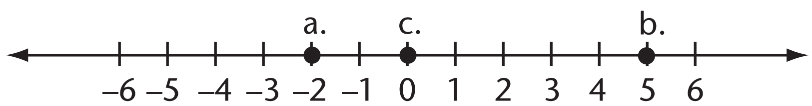


2. a. 3
   b. 4
   c. (3, 4)

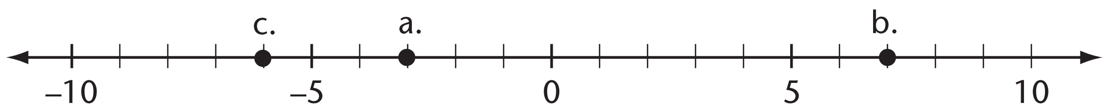
3. The Cartesian plane is an example of a **coordinate** system.

Exercises 1.2

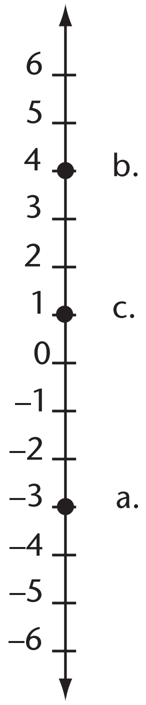
1.



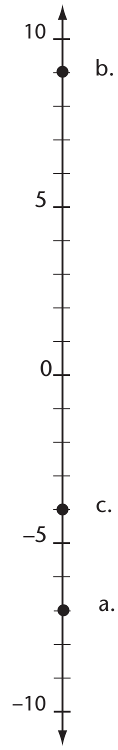
2.



3.

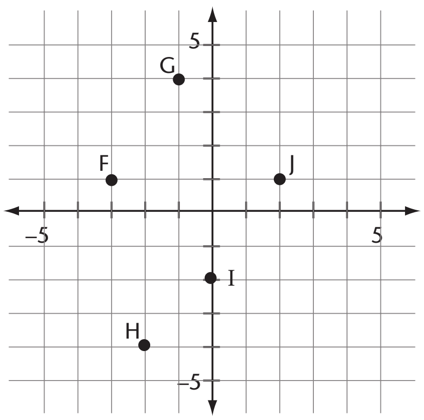


4.



5. A(3, 4)
   B (0, –2)
   C (1, –4)
   D (–2, 5)
   E (–1, –3)

6.



F (–3, 1)

G (–1, 4)

H (–2 , –4)

I (0, –2)

J (2, 1)

Exercises 1.3

1. A - Quadrant II
   B - Quadrant IV
   C - Quadrant I
   D - Quadrant III
   E - Quadrant IV
   F - Quadrant II

2. E (1, –1) and B (4, –2)

3. The x-coordinate of all the points in Quadrant IV is positive.
   The y-coordinate of all the points in Quadrant IV is negative.

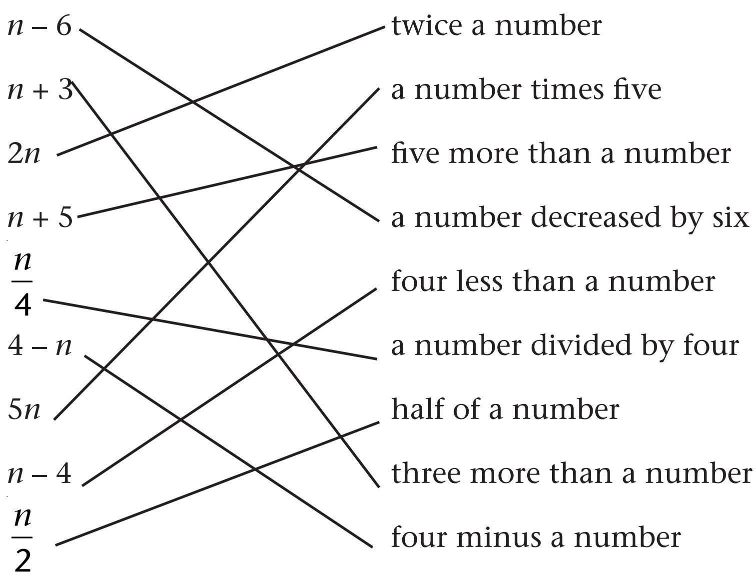
Lesson 2: Expressions

Exercises 2.1

1. a. 12
   b. 6
   c. 4
   d. –4

2. Darian walks 6 blocks farther than Teagan.

Exercises 2.2



Exercises 2.3

1.

	How many terms?	What are the variables?	List any coefficients.	Is there a constant term? What is it?
$m + 4$	2	**m**	**1**	4
$2x - 9$	**2**	**x**	**2**	**–9**
$-3s$	**1**	**s**	**–3**	**no constant term**
$5 + p$	**2**	**p**	**1**	**5**
$x + 3y + 7$	**3**	**x, y**	**1, 3**	**7**
$b + 2$	**2**	**b**	**1**	**2**
$4t$	**1**	**t**	**4**	**no constant term**

2. $7w + 5$ or $5 + 7w$

Exercises 2.4

1. a. Augustin's age is 6 less than Maggie's age. Use m as a variable meaning Maggie's age.
   Augustin's age is $m - 6$
   b. Maggie's age is 6 more than Augustin's age. Use a as a variable meaning Augustin's age.
   Maggie's age is $a + 6$.

2. Use p as a variable meaning the number of people in the theatre. Quyen can expect to sell $\frac{1}{2}p$ bags of popcorn.

3. a. $n - 4$
   b. $n + 3$
   c. $2n$
   d. $2n + 1$
   e. $n - 2$
   f. $\frac{n}{5}$
   g. $2n - 3$

4. Answers may vary but should resemble one of these options.
   a. three times a number
      triple a number

   b. five more than a number
      a number increased by five
      a number plus five

   c. two less than a number
      a number decreased by two

   d. a number divided by three
      one-third of a number

Exercises 2.5

1. a. 2
   b. x
   c. 2
   d. 5

2. a. $j + 3$
   b. $3j$
   c. $3j + 1$

3. a. 12
   b. 6
   c. –4
   d. 11
   e. 5
   f. 32
   g. 6
   h. 35
   i. 8
   j. 4
   k. 3
   l. 14
   m. 5
   n. 3
   o. –5
   p. 27
   q. 2
   r. 4
   s. 120
   t. –11

Exercises 2.6

1. a. $(4) - 7$
      $= -3$

   b. $3 + 2(4)$
      $= 3 + 8$
      $= 11$

   d. $3(4) - 4$
      $= 12 - 4$
      $= 8$

   e. $6(4)$
      $= 24$

c. $6 + (4)$
$= 10$

f. $9 - 5(4)$
$= 9 - 20$
$= -11$

2. a. $4 + 2(3)$
$= 4 + 6$
$= 10$

b. $4 + 2(5)$
$= 4 + 10$
$= 14$

c. $4 + 2(0)$
$= 4 + 0$
$= 4$

d. $4 + 2(2.3)$
$= 4 + 4.6$
$= 8.6$

e. $4 + 2\left(\frac{3}{4}\right)$
$= 4 + \frac{6}{4}$
$= 4 + \frac{3}{2}$
$= \frac{8}{2} + \frac{3}{2}$
$= \frac{11}{2}$
$= 5\frac{1}{2}$

3. a. $(3) + 2(1)$
$= 3 + 2$
$= 5$

b. $(-2) + (1) + 4$
$= -2 + 1 + 4$
$= -1 + 4$
$= 3$

c. $2(3) - 3(1)$
$= 6 - 3$
$= 3$

d. $-8(3) + 4(1) -3$
$= -24 + 4 - 3$
$= -20 - 3$
$= -23$

e. $(-2) + 5(1) + 6.8$
$= -2 + 5 + 6.8$
$= 3 + 6.8$
$= 9.8$

4. a. $3(3.4) - 2(4)$
   $= 10.2 - 8$
   $= 2.2$

   b. $3(7) - 2(5.6)$
   $= 21 - 11.2$
   $= 9.8$

   c. $3(1.4) - 2(2.1)$
   $= 4.2 - 4.2$
   $= 0$

5. $2.50(5) + 1.25(3)$
   $= 7.50 + 3.75$
   $= 11.25$

   Five hotdogs and three drinks cost $11.25

Lesson 3: Describing Relationships

Exercises 3.1

Answers will vary. Some examples of correct answers are:

1. Subtract 4 from each x-value to get the y-value. Each x is 4 more than y. $y = x - 4$.
2. It costs $5 to attend the dance. Each snack cost $1.
3. Multiply r by –3 to get t. $t = -3r$

Exercises 3.2

1. This graph touches the y–axis at <u>–4</u>

   Moving from left to right: <u>For every one that x goes to the right, y goes up one</u>

x	–2	0	2	4
y	–6	–4	–2	0

   Describe the relationship from the table of values: <u>For every one that x increases, y increases by one. Or, y is 4 less than x.</u>

2. This graph touches the y–axis at <u>0</u>

   Moving from left to right:
   <u>For every one that x goes to the right, y goes down by 4</u>

x	–1	0	1	2
y	4	0	–4	–8

   Describe the relationship from the table of values: <u>Whenever x goes up by 1, y goes down by 4. Or, y is –4 times x.</u>

3. This graph touches the y–axis at <u>5</u>

   Moving from left to right: <u>For every one that x goes to the right, y goes up by 2</u>

x	–2	–1	0	1
y	1	3	5	7

   Describe the relationship from the table of values: <u>Whenever x goes up by 1, y goes up by 2. Or, y is 5 more than twice x.</u>

Lesson 4: Algebra, Part 1

Exercises 4.1

1. The equation is: 7 = 3 + 4
2. The equation is: 8 + 4 = 12
3. There are a variety of methods.

   Did you try adding some numbers to 13 until you got 25?

   For example, 13 + 10 = 23, 13 + 11 = 24, 13 + 12 = 25, so the value is 12.

   Or did you subtract 13 from 25 to get 12 (25 – 13 = 12)?

   The value of the block is 12.

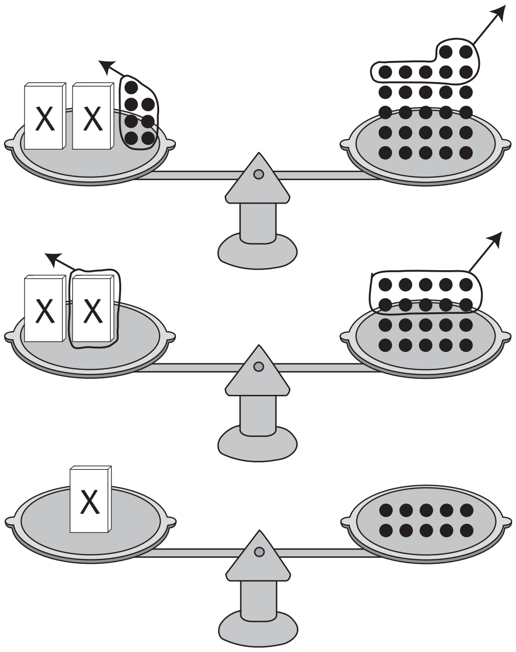
   25 = 13 + 12

Exercises 4.2

1. a. Equal
   b. Equal
   c. Not Equal
   d. Equal
   e. Equal
   f. Not Equal
   g. Equal
   h. Equal
   I. Not Equal

2. a. 6 counters
   b. 7 counters need to be removed from the left hand side.

   1 block equals 10 counters



   c. 30
   d. 5

3. Answers will vary—some examples might be:
   6 + 6 = 12
   2(6) = 12
   14 – 2 = 12

Exercises 4.3

a. expression
b. equation
c. expression
d. expression
e. equation
f. equation
g. equation
h. expression

Exercises 4.4

WORD	OPERATION	SYMBOL	EXAMPLE	LET n = NUMBER
a. plus	add	+	a number plus five	$n + 5$
b. minus	subtract	–	a number minus three	$n - 3$
c. more than	add	+	six more than a number	$n + 6$
d. less than	subtract	–	two less than a number	$n - 2$
e. increased by	add	+	a number increased by four	$n + 4$
f. decreased by	subtract	–	a number decreased by one	$n - 1$
g. the sum of	add	+	the sum of a number and seven	$n + 7$
h. the difference between	subtract	–	the difference between a number and one	$n - 1$
i. the product of	multiply	×	the product of a number and four	$4n$ or $4 \times n$
j. the quotient of	divide	÷	the quotient of a number and five	$\frac{n}{5}$ or $n \div 5$
k. twice	multiply by 2	× 2	twice a number	$2n$ or $2 \times n$
l. triple	multiply by 3	× 3	triple the sum of a number and four	$3(n + 4)$
m. half	multiply by ½ or divide by 2	× ½ or ÷ 2	half a number	$\frac{n}{2}$ or $\frac{1}{2} \times n$
n. of	multiply	×	one-third of a number	$\frac{n}{3}$ or $\frac{1}{3} \times n$

Exercises 4.5

(For 1–4, remember, there is more than one way to translate an equation into words, as long as both sides remain equal.)

1. a. Four plus a number is 6.
   b. Three less than a number equals nine, or a number minus three is nine.
   c. Two plus a number is seven.
   d. Three times a number is 12.
   e. The product of 5 and a number plus one equals 37.

2. b. $6 + x = 11$
   c. $n - 7 = 5$
   h. $5x = 35$
   g. $w + 2 = 8$
   d. $\frac{1}{2}h = 12$
   f. $4 + x = 16$
   a. $4(2x) = 16$
   e. $2(x + 3) = 18$

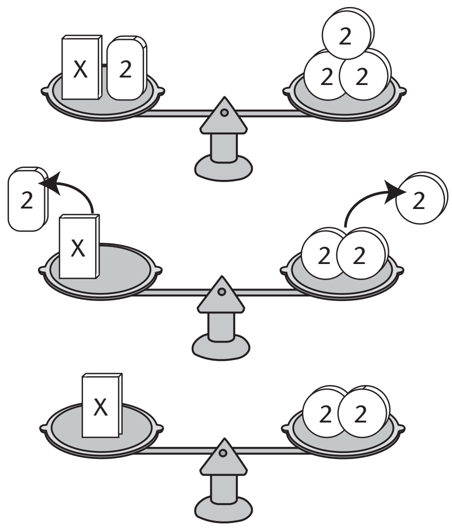
3. b. A number decreased by five is three.
   a. A number added to seven equals twelve.
   g. A number divided by 4 is 2.
   c. A number times 3 equals 0.
   f. The product of 8 and a number is the same as six plus two.
   e. Five multiplied by the product of 2 and a number equals 40.
   d. Three times the difference of a number and ten is 15.

Lesson 5: Algebra, Part 2

Exercises 5.1

1. a. $x + 3 = 21$
   b. $12 = 2 + x$

2. Scales / Equation



$x + 2 = 6$

$x + 2 = 6$
$(-2) \quad (-2)$

$x = 4$

Exercises 5.2

1. $c - 3 = 7$
   $(+3) \quad (+3)$
   $c = 10$

2. $x + 3 = 11$
   $(-3) \quad (-3)$
   $x = 8$

3. $p - 3 = 10$
   $(+3) \quad (+3)$
   $p = 13$

4. $m + 14 = 21$
   $(-14) \quad (-14)$
   $x = 7$

Exercises 5.3

1. a. 10
   b. 5
   c. 28

2. a. yes
   b. no
   c. no

3. a.
$$\begin{aligned} 3+x&=8 \\ 3+x&=8 \\ (-3)\quad &(-3) \\ x&=5 \end{aligned}$$

b.
$$\begin{aligned} q-11&=18 \\ q-11&=18 \\ (+11)\quad &(+11) \\ q&=29 \end{aligned}$$

c.
$$\begin{aligned} x-10&=2 \\ x-10&=2 \\ (+10)\quad &(+10) \\ x&=12 \end{aligned}$$

d.
$$\begin{aligned} 3+x&=19 \\ 3+x&=19 \\ (-3)\quad &(-3) \\ x&=16 \end{aligned}$$

e.
$$\begin{aligned} w-25&=50 \\ w-25&=50 \\ (+25)\quad &(+25) \\ w&=75 \end{aligned}$$

f.
$$\begin{aligned} a-8&=8 \\ a-8&=8 \\ (+8)\quad &(+8) \\ a&=16 \end{aligned}$$

g.
$$\begin{aligned} x+15&=27 \\ x+15&=27 \\ (-15)\quad &(-15) \\ x&=12 \end{aligned}$$

h.
$$\begin{aligned} x-47&=62 \\ x-47&=62 \\ (+47)\quad &(+47) \\ x&=109 \end{aligned}$$

Exercises 5.4

1. a.

$$\begin{aligned} x-6&=8 \\ x-6&=8 \\ (+6)\quad &(+6) \\ x&=14 \end{aligned}$$

Check:

$$\begin{aligned} x-6&=8 \\ (14)-6&=8 \\ 8&=8 \end{aligned}$$

b.

$$\begin{aligned} x+3&=12 \\ x+3&=12 \\ (-3)\quad &(-3) \\ x&=9 \end{aligned}$$

Check:

$$\begin{aligned} x+3&=12 \\ (9)+3&=12 \\ 12&=12 \end{aligned}$$

c.

$$4+x=25$$
$$4+x=25$$
$$(-4)\quad(-4)$$
$$x=21$$

Check:

$$4+x=25$$
$$4+(21)=25$$
$$25=25$$

d.

$$x+11=19$$
$$x+11=19$$
$$(-11)\quad(-11)$$
$$x=8$$

Check:

$$x+11=19$$
$$(8)+11=19$$
$$19=19$$

2.

$$x-7=31$$
$$x-7=31$$
$$(+7)\quad(+7)$$
$$x=38$$

Check:

$$x-7=31$$
$$(38)-7=31$$
$$31=31$$

Alex started with 38 hockey cards.

Exercises 5.5

1. a. 3
   b. one block = 13

2. a.
$$3x=24$$
$$3x=24$$
$$(\div 3)\quad(\div 3)$$
$$x=8$$

   b.
$$2x=14$$
$$2x=14$$
$$(\div 2)\quad(\div 2)$$
$$x=7$$

   c.
$$4y=160$$
$$4y=160$$
$$(\div 4)\quad(\div 4)$$
$$y=40$$

   d.
$$5x=50$$
$$5x=50$$
$$(\div 5)\quad(\div 5)$$
$$x=10$$

Exercises 5.6

1. a. 36 counters fill one cup
   b. 168 circles

2. a.
$$\frac{n}{3}=12$$
$$(3)\frac{n}{3}=12(3)$$
$$n=\mathbf{36}$$

b.
$$\frac{n}{4}=72$$
$$(4)\frac{n}{4}=72(4)$$
$$n=\mathbf{288}$$

c.
$$\frac{n}{5}=35$$
$$(5)\frac{n}{5}=35(5)$$
$$n=\mathbf{175}$$

d.
$$\frac{n}{8}=56$$
$$(8)\frac{n}{8}=56(8)$$
$$n=\mathbf{448}$$

e.
$$\frac{n}{6}=36$$
$$(6)\frac{n}{6}=36(6)$$
$$n=\mathbf{216}$$

3. a. 3
   b. 6
   c. 45
   d. 28
   e. 0

Exercises 5.7

1. a. 20
   b. 5
   c. 1
   d. 20 – 8 = 12

2. a. yes
   b. no
   c. yes
   d. yes

3.
$$3x=12$$
$$\frac{3x}{3}=\frac{12}{3}$$
$$x=4$$

4. a.
$$4a=32$$
$$\frac{4a}{4}=\frac{32}{4}$$
$$a=\underline{8}$$

b.
$$\frac{1}{3}a=21$$
$$(3)\frac{1}{3}a=21(3)$$
$$a=\underline{63}$$

c.
$$8x = 32$$
$$\frac{8x}{8} = \frac{32}{8}$$
$$x = \underline{4}$$

d.
$$\frac{1}{2}b = 6$$
$$(\underline{2})\frac{1}{2}b = 6(\underline{2})$$
$$b = \underline{12}$$

5. a. 12
   b. 2
   c. 84
   d. 2

Exercises 5.8

1. a. $\underline{2x} + 7$
   b. $\underline{b} - 3$
   c. $4 + \underline{6q}$
   d. $37 + \underline{86t}$
   e. $\frac{1}{2}y - 1$
   f. $7 + \underline{2x}$
   g. $-13 + \underline{9q}$

2. a. $\mathbf{2}x + 7$
   b. $b - 3$ **coefficient = 1**
   c. $4 + \mathbf{6}q$
   d. $37 + \mathbf{86}t$
   e. $\frac{\mathbf{1}}{\mathbf{2}}y - 1$
   f. $7 + \mathbf{2}x$
   g. $-13 + \mathbf{9}q$

Exercises 5.9

1. a. equation
   b. solution
   c. inverse
   d. division
   e. substituting
   f. other
   g. isolate

2. Let x = 1 cup

Equation:

$$3x + 7 = 22$$
$$3x + 7 = 22$$
$$(-7)\ (-7)$$
$$3x = 15$$
$$(\div 3)\ (\div 3)$$
$$x = 5$$

Verify:

$$3x + 7 = 22$$
$$3(5) + 7 = 22$$
$$15 + 7 = 22$$
$$22 = 22$$

Smile!!

Exercises 5.10

1. $2x - 5 = 7$

Solve:

Step 1:
$$\begin{aligned} 2x - 5 &= 7 \\ +5 \quad &\ \ +5 \\ 2x &= 12 \end{aligned}$$

Step 2:
$$\begin{aligned} 2x &= 12 \\ \div 2 \quad &\ \ \div 2 \\ x &= 6 \end{aligned}$$

Check your answer:
$$\begin{aligned} 2x - 5 &= 7 \\ 2(6) - 5 &= 7 \\ 12 - 5 &= 7 \\ 7 &= 7 \end{aligned}$$

2. $4y + 6 = 18$

Solve:

Step 1:
$$\begin{aligned} 4y + 6 &= 18 \\ (-6) \quad &\ \ (-6) \\ 4y &= 12 \end{aligned}$$

Step 2:
$$\begin{aligned} 4y \div 4 &= 12 \div 4 \\ y &= 3 \end{aligned}$$

Check your answer:
$$\begin{aligned} 4y + 6 &= 18 \\ 4(3) + 6 &= 18 \\ 12 + 6 &= 18 \\ 18 &= 18 \end{aligned}$$

3. $y \div 4 - 2 = 14$

Solve:
$$\begin{aligned} \frac{y}{4} - 2 &= 14 \\ (+2) \quad &\ \ (+2) \\ \frac{y}{4} &= 16 \\ \frac{y}{4} &= 16 \\ (\times 4) \quad &\ \ (\times 4) \\ y &= 64 \end{aligned}$$

Check your answer:
$$\begin{aligned} \left(\frac{64}{4}\right) - 2 &= 14 \\ \frac{64}{4} - 2 &= 14 \\ 16 - 2 &= 14 \\ 14 &= 14 \end{aligned}$$

4. $\frac{x}{3} + 6 = 10$

Solve:

$$\frac{x}{3} + 6 = 10$$

Step 1:

$$\frac{x}{3} + 6 = 10$$
$$(-6) \quad (-6)$$
$$\frac{x}{3} = 4$$

Step 2:

$$\frac{x}{3} = 4$$
$$(\times 3) \quad (\times 3)$$
$$x = 12$$

Substitute:

$$\frac{x}{3} + 6 = 10$$
$$\frac{(12)}{3} + 6 = 10$$
$$4 + 6 = 10$$
$$10 = 10$$

Exercises 5.11

1. a.

$$\begin{aligned} 2n + 8 &= 30 \\ 2n + 8 &= 30 \\ (-8) \quad & (-8) \\ 2n &= 22 \\ 2n &= 22 \\ (\div 2) \quad & (\div 2) \\ n &= 11 \end{aligned}$$

Verify:

$$\begin{aligned} 2n + 8 &= 30 \\ 2(11) + 8 &= 30 \\ 22 + 8 &= 30 \\ 30 &= 30 \end{aligned}$$

b.

$$\begin{aligned} 5n - 4 &= 11 \\ 5n - 4 &= 11 \\ (+4) \quad & (+4) \\ 5n &= 15 \\ 5n &= 15 \\ (\div 5) \quad & (\div 5) \\ n &= 3 \end{aligned}$$

Verify:

$$\begin{aligned} 5n - 4 &= 11 \\ 5(3) - 4 &= 11 \\ 15 - 4 &= 11 \\ 11 &= 11 \end{aligned}$$

c.

$$\begin{aligned} \frac{n}{3} + 6 &= 10 \\ \frac{n}{3} + 6 &= 10 \\ (-6) \quad & (-6) \\ \frac{n}{3} &= 4 \\ \frac{n}{3} &= 4 \\ (\times 3) \quad & (\times 3) \\ n &= 12 \end{aligned}$$

Verify:

$$\begin{aligned} \frac{n}{3} + 6 &= 10 \\ \frac{12}{3} + 6 &= 10 \\ 4 + 6 &= 10 \\ 10 &= 10 \end{aligned}$$

2. a.

$$2x + 4 = 10$$
$$2x + 4 = 10$$
$$(-4) \quad (-4)$$
$$2x = 6$$
$$2x = 6$$
$$(\div 2) \quad (\div 2)$$
$$x = 3$$

Verify:

$$2x + 4 = 10$$
$$2(3) + 4 = 10$$
$$6 + 4 = 10$$
$$10 = 10$$

b.

$$3x - 5 = 4$$
$$3x - 5 = 4$$
$$(+5) \quad (+5)$$
$$3x = 9$$
$$3x = 9$$
$$(\div 3) \quad (\div 3)$$
$$x = 3$$

Verify:

$$3x - 5 = 4$$
$$3(3) - 5 = 4$$
$$9 - 5 = 4$$
$$4 = 4$$

c.

$$\frac{1}{2}x + 4 = 10$$
$$\frac{1}{2}x + 4 = 10$$
$$(-4) \quad (-4)$$
$$\frac{1}{2}x = 6$$
$$\frac{1}{2}x = 6$$
$$(\text{x}2) \quad (\text{x}2)$$
$$x = 12$$

Verify:

$$\frac{1}{2}x + 4 = 10$$
$$\frac{1}{2}(12) + 4 = 10$$
$$6 + 4 = 10$$
$$10 = 10$$

d.

$$\frac{1}{3}x - 2 = 4$$
$$\frac{1}{3}x - 2 = 4$$
$$(+2) \quad (+2)$$
$$\frac{1}{3}x = 6$$
$$\frac{1}{3}x = 6$$
$$(\text{x}3) \quad (\text{x}3)$$
$$x = 18$$

Verify:

$$\frac{1}{3}x - 2 = 4$$
$$\frac{1}{3}(18) - 2 = 4$$
$$6 - 2 = 4$$
$$4 = 4$$

3. $5x + 5 = 100$

$$5x + 5 = 100$$
$$(-5) \ (-5)$$
$$5x = 95$$

$$5x = 95$$
$$(\div 5)(\div 5)$$
$$x = 19$$

Each block is 19 cm in height.

4. Let x = number of leaves

$$3x - 5 = 46$$

$$3x - 5 = 46$$
$$(+5) \ (+5)$$
$$3x = 51$$

$$3x = 51$$
$$(\div 3)(\div 3)$$
$$x = 17$$

Check:

$$3x - 5 = 46$$
$$3(17) - 5 = 46$$
$$51 - 5 = 46$$
$$46 = 46$$

Stephanie has 17 leaves in her collection.