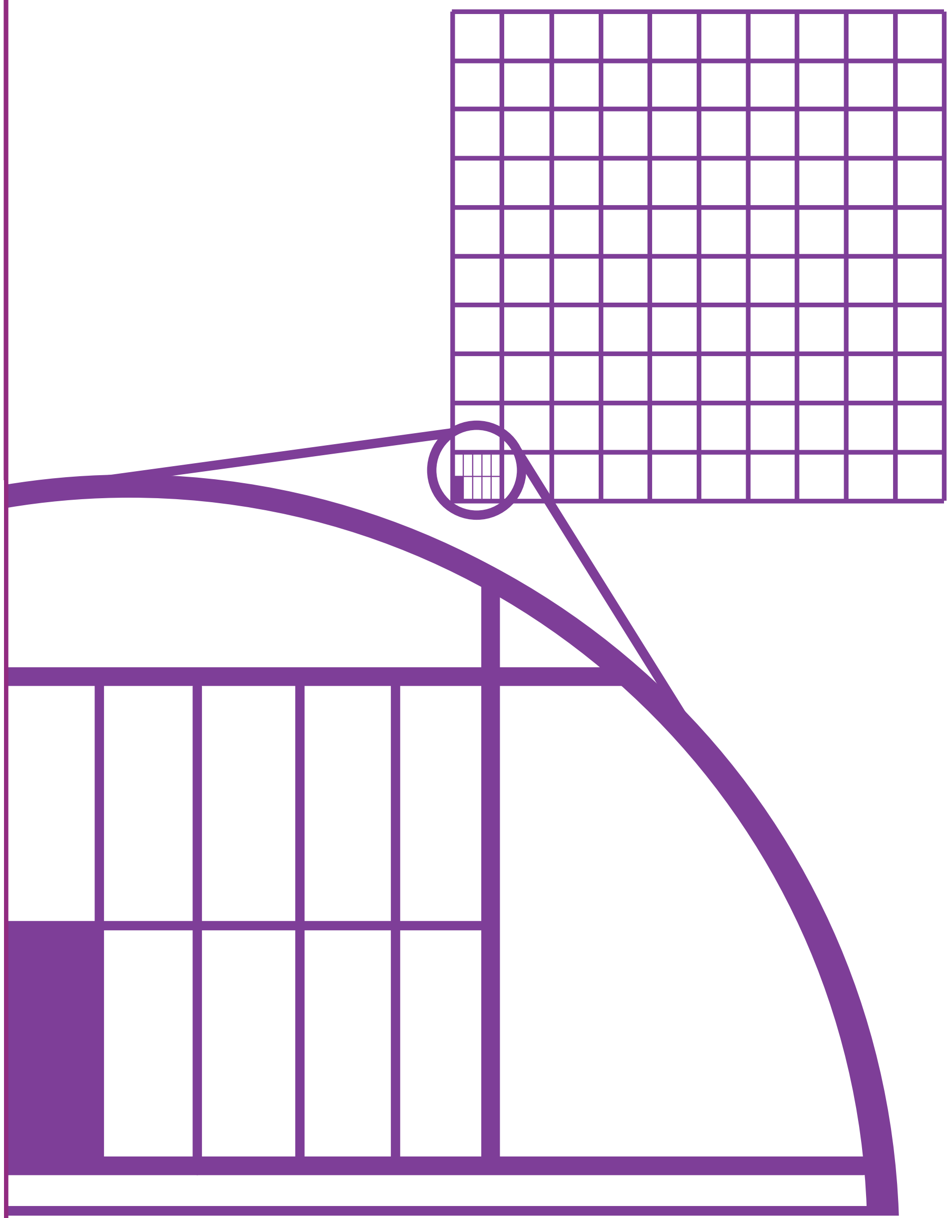
LITERACY FOUNDATIONS MATH



LEVEL 5
NUMBER SENSE

To the Student

This resource covers topics from the British Columbia Ministry of Education's Literacy Foundations Math Level 5. You may find this resource useful if you're a Literacy Foundations Math student, or a K-12 student in grades 7 – 9.

We have provided learning material, exercises, and answers for the exercises, which are located at the back of each set of related lessons. We hope you find it helpful.

Literacy Foundations Math Prescribed Learning Outcomes

The Literacy Foundations Math Prescribed Learning Outcomes (PLOs) are grouped into four areas: Number (A), Patterns and Relations (B), Shape and Space (C), and Statistics and Probability (D). For a complete list of the PLOs in Level 5, search for Literacy Foundations Math curriculum on the BC Ministry of Education's website.

PLOs Represented in This Resource

The PLOs represented in this Level 5 resource are as follows:

Number
All topics, A1 – A12

Patterns and Relations
All topics, B1 – B6

Shape and Space
All topics, C1 – C3

Statistics and Probability
D2

PLOs Not Represented in This Resource

The PLOs for which no material is included in this resource are as follows:

Statistics and Probability
There is no material for D1, line graphs from data sets.

Acknowledgements and Copyright

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Table of Contents

Lesson 1

Fractions

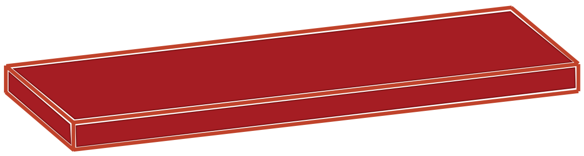
Learning Outcomes

By the end of this section you will be better able to:

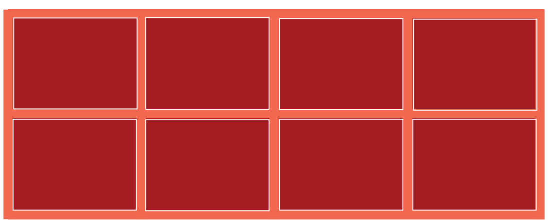
- convert between improper fractions and mixed numbers
- multiply and divide with proper fractions and mixed numbers
- convert between standard fractions and decimal fractions
- compare fractions, decimals, and whole numbers

Vocabulary of Mixed Numbers and Improper Fractions

Let's begin with a review of fractions. We can think of any object as being "whole." For example, the picture below shows one whole chocolate bar.



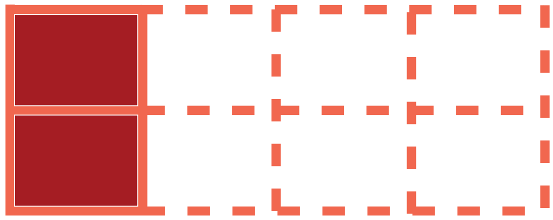
We can also think of it as being one whole object made of any number of equal parts. Think of dividing the chocolate bar into equal pieces to share with your friends. Before you share the chocolate bar, you divide it into 8 pieces. You still have one whole chocolate bar, because you still have 8 out of 8 pieces. In this case the whole object can be viewed as a fraction that equals "1".



$$1 = \frac{8}{8}$$

If you were to share with three of your friends, you might give each person 2 pieces, leaving only 2 pieces for yourself.

$$\frac{2}{8} = \frac{\textbf{numerator}}{\textbf{denominator}} = \frac{\text{number of equal parts I have}}{\text{total number of equal parts}}$$



In a **proper fraction** the numerator is smaller than the denominator.

Example:

$$\frac{2}{3}$$

In an **improper fraction**, the numerator is bigger than the denominator.

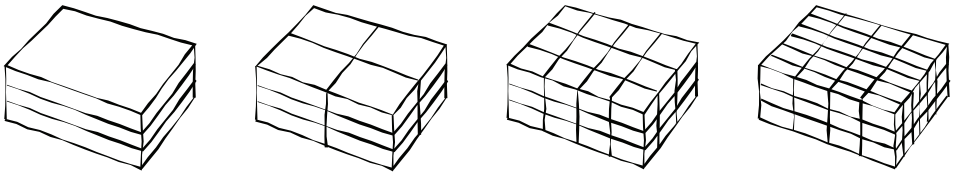
Example:

$$\frac{5}{3}$$

Fractions that have the same value or represent the same ratio are called **equivalent fractions**.

Example:

$$\frac{3}{4} = \frac{9}{12} = \frac{18}{24}$$



A **mixed number** is made of a whole number and a fraction. It is really a short hand notation for the addition of a whole number and a fraction.

Example:

$$2\frac{1}{3} = 2 + \frac{1}{3}$$

Exercises 1.1

Match the term on the left to the example on the right.

______ 1. denominator

______ 2. improper fraction

______ 3. mixed number

______ 4. numerator

______ 5. proper fraction

a. the 3 in $\frac{2}{3}$

b. $\frac{4}{7}$

c. $2\frac{1}{5}$

d. $\frac{9}{4}$

e. the 2 in $\frac{2}{5}$

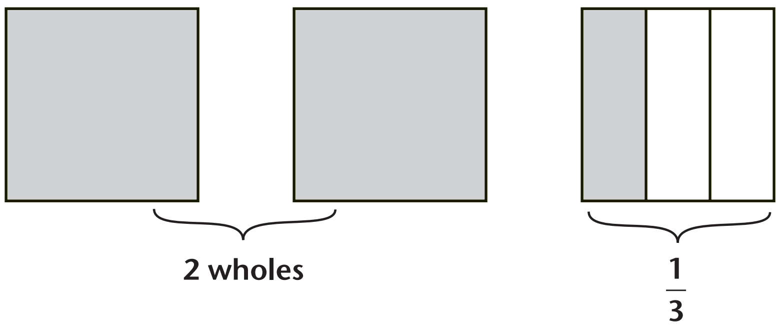
Turn to the Answer Key at the end of the module to check your work.

Proper and Improper Fractions and Mixed Numbers

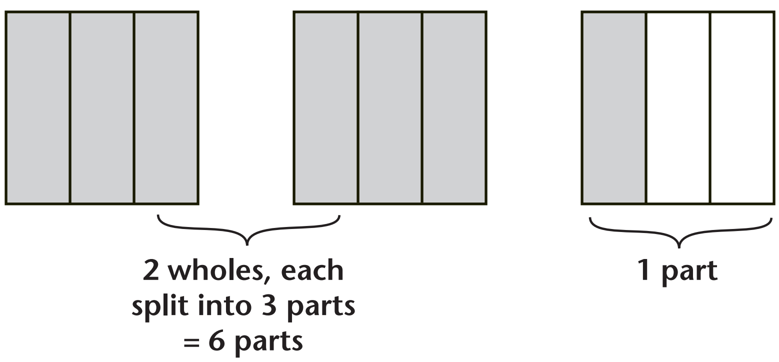
When we're working with fractions, we often see mixed numbers. It can be difficult to perform operations (addition, subtraction, multiplication and division) on mixed numbers. In order to work with fractions, we have to know how to convert from a mixed number to an improper fraction.

Let's work through an example. We'll change $2\frac{1}{3}$ to an improper fraction.

First let's draw a picture of the mixed number.



We can divide up the wholes into the same number of parts as the fraction: 3 parts for each whole.



Now, we can find the total number of parts from the picture. One way is just to count them. Another way is to do the following operation:

$$\begin{aligned}&(2 \text{ wholes} \times 3 \text{ parts in each}) + 1 \text{ part}\\ &= 2 \times 3 + 1\\ &= 6 + 1\\ &= 7\end{aligned}$$

So, we have seven parts. Remember that the “parts” are actually thirds. So we have seven thirds or $\frac{7}{3}$.

$$2\frac{1}{3} = \frac{7}{3}$$

The Steps

For any mixed number, you can follow these steps to convert to an improper fraction.

Step 1: Multiply the whole number by the denominator of the fraction.

Step 2: Add the numerator of the fraction to this product.

Step 3: Put the sum on top of the denominator.

Let’s look at one more example.

Write $6\frac{3}{5}$ as an improper fraction.

Step 1: Multiply the whole number by the denominator of the fraction.

$$6\frac{3}{5}$$

$6 \times 5 = 30$

Step 2: Add the numerator of the fraction to this product.

$$6\frac{3}{5}$$

$30 + 3 = 33$

Step 3: Put the sum on top of the denominator.

$$\frac{33}{5}$$

So, $6\frac{3}{5} = \frac{33}{5}$.

Exercises 1.2

1. Convert the following mixed numbers into improper fractions using pictures for each.

   a. $4\frac{2}{3}$

   b. $5\frac{1}{6}$

   c. $2\frac{3}{7}$

2. Convert the following mixed numbers into improper fractions.

   a. $8\frac{1}{5}$

   b. $6\frac{2}{9}$

   c. $3\frac{1}{4}$

Turn to the Answer Key at the end of the module to check your work.

Converting from an Improper Fraction to a Mixed Number

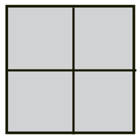
After you perform operations on fractions, you will often be asked to express your answer as a proper fraction or a mixed number. If your answer is an improper fraction, you will have to convert it to a mixed number.

Lets try an example. We'll change the improper fraction $\frac{9}{4}$ to a mixed number.

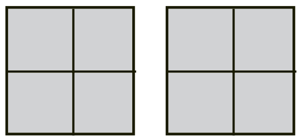
Let's try drawing a picture.

First of all, we know that our "wholes" have been cut into quarters. We'll draw quarters until we have 9 of them.

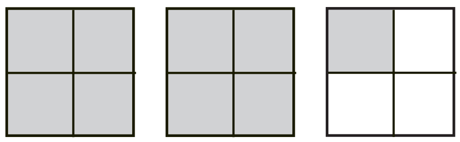
Here are 4 quarters:



Here are 8 quarters:



Now we only need one more quarter to make 9. We'll draw another square, but shade in only one part.



You can see from the drawing that we have two wholes and one quarter, so our mixed number is $2\frac{1}{4}$.

$$\frac{9}{4} = 2\frac{1}{4}$$

The Steps

For any improper fraction, you can follow these steps to convert to a mixed number.

Step 1: Divide the numerator by the denominator. Make note of the whole number and the remainder.

Step 2: The whole number from the division becomes the whole number in your mixed number.

Step 3: The remainder becomes the numerator of the fraction in your mixed number.

Step 4: The denominator of this fraction is the same as the denominator of the improper fraction you started with.

Let's work through an example. We'll change $\frac{13}{5}$ to a mixed number.

Step 1: Divide the numerator by the denominator. Make note of the whole number and the remainder.

$$\begin{array}{r} 2 \\ 5\overline{)13} \\ -10 \\ \hline 3 \end{array}$$

Step 2: The whole number from the division becomes the whole number in your mixed number.

$$\begin{array}{r} 2 \text{ whole} \\ 5\overline{)13} \\ -10 \\ \hline 3 \end{array} \quad \rightarrow 2\frac{\square}{\square}$$

Step 3: The remainder becomes the numerator of the fraction in your mixed number.

$$\begin{array}{r} 2 \text{ whole} \\ 5\overline{)13} \\ -10 \\ \hline 3 \text{ remainder} \end{array} \quad \rightarrow 2\frac{3}{\square}$$

Step 4: The denominator of this fraction is the same as the denominator of the improper fraction you started with.

$$\begin{array}{r} 2 \text{ whole} \\ 5\overline{)13} \\ -10 \\ \hline 3 \text{ remainder} \end{array} \quad \rightarrow 2\frac{3}{5}$$

The improper fraction we started with was $\frac{13}{5}$, so our denominator is 5.

So $\frac{13}{5} = 2\frac{3}{5}$.

Exercises 1.3

Convert the following improper fractions to mixed numbers and reduce when necessary:

1. $\frac{53}{8}$

2. $\frac{35}{5}$

3. $\frac{42}{12}$

4. $\frac{54}{13}$

5. $\frac{20}{6}$

Turn to the Answer Key at the end of the module to check your work.

Multiplying Fractions

When we multiply fractions, we can use the following rule:

$$\frac{\text{numerator A}}{\text{denominator A}} \times \frac{\text{numerator B}}{\text{denominator B}} = \frac{\text{numerator A} \times \text{numerator B}}{\text{denominator A} \times \text{denominator B}}$$

Let's see this rule in action. We'll work through three multiplication problems together.

Example 1:

$$\frac{3}{4} \times \frac{1}{3}$$

Multiply the two numerators, and multiply the two denominators.

$$= \frac{3 \times 1}{4 \times 3}$$
$$= \frac{3}{12}$$

Don't forget to reduce your answer to lowest terms.

$$= \frac{3 \div 3}{12 \div 3}$$
$$= \frac{1}{4}$$

Example 2:

$$\frac{1}{4} \times \frac{3}{5}$$

Multiply the two numerators, and multiply the two denominators.

$$= \frac{1 \times 3}{4 \times 5}$$
$$= \frac{3}{20}$$

This fraction cannot be reduced any further, so this is your final answer.

Example 3:

$$\frac{11}{4} \times \frac{2}{5}$$

Multiply the two numerators, and multiply the two denominators.

$$= \frac{11 \times \cancel{2}^{1}}{\cancel{4}_{2} \times 5}$$

$$= \frac{11}{10}$$

Simplify by dividing 2 and 4 by a common factor of 2.

This is an improper fraction. You should always leave your answers as proper fractions or mixed numbers. If we convert this to a mixed number we get:

$$1\frac{1}{10}$$

Now it's your turn to try multiplying fractions.

Exercises 1.4

Complete the following multiplication problems. Your answers should be left as either *proper fractions* or *mixed numbers*, and should be *reduced* to lowest terms.

1. $\frac{2}{3} \times \frac{4}{9}$

2. $\frac{1}{5} \times \frac{3}{4}$

3. $\frac{4}{5} \times \frac{6}{13}$

4. $\frac{2}{7} \times \frac{1}{5}$

5. $\frac{8}{3} \times \frac{5}{7}$

Turn to the Answer Key at the end of the module to check your work.

Multiplying a Fraction by a Whole Number or a Mixed Number

Now that we know how to multiply two proper fractions, what happens when we have a fraction multiplied by a whole number? What about a fraction multiplied by a mixed number? Let's work through some examples.

Multiplying by a Whole Number

Example 1

$$\frac{2}{3} \times 4$$

Remember, we can write any whole number as an improper fraction. 4 is the same as $\frac{4}{1}$. Let's rewrite the expression, and solve the problem.

$$\frac{2}{3} \times 4$$

$$= \frac{2}{3} \times \frac{4}{1}$$

Now, this looks familiar! We can use the multiplication rule for fractions.

$$= \frac{2 \times 4}{3 \times 1}$$

$$= \frac{8}{3}$$

We shouldn't leave the answer as an improper fraction. Convert to a mixed number.

$$= 2\frac{2}{3}$$

Example 2

$$8 \times \frac{3}{5}$$

We can write the whole number as an improper fraction. 8 is the same as $\frac{8}{1}$. Let's rewrite the expression, and solve the problem.

$$\begin{aligned} & 8 \times \frac{3}{5} \\ &= \frac{8}{1} \times \frac{3}{5} \\ &= \frac{8 \times 3}{1 \times 5} \\ &= \frac{24}{5} \\ &= 4\frac{4}{5} \end{aligned}$$

Multiplying by a Mixed Number

Example 1

$$1\frac{1}{5} \times \frac{3}{4}$$

Remember: we can convert any mixed number to an improper fraction. $1\frac{1}{5}$ is the same as $\frac{6}{5}$. Let's rewrite the expression, and solve the problem.

$$\begin{aligned} & 1\frac{1}{5} \times \frac{3}{4} \\ &= \frac{6}{5} \times \frac{3}{4} \\ &= \frac{\cancel{6}^{3} \times 3}{5 \times \cancel{4}_{2}} \\ &= \frac{9}{10} \end{aligned}$$

Now, this looks familiar! We can use the multiplication rule for fractions.

Simplifying before multiplying means we don't have to reduce our answer. It also makes the multiplication easier!

Example 2

$$\frac{6}{11} \times 4\frac{1}{2}$$

We can write the mixed number as an improper fraction. $4\frac{1}{2}$ is the same as $\frac{9}{2}$. Let's rewrite the expression, and solve the problem.

$$\frac{6}{11} \times 4\frac{1}{2}$$
$$= \frac{6}{11} \times \frac{9}{2}$$
$$= \frac{\cancel{6}^{3} \times 9}{11 \times \cancel{2}_{1}}$$
$$= \frac{27}{11}$$
$$= 2\frac{5}{11}$$

Convert this improper fraction into a mixed number.

Now it's time for you to try. Look back at the examples at any time if you need a hint.

Exercises 1.5

Complete the following multiplication problems. Your answers should be left as either proper fractions or mixed numbers, and should be reduced to lowest terms.

1. $\frac{3}{7} \times 3$

2. $\frac{4}{5} \times 3\frac{3}{4}$

3. $\frac{9}{10} \times 8$

4. $\frac{8}{5} \times 7$

5. $2\frac{1}{4} \times \frac{6}{11}$

6. $2\frac{1}{2} \times 1\frac{1}{9}$

7. $\frac{4}{5} \times 6$

Turn to the Answer Key at the end of the module to check your work.

Dividing Fractions

The rule for dividing fractions is: multiply the first fraction by the **reciprocal** of the second fraction.

You're probably wondering, "What is a reciprocal?" A reciprocal is a number that you multiply a fraction by so that the result equals one. The easiest way to find it is to just flip the fraction over.

Here are two examples:

What is the reciprocal of $\frac{4}{5}$?

We flip the fraction to find that the reciprocal is $\frac{5}{4}$

What is the reciprocal of 3?

3 is the same as $\frac{3}{1}$, so we flip and the reciprocal is $\frac{1}{3}$.

In general, the rule for division of fractions looks like this:

$$\begin{aligned} &\frac{a}{A} \div \frac{b}{B} \\ &= \frac{a}{A} \times \frac{B}{b} \\ &= \frac{a \times B}{A \times b} \end{aligned}$$

Now that we have a rule for division, let's work through an example so you know how to use it.

$$\frac{1}{4} \div 2 = ?$$

Following the rule, we'll rewrite the first fraction, and then multiply it by the reciprocal of the second.

$$\frac{1}{4} \div 2$$
$$= \frac{1}{4} \div \frac{2}{1}$$
$$= \frac{1}{4} \times \frac{1}{2}$$
$$= \frac{1 \times 1}{4 \times 2}$$
$$= \frac{1}{8}$$

2 is the same as $\frac{2}{1}$. Flip it to get the reciprocal.

Now you have a multiplication problem—we have already done lots of these!

It may seem a bit strange to be changing operations and flipping fractions. To help you understand this process, think about two people sharing a quarter of a pizza. Sam and his sister were sharing a quarter of a pizza. We can think of this scenario in two ways:

We must split the leftover pizza into two parts.	**OR**	Each person gets half of the leftover pizza.

We can translate these sentences into mathematical language. Remember, there was a quarter of the pizza left in the fridge for Sam and his sister to share equally.

the leftover pizza divided by 2	**OR**	half of the leftover pizza
$\frac{1}{4} \div 2$		$\frac{1}{2} \times \frac{1}{4}$

With a little shuffling around, you'll find our division rule!

$$\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2}$$

Remember, you can multiply two numbers in any order. $\frac{1}{4} \times \frac{1}{2}$ is the same as $\frac{1}{2} \times \frac{1}{4}$

Let's try another example using the division rule.

$$\frac{2}{3} \div \frac{3}{4} = ?$$

Rewrite the first fraction, change the operation to multiplication, and flip the second fraction.

$$\begin{aligned} &\frac{2}{3} \div \frac{3}{4} \\ &= \frac{2}{3} \times \frac{4}{3} \\ &= \frac{2 \times 4}{3 \times 3} \\ &= \frac{8}{9} \end{aligned}$$

The answer is a proper fraction, and can't be reduced, so we can leave it as it is.

Mixed Numbers

Let's look at an example where one of the fractions is a mixed number.

$$\frac{5}{6} \div 1\frac{2}{3} = ?$$

By the division rule, we know we need to find the reciprocal of the second fraction. Before we can do this, we need to convert the mixed number to an improper fraction.

$$\begin{aligned} &\frac{5}{6} \div 1\frac{2}{3} \\ &= \frac{5}{6} \div \frac{5}{3} \end{aligned}$$

Now you can flip the second fraction. The reciprocal of $\frac{5}{3}$ is $\frac{3}{5}$.

$$\begin{aligned} &= \frac{5}{6} \times \frac{3}{5} \\ &= \frac{\cancel{5}^1 \times \cancel{3}^1}{\cancel{6}_2 \times \cancel{5}_1} \end{aligned}$$

Simplify before you multiply.

$$= \frac{1}{2}$$

Exercises 1.6

1. Write the reciprocal of each of the following numbers.

   a. $\frac{2}{3}$

   b. $\frac{1}{4}$

   c. 7

   d. $2\frac{4}{5}$

2. Complete the following division problems. Your final answers should be either proper fractions or mixed numbers, and should be reduced to lowest terms.

   a. $\frac{7}{5} \div 3$

   b. $\frac{5}{6} \div 4$

   c. $\frac{1}{2} \div \frac{2}{3}$

   d. $\frac{1}{5} \div 9$

   e. $5\frac{1}{5} \div \frac{1}{5}$

   f. $\frac{1}{4} \div 2$

   g. $\frac{1}{5} \div \frac{1}{4}$

   h. $\frac{2}{5} \div \frac{2}{3}$

   i. $\frac{11}{9} \div \frac{1}{5}$

   j. $7 \div 2\frac{3}{4}$

Turn to the Answer Key at the end of the module to check your work.

Decimal Fractions

Terminating and Repeating Decimals

Terminating means ending. A **terminating decimal** has an end.

Here are some examples:

$\frac{1}{2} = 0.5$

$\frac{2}{10} = 0.2$

$\frac{223}{100} = 2.23$

Repeating decimals have a pattern that keeps repeating (they never end).

Here are some examples:

$\frac{1}{3} = 0.33333333333...$

The 3 repeats, so we write $\frac{1}{3} = 0.\dot{3}$.

A dot on top of a digit means that digit repeats.

$\frac{9}{7} = 1.285714285714285714....$

The 285714 repeats, so we write $\frac{9}{7} = 1.\overline{285714}$.

$\frac{5}{99} = 0.0505050505....$

The 05 repeats, so we write $\frac{5}{99} = 0.\overline{05}$.

A line on top of digits means those digits repeat.

Exercises 1.7

Classify each decimal as terminating or repeating by writing "terminating" or "repeating" to the right of the decimal.

1. 34.3728 ________________

2. 0.83838383838... ________________

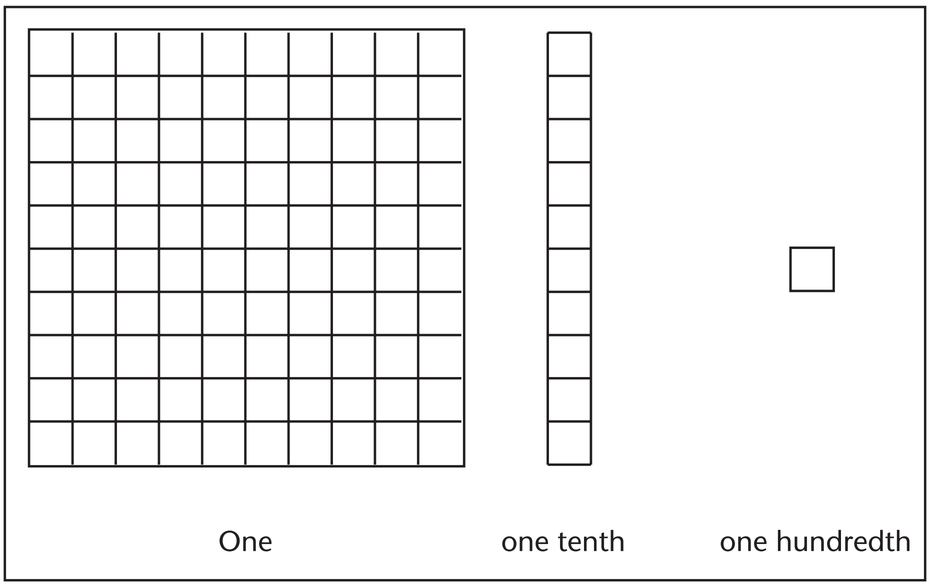
3. 0.943 ________________

4. 0.55555555... ________________

Turn to the Answer Key at the end of the module to check your work.

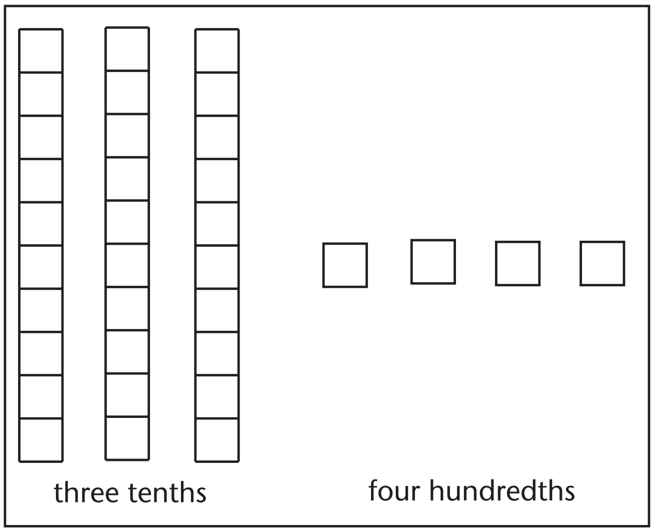
A Review of Decimal Fractions

Let's have a look at a hundredths block to help us better understand decimals and fractions.



You can see that the big square represents one whole. It has been divided into 100 small squares. Each of these small squares represents one hundredth. Ten of these small squares together represent one tenth.

If we had three tenths and four hundredths it would look like this:



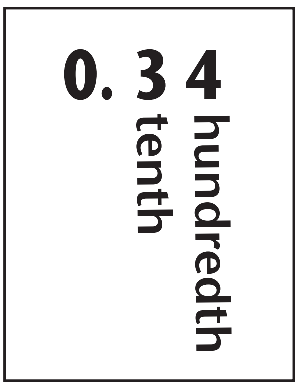
Let's write this as a fraction:

Three tenths = $\frac{3}{10}$

Four hundredths = $\frac{4}{100}$

So three tenths and four hundredths = $\frac{3}{10} + \frac{4}{100} = \frac{30}{100} + \frac{4}{100} = \frac{34}{100}$

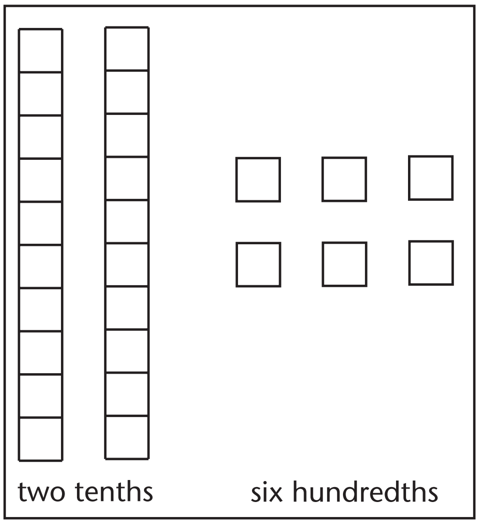
We can also write this as a decimal:



So we can see that $\frac{34}{100} = 0.34$.

Let's try another question.

Write 2 tenths and 6 hundredths as a fraction and into a decimal.



Fraction:

$$\frac{2}{10} + \frac{6}{100} = \frac{20}{100} + \frac{6}{100} = \frac{26}{100}$$

Decimal:

0. 2 6

tenth

hundredth

So $\frac{26}{100} = 0.26$

Let's try one last example.

Write 9 tenths, 2 hundredths, and 8 thousandths as a fraction and as a decimal.

Fraction:

$$\frac{9}{10}+\frac{2}{100}+\frac{8}{1000}$$
$$=\frac{900}{1000}+\frac{20}{1000}+\frac{8}{1000}$$
$$=\frac{928}{1000}$$

Decimal:

0.928

tenth

hundredth

thousandth

So $\frac{928}{1000}=0.982$.

Exercises 1.8

Write the following as a fraction and a decimal.

	Fraction	Decimal
1. 4 tenths, 9 hundredths		
2. 7 tenths, 3 hundredths, 5 thousandths		
3. 6 tenths, 4 hundredths, 2 thousandths		
4. 3 tenths, 2 hundredths, 1 thousandth		

Turn to the Answer Key at the end of the module to check your work.

Writing Fractions as Decimal Fractions

A **power** is a number that has been multiplied by itself. Examples of powers of 10 are 10, 100, 1000, and 10 000. You will learn more about powers in another lesson.

You might have noticed that a terminating decimal can be written as a fraction with a denominator that is a power of 10. The number of place values in the decimal tells us how many 0's are needed in the denominator of the fraction.

Look at the examples below and notice the pattern:

$$0.3 = \frac{3}{10}$$

$$0.36 = \frac{36}{100}$$

$$0.361 = \frac{361}{1000}$$

Just like when we were working with fractions before, we often want to simplify our final answer. This means reducing fractions to their lowest terms. Can we simplify any of the fraction answers above? Let's check.

$$0.3 = \frac{3}{10}$$

3 and 10 don't have any common factors. This fraction is already in lowest terms.

$$0.36 = \frac{36}{100}$$

36 and 100 can both be divided by 4—that means we can reduce this fraction.

$$\frac{36}{100} = \frac{(36 \div 4)}{(100 \div 4)} = \frac{9}{25}$$

9 and 25 don't have any common factors, so this fraction is simplified.

$$0.361 = \frac{361}{1000}$$

Now our numbers are getting bigger. Do 361 and 1000 have any common factors? The only factors of 1000 are 2 and 5. All multiples of 2 are even numbers, so 2 is not a factor of 361. All multiples of 5 end in 0 or 5, so 5 is not a factor of 361. This fraction is already in lowest terms.

So what happens when you see fractions like $\frac{1}{2}$ and $\frac{7}{25}$? They can be written as terminating decimals, but they don't have denominators that are powers of 10. We can rewrite them as equivalent fractions with denominators that are powers of 10.

Let's try it.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$$

$$\frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 0.28$$

Here's another example. Let's convert $\frac{1}{4}$ into a fraction with a denominator of 100 to find the equivalent decimal.

$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100}$ Since $\frac{25}{100} = 0.25$ we know that $\frac{1}{4} = 0.25$.

Exercises 1.9

1. Rewrite each of the following as a fraction with the denominator as a power of 10. Then write the fraction as a decimal. The first one is done for you.

Reduced Fraction	Equivalent Fraction With a Power of 10 as Its Denominator	Decimal
$\frac{3}{5}$	$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$	0.6
a. $\frac{2}{5}$		
b. $\frac{16}{25}$		
c. $\frac{7}{20}$		
d. $\frac{123}{500}$		

Turn to the Answer Key at the end of the module to check your work.

Writing Repeating Decimals and Fractions

Remember: to write fractions as decimals, divide the numerator by the denominator.

Example

$$\frac{1}{3} = 1 \div 3 = 0.3333.... = 0.\overline{3}$$

Now, look at the pattern below. Without using a calculator, can you write $\frac{4}{9}$ as a decimal?

$$\frac{1}{9} = 0.11111....$$
$$\frac{2}{9} = 0.2222.....$$
$$\frac{3}{9} = 0.3333.....$$
$$\frac{4}{9} =$$

Write a rule for converting repeating decimals like 0.777777.... into a fraction.

What happens if the denominator is 99? Let's find out.

$$\frac{1}{99} = 0.0101010101....$$
$$\frac{2}{99} = 0.02020202.....$$
$$\frac{3}{99} = 0.03030303.....$$
$$\frac{15}{99} = 0.15151515.....$$
$$\frac{83}{99} = 0.8383838.....$$
$$\frac{65}{99} = 0.65656565...$$

Have you figured out the pattern? How do you write a repeating decimal as a fraction?

Here are the steps:

	Example
1. Find out which digits repeat.	In 0.123123123123...., 123 repeats.
2. Write the repeating part as the numerator.	$\frac{123}{?}$
3. Count how many digits repeat; this will tell you how many 9s will be needed in the denominator.	123 repeats (this is three digits,) so 999 will be the denominator $\frac{123}{999}$
4. Be sure to reduce your fraction.	$\frac{123}{999} = \frac{123 \div 3}{999 \div 3} = \frac{41}{333}$

Can you think of an example where we would want to round our decimal answer?

A store is selling 3 apples for $0.98. You only want one apple. How much will the apple cost?

$$\frac{\$0.98}{3} = \$0.3266666...$$

You need to round your answer to the nearest penny.

One apple would cost $0.33.

Make sure you always round your answer to two decimal places when working with money.

Exercises 1.10

1. Without using a calculator, write each of these fractions as a repeating decimal.

   a. $\frac{8}{99} =$

   b. $\frac{35}{99} =$

   c. $\frac{15}{99} =$

   d. $\frac{84}{99} =$

2. Write a rule for converting repeating decimals where two digits repeat like 0.67676767.... into a fraction.

3. Convert the following repeating decimals to fractions. Make sure your fractions are reduced.

   a. 0.5555...

   b. 0.343434...

   c. 0.789789789....

   d. 0.24682468...

   e. 0.015015015.....

4. Let's convert fractions into decimals. This time our decimal number will

represent money. Think of each fraction as a division question, and use your calculator to find the decimal nuber.

Fraction	Decimal Not Rounded	Rounded
$\frac{10}{33}$	10 ÷ 33 =0.30303030...	**$0.30**
a. $\frac{16}{25}$		$
b. $\frac{80}{120}$		$
c. $\frac{17}{200}$		$
d. $\frac{125}{300}$		$

Turn to the Answer Key at the end of the module to check your work.

Comparing Fractions and Decimals

Remember that equivalent means “the same.” Equivalent fractions have equal value, but have different numerators and denominators.

Equivalent fractions can be found by either:

- multiplying the numerator AND denominator of a fraction by the same number

OR

- dividing the numerator AND denominator of a fraction by a common factor

In this example, we’ll multiply the top and bottom of the fraction by 5.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

Exercises 1.11

1. What would the equivalent fraction of $\frac{1}{2}$ be if you multiplied by 4 instead?

2. Match each of the ten fractions in the top row with an equivalent fraction in the bottom row. The first one is done for you.

$$\frac{1}{10}\quad\frac{2}{10}\quad\frac{3}{10}\quad\frac{4}{10}\quad\frac{5}{10}\quad\frac{6}{10}\quad\frac{7}{10}\quad\frac{8}{10}\quad\frac{9}{10}\quad\frac{10}{10}$$

$$\frac{9}{15}\quad\frac{24}{80}\quad\frac{18}{20}\quad\frac{35}{50}\quad\frac{7}{7}\quad\frac{7}{70}\quad\frac{1}{5}\quad\frac{2}{5}\quad\frac{1}{2}\quad\frac{4}{5}$$

Turn to the Answer Key at the end of the module to check your work.

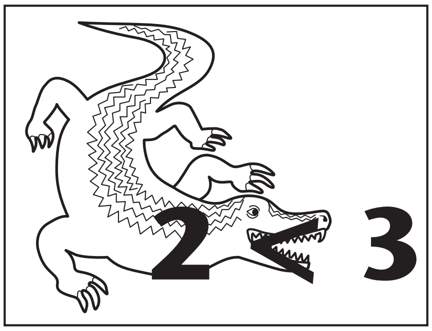
Inequalities

We will be comparing numbers in this lesson, so let's review the inequality signs.

One way to remember which way the inequality goes is to think of the sign as a crocodile and the number as an amount of fish.

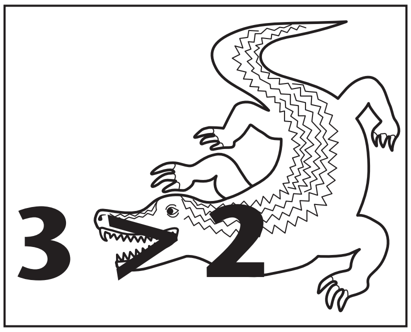
< means less than

For example, 2 < 3.



> means greater than

For example, 3 > 2.



If you were a hungry crocodile, would you want to eat 5 fish or 10 fish?

You would want to eat 10 fish. So 5 < 10

Another way to look at < is think of it as an L. The L stands for "less than." So we know 5 < 10 means 5 is less than 10.

When comparing decimal numbers that are less than 1, make sure you compare digits from left to right. Compare the tenths values; if they are the same, then compare the hundredths, and then continue moving until a digit is different.

Don't be fooled. Just because a number is longer does not mean that it is larger. For instance, 0.1000001 is longer than 0.1001 but its value is less.

Compare 0.5 and 0.4. 5 is greater than 4. So, 0.5 > 0.4

Compare 0.54 and 0.5. There is a 5 in both tenths places, so let's look at the hundredths. 4 is bigger than nothing (0), so 0.54 > 0.5.

Try some inequalities. Write the inequality for each pair of numbers below. The first one is done for you.

a. 4 < 9

b. 9 __ 8

c. 21 __ 86

d. 12 __ 65

e. 12 __ 43

f. 0.5 __ 0.8

g. 0.12 __ 0.36

h. 0.123 __ 0.2

i. 0.49 __ 0.4

j. 0.55 __ 0.551

Compare your answers with the ones below.

Answers

a. <
b. >
c. <
d. <
e. <
f. <
g. <
h. <
i. >
j. <

Before comparing fractions and decimals, all of the numbers need to be in the same form.

- convert all the numbers to decimals

OR

- convert all the numbers to fractions

And then compare them.

Comparing Decimals

Compare 0.36 and $\frac{23}{25}$.

First, let's try changing everything into a decimal.

To compare 0.36 and $\frac{23}{25}$ we can change $\frac{23}{25}$ into a decimal.

Method 1: using a common factor:

$$\frac{23}{25} = \frac{23 \times 4}{25 \times 4} = \frac{92}{100} = 0.92$$

or

Method 2: dividing the numerator by the denominator:

$$\frac{23}{25} = 23 \div 25 = 0.92$$

Which is smaller? 0.36 or 0.92? Since 0.36 is smaller than 0.92, we know $0.36 < \frac{23}{25}$.

Comparing Fractions

Compare 0.36 and $\frac{23}{25}$.

This time let's convert everything into a fraction.

$0.36 = \frac{36}{100}$. To compare these two fractions we need both of them over the same denominator. 100 is the lowest common denominator of $\frac{36}{100}$ and $\frac{23}{25}$. So let's change $\frac{23}{25}$ into its equivalent fraction of $\frac{92}{100}$.

Which is smaller? Since $\frac{36}{100}$ is smaller than $\frac{92}{100}$ we know $0.36 < \frac{23}{25}$.

Exercises 1.12

Now it's your turn.

1. Convert all of the fractions into decimals. Decide which number is the smallest, then write an inequality.

a. 1.34, $\frac{27}{20}$

$\frac{27}{20} = 27 \div 20 = 1.35$

$1.34 < \frac{20}{27}$

b. 0.78, $\frac{3}{5}$

c. 0.96, $\frac{19}{20}$

d. 1.27, $\frac{38}{25}$

e. 3.45, $\frac{162}{50}$

2. Convert all of the decimals into fractions. Decide which number is the smallest, then write an inequality.

   a. $0.82, \frac{4}{5}$ $0.82 = \frac{82}{100}$ $\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100}$ $\frac{82}{100} > \frac{80}{100}$ $0.82 > \frac{4}{5}$

   b. $0.2, \frac{6}{25}$

   c. $0.4, \frac{7}{20}$

   d. $0.78, \frac{37}{50}$

   e. $0.93, \frac{19}{20}$

3. Which method did you like using best? Did one method take longer than another method?

Turn to the Answer Key at the end of the module to check your work.

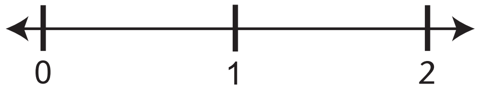
Placing Fractions and Decimals on a Number Line

Now that we know how to compare fractions and decimals we can place them on the number line. We can convert all of the numbers into decimals, or into fractions with common denominators. In this lesson we'll convert everything into decimals to compare and place on a number line. We'll use this method since it often takes longer to change everything into common fractions with the same denominators. If you prefer another method, you can use it in your own work.

Example

Place $\frac{6}{5}$, 0.92, and 1 on the number line

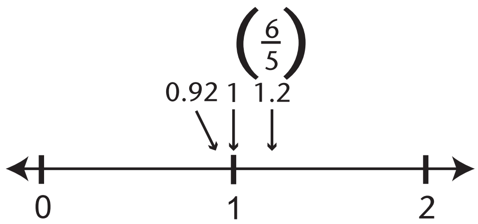
Let's compare the numbers on a number line like the one below.



We can convert $\frac{6}{5}$ into a decimal.

$$\frac{6}{5} = 6 \div 5 = 1.2$$

Now place 1.2, 0.92, and 1 on the number line.



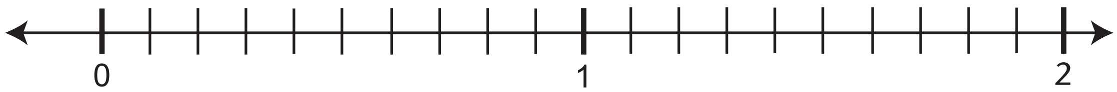
Now that the numbers are on the number line, you can compare them easily:

- 0.92 is furthest left, so it's the smallest number in the set.
- 1.2 (or $\frac{6}{5}$) is furthest right, so it's the largest number in the set.

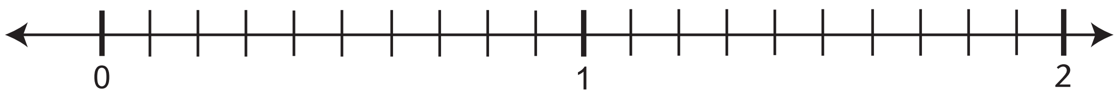
Exercises 1.13

Convert these lists of numbers into decimals, and then place them on the number line.

1. 1.3, 0.2, 1.5, $\frac{3}{4}$, 1, $1\frac{4}{5}$



2. 1.75, 0.4, $\frac{3}{5}$, 0.9, $\frac{7}{10}$, $\frac{1}{2}$



Turn to the Answer Key at the end of the module to check your work.

Lesson 2

Ratios and Rates

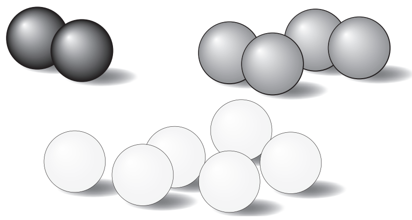
Learning Outcomes

By the end of this lesson you will be better able to:

- use ratios to find unit rates
- apply ratios and rates to solving problems

Ratios

Look at the picture below.



We can see from this picture that:

- There are 2 black marbles.
- There are 4 grey marbles.
- There are 6 white marbles.
- There are 12 marbles all together.

Using this information, we can make some comparisons. For example, we can compare:

- the number of grey marbles to the number of white marbles
- the number of black marbles to the number of grey marbles to the number of white marbles
- the number of white marbles to the total number of marbles

In math, we can describe these comparisons using ratios. A **ratio** is a comparison of two or more numbers. We can write each comparison listed above as a ratio by separating the numbers with a colon.

the number of grey marbles to the number of white marbles	4:6
the number of black marbles to the number of grey marbles to the number of white marbles	2:4:6
the number of white marbles to the total number of marbles	6:12

Notice that the order in which the numbers appear is very important. Write the numbers in the ratio in the same order that they are listed in the words.

grey to white

4 : 6

Each number in a ratio is called a *term*. The ratio 4:6 is a *two-term* ratio because it contains two terms, 4 and 6. The ratio 2:4:6 is a *three-term* ratio because it contains three terms, 2, 4, and 6.

Part-to-Part Ratios

A *part-to-part* ratio describes certain parts of a group, or certain parts of a whole. In the marble example above, the ratio of grey marbles to white marbles (4:6) is a part-to-part ratio. It compares different parts of a collection of marbles. The three-term ratio 2:4:6 is also a part-to-part ratio. It describes three parts of the collection.

Part-to-Whole Ratios

A *part-to-whole* ratio describes a part of a group in comparison to the whole group. In the marble example above the ratio of white marbles to the total number of marbles (6:12) is a part-to-whole ratio. It compares a specific part of the group to the whole group.

Part-to-whole ratios can also be written as fractions. For example, we could write the ratio of white marbles to the total number of marbles as 6:12 or $\frac{6}{12}$.

Proportions

Let's look at the ratio of white marbles to the total number of marbles again. This ratio can be written as the fraction $\frac{6}{12}$. Remember that we can create equivalent fractions by multiplying or dividing the numerator and the denominator by a common factor. In this case:

$$\frac{6}{12} \xrightarrow[\div 6]{\div 6} \frac{1}{2}$$

$\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions.

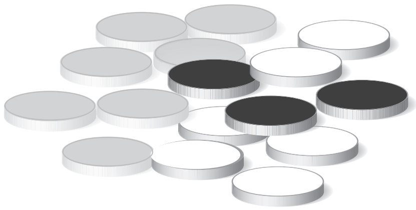
We can do the same thing with ratios. 6:12 can be written as 1:2.

$$6:12 \xrightarrow[\div 6]{\div 6} 1:2$$

A pair of equivalent ratios is called a **proportion**. We say that these two ratios are *proportional* to each other.

Exercise 2.1

1. Look at the counters below. The ratios below describe how the coloured counters relate to each other. Explain the relation for each ratio. The first one has been done for you.



   a. 3:7 Number of black counters to the number of grey counters

   b. 3:6 ______________________________

   ____

   c. 7:16 ______________________________

   ____

   d. 3:6:7 ______________________________

   ____

   e. 3:8 ______________________________

   ____

2. Classify each ratio in question 1 as either a part-to-part ratio or a part-to-whole ratio.

   a.

   b.

   c.

   d.

   e.

3. Write a part-to-part ratio for each of the comparisons below.

   a. There are 12 boys and 15 girls in a math class.

   b. To prepare pancakes from a packaged mix, you need 1 cup of water and 2 cups of pancake mix.

   c. This week, the forecast calls for three days of sunshine and four days of rain.

   d. In your dresser drawer you have one pair of pants, three pairs of shorts, and four T-shirts.

4. For each of the comparisons in question 3, write a part-to-whole ratio.

   a.

   b.

   c.

   d.

5. Which of the following pairs of ratios are proportional? How do you know?

   a. 2:4 and 6:12

   b. 1:3 and 4:15

   c. 16:30 and 8:15

Turn to the Answer Key at the end of the module to check your work.

Rates

Ratios compare quantities of the same kind: numbers of girls and boys (people), numbers of rainy days and sunny days (days), numbers of pants, shorts, and shirts in a dresser drawer (articles of clothing). But what if we want to compare different types of things?

A rate is a way of comparing two measurements or quantities. For example, speed is a rate. The rate 50 km/h compares distance and time. The rate means that you can travel 50 kilometres in one hour.

Other examples of rates are:

- the number of litres of water you use in the shower every week
- the amount of rain that falls in a year
- the amount of money paid for every hour you work
- the distance you can travel in a vehicle with a certain amount of fuel

When we worked with ratios, we did not include units. When we work with rates, the units are very important. If your friend told you that oranges were on sale for 1.99/1, you would probably ask for more information. You might assume that your friend meant $1.99, since they're talking about price, but you wouldn't know if they meant $1.99 per orange or $1.99 per pound of oranges, or $1.99 per kilogram of oranges. The rate 1.99/1 is not as meaningful as the rate $1.99/kg.

Unit Rates

In rates, as in ratios, equivalent fractions play a very important role. We change ratios and rates into fractions, then use our fraction knowledge to find equivalent forms. For example, if you get paid $36.00 for 4 hours of work on your part time job, what is your hourly wage?

Your rate of pay is $36.00/4 hours. To figure out your hourly wage, create equivalent fractions.

$$\frac{\$36.00}{4\text{ hours}} = \frac{?}{1\text{ hour}}$$

$$\div 4$$

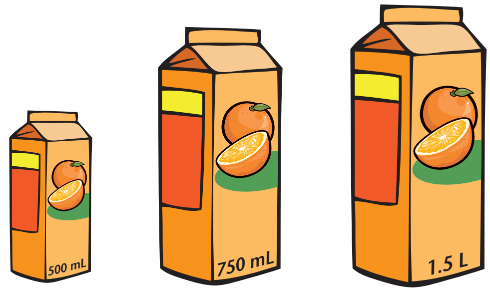
$$\frac{\$36.00}{4\text{ hours}} = \frac{\$9.00}{1\text{ hour}}$$

$$\div 4$$

So, your wage is $9.00/h.

Notice that the second term in this rate is 1. A rate that has 1 as its second term is called a **unit rate**. Unit rates are often used to make comparisons. For example, if you were grocery shopping you might want to compare the prices of two different brands. Or, you might want to compare the prices of the same item at different stores.

Which carton of juice is the best buy?

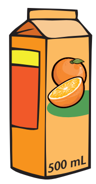


\$1.89 **\$2.45** **\$4.29**

If we can find the price per unit volume of each carton, we can compare them to figure out which is the best deal.

If we buy the smallest carton, we get 500 mL of juice for \$1.89.

$$\begin{aligned}\text{unit price} &= \frac{\text{cost}}{\text{volume}}\\ &= \frac{\$1.89}{500\text{ mL}}\\ &= \$0.00378/\text{mL}\end{aligned}$$

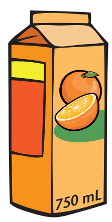


We want to create a unit rate, so we know that we want the denominator to be 1. Use a calculator to divide \$1.89 by 500.

The unit price of this carton is \$0.00378/mL.

If we buy the medium-sized carton, we get 750 mL of juice for \$2.45.

$$\begin{aligned}\text{unit price} &= \frac{\text{cost}}{\text{volume}}\\ &= \frac{\$2.45}{750\text{ mL}}\\ &= \$0.00327/\text{mL}\end{aligned}$$



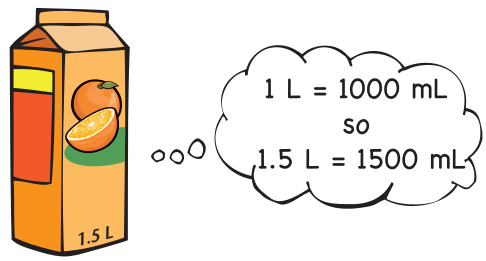
Fractions mean dividing. To find this unit rate divide, we divide just as we did in the previous example.

Use a calculator to find \$2.45 ÷ 750.

The unit price of this carton is \$0.00327/mL.

If we buy the large carton, we get 1.5 L of juice for \$4.29. To compare this price to the others, we need the unit price to be in the same units. To convert litres to millilitres, multiply by 1000.

$$\begin{aligned}\text{unit price} &= \frac{\text{cost}}{\text{volume}}\\ &= \frac{\$4.29}{1500\text{ mL}}\\ &= \$0.00286/\text{mL}\end{aligned}$$



The unit price of this carton is \$0.00286/mL.

The largest container has the lowest unit price, so it is the best value.

Exercises 2.2

1. Write a rate for each sentence below.

   a. Adrian travelled 110 kilometres in two hours.

   b. Angelique paid $11.19 for three kilograms of apples.

   c. David took his heart rate after jogging. He counted 30 beats in a 10-second time period.

2. Write a description for each rate below.

   a. $\frac{400\text{ km}}{28\text{ L}}$

   b. 70 km/h

   c. $\frac{\$72}{5\text{ h}}$

3. Calculate the unit rate for each of the rates described in question 1.

   a.

   b.

   c.

4. You can buy a package of four batteries for $6.67 or a package of 10 for $13.90. Which package is the best buy?

Turn to the Answer Key at the end of the module to check your work.

Problem-solving with Rates and Ratios

Let's apply our knowledge of ratios and rates to solve some problems.

Problem 1

At a hockey game, your favourite team out shot their opponent 2 to 1. If your team made 30 shots, how many shots did their opponent make?

The ratio of shots made by your team to the number of shots made by their opponent is 2:1. That means that for every two shots your team made, their opponent only made one. We can use proportions to solve this problem.

$$2:1 = 30:\square$$

$$\times 15$$

$$2:1 = 30:\square$$

$$\times 15$$

$$2:1 = 30:15$$

We could also set up this proportion using fractions.

$$\times 15$$

$$\frac{2}{1} = \frac{30}{\square}$$

$$\times 15$$

$$\frac{2}{1} = \frac{30}{15}$$

Your favourite team's opponent made 15 shots on goal.

Problem 2

Jillian works at a coffee shop. Last week she worked 25 hours and earned \$225.

a. What is her hourly rate of pay?

b. She is scheduled to work 31 hours next week. How much money will she earn?

To answer part (a), we need to find out how much Jillian makes in one hour.

$$\begin{aligned}\text{unit rate} &= \frac{\text{amount earned}}{\text{hours worked}}\\ &= \frac{\$225}{25\text{ h}}\\ &= \$9/\text{h}\end{aligned}$$

Jillian's rate of pay is \$9 per hour.

To answer part (b), we need to figure out how much she'd earn if she worked 31 hours. We can use the unit rate we found in part (a) and multiply it by the number of hours she is scheduled to work next week.

$$\underset{\times 31}{\overset{\times 31}{\frac{\$9}{1\text{ h}} = \frac{\square}{31\text{ h}}}}$$

$$\frac{\$9}{1\text{ h}} = \frac{\$274}{31\text{ h}}$$

Jillian will earn \$274 next week if she works all her scheduled hours.

Problem 3

Stephen consulted his map to find the distance between Nanaimo and Courtenay. He used a ruler to measure the distance on the map: 8.25 cm. "Great!" he thought, "Now I'll just look at the scale."

Unfortunately, the bottom of the map was ripped, and the scale was missing. Stephen was discouraged for a moment, but then he had an idea. "I know that it's about 20 km from Nanaimo to Ladysmith. I'll measure that distance on the map and make my own scale!" Stephen found the distance between Nanaimo and Ladysmith to be 1.5 cm on the map.

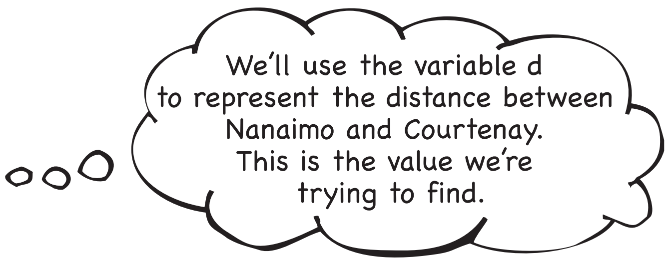
Set up a proportion and find the distance between Nanaimo and Courtenay using Stephen's scale.

Maps are created using a scale. This means that any distance shown on a map is proportional to the actual distance.

$$\frac{\text{distance on map}}{\text{actual distance}}$$

We can use this ratio to set up a proportion with the information in the problem.

$$\frac{1.5 \text{ cm}}{20 \text{ km}} = \frac{8.25 \text{ cm}}{d}$$



$$\frac{1.5\text{ cm}}{20\text{ km}} \overset{\times \square}{=} \frac{8.25\text{ cm}}{d}$$

What can we multiply 1.5 by to get 8.25?

$$\frac{1.5\text{ cm}}{20\text{ km}} \overset{\times 5.5}{=} \frac{8.25\text{ cm}}{d}$$

Divide to find what number multiplied by 1.5 gives 8.25.
8.25 ÷ 1.5 = 5.5

From the proportion you can see that

$d = 20 \times 5.5$

$d = 110$

So the distance from Nanaimo to Courtenay is 110 km.

The Cross-Product Method

In some proportions, it's easy to determine what factor you should multiply or divide by to find the missing number. As you saw in Problem 3 (above) it's not always so easy. We'll try solving Problem 3 again, using a different method. But first, let's look at a simple proportion.

$$\frac{1}{2} = \frac{2}{4}$$

Try this:

Multiply the numerator of the first fraction by the denominator of the second fraction.

$$\frac{1}{2} = \frac{2}{4}$$

1 x 4 = 4

Multiply the denominator of the first fraction by the numerator of the second fraction.

$$\frac{1}{2} = \frac{2}{4}$$

2 x 2 = 4

Notice that you get the same answer for both. These are called *cross-products*. In any proportion, the cross-products are equal.

If $\frac{a}{A} = \frac{b}{B}$ then $aB = Ab$

We can use this to help us solve proportion problems.

Let's go back to Problem 3 (the map problem). Here's our proportion:

$$\frac{1.5\text{ cm}}{20\text{ km}} = \frac{8.25\text{ cm}}{d} \qquad \frac{1.5\text{ cm}}{20\text{ km}} = \frac{8.25\text{ cm}}{d}$$

We can write the cross products as an equation.

Remember, side-by-side brackets mean multiply.

$$(1.5\text{ cm})(d) = (20\text{ km})(8.25\text{ cm})$$

Now, solve the equation.

To undo multiplying by 1.5 cm, divide by 1.5 cm.

What you do to one side of the equation, you must also do to the other side.

$$\frac{(1.5\text{ cm})(d)}{1.5\text{ cm}} = \frac{(20\text{ km})(8.25\text{ cm})}{1.5\text{ cm}}$$

$$d = \frac{(20\text{ km})(8.25\text{ cm})}{1.5\text{ cm}}$$

$$d = 110\text{ km}$$

We got the same answer as we did before; the actual distance from Nanaimo to Courtenay is 110 km.

When you're solving problems, you can use whichever method works best for you.

Exercises 2.3

1. Cassie rides her bike to school. The school is 8.5 km away from her house, and it usually takes her 30 minutes to get there. What is Cassie's rate of speed on her bike (in km/h)?

2. If a can of paint covers 9 square metres, how many cans of paint does it take to paint a room which has 27 square metres of wall area?

3. Chris is offered two different jobs.
   - The first job is working in a hardware store. The manager says he will pay her $440/week if she works 40 hours a week. He would pay her the same hourly wage if she wants fewer hours.
   - The second job is at the library. The librarian says she will pay Chris $350/week for 25 hours of work per week. The librarian is not flexible about the number of hours Chris can work.

   a. Calculate the hourly rate of pay for each job.

b. Which job should Chris take? Explain your answer.

4. Marcel's BC Hydro bill arrived in the mail. The bill showed that he used 194 kWh of electricity for which he was charged $11.47. If he uses 230 kWh next month, how much will his bill be?

Turn to the Answer Key at the end of the module to check your work.

Lesson 3

Percent

Learning Outcomes

By the end of this lesson you will be better able to:

- describe a percentage with a fraction, a ratio, or a decimal number
- understand percentages less than 1% and greater than 100%
- solve problems involving percent

Drawing Percentages

Draw a ten by ten square on the grid below. Shade it in lightly.

We can calculate the area of this square as follows:

A square is a type of rectangle.
A = l x w

A = *length* × *width*

A = 10 × 10

A = 100 square units

Next draw a five by five square, starting in one of the corners of the ten by ten square. Shade this square a different colour. Calculate the area of this square as follows:

A = *length* × *width*

A = 5 × 5

A = 25 square units

Let's compare the ratio of the area of the smaller square to the area of the bigger square. We can think of the big square as a whole, and the smaller square as a part of the whole. The ratio of their areas, then, will be a part-to-whole ratio. We can write this in three ways:

25 to 100 25 : 100 $\frac{25}{100}$

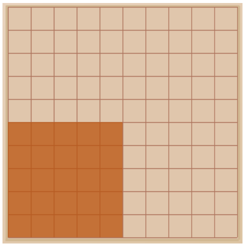
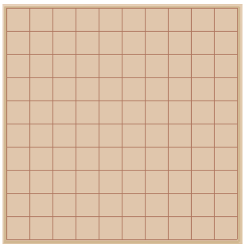
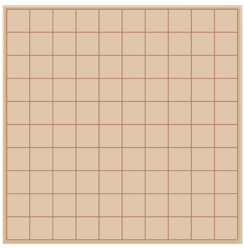
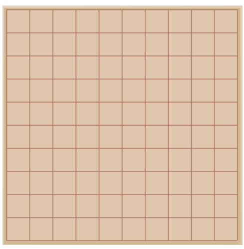
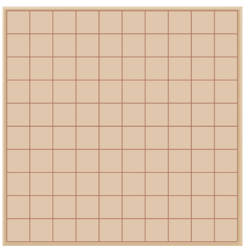
You may remember that a fraction of a whole, expressed as a fraction of 100, is called a percent (or a percentage). So, we can actually write our area ratio above in two more ways: as a percent and as a decimal.

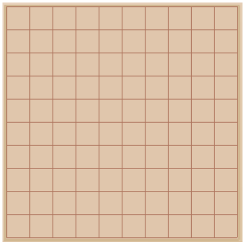
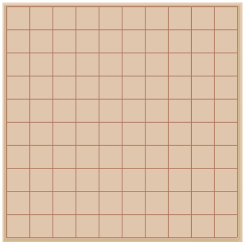
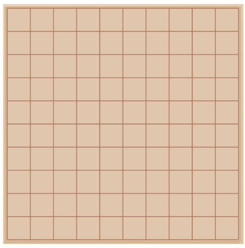
25 to 100 25 : 100 $\frac{25}{100}$ 25% 0.25

The % symbol is important! 25% is not the same as 25.

Exercises 3.1

Following the example given in the first row, complete the table.

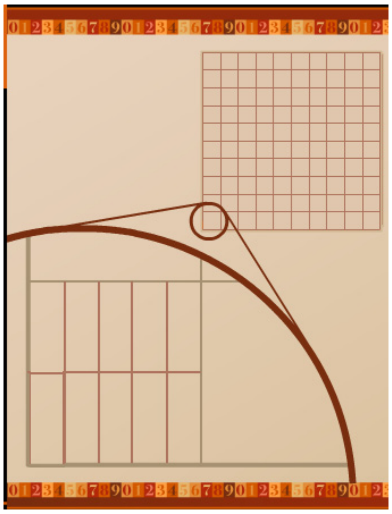
Picture	Percent	Fraction	Ratio	Decimal
	25%	$\frac{25}{100}$	25:100	0.25
	64%		64:100	
		$\frac{12}{100}$		
				0.04
				0.2625

	52%		52:100	
				0.765
		$\frac{90}{100}$		

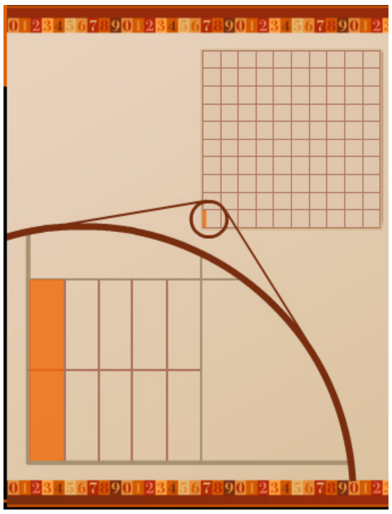
Turn to the Answer Key at the end of the module to check your work.

Percentages Less Than 1%

In the ten-by-ten grid we've been working with, each square represents 1%. What happens if we divide that 1% square into smaller pieces?



Here you can see that the 1% square is divided into ten pieces (or tenths). If you shade in two of those pieces, what percent do you have?



You have:

two tenths of one percent

$$\frac{2}{10} \times \frac{1}{100}$$

This is the same as $\frac{2}{1000}$.

To write this as a percent, we need to convert it to a fraction with a denominator of 100.

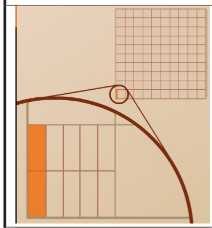
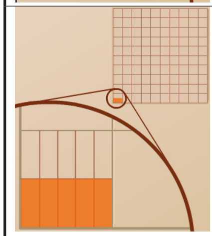
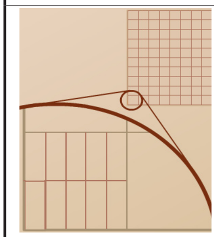
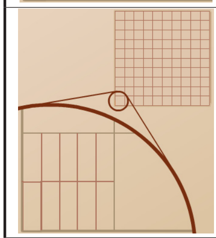
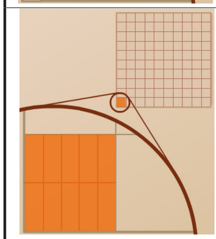
$$\frac{2}{1000} \overset{\div 10}{\underset{\div 10}{=}} \frac{?}{100}$$

$$\frac{2}{1000} = \frac{0.2}{100} = 0.2\%$$

We can write fractional percents as percents, fractions, ratios, and decimals. Give it a try in the next activity.

Exercises 3.2

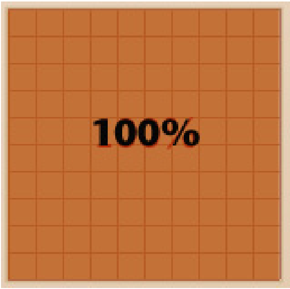
Following the example given in the first row, complete the table.

Picture	Percent	Fraction	Ratio	Decimal
	0.2%	$\frac{2}{1000}$	2:1000	0.002
		$\frac{5}{1000}$		
	0.6%			
				0.003
				

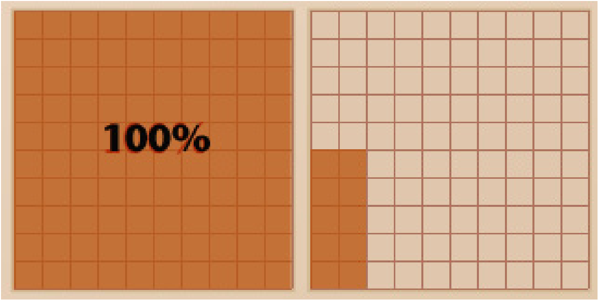
Turn to the Answer Key at the end of the module to check your work.

Percentages Greater Than 100%

One whole ten by ten grid represents 100%.



We can use multiple ten-by-ten grids to represent percentages that are greater than 100%.



1st grid has 100 shaded squares = 100%

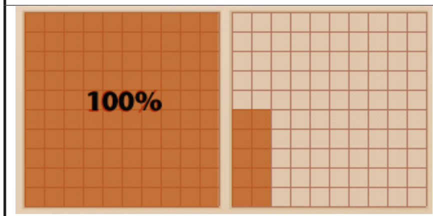
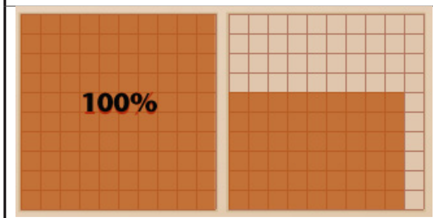
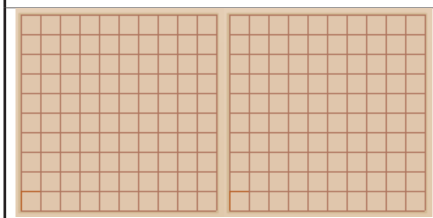
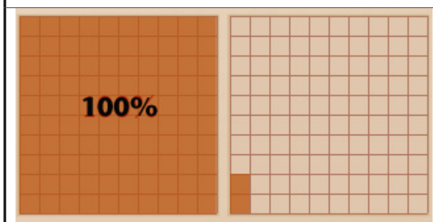
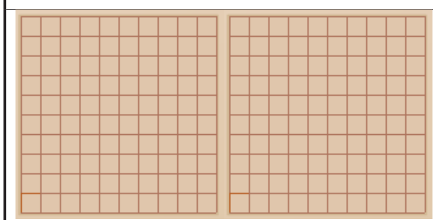
2nd grid has 10 shaded squares = 10%

Total = 110 shaded squares = 110%

We can write percentages that are greater than 100% as percents, fractions, ratios, and decimals. Give it a try in the next activity.

Exercises 3.3

Following the example given in the first row, complete the table.

Picture	Percent	Fraction	Ratio	Decimal
	110%	$\frac{110}{100} = 1\frac{10}{100}$	110:100	1.1
			154:100	
	127%			1.27
				
				1.81

Turn to the Answer Key at the end of the module to check your work.

Calculations with Percents

You have seen that percents can range from 0% to more than 100%. In the next lesson we'll explore a number of situations in which you might find percents, and we'll solve several problems. For now, we'll work on our calculation skills, and get some practice converting between percents, fractions, ratios, and decimals.

Let's work through several examples together.

Example 1

Write the following fractions as percents.

a. $\frac{14}{20}$

b. $\frac{3}{250}$

c. $1\frac{3}{5}$

To write the fractions as percents, create equivalent fractions out of 100. This is a proportion. You can solve proportions by figuring out what factor to multiply the numerator and denominator by to create the equivalent fraction, or you can use the cross product method.

a. $$\frac{14}{20} \overset{\times 5}{=} \frac{x}{100}$$

$$\frac{14}{20} = \frac{70}{100} = 70\%$$

b. $$\frac{3}{250} = \frac{x}{100}$$

$$(3)(100) = (250)(x)$$

$$\frac{(3)(100)}{250} = \frac{(250)x}{250}$$

$$1.2 = x$$

So,

$$\frac{3}{250} = \frac{1.2}{100} = 1.2\%$$

If you're not sure what to multiply or divide by, use the cross-product method.

c. $1\frac{3}{5} = \frac{x}{100}$

First change this to an improper fraction.

$$\frac{8}{5} \overset{\times 20}{=} \frac{x}{100}$$

$$\frac{8}{5} = \frac{160}{100}$$

$$\frac{8}{5} = 160\%$$

Example 2

Write each decimal as a percent.

a. 0.35

b. 0.0001

c. 1.67

To convert from decimals to percents, simply multiply by 100 and add the percent symbol. A quick way to do this is to move the decimal point two places to the right.

a. $0.35 \times 100 = 35$

So, $0.35 = 35\%$

Don't forget the percent symbol!

b. $0.0001 \times 100 = 0.01$

So, $0.0001 = 0.01\%$

c. $1.67 \times 100 = 167$

So, $1.67 = 167\%$

Example 3

Write each part-to-whole ratio as a percent.

a. 1:200

b. 2:10

c. 8:3

Remember that part-to-whole ratios can be written as fractions. We can set up proportions to find an equivalent fraction with a denominator of 100.

a. $1:200 = \dfrac{1}{200}$

$$\frac{1}{200} \underset{\div 2}{\overset{\div 2}{=}} \frac{x}{100}$$

$$\frac{1}{200} = \frac{0.5}{100} = 0.5\%$$

b. $2:10 = \dfrac{2}{10}$

$$\frac{2}{10} \underset{\times 10}{\overset{\times 10}{=}} \frac{x}{100}$$

$$\frac{2}{10} = \frac{20}{100} = 20\%$$

c. $8:3 = \dfrac{8}{3}$

$$\frac{8}{3} = \frac{x}{100}$$

Use the cross-product method to solve this one.

$$(8)(100) = (3)(x)$$

$$\frac{(8)(100)}{3} = \frac{(3)(x)}{3}$$

$$266.\overline{6} = x$$

So,

$$8:3 = 266.\overline{6} \approx 266.7\%$$

Example 4

Write each of the following percents as a decimal and as a fraction.

a. 41.5%

b. 140%

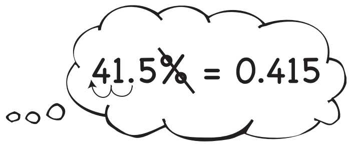
c. $\dfrac{3}{4}\%$

To convert a percent to a decimal, simply divide by 100 and remove the percent sign. A quick way to do this is to move the decimal point two places to the left.

Once you have a decimal, you can easily convert to a fraction. Don't forget to reduce to lowest terms.

a. $41.5 \div 100 = 0.415$

so $41.5\% = 0.415$



To convert the decimal to a fraction, start by checking the place value of the last digit. The 5 is in the thousandths place, so

$$0.415 = \frac{415}{1000}$$

Now reduce the fraction to lowest terms.

$$\frac{415}{1000} \underset{\div 5}{\overset{\div 5}{=}} \frac{83}{200}$$

So, $41.5\% = \frac{83}{200}$.

b. $140 \div 100 = 1.4$

so $140\% = 1.4$

Now convert the decimal to a fraction. We have one whole, and four tenths so

$$1.4 = 1\frac{4}{10}$$

Now reduce the fraction to lowest terms.

$$1\frac{4}{10} \underset{\div 2}{\overset{\div 2}{=}} 1\frac{2}{5}$$

So, $140\% = 1\frac{2}{5}$.

c. Start by writing $\frac{3}{4}\%$ as a decimal percentage.

$\frac{3}{4}\% = 0.75\%$

$3 \div 4 = 0.75$

Now convert the percent to a decimal.

$0.75 \div 100 = 0.0075$

So, $0.75\% = 0.0075$

We can also move the decimal point two places to the left.

$0.75\cancel{\%} = 0.0075$

The 5 is in the ten thousandths place, so

$$0.0075 = \frac{75}{10\,000}$$

Reduce the fraction to the lowest terms.

$$\frac{75}{10\,000} \xrightarrow{\div 25} \frac{3}{400}$$

So, $0.75\% = \frac{3}{400}$.

Now it's your turn to practice converting between percents, decimals, fractions, and ratios.

Exercises 3.4

1. Write the following fractions as percents.

   a. $\frac{171}{300}$

   b. $\frac{41}{20}$

   c. $\frac{3}{125}$

   d. $1\frac{7}{15}$

2. Write the following decimals as percents.

   a. 0.14

   b. 0.005

   c. 0.1

   d. 1.23

3. Write each of the following percents as a decimal and as a fraction.

	Decimal	Fraction
a. 0.07%		
b. $23\frac{1}{2}\%$		
c. 325%		

4. Write the following ratios as percents.

   a. 7:4

   b. 1:500

Turn to the Answer Key at the end of the module to check your work.

Solving Problems with Percents

You were so excited to buy that new hoodie! But when you went to pay, the total that came up on the cash register was more than the price on the tag. What happened?

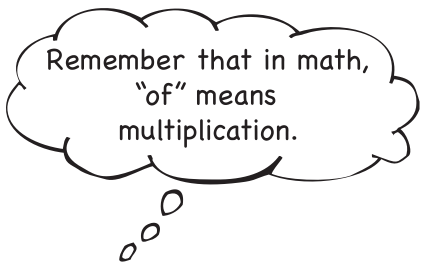
In some provinces two taxes are added to purchases: the Goods and Services Tax (GST) and the Provincial Sales Tax (PST). Other provinces have just one tax, the Harmonized Sales Tax (HST). BC may soon adopt the HST, but in this lesson we'll explore how to work with two taxes. The current rates are:

- GST 5%
- PST 7%

Example 1

A skateboard has a displayed price of $140.00. What is the total amount of taxes on this item? What is the check out price?

$5\% = \frac{5}{100} = 0.05$ $7\% = \frac{7}{100} = 0.07$



GST = 5% of \$140.00 = 0.05 × 140.00 = \$7.00

PST = 7% of \$140.00 = 0.07 × 140.00 = \$ 9.80

The total amount of taxes = \$7.00 + \$9.80 = \$16.80

The check out price is: \$140.00 + \$16.80 = \$ 156.80

Note: The GST and PST are each calculated separately—the calculation of one does not affect the calculation of the other. We could perform these calculations in any order. In fact, we could combine the GST and PST into one total percentage (5% + 7% = 12%).

Working Backward

Sometimes we might want to find the price of an item before taxes. To do this we can work backward from the cost of the taxes.

Example 2

For example, if we know the GST on an item is \$9.00, we can find the original price.

We know the percentage and the cost of the tax, but we don't know the original amount. In the previous example, we used the following expression to find the tax cost.

original price × percentage tax = tax cost

We can use this same expression, substituting what we know.

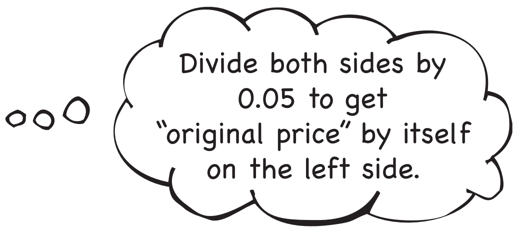
original price × 5% = \$9.00

OR

original price × 0.05 = \$9.00

Solve for the original price:

$$\frac{\text{original price} \times 0.05}{0.05} = \frac{\$9.00}{0.05}$$



original price = \$180.00

Example 3

You pay \$3.00 in taxes on a T-shirt. How much did the T-shirt cost before taxes?

Total taxes include 5% GST + 7% PST = 12%

original price × 12% = \$3.00

original price × 0.12 = \$3.00

$$\frac{\text{original price} \times 0.12}{0.12} = \frac{\$3.00}{0.12}$$

original price = \$25.00

The T-shirt cost \$25.00 before taxes.

Exercises 3.5

For the questions below, assume that the GST is 5% and the PST is 7%.

1. If the display price of an item is $120.00, what is the

    a. GST paid on the item?

    b. PST paid on the item?

2. If the price tag on a pair of jeans reads $70.00, what is the total amount of taxes? What is the total price?

3. If the GST paid on a new collector's edition of a video game is \$3.40, what was the original price?

4. If the PST paid on an item is \$10.00, what was the original price of the item?

5. If the total tax on an item is \$9.60, what was the original price of the item?

Turn to the Answer Key at the end of the module to check your work.

Profits, Taxes, and Discounts

Stores mark up the price on their merchandise to cover expenses, wages, and make a profit. Sometimes a store will decide to mark up an item based on a specific dollar amount. This is called simple markup. Other times, a store will decide on a percentage markup, and apply the same percent markup to similar items. The price that a store sells items at is called the retail price.

Stores often offer discounts in order to get rid of old merchandise or to encourage people to buy more items. There are many ways to offer a discount. One of the most common is a percent discount.

Let's work through some examples.

Example 1 (Simple Markup)

A store buys a box of Wii® accessories for \$200.00. The store plans to sell the accessories and wishes to mark it up by \$25.00. If you were to purchase the accessories from this store, how much would you pay including taxes?

Solution:

Since the store wants to markup the Wii® accessories by \$25.00, the price tag will read \$225.00 (\$200.00 cost + \$25.00 markup = \$225.00).

When you purchase the accessories, you also have to pay GST (5%) and PST (7%).

GST = 5% of \$225.00 = 0.05 × \$225.00 = \$ 11.25

PST = 7% of \$225.00 = 0.07 × \$225.00 = \$15.75

So the total cost for you = \$225.00 + \$11.25 + \$15.75 = \$252.00.

Note: The GST and PST are each calculated separately—the calculation of one does not affect the calculation of the other. We could perform these calculations in any order. In fact, we could combine the GST and PST into one total percentage (5% + 7% = 12%).

To make sure this works, check that you get the same answer.

12% of \$225.00 = 0.12 × 225.00 = \$27.00

Previously we calculated that the GST was \$11.25 and the PST was \$15.75.

Combining these, you can see that we get \$27.00, which is the same answer.

Example 2 (Percent Markup)

A store buys hoodies at a wholesale price of \$60.00 each. They usually mark up the price of a clothing item by 35%. What is the retail price for the hoodies?

Solution:

The markup is 35% of the wholesale cost.

$35\% \times \$60.00 = 0.35 \times \$60.00 = \$21$

$$\begin{aligned} \text{retail price} &= \text{cost of item} + \text{markup} \\ &= \$60.00 + \$21 \\ &= \$82.00 \end{aligned}$$

The retail price of the hoodies is \$82 each.

Example 3 (Percent Discount)

A CD regularly sells for \$16. You can buy it on sale for 15% off. What is the sale price?

Solution:

$$\begin{aligned} \text{sale price} &= \text{original price} - \text{discount amount} \\ &= \$16 - (15\% \text{ of } \$16) \\ &= \$16 - (0.15 \times \$16) \\ &= \$16 - \$2.40 \\ &= \$13.60 \end{aligned}$$

The sale price is \$13.60.

Another way to approach this problem is to consider how much of the original retail price you will be paying. If you are getting a 15% discount, then you are paying 85% of the original price.

100% – 15% = 85%

Then,

$$
\begin{aligned}
\text{sale price} &= 85\% \text{ of the original price}\\
&= 85\% \times \$16\\
&= 0.85 \times \$16\\
&= \$13.60
\end{aligned}
$$

You get the same answer; the discounted price is $13.60.

Example 4 (Percent Discount: Working Backwards)

A pair of jeans is marked down by 20%, and a sale tag now advertises the sale price is $46.40. What was the original price?

Solution:

Sale price = Original Price – (20% of the original price)

Let x represent the original price, and substitute the values we know.

$$
\begin{aligned}
\$46.80 &= x - (20\% \text{ of } x)\\
\$46.80 &= x - 0.20x\\
\$46.80 &= 0.80x\\
\frac{\$46.40}{0.80} &= \frac{0.80x}{0.80}\\
\$58 &= x
\end{aligned}
$$

Simplify like terms.
x = 1x and
1x – 0.20x = 0.80x.

The original price of the pair of jeans was $58.00.

We can approach this problem another way.

The original price for the item can be considered 100%. If 20% is taken off, that would leave 80% of the price (100% – 20% = 80%).

So, the expression we can use is:

80% of the original price is $46.40.

Try using this expression to solve the problem. You should get the same answer.

Example 5 (Combined Discounts)

A portable DVD player usually sells for \$150.00 at a local store. The weekend flyer had an advertisement for a 10% discount. You go to check it out and find out that the store is giving a further discount of 20% off any discounted price! What will be the new ticket price?

Solution:

Price after 10% discount = \$150.00 – 10% of \$150.00

= \$150.00 – 0.10 × 150.00

= \$150.00 – \$15.00

= \$135.00

Price after a further 20% discount = \$135.00 – 20% of \$135.00

= \$135.00 – 0.20 × \$135.00

= \$135.00 – \$27.00

= \$108.00

So the overall discounted price is \$108.00.

Note: In Example 1 we found that we could combine the two taxes before figuring out the cost of the taxes. It didn't matter which order we calculated the taxes, or if we combined them first.

Example 5 is different. In Example 5, there is a discount on a discount. This is called a compounding percent. The order that you calculate these percents is very important, and you cannot simply add the percents together.

Other Problems

We have solved several percent problems related to shopping: profits, taxes, and discounts. There are many other applications of percents. The next activity will ask you to solve a number of problems. Some will be similar to the ones we solved in this Explore, but some will be a bit different. Think through the problems carefully, and use what you know about percents, ratios, and proportions to help you.

Exercises 3.6

For the questions below, assume that the GST is 5% and the PST is 7%.

1. A street vendor buys a pair of jeans wholesale for $90.00 and sells it for $120.00 including taxes. What is the profit amount for the vendor? (GST is 5% and PST is 7%)

2. Cole bought a Blackberry for $300.00 after a 20% discount. What was the original listed price?

3. A classic video game discounted by 10% has been advertised for a further 15% discount. If the original price was $80.00, what was the price of the game after both discounts?

4. A winter jacket has a listed price of \$160.00. If the store advertises a discount of 30%, how much does it cost after the discount and the taxes are added? (GST is 5% and PST is 7%)

5. In Grade 10, students face their first provincial exams. The provincial exam is worth 20% of their final mark, the remainder comes from their class grade. Alex is a student in 10th grade. If he has 70% in his class mark and 73% on his provincial exam, what mark does he get for a final grade in the course?

6. Most of the water on Earth is saltwater. Only approximately 2.5% of the water on Earth is freshwater. Two thirds of that freshwater is frozen in icecaps and glaciers. Our drinking water comes from freshwater sources such as groundwater, rivers, and lakes.

   a. What percent of the Earth's freshwater is frozen? (Express your answer to the nearest hundredth.)

b. What percent of the Earth's water is available to us for use? (Express your answer to the nearest hundredth.)

Turn to the Answer Key at the end of the module to check your work.

Lesson 4
Integers, Part 1

Learning Outcomes

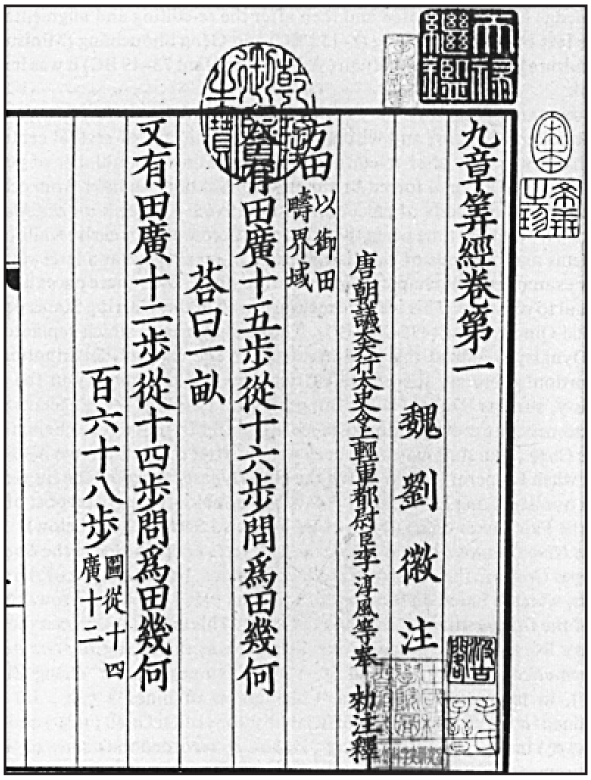
By the end of this lesson you will be better able to:

- describe situations using integers (positve and negative numbers)
- add, subtract, multiply, and divide with integers

Introduction to Integers

Mathematicians in China were contemplating the meaning and use of negative numbers as early as 100 BC.

This image is from the famous Chinese book the *Jiu zhang suanshu* or the *Nine Chapters on the Mathematical Art*. In the book there is a description of using different coloured counting rods—red for positive numbers and black for negative numbers.


九章筭經卷第一
魏 劉 徽 注
唐朝議大夫行太史令上輕車都尉臣李淳風等奉 勅注釋
方田 以御田疇界域
今有田廣十五步從十六步問爲田幾何
荅曰一畝
又有田廣十二步從十四步問爲田幾何
百六十八步 圖從十四廣十二

Many years later in India and in Europe, negative numbers were used in banking and trade to represent money lost and debts owed.

Positive and negative numbers allow us to give numbers a direction. Temperature can go up and temperature can go down. When you're walking on the sidewalk, you can go forward or you can go backward.

If you have money in the bank, you have a positive balance in your account. If your account is overdrawn, the balance is negative.

An **integer** is a whole number that has a direction, either positive or negative. When you hear that the temperature has changed by 5°C, you want to know whether the temperature went up or down. Earning $1000 is not the same as spending $1000.

When we read questions about integers, there are words that we can use as clues to help us know whether an integer is positive or negative.

When you see a negative sign, what does it mean to you?

It is used to represent anything that is taken away, subtracted, or lost. A positive sign represents things being added together, or increasing in value.

Look at this Example

10 degrees below zero.

When I read this, the word that stands out to me is "below." I use this clue to help me decide what sign to attach to the number. I know "below" means negative, so:

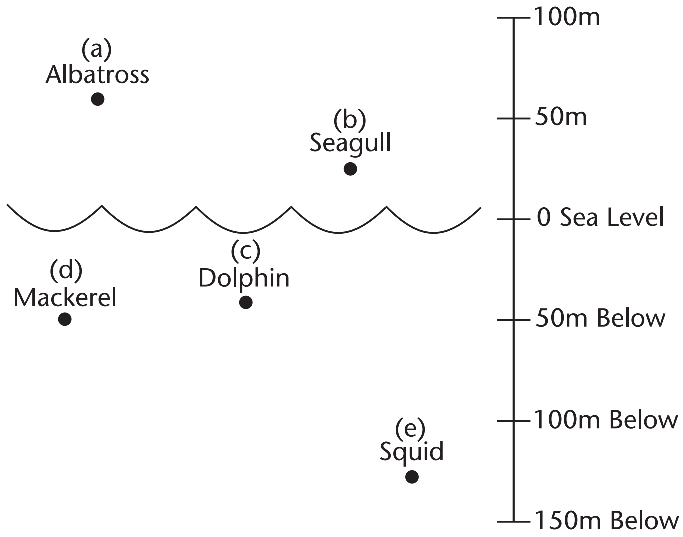
10 degrees below zero = –10°

Read the following statements and find the clue word that will help you understand if the integer is positive or negative. Fill in the chart with other thinking that helps you determine if the integer is positive or negative. The first one is done for you.

STATEMENT	CLUE WORD	POSITIVE OR NEGATIVE?	INTEGER
10 below zero	below	negative	-10
25 meters above sea level			
a gain of 5 kilograms			
a debt of 11 dollars			
a loss of 6 dollars			

"Above" and "gain" are words that mean *positive*. "Debt" and "loss" are words that mean *negative*. Keep thinking about this idea until your answers match these: –10, +25, +5, –11, –6.

Exercises 4.1



1. Write an integer for the level at which each animal flies or swims.

   a. __________

   b. __________

   c. __________

   d. __________

   e. __________

2. Write an integer to represent each quantity.

    a. A temperature of 50 degrees below zero __________

    b. A temperature of 10 degrees above zero __________

    c. A depth of two hundred meters below sea level __________

    d. A library fine of three dollars __________

    e. A gain of five kilograms __________

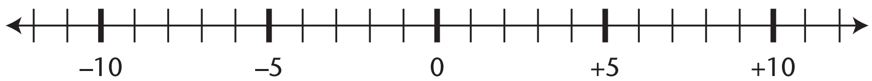
    f. A loss of six dollars __________

Turn to the Answer Key at the end of the module to check your work

Number Lines

Number lines are useful tools to help us “see” integers. Just like a thermometer, we can see both positive and negative numbers on the number line.

Number lines can help us put integers in order. You have lots of experience with positive integers. But how do we put negative integers in order?



Look at the number line above. Write 3 numbers that are GREATER than zero.

1.

2.

3.

Look at the number line and write 3 numbers that are LESS than zero.

1.

2.

3.

Positive numbers are greater than zero and negative numbers are less than zero.

An integer is GREATER THAN another integer the FURTHER RIGHT it is on the number line.

An integer is LESS THAN another integer the FURTHER LEFT it is on the number line.

Comparing Integers

Circle the LEAST integer in the pair, and write down your thinking as you go.

–3 or +5

Now you try one.

Circle the LEAST integer in the pair, and write down your thinking as you go.

–7 or –2

Did you circle –7? You are ready to practice some on your own.

Exercises 4.2

1. Circle the **greatest** integer.
   a. –3 or +5
   b. +9 or –3
   c. –7 or –2

2. Circle the **least** integer.
   a. +3 or +8
   b. –7 or –11
   c. –21 or –3

3. Put these integers in order from **least** to **greatest**.
   +5, –2, +12, 0, –8

Turn to the Answer Key at the end of the module to check your work.

Adding Integers

Introduction

In this lesson we're going to explore how to add integers. We'll look at a variety of methods you can use to help make adding integers as easy as possible! As you go through the lesson, think about which methods work best for you.

The key to adding integers is to know about a few little hints. There are some basic rules that we follow when working with integers that will help us when we need to add them. These rules are slightly different from the rules we normally follow when adding, so let's look at these helpful hints before we go further.

Hint #1: The sign leads the way!

You might notice when adding integers, that there seems to be signs everywhere! It helps to keep them organized. First, figure out which signs are stuck to which number. If you can remember that the sign leads the way, it helps make the questions and **equations** seem a little less crazy!

$$\boxed{+2 + -3}$$

In this example, the + sign is in front of the 2. We know that it is stuck to the 2 because the sign leads the way.

In this example the 3 has "+" and "–" signs in front of it. The sign leads the way: we know that the sign that sticks to the 3 is the one right in front. The value of the 3 is (–3).

Hint #2: Brackets keep it all together!

The second important hint to remember when working with integers is knowing how to keep the signs organized.

	WHAT IS IT?	EXAMPLE
Integer signs	Integer signs are the signs that tell us whether an integer is negative or positive. Think of a number line; the sign tells us which direction to go. Right for positive, or left for negative.	(+2) (–3)
		Think of your own example here:
Operation signs	Operation signs tell us what to do. Addition and Subtraction are both operations signs.	2 + 5 = 10 – 8=
		Think of your own example here:

Once we know which sign is attached to which number, the brackets keep it all together. The brackets keep the numbers and signs together, just like the drawer in your dresser is a place to keep all your socks together.

$$\boxed{+2 + -3 =}$$

We need to figure out which signs are integer signs and which are **operation signs**. Remember the hints?

1. The sign leads the way
2. Brackets keep it all together. Put the brackets around each integer. This includes the sign directly in front of the number. The number and the sign are stuck together.

If we follow these hints, we can rewrite the equation to look like this:

$$(+2) + (-3) =$$

Hint #3: Be positive that it's positive!

The last thing to remember about integers is that sometimes questions look like they're missing signs. You might see questions that look like this:

$$\boxed{2 + (-3) =}$$

At first glance it looks like the 2 doesn't have a sign because there's no sign in front leading the way. When there is no sign, it means that the integer is positive. So another way to write this equation would be:

$$(+2) + (-3) =$$

These concepts become extra helpful when there are several integers. For Example

$$7 + -3 + 2 =$$

$$(+7) + (-3) + (+2) =$$

It can be helpful to add brackets and positive signs when they are missing to keep things organized when you are working.

Take a look at all 3 hints. Remember, you can always come back and check on them if you are stuck or feel like you need a hint.

1. The sign leads the way—the sign immediately in front of a number is the sign that is stuck to it.

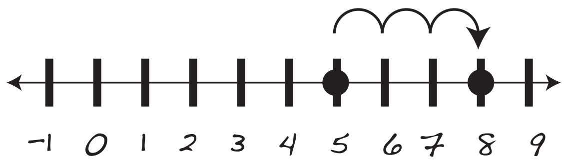
2. Brackets keep it all together—brackets go around a number and the sign in front of it.

3. Be positive that it's positive—when a number has no sign leading it, it means that the number is (+).

Exercises 4.3

1. Answer the following questions by using a number line. First, mark your starting point, then mark the change. The first one is done for you.

   a. 5 + 3 = 8

   b. 6 – 2 =

   c. 10 – 4 =

   d. 5 + 7 =

2. Practice writing brackets around the integers in these sums.

   a. –6 + – 2 =

   b. 7 + –3 =

   c. 9 + –6 =

   d. –12 + 3 + –7 =

   e. –64 + 32 + 11 =

Turn to the Answer Key at the end of the module to check your work.

Adding Integers—Using the Zero Principle

There are lots of ways that we can add integers. One is by following the **zero principle**: the **sum** of two **opposite integers** will always be zero. What does that mean? Let's break it apart.

LOOK AT THE WORD:	I KNOW IT MEANS:
Sum	This means "to add."
Opposite integers	Opposite integers have opposite signs. Also, they are the same distance from zero on the number line, but in opposite directions. Examples: (+1) and (–1) are opposite integers. (–4) and (+4) are opposite integers.
Always be zero	I know that "always" means it's a rule. And "be zero" means that it will equal zero.

For example, look at the following equation:

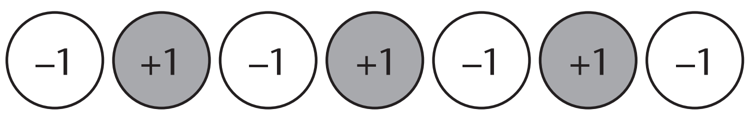
$$\boxed{(1) + (-1) = 0}$$

The opposite integers are (+1) and (–1). And when we add them together, it will equal zero. This pair of opposite integers is sometimes called a **zero pair.**

Write the zero principle in your own words. If it helps, draw a picture in your thinking space.

Drawing pictures is a great way to understand the zero principle. Let's sketch it out!

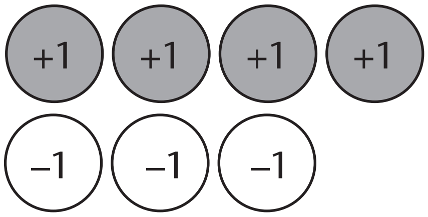
We can use coloured chips to represent positive and negative numbers. Here we'll use grey chips to represent positive numbers and white chips to represent negative numbers. (You could choose other colours if you want.)



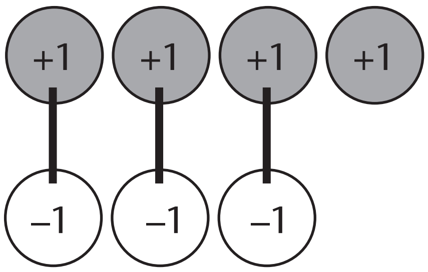
The zero principle says that every zero pair equals zero.

Look at the chips on the previous page. Let's add them together following the zero principle.

Step 1: Line the chips up into zero pairs

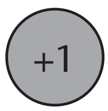


Step 2: Now let's draw lines between all the zero pairs



Step 3: Now see what's left over

We have 1 positive chip left over.



If we were to write this example as an equation, it would be:

$$(+4) + (-3) = (+1)$$

Exercises 4.4

1. Use the zero principle to find the sums.

   a. (+5) + (–3) =

   b. (–2) + (+3) =

   c. (–3) + (+2) =

   d. (–2) + (–1) =

   e. (–4) + (+3) =

   f. (+1) + (–3) + (+4) =

2. Find the sums. Remember to put in brackets first.

   a. –4 + 5 =

   b. 2 + –3 =

   c. +4 + –2 =

   d. –3 + –1 =

Turn to the Answer Key at the end of the module to check your work.

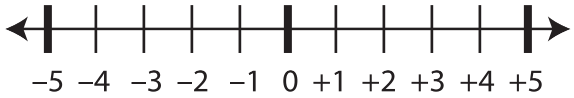
Adding Integers—Using a Number Line

Look at this example again:

$$\boxed{(+4) + (-3) =}$$

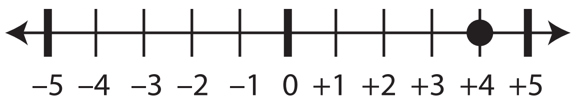
This time we will use a number line to solve it.

Step 1: Draw the number line



Step 2: Mark the starting point on the number line (this is the first integer)

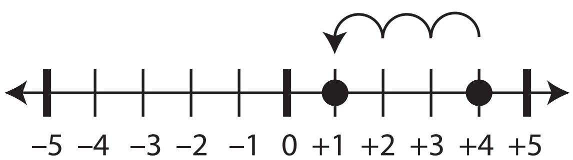
Look at the first integer, +4, and mark that with a dot on a number line.



Step 3: Now mark the change. How does the next integer affect the first one?

Okay, now let's add –3. Usually when we add, we move to the right on the number line. With integers, we have to be careful. The integer sign is a hint: the sign is negative, so we move the other way, in the negative direction. Start at the starting point, +4 and move 3 units left.

Remember: When we are deciding which way to draw the arrow, (+) means in the positive direction or right. When an integer is (–), we go in the negative direction, or left on the number line.

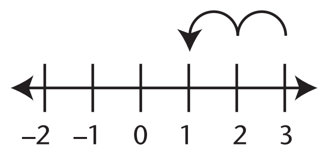


Step 4: See where we end up

On the number line we can see that after adding the second integer, we end up at (+1). Just like when we used the chips! Which method do you like? Have a look at the next example and decide which method works for you.

Example Adding Integers

(+3) + (–2) =

Zero Principle:	**Number Line:**
(+1) (+1) (+1) (+3) (–1) (–1) (–2) +1 left over	 +1

Excellent job! Which method did you like best? Why did you choose it? Thinking about *why* you do things is a good way to learn.

Exercises 4.5

1. a. $(-4) + (-1) =$

   b. $(+2) + (+6) =$

   c. $(+8) + (-3) =$

   d. $(-7) + (+4) =$

   e. $(+6) + (-3) =$

   f. $(-9) + (+4) =$

2. a. $(-2) + (-3) + (-2) =$

   b. $(+2) + (-4) + (+3) =$

3. a. $(\quad\quad) + (-5) = +5$

   b. $(\quad\quad) + (-8) = -6$

4. The sum of two integers is –7. Give four possible equations.

Turn to the Answer Key at the end of the module to check your work.

Adding Integers—Far From Zero

The same methods can be used when adding integers with larger values. We can use the zero pair principle or a number line to solve these equations just as we did before. Take a look at this example; we will use the zero pair principle to solve it.

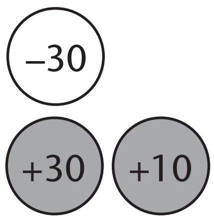
Using the Zero Pair Principle

Let's use this Example

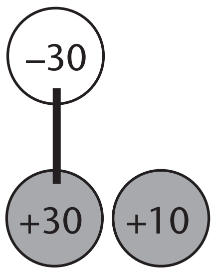
$$\boxed{(+40) + (-30) =}$$

Step 1: Line up integer chips into zero pairs

We don't need to draw 40 positive chips and 30 negative chips.
We need 30 positive chips to cancel out all the negative chips. Break up the (+40) into (+30) and (+10).

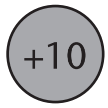


Step 2: Draw a line to each zero pair



Every zero pair equals zero.

Step 3: See what is left over



(+10) is left over after cancelling out the zero pairs.

$$(+40) + (-30) = (+10)$$

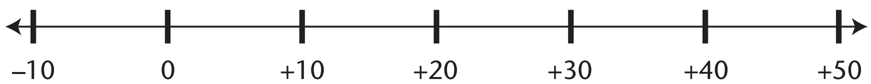
We didn't need to draw every chip for the zero pairs method, and we don't need to mark every number on the number line. Make a number line with +10, +20, etc.

Now let's try the same question using a number line.

Using a Number Line

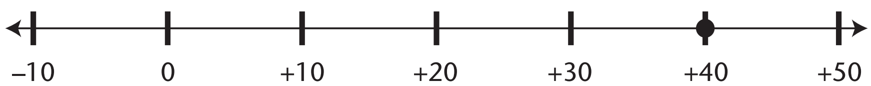
Step 1: Draw the number line

$$(+40) + (-30) =$$



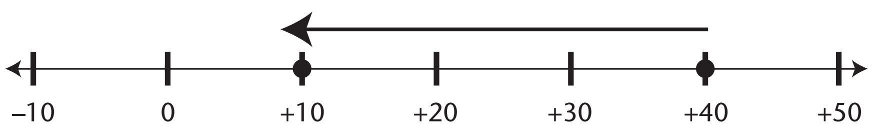
Step 2: Mark the first integer on the number line as the starting point

Mark +40 with a dot on a number line.



Step 3: Now mark the change

OK, now let's add –30. Remember: the integer sign is a hint.
Which direction should we move?



Step 4: Where do we end up?

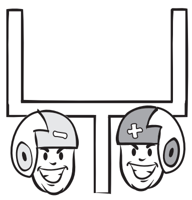
On the number line we can see that after adding the second integer, we end up on (+10).

Just like smaller integers, we can add larger numbers on a number line. We just have to create a number line that includes the larger values in the question.

Adding Integers Far From Zero —The Scoreboard Method

Here is another method to try. It is called the scoreboard method. You have seen scoreboards before. They are used in football games, hockey games, and other sports. Imagine there are two teams playing against each other:

THE NEGATIVE TEAM	THE POSITIVE TEAM



An integer addition question is just like a scoreboard for a game. First you draw a score board, then record what happens in the game.

Let's try one:

$$\boxed{(-22) + (+14) =}$$

The Positive Team gets 14 points. The Negative Team gets 22 points.

THE NEGATIVE TEAM	THE POSITIVE TEAM
22	14

Who won? By how much did they win? In this example the Negative Team won by 8 points. So our answer would be -8.

Try this one:

$$\boxed{(-15) + (-10) =}$$

For this example, we have to record two integers in the Negatives. Imagine they had to play two periods and those were the scores. It would look like this:

THE NEGATIVE TEAM	THE POSITIVE TEAM
15	0
10	0

The Negatives won again, but this time they got a shut out! Add up all the scores from the periods, and you will get the answer (–25).

Now you give these a try:

$$\boxed{(-4) + (7) =}$$

THE NEGATIVE TEAM	THE POSITIVE TEAM

$$(-25) + 45 =$$

THE NEGATIVE TEAM	THE POSITIVE TEAM

Check your answers using one of the other methods.

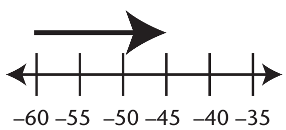
Did you get these answers?

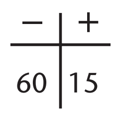
$$(-4) + (+7) = +3$$

$$(-25) + (+45) = +20$$

Example Adding Integers

$$(-60) + (+15) =$$

<table>
<tr><td colspan="3">1. Choose a method.</td></tr>
<tr>
<td>Zero Principle:
1. Line up integer chips into zero pairs. Colour the positive chips red and the negative chips blue, if it helps.
2. Draw a line between each zero pair.
3. See what is left over.</td>
<td>Number Line:
1. Make the number line.
- where is the zero?
- what is the interval?
2. Mark the first integer on the number line.
3. Mark the change using an arrow.
4. See where you end up.</td>
<td>The Scoreboard:
1. Draw the scoreboard.
2. Report the scores for each team game.
3. Find out which team won and by how much.</td>
</tr>
<tr><td colspan="3">2. Solve.</td></tr>
<tr><td colspan="3">Once you have chosen a method, solve the equation here. Compare your answer to the solutions below.</td></tr>
<tr><td colspan="3">3. Compare your solution.</td></tr>
<tr>
<td>Zero Principle:
(–15) (–45)
(+15)
–45 left over</td>
<td>Number Line:


–45 left over</td>
<td>The Scoreboard:


The Negative Team won by 45 points.</td>
</tr>
</table>

How did you do? Which method works best for you?

Are you ready to try some on your own? Look back at these examples if you are stuck or need a hint.

Exercises 4.6

1. Predict whether the sum will be positive or negative.
   a. (–50) + (–20) will be
   b. (–50) + (+20) will be

2. Calculate.
   a. (+5) + (+3) =
   b. (–5) + (–3) =
   c. (–60) + (+20) =
   d. (–10) + (–15) =

3. Calculate.
   a. (–25) + (+35) =
   b. (–15) + (+38) =

4. Arrange the temperatures in order from coldest to warmest.
   –17°C, 27°C, –6°C, 0°C, 16°C, 2°C, 22°C

5. If the temperature is –15°C, what will the temperature be if it:
   a. increases 20°C

   b. increases 15°C

   c. increases 5°C

6. Ryan walks up and down a staircase. He starts on the 5th step and walks:
   - up 2 steps
   - down 3 steps
   - up 4 steps
   - down 5 steps

   What step does he finish on?

7. Add.
   a. $(+1) + (+2) =$

   b. $(-3) + (-2) =$

   c. $(+5) + (-4) =$

   d. $(-6) + (+2) =$

   e. $(-5) + (-3) =$

   f. $(+4) + (-4) =$

8. Find the missing numbers.

   a. $(+7) + (\quad) = (+5)$

   b. $(-3) + (\quad) = (+1)$

   c. $(-4) + (\quad) = (-6)$

   d. $(\quad) + (+2) = (-4)$

9. a. $(-12) + (-6) + (-18) =$

   b. $(-37) + (-20) + (+12) =$

10. Predict whether the sum will be positive or negative.

    a. $(-20) + (+50)$ will be ____________________

    b. $(-20) + (-50)$ will be ____________________

11. Calculate.

    a. $(-10) + (-15) =$

    b. $(-15) + (+10) =$

    c. $(+100) + (-80) =$

    d. $(+125) + (-52) =$

    e. $(+125) + (-32) =$

Turn to the Answer Key at the end of the module to check your work.

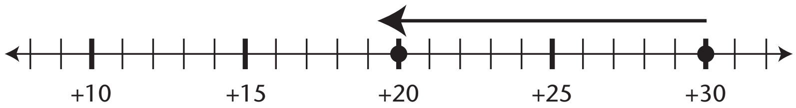
Subtracting Integers

Introduction

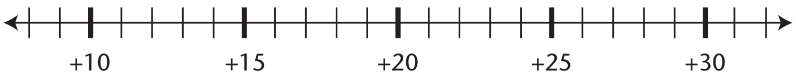
The great news about this lesson is that you will learn about how we can make subtraction disappear! Well, it still exists, but we're going to look at it in a new way. In this lesson you will learn about how to use integers to turn all subtraction questions into addition questions.

1. Use the number line to subtract. Look closely at the intervals on the number lines. The first one is done for you.

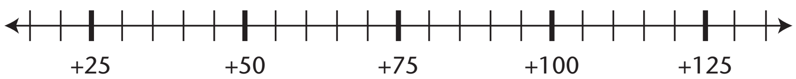
   a. 30 – 10 = 20



   b. 25 – 5 =



   c. 120 – 75 =



2. Subtract.

   a. 37 – 24 =

   b. 54 – 8 =

   c. 317 – 97 =

   d. 1072 – 67 =

   e. 47 – 39 =

   f. 515 – 11 =

Answers

1. b. 20 c. 45
2. a. 13 b. 46 c. 220 d. 1005 e. 8 f. 504

Subtracting Integers

Have you ever received a gift card for a birthday present? Gift cards are a perfect example of how subtracting integers works.

Imagine that you could spend the balance, and more than the balance, as long as you reloaded your gift card back to zero. Take a look at this example.

a. Gift card balance is $10.00
b. You buy a CD for $15.00
c. Your new gift card balance is –$5.00 because you spent more than the original balance.

If you bought another CD for $15.00, you would be subtracting $15.00 from an already negative balance. In other words, we are subtracting $15.00 from the balance of the card.

$$(-5) - (+15) =$$

a 5 dollar debt minus 15 dollars

You can look at it another way:

$$(-5) + (-15) =$$

a 5 dollar debt plus another 15 dollar debt

There is already a negative balance and we are adding more debt.

$$(-5) - (+15) = (-5) + (-15) = -20$$

Let's try some more examples:

$$\boxed{(4) - (-6) =}$$

For this example, the integer following the subtraction sign is (–6). When we change the operation sign from subtraction to addition, the sign of the integer must also change.

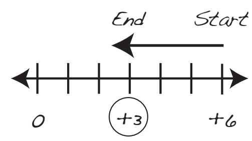
$$(4) + (+6) =$$

$$(4) + (+6) = +10$$

Practice turning these equations into addition questions.

	STEP 1	**STEP 2**	**STEP 3**
	Add brackets and integer signs, if necessary.	Do the switch (add the opposite).	Solve using addition. (Use any method you like: zero pairs, number lines, or the scoreboard.)
(5) – (+8) =	(+5) – (+8) =	(+5) + (–8) =	− | + win by 3 8 | 5 −3
(–7) – (–2) =			
(6) – (3) =			
7 – (+4) =			
–3 – 2 =			

Great job, now compare your answers to the solutions below.

	STEP 1	STEP 2	STEP 3
	Add brackets and integer signs, if necessary.	Do the switch (add the opposite).	Solve using addition. (Use any method you like: zero pairs, number lines, or the scoreboard.)
(5) – (+8) =	(+5) – (+8) =	(+5) + (–8) =	– | + win by 3 8 | 5 –3
(–7) – (–2) =	(–7) – (–2) =	(–7) + (+2) =	–2 –5 left over +2 –5
(6) – (3) =	(+6) – (+3) =	(+6) + (–3) =	
7 – (+4) =	(+7) – (+4) =	(+7) + (–4) =	– | + win by 3 4 | 7 +3
–3 – 2 =	(–3) – (+2) =	(–3) + (–2) =	– | + win by 5 3 | 0 2 | 0 –5

Exercises 4.7

1. Solve.

   a. (+3) – (–2) =

   b. (–7) – (–3) =

   c. (–2) – (+6) =

   d. (–1) – (+4) =

2. Solve.

   a. (+9) – (+2) – (+4) =

   b. (–6) – (–4) – (–3) =

   c. (+3) – (–1) – (+4) =

   d. (–4) – (–2) – (+3) =

3. A valley is 200 metres below sea level and the top of a mountain is 2000 metres above sea level. Cullen says the difference is 2200 metres. Ann says the difference is 1800 metres. Who is right?

4. Complete.

   a. (+3) – (+7) =

   b. (–3) – (–6) =

   c. (+3) – (–5) =

   d. (–5) – (+3) =

5. Complete.
   a. $(\quad) - (+1) = +4$
   b. $(+2) - (\quad) = -3$
   c. $(-3) - (\quad) = +2$
   d. $(\quad) - (-4) = 0$

6. An eagle is flying 3 metres above the ocean and spots a salmon swimming 1 metre below the water. How far apart are the eagle and the salmon?

   Hint: "How far apart" means you should subtract to find the answer.

   Drawing a picture might help you!

7. Mount Everest is 8848 metres above sea level. The Dead Sea is 411 metres below sea level. What is the difference between the two elevations?

   Be sure to show your work and answer in a complete sentence.

Turn to the Answer Key at the end of the module to check your work.

Lesson 4
Integers, Part 2

Integers are numbers that have direction. Every value has a **size** and a **direction**.

For Example

–3	direction: negative size: 3
+2	direction : positive size 2

When multiplying and dividing with integers, the **size** of the answer is exactly the same as it always has been. In this lesson you will learn about finding the **direction** of the answer.

Practise thinking about the **size** of the answer with these multiplication and division questions.

Multiplying and Dividing

1. $4 \times 5 =$
2. $21 \div 7 =$
3. $3 \times 8 =$
4. $63 \div 9 =$
5. $2 \times 7 =$
6. $18 \div 6 =$
7. $4 \times 4 =$
8. $15 \div 5 =$
9. $6 \times 4 =$
10. $72 \div 9 =$

Answers

1. 20 2. 3 3. 24 4. 7 5. 14 6. 3 7. 16 8. 3 9. 24 10. 8

Multiplication

You know lots about multiplication already.

If you have 4 groups with 3 items in each group, you can figure out the total number of items by multiplying.

$$4 \times 3 = 12$$

There are twelve items all together.

Perhaps in one of the other math courses you've taken, you've learned how to do problems like this one.

Example

Susan worked at a tulip farm last spring, packaging bulbs in boxes before they were sent to the store. She put 15 bulbs in every box. On her most productive day, she filled 42 boxes. How many bulbs did she pack?

Groups:	boxes
Items:	bulbs
# of groups:	Susan packed 42 boxes.
# of items in one group:	There were 15 bulbs in each box.
Total items:	We don't know. Multiply to find out.
	$42 \times 15 = 630$
	Susan packed 630 bulbs.

Use this structure to solve the problem.

Groups: ____________________

Items: ____________________

# of groups: ______________________________

# of items in one group: ______________________________

Total items: __________

Exercises 4.8

These are all multiplication problems.

1. This question is very similar to the example. Follow the example if you need help.

   Susan worked at a tulip farm last spring, packaging bulbs in boxes before they were sent to the store. She put 24 bulbs in every box. On her most productive day, she filled 370 boxes. How many bulbs did she pack?

   Fill in the blanks with the correct numbers.

Groups: boxes
Items: bulbs
# of groups: Susan packed ____________ boxes.

# of items in one group: There were ____________ bulbs in each box.

Total items: We don't know. Multiply to find out

________ × ________ = ________

Susan packed ________ bulbs.

2. In the winter, Amir feeds his cows four bales of hay every day. Spring is coming and he thinks that he will be able to put the cows out on the pasture in 45 days. How many bales of hay does he need?

   Fill in the blanks with the correct numbers.

Groups: days
Items: bales of hay
# of groups: Amir needs hay for ____________ days.

# of items in one group: He needs ____________ bales of hay each day.

Total items: We don't know. Multiply to find out.

________ × ________ = ________

Amir needs ________ bales of hay.

3. A box of collectible trading cards contains 24 packs. Each pack has 15 trading cards. How many cards are in a box?

   Draw a picture that describes this situation.

   Fill in the blanks with the correct numbers.

Groups:	<u>packs of trading cards</u>
Items:	<u>cards in each pack</u>
# of groups:	________ packs
# of items in one group:	________ cards in each pack
Total items:	We don't know. Multiply to find out.
	________ × ________ = ________
	There are ________ trading cards in a box.

4. Alexis got a beading kit for her birthday. The kit contains six pouches of coloured of beads, and there are 22 beads in each pouch. How many beads are in the kit?

   Draw a picture that describes this situation.

   Fill in the blanks with the correct numbers.

   Groups: pouches of beads
   Items: beads

   # of groups: ________ pouches of beads

   # of items in one group: ________ beads in each pouch

   Total items: We don't know. Multiply to find out.

   ________ × ________ = ________

   There are ________ beads in the kit.

5. Chris earns $9 per hour at the Burger Hut. He worked 21 hours last week. How much money did he make?

Draw a picture that describes this situation.

Fill in the blanks.

Groups: ____________________

Items: ____________________

# of groups: ________

# of items in one group: ________

Total items: We don't know. Multiply to find out.

________ × ________ = ________

Chris earned $ ________ last week.

6. Jamie's band gets paid 5¢ every time someone buys one of their songs from an Internet music store. Their latest hit has been downloaded 3726 times since it was posted this morning. How much money have they made?

   Think about groups, items, and total when you solve the problem.

Turn to the Answer Key at the end of the module to check your work.

Division

How did that go? Make sure you have checked your answers before you move on.

Division is the opposite of multiplication. If you have twelve items in four equal groups, you can figure out the number of items in each group by dividing.

$$12 \div 4 = 3$$

There are three items in each group.

When you work on this next set of questions, think about how they are different from the last set of questions.

Exercises 4.9

These are all division problems.

1. One day at the tulip farm, Susan’s boss bought pizza for everyone. They packed 60 000 tulips yesterday! With 24 tulips in each box, how many boxes did they fill?

Groups: <u>boxes</u>

Items: <u>bulbs</u>

Fill in the blanks with the correct numbers.

# of groups: We don’t know. Divide to find out.

# of items in one group: There were ________ bulbs in each box.

Total items: There were ________ bulbs altogether.

________ ÷ ________ = ________

They filled ________ boxes with bulbs.

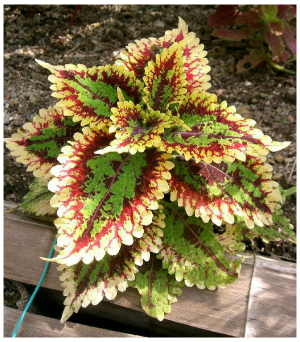
2. The calendar fundraiser is going well. The class keeps $3 for every calendar that they sell. They have set a fundraising goal of $465. How many calendars do they need to sell?

   Draw a picture that describes this situation.

Groups:	calendars
Items:	dollars for each calendar that they sell
# of groups:	We don't know. Divide to find out.
# of items in one group:	The class earns $_______ for each calendar that they sell.
Total items:	The class wants to raise $_______.
	_______ ÷ _______ = _______
	They need to sell _______ calendars to reach their goal.

3. The landscaper for a housing development has 585 coleus seedlings ready to be transplanted.

   How many seedlings can she plant at each one of 13 new houses?



Groups: <u>new houses</u>

Items: <u>seedlings</u>

\# of groups: There are ________ new houses.

\# of items in one group: We don't know. Divide to find out.

Total items: There are ________ seedlings.

________ ÷ ________ = ________

The landscaper can plant ________ coleus seedlings in each yard.

4. Chris needs $648 to buy a new guitar. How many hours does he need to work at the Burger Hut, where he earns $9 per hour, to make that much money?

   Fill in the blanks.

Groups: ____________

Items: ____________

\# of groups: We don't know. Divide to find out.

\# of items in one group: ____________

Total items: ____________

________ ÷ ________ = ________

Chris needs to work for ________ hours to earn the money to buy the guitar.

5. Alexis has 56 beads left in her beading kit. She has worked out a design that she likes for a bracelet with 7 beads. How many bracelets can she make with the beads that she has left?

   Draw a picture that describes this situation.

   Fill in the blanks.

   Groups: ____________

   Items: ____________

   # of groups: We don't know. Divide to find out.

   # of items in one group: ____________

   Total items: ____________

   ________ ÷ ________ = ________

   Alexis can make ________ bracelets.

6. Nancy has noticed that she is nearly out of one brand of collectible trading cards at her store. One box, which contains 24 packs of cards, costs $18. How much does she pay for each pack of cards?

   Think about groups, items, and total when you solve the problem.

Turn to the Answer Key at the end of the module to check your work.

Deciding Whether to Use Multiplication or Division

How are multiplication questions different from division questions?

Think about groups, items in a group, and total.

If you need to find the total, you're doing a multiplication question.

If you know the total, you're doing a division question.

Exercises 4.10

There are multiplication and division problems here.

1. Amir has 192 bales of hay. If he feeds his cows 4 bales every day, how many days will his hay last?

   Think about groups, items, and total when you solve the problem.

   Draw a picture that describes this situation.

   Fill in the blanks with the description and the correct number, or write "We don't know."

   Groups: ____________________

   Items: ____________________

   Total: ____________________

   ________ ÷ ________ = ________

   ____

   ____

2. The class has sold 32 calendars so far in this year's fundraiser. The calendars sell for $14. How much money have they collected?

   Think about groups, items, and total when you solve the problem.

   Draw a picture that describes this situation.

   Fill in the blanks with the description and the correct number, or write "We don't know."

   Groups: ____________________

   Items: ____________________

   Total: ____________________

   ______ ÷ ______ = ______

   ____

   ____

3. The 5 members of Jamie's band are celebrating. They have earned $700 selling their songs at an Internet music store. How much money do they each get?

   Think about groups, items, and total when you solve the problem.

   Draw a picture that describes this situation.

   Fill in the blanks with the description and the correct number, or write "We don't know."

   Groups: ____________________

   Items: ____________________

   Total: ____________________

   ________ ÷ ________ = ________

   ____

   ____

4. The new housing development is almost finished. There are 13 new houses. The landscaper wants to put a cedar hedge along the driveway of each new home. She needs 8 plants for each hedge. How many cedar plants does she need?

   Think about groups, items, and total when you solve the problem.

   Draw a picture that describes this situation.

   Fill in the blanks with the description and the correct number, or write "We don't know."

   Groups: ______________________

   Items: ______________________

   Total: ______________________

   ________ ÷ ________ = ________

   ____

   ____

Turn to the Answer Key at the end of the module to check your work.

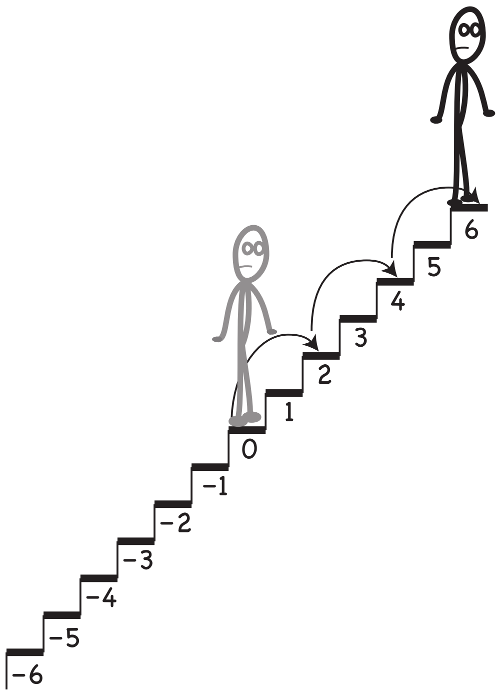
Multiplying and Dividing with Integers

× and ÷ with a Positive Number and a Negative Number

There are lots of ways to think about positive and negative numbers. Perhaps you like thinking about temperature. The temperature can be +3° C. The temperature can be –4° C. The temperature can go up (move in a positive direction). The temperature can go down (move in a negative direction).

Maybe the money analogy is your favourite. You have \$10 (that's +10). You owe \$10 (that's –10). You earn money (move in a positive direction) and you spend money (move in a negative direction).

In this lesson we're going to think about stairs.



Start at 0.

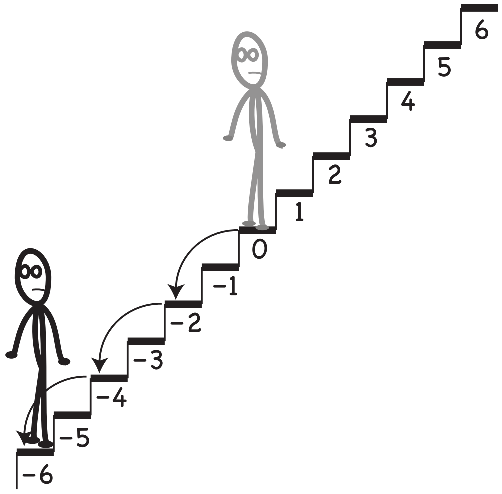
Go up two stairs.

Do that three times.

Where are you?

+6

$2 \times 3 = 6$



Go back to 0.

Go down two stairs.

Do that three times.

Where are you?

–6

$-2 \times 3 = -6$

When multiplying or dividing,
if the signs are DIFFERENT (one +, one –)
the answer is NEGATIVE.

Exercises 4.11

1. Remember: If the signs are different, the answer is negative.

a. $4 \times (-3) =$

b. $-4 \times 3 =$

c. $12 \div 1 =$

d. $-1 \times 12 =$

e. $12 \div (-4) =$

f. $-12 \div 4 =$

g. $2 \times (-6) =$

h. $-2 \times 6 =$

i. $-12 \div 3 =$

j. $12 \div (-3) =$

k. $12 \div 3 =$

l. $-12 \times 1 =$

m. $12 \times (-1) =$

n. $12 \div 2 =$

o. $12 \div (-2) =$

p. $12 \div (-6) =$

q. $-12 \div 6 =$

r. $3 \times (-4) =$

s. $4 \times 3 =$

t. $6 \times 2 =$

u. $-6 \times 2 =$

v. $6 \times (-2) =$

2. Winter is coming and the temperature is dropping. The weather forecast says to expect the temperature to go down by 3°C every day for the next 5 days. How much colder will it be on the fifth day than it is today?

3. Margaret, Halim, and André have decided to close the store that they owned together. Their company is $900 in debt. They want to split the debt equally between the three of them. How much does each of them owe?

Turn to the Answer Key at the end of the module to check your work.

× and ÷ with Two Negative Numbers

Do you remember doing "fact families"?

$$2 \times 3 = 6$$

$$3 \times 2 = 6$$

$$6 \div 2 = 3$$

$$6 \div 3 = 2$$

Let's look at the fact family that goes with 2 × (-3).

$$2 \times (-3) = -6$$

$$(-3) \times 2 = -6$$

$$-6 \div 2 = (-3)$$

$$-6 \div (-3) = 2$$

The first three facts in that list follow the rule we just learned. When the signs are different, the answer is negative.

Look at the last fact. A negative number divided by a negative number is a positive number.

Multiplication and division of integers have the same rules for signs.

When multiplying or dividing,
if the signs are the SAME (both + or both –)
the answer is POSITIVE.

Exercises 4.12

When multiplying or dividing,
if the signs are the SAME (both + or both –)
the answer is POSITIVE.

1. $4 \times 5 =$
2. $-4 \times (-5) =$
3. $4 \times (-5) =$
4. $20 \div 4 =$
5. $-20 \div (-5) =$
6. $-10 \times (-2) =$
7. $-20 \div (-2) =$
8. $20 \div 2 =$
9. $-1 \times (-20) =$
10. $3 \times (-8) =$
11. $-24 \div 8 =$
12. $-4 \times 6 =$
13. $24 \times (-1) =$
14. $24 \div (-4) =$
15. $2 \times (-12) =$
16. $-24 \div 3 =$

17. $-24 \div 1 =$

18. $(-7) \times (-7) =$

19. $7 \times 7 =$

20. $49 \div (-7) =$

21. $49 \div 7 =$

22. $-5 \times (-5) =$

23. $5 \times 5 =$

24. $-25 \div (-25) =$

25. $-1 \times (-5) =$

26. $-4 \times (-4) =$

27. $4 \times 4 =$

28. $16 \div (-4) =$

29. $-16 \div 4 =$

30. $4 \div 2 =$

31. $4 \div (-2) =$

32. $-4 \div 2 =$

33. $-4 \div (-2) =$

34. $56 \div 8 =$

Turn to the Answer Key at the end of the module to check your work.

× and ÷ with More Than Two Numbers

What about questions with more than two numbers?

$$2 \times 3 \times (-4) \qquad (2)(-3)(-4) \qquad \frac{12(-3)}{4}$$

$$(3)(-1)(-2)(-5)$$

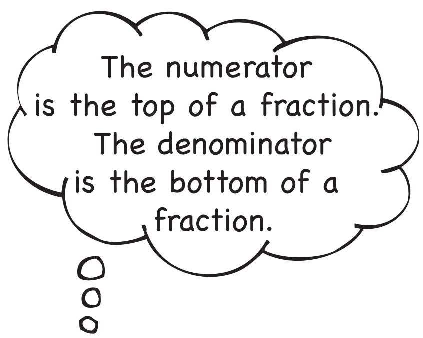
$$\frac{12(-3)(5)}{(-6)(-15)} \qquad \frac{5(-2)(-1)(6)}{15}$$

In the last exercise, you did several multiplication and division questions with two negative numbers. The signs were the same, so the answer was always positive. If there is an *even* number of negative signs, the answer is *positive*. If there is an *odd* number of a negative signs, the answer is *negative*.

When an expression contains only the operations of multiplication and division, you can do the operations in any order that you like.

Let's look at a question with more operations.

$$\frac{12(-3)(5)}{(-6)(-15)}$$



Will this answer be positive or negative? The **numerator** will be negative and the **denominator** will be positive. A negative divided by a positive is negative. This answer will be negative.

There are a number of different ways to do this question. We're going to look at two of them.

$\dfrac{12(-3)(5)}{(-6)(-15)}$ Multiply everything that is in the numerator.
Multiply everything that is in the denominator.

Remember: the fraction bar means "divided by". $\dfrac{-180}{90}$ means $-180 \div 90$.

$= \dfrac{-180}{90}$ Divide.

$= -2$

That method works, but sometimes the numbers get pretty big after the multiplication step. This time, let's simplify by doing some of the division first.

$\dfrac{12(-3)(5)}{(-6)(-15)}$ There are an odd number of negative signs, so the answer will be negative.

$= -\dfrac{12(3)(5)}{(6)(15)}$

$= -\dfrac{{}^{2}\cancel{12}(3)(5)}{{}_{1}(\cancel{6})(15)}$ 6 divides into 12 twice. 6 divides into 6 once.

$= -\dfrac{2(3)(\cancel{5})^{1}}{(1)(\cancel{15})_{3}}$ 5 divides into both 5 and 15.

$= -\dfrac{2(\cancel{3})^{1}(1)}{(1)(\cancel{3})_{1}}$ 3 divides into both 3 and 3.

$= -\dfrac{2(1)(1)}{(1)(1)}$

$= -2$

Exercises 4.13

When multiplying or dividing,

if there are an EVEN number of negative signs, the answer is POSITIVE.

If there are an ODD number of negative signs, the answer is NEGATIVE.

1. $\frac{12}{(3)(-1)(2)}$

2. $\frac{(3)(-3)(-7)}{(-9)}$

3. $(4)(5)(-1)$

4. $\frac{(-16)(25)(-2)}{(10)(-4)}$

5. $(2)(-5)(7)(-2)$

6. $\frac{(6)(-4)(2)}{-12}$

7. $\frac{(24)(-14)}{(-8)(-7)(-1)}$

8. $\frac{(8)(-7)}{4(14)}$

9. $(-1)(2)(-3)(4)(-5)$

10. $\dfrac{(-15)(6)}{-9}$

11. $(5)(-3)(2)$

12. $\dfrac{(-21)(9)}{(7)(-3)}$

Turn to the Answer Key at the end of the module to check your work.

Lesson 5
Powers

Learning Outcomes

By the end of this lesson you will be better able to:

- write powers as the product of factors and explain their meaning
- evaluate expressions involving powers with integer bases
- use the exponent rules for multiplying and dividing powers

Exponents

Exponents, those tiny raised numbers, are a clearer way to communicate multiplying over and over with the same number.

Instead of writing:

$$3 \times 3$$

we usually write:

$$3^2$$

which means "two 3s multiplied together". When you read it, you can say "three squared" or "three to the power of two" or "three to the second power".

The expression

$$3 \times 3 \times 3 \times 3 \times 3$$

can be written more clearly using an exponent. That expression has five 3s multiplied together. It can be written as:

$$3^5$$

When you read it, say "three to the power of five" or "three to the fifth power".

The tiny raised 5 is called an **exponent**.

The 3 is called the **base**.

The entire expression, and the number it represents, is called a **power** of 3.

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

243 is a power of 3. It is the fifth power of 3.

Here are some examples that use exponents to write expressions in a simpler, clearer way.

$$2 \times 2 \times 2 \times 2 = 2^4$$

$$7 \times 7 \times 7 = 7^3$$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3$$

$$2.4 \times 2.4 = 2.4^4$$

$$10 \times 10 \times 10 \times 10 \times 10 = 10^5$$

Evaluating Exponential Expressions

When you want to find the value of an exponential expression, it can be helpful to **expand** the expression. Expanding means writing it out to show all of the repeated multiplications.

$$\begin{aligned} 2^4 &= 2 \times 2 \times 2 \times 2 \\ &= 4 \times 2 \times 2 \\ &= 8 \times 2 \\ &= 16 \end{aligned}$$

Remember that multiplication doesn't care about order, so you can perform the multiplication steps however you like. Here's the same example from above, worked out slightly differently.

$$\begin{aligned} 2^4 &= 2 \times 2 \times 2 \times 2 \\ &= 4 \times 4 \\ &= 16 \end{aligned}$$

Exercises 5.1

1. Rewrite these expressions using exponents.

   a. 4×4

   b. $9 \times 9 \times 9$

   c. $2 \times 2 \times 2 \times 2 \times 2$

   d. $7 \times 7 \times 7 \times 7$

   e. $10 \times 10 \times 10 \times 10$

   f. $\frac{2}{5} \times \frac{2}{5}$

   g. $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

   h. $\frac{3}{4} \times \frac{3}{4}$

2. Expand these powers and evaluate.

For example $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

a. 2^3

b. 5^2

c. 3^3

d. 10^2

e. 10^3

f. $\left(\frac{1}{3}\right)^2$

g. $\left(\frac{1}{2}\right)^3$

h. $\left(\frac{3}{4}\right)^3$

Turn to the Answer Key at the end of the module to check your work.

Multiplying with Exponents

Multiplying with powers of the same number can be simplified by using exponents. Follow the example of the first question below to complete the next two questions.

$$10^2 \times 10^3 = \underline{\ 100\ } \times \underline{\ 1000\ } \times \underline{\ 100\,000\ } = \underline{\ 10^5\ }$$

$$10^3 \times 10^1 = \underline{\qquad} \times \underline{\qquad} \times \underline{\qquad\quad} = \underline{\qquad}$$

$$10^4 \times 10^2 = \underline{\qquad} \times \underline{\qquad} \times \underline{\qquad\quad} = \underline{\qquad}$$

Check that you have these answers:

$$10^3 \times 10^1 = \underline{\ 1000\ } \times \underline{\ 10\ } \times \underline{\ 10\,000\ } = \underline{\ 10^4\ }$$

$$10^4 \times 10^2 = \underline{\ 10000\ } \times \underline{\ 100\ } \times \underline{\ 100\,000\ } = \underline{\ 10^6\ }$$

There is a relationship between the exponents in the original question and the exponent in the answer. Study the examples and see if you can find the pattern.

When powers of the **same** base are multiplied, the product is also a power of that same base. You can multiply by adding the exponents.

Now look again at our three examples:

$$10^2 \times 10^3 = 10^{2+3} = 10^5$$

$$10^3 \times 10^1 = 10^{3+1} = 10^4$$

$$10^4 \times 10^2 = 10^{4+2} = 10^6$$

We can also use this method with bases other than 10. The only thing we must make sure of is that powers of the **same** base are multiplied.

$3^2 \times 5^3 = ?$

$\uparrow \quad \uparrow$

The bases are 3 and 5. We **cannot** use our property in this question.

So $3^2 \times 5^3 = 9 \times 125 = 1125$

$2^3 \times 2^4 = ?$

$\uparrow \quad \uparrow$

The bases are the same.

By the long method, $2^3 \times 2^4 = 8 \times 16 = 128 = 2^7$

By the property, $2^3 \times 2^4 = 2^{3+4} = 2^7$

Exercises 5.2

Evaluate each expression.

- If the bases are the SAME, add the exponents and leave your answer in exponential form.
- If the bases are DIFFERENT, you need to evaluate each power and then multiply.

For example

$3^2 \times 3^3$	$3^2 \times 2^3$
$= 3^5$	$= 9 \times 8$
	$= 72$

1. $10^5 \times 10^2 =$

2. $3^4 \times 3^2 =$

3. $2^2 \times 2^3 =$

4. $2^2 \times 3^2 =$

5. $2^2 \times 2^4 =$

6. $10^1 \times 10^2 =$

7. $2^4 \times 2^5 =$

8. $6^2 \times 6^2 \times 6^2 =$

9. $3^2 \times 3^3 =$

10. $5^1 \times 5^2 =$

11. $2^2 \times 2^1 \times 2^2 =$

12. $10^3 \times 10^1 =$

13. $6^2 \times 2^2 =$

14. $3^2 \times 5^2 =$

15. $2^4 \times 2^4 =$

16. $3^2 \times 4^3 =$

Turn to the Answer Key at the end of the module to check your work.

Dividing with Exponents

Dividing with powers of the same number can also be simplified by the use of exponents. Follow the example of the first question below to complete the next two questions.

$$10^5 \div 10^3 = \frac{10^5}{10^3} = \frac{\cancel{10}^1 \times \cancel{10}^1 \times \cancel{10}^1 \times 10 \times 10}{\cancel{10}_1 \times \cancel{10}_1 \times \cancel{10}_1} = 10 \times 10 = 100 = 10^2$$

$$10^4 \div 10^1 = \_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_ = \_\_\_\_$$

$$10^3 \div 10^2 = \_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_ = \_\_\_\_$$

Check that you have these answers:

$$10^4 \div 10^1 = \frac{10^4}{10^1} = \frac{\cancel{10}^1 \times 10 \times 10 \times 10}{\cancel{10}_1} = 10 \times 10 \times 10 = 1000 = 10^3$$

$$10^3 \div 10^2 = \frac{10^3}{10^2} = \frac{\cancel{10}^1 \times \cancel{10}^1 \times 10}{\cancel{10}_1 \times \cancel{10}_1} = 10 = 10 = 10^1$$

When dividing powers with the SAME base,
you can subtract the exponents.

Now look again at our three examples:

$$10^5 \div 10^3 = 10^{5-3} = 10^2$$

$$10^4 \div 10^1 = 10^{4-1} = 10^3$$

$$10^3 \div 10^2 = 10^{3-2} = 10^1$$

We can also use this method with bases other than 10. But once again, we must be sure that powers of the **same** base are divided.

$3^4 \div 2^3 = ?$

$\uparrow \quad \uparrow$

The bases are 3 and 2. We **cannot** use our property in this question.

So $\dfrac{3^4}{2^3} = \dfrac{81}{8} = 10\dfrac{1}{8}$

$3^5 \div 3^4 = ?$

$\uparrow \quad \uparrow$

The bases are the same.

$$3^5 \div 3^4 = \frac{3^5}{3^4} = \frac{\cancel{3}^1 \times \cancel{3}^1 \times \cancel{3}^1 \times \cancel{3}^1 \times 3}{\cancel{3}_1 \times \cancel{3}_1 \times \cancel{3}_1 \times \cancel{3}_1} = 3^1$$

or $3^5 \div 3^4 = 3^{5-4} = 3^1 = 3$

Exercises 5.3

1. $10^5 \div 10^2 =$

2. $3^4 \div 3^2 =$

3. $8^7 \div 8^5 =$

4. $10^6 \div 10^4 =$

5. $9^5 \div 9^4 =$

Turn to the Answer Key at the end of the module to check your work.

One and Zero as Exponents

Note that $10 = 10^1$. It is a good idea to write in a little 1 when you see $10^5 \times 10 = ?$ or $3^4 \div 3 = ?$

Then $10^5 \times 10^1 = 10^{5+1} = 10^6$ and $3^4 \div 3^1 = 3^{4-1} = 3^3$

Zero as an Exponent

We define any number, except zero, raised to the zero power, as 1. For example, $5^0 = 1$, $0.2^0 = 1$, and $10^0 = 1$. There is a reason for this!

Consider this division question:

$$10^3 \div 10^3 = \frac{10^3}{10^3} = \frac{\cancel{10}^1 \times \cancel{10}^1 \times \cancel{10}^1}{\cancel{10}_1 \times \cancel{10}_1 \times \cancel{10}_1} = 1$$

If we use the property for dividing with exponents for questions in which the exponents are equal, our answer must be the same as when we do it the long way, as above.

$$10^3 \div 10^3 = 10^{3-3} = 10^0$$

It follows that 10^0 must equal 1, because the quotient for $10^3 \div 10^3$ can only have the value of 1, because 10^3 goes into 10^3 one time.

This reasoning will apply to any number, except zero. Zero to the zero power has no meaning.

$$\boxed{\text{For any number, } n\text{, except zero, } n^0 = 1.}$$

Exercises 5.4

1. Evaluate:

   a. 8^0

   b. $\frac{3^0}{11} =$

   c. $10^2 \times 10 =$

   d. $\frac{3^4 \times 3}{3^5} =$

2. Evaluate:

   a. $10^2 \times 10^5 =$

   b. $10^3 \times 10 =$

   c. $2^4 \times 2^2 =$

   d. $3^2 \times 3^0 =$

   e. $8^2 \times 8 =$

3. Express each quotient in exponential form, and then work out the answer. For example, $2^5 \div 2^3 = 2^2 = 4$

   a. $9^5 \div 9^3 =$ ______________________________

   b. $10^5 \div 10^2 =$ ______________________________

   c. $5^4 \div 5^4 =$ ______________________________

   d. $10^4 \div 10 =$ ______________________________

   e. $8^{10} \div 8^8 =$ ______________________________

4. Solve for *n*. (What number must *n* equal to make the following true?)

   a. $3^5 \times 3^4 = 3^n$ $n =$ ____________

   b. $10^2 \times 10^n = 10^5$ $n =$ ____________

   c. $2^n \times 2^7 = 2^7$ $n =$ ____________

   d. $n^5 \times n^2 = 6^7$ $n =$ ____________

e. $10^n \div 10^1 = 1$ $n =$ ____________

f. $8^6 \div 8^2 = 8^n$ $n =$ ____________

g. $5^4 \div 5^n = 5$ $n =$ ____________

h. $7^n \div 7^4 = 7^3$ $n =$ ____________

i. $3^4 \div 3^n = 3^0$ $n =$ ____________

j. $5^4 \div 5 = n^3$ $n =$ ____________

Turn to the Answer Key at the end of the module to check your work.

Negative Exponents

Just as it is difficult to think about zero factors of 2 multiplied together in the case of the expression 2^0, it is perhaps even more difficult to think about negative three factors of 2 multiplied together. That just doesn't make sense!

Look at the following chart. Do you see that each time you decrease the exponent by 1, you are dividing by a factor of 2?

2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
$2^2=\frac{2^3}{2}$ $=\frac{2\times2\times2}{2}$ $=4$	$2^1=\frac{2^1}{2}$ $=\frac{2\times2}{2}$ $=2$	$2^0=\frac{2^1}{2}$ $=1$	$2^{-1}=\frac{2^0}{2}$ $=\frac{1}{2}$	$2^{-2}=\frac{2^{-1}}{2}$ $=\frac{1}{2}\div2$ $=\frac{1}{2\times2}$	$2^{-3}=\frac{2^{-2}}{2}$ $=\frac{1}{2\times2}\div2$ $=\frac{1}{2\times2\times2}$
2^2	2^1	2^0	$=\frac{1}{2^1}$	$=\frac{1}{2^2}$	$=\frac{1}{2^3}$

A negative exponent means to divide by that number of factors instead of multiplying! So 5^{-3} is the same as $\frac{1}{5^3}$. This pattern reveals the general rule that $a^{-m}=\frac{1}{a^m}$, where $a\neq0$.

Another Way to Understand Negative Exponents

Two numbers are reciprocals of each other if their product is equal to one. For example, since $3\times\frac{1}{3}=1$, 3 and $\frac{1}{3}$ are reciprocals of each other.

$$\begin{aligned}\text{since}\quad & a^m\times a^{-m}\\ &=a^{m+(-m)}\\ &=a^0\\ &=1\end{aligned}$$

a^m and a^{-m} are reciprocals of each other.

Therefore,

$$a^{-m}=\frac{1}{a^m}$$

The following examples show a practical way of dealing with negative exponents.

Example 1

Evaluate 2^{-3}.

Solution

$$\begin{aligned} 2^{-3} &= \frac{1}{2^3} \\ &= \frac{1}{2 \times 2 \times 2} \\ &= \frac{1}{8} \end{aligned}$$

OR

$2^3 = 8$

The reciprocal of 8 is $\frac{1}{8}$.

Therefore, $2^{-3} = \frac{1}{8}$.

Example 2

Evaluate 5^{-2}.

Solution

$$\begin{aligned} 5^{-2} &= \frac{1}{5^2} \\ &= \frac{1}{5 \times 5} \\ &= \frac{1}{25} \end{aligned}$$

OR

$5^2 = 5 \times 5 = 25$

The reciprocal of 25 is $\frac{1}{25}$.

Therefore, $5^{-2} = \frac{1}{25}$.

Example 3

Evaluate 10^{-3}.

Solution

$$\begin{aligned} 10^{-3} &= \frac{1}{10^3} \\ &= \frac{1}{10 \times 10 \times 10} \\ &= \frac{1}{1000} \\ &= 0.001 \end{aligned}$$

OR

$10^3 = 1000$

$\frac{1}{1000}$ is the reciprocal of 1000, so

$10^{-3} = \frac{1}{1000}$

and

$10^3 = 0.001$

Powers of 10 with Negative Exponents

Powers of 10 are important because they're used for scientific notation. Here are a few examples of powers of 10 with negative exponents.

Example

Evaluate 10^{-1} and express the answer as a decimal.

Solution

$$\begin{aligned} 10^{-1} &= \frac{1}{10^1} \\ &= \frac{1}{10} \\ &= 0.1 \end{aligned}$$

OR

$10^1 = 10$

The reciprocal of 10 is $\frac{1}{10}$, so

$10^1 = \frac{1}{10}$

$\frac{1}{10} = 0.1$

Example

Evaluate 10^{-2} and express the answer as a decimal.

Solution

$$\begin{aligned} 10^{-2} &= \frac{1}{10^2} \\ &= \frac{1}{10 \times 10} \\ &= \frac{1}{100} \\ &= 0.01 \end{aligned}$$

Example

Evaluate 10^{-4} and express the answer as a decimal.

Solution

$$\begin{aligned} 10^{-4} &= \frac{1}{10^4} \\ &= \frac{1}{10 \times 10 \times 10 \times 10} \\ &= \frac{1}{10\ 000} \\ &= 0.000\ 1 \end{aligned}$$

From these examples, fill in the missing values. See if you can predict the next few values.

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$

$$10^{-3} = \underline{\qquad\qquad} = \underline{\qquad\qquad}$$

$$10^{-4} = \frac{1}{10 \times 10 \times 10 \times 10} = 0.000\ 1$$

$$10^{-5} = \underline{\qquad\qquad\qquad\qquad} = \underline{\qquad\qquad}$$

$$10^{-6} = \underline{\qquad\qquad\qquad\qquad\qquad} = \underline{\qquad\qquad}$$

$$10^{-7} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad} = \underline{\qquad\qquad}$$

Answers

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$

$$10^{-3} = \frac{1}{10 \times 10 \times 10} = 0.001$$

$$10^{-4} = \frac{1}{10 \times 10 \times 10 \times 10} = 0.000\ 1$$

$$10^{-5} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10} = 0.000\ 01$$

$$10^{-6} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = 0.000\ 001$$

$$10^{-7} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10} = 0.000\ 000\ 1$$

You may have noticed the pattern that tells you how to evaluate powers of 10 with negative exponents.

For a power of 10, a negative exponent tells the number of places after the decimal point.

Exercises 5.5

Evaluate the following. Express the answer as a fraction and if it is a power of 10, a decimal.

1. 3^{-2}

2. 5^{-1}

3. 10^{-8}

4. 9^{-2}

5. 10^{-9}

Try the following multiplication questions. Remember that to multiply powers, check for the same base, then add the exponents. Express the answer as a number (not a power).

6. $2^{6} \times 2^{-4}$

7. $3^{-7} \times 3^{10}$

8. $10^{2} \times 10^{-5}$

9. $10^{-3} \times 10^{-6}$

10. $10^{5} \times 10^{-2}$

Turn to the Answer Key at the end of the module to check your work.

Lesson 6
Scientific Notation

Learning Outcomes

By the end of this lesson you will be better able to:

- write very large and very small numbers using scientific notation
- use scientific notation to do calculations with very large and very small numbers

Previously, you learned the basic rules on exponents. Now you will learn how to use these rules when writing very large and very small numbers. These numbers can be written in scientific notation.

First, let's quickly review the exponent rules, using a base of 10.

Rule 1: A positive integer used as an exponent indicates the number of factors that form the product.

$$\begin{aligned} 10^5 &= 10 \times 10 \times 10 \times 10 \times 10 \\ &= 100\ 000 \end{aligned}$$

Rule 2: The integer 0, used as an exponent, always results in the value 1.

$$\begin{aligned} 10^0 &= 1 \\ 10\ 000^0 &= 1 \end{aligned}$$

Rule 3: In multiplying numbers with exponents of the same base, add the exponents.

$$10^2 \times 10^3 = 10^{2+3} = 10^5$$

Rule 4: In dividing numbers with exponents of the same base, subtract the exponents.

$$\frac{10^6}{10^4} = 10^{6-4} = 10^2$$

Rule 5: A base without an exponent is the same as a base raised to the power of 1.

$$\begin{aligned} 10 &= 10^1 \\ 10^2 \times 10 &= 10^2 \times 10^1 \\ &= 10^{2+1} = 10^3 \end{aligned}$$

Rule 6: A negative exponent tells the number of places after the decimal point.

$$\begin{aligned} 10^{-1} &= 0.1 \\ 10^{-3} &= 0.001 \end{aligned}$$

Exercises 6.1

1. Express each of the following in exponential form.

   a. $10^3 \times 10^6 =$

   b. $10^8 \div 10^7 =$

   c. $10^5 \div 10^2 =$

   d. $10^5 \times 10 =$

   e. $10^9 \div 10 =$

2. Find a value of *n* that makes each equation true.

   a. $10^5 \times 10^n = 10^6$

   b. $10^7 + 10^n = 10^4$

   c. $10^n \times 10 = 10^6$

   d. $10^5 + 10^n = 10^5$

   e. $n^3 \times n^3 = 10^6$

3. Express each of the following as a number in decimal form.

   a. $10^{-4} =$

   b. $10^{2} =$

   c. 10^{-3}

   d. $10^{0} =$

   e. $10^{5} =$

4. Express each of the following as a power of base 10.

   a. 0.001 =

   b. 10 000 =

   c. 0.1 =

   d. 100 =

   e. 1 =

Turn to the Answer Key at the end of the module to check your work.

Scientific Notation: Large Numbers

In science, we often have to use very large numbers. For example, the distance to the sun from earth is approximately 147 000 000 km.

Chemists count atoms in groups called 'moles', just like we count eggs in groups called dozens. There are 602 214 179 000 000 000 000 000 atoms in a mole.

With very very large numbers, you loose the sense of how big the number really is. It becomes just a big pile of digits.

Scientific notation makes is easier for scientists and technicians to communicate and calculate using very large numbers.

To write 147 000 000 in scientific notation, we move the decimal point as shown below:

$$\begin{aligned} 147\,000\,000 &= 1.47 \times 100\,000\,000 \\ &= 1.47 \times 10^8 \end{aligned}$$

Notice that the decimal point has been moved 8 places to the left, and the exponent of base 10 is 8.

Also notice that there is only one digit to the left of the decimal point. This digit may be from 1 to 9.

Example

Express 625 000 in scientific notation.

$$\begin{aligned} 625\,000 &= 6.25 \times 100\,000 \\ &= 6.25 \times 10^5 \end{aligned}$$

Try these two examples. Express in scientific notation:

1. 2560

2. 2 350 000

Answers

1. 2.56×10^3 2. 2.35×10^6

Often you will be given a number written in scientific notation and you will be asked to write it in standard notation. Remember the following rule:

The exponent of 10 is the number of places that the decimal point has to be moved.

Example

Express 6.3×10^7 in standard notation.

$$\begin{aligned} 6.3 \times 10^7 &= 6.3 \times 10\,000\,000 \\ &= 63\,000\,000 \end{aligned}$$

Some calculators have scientific notation built into them. For example, the number 35 000 could appear as $\boxed{3.5\;\;04}$ which is the same as 3.5×10^4 Note the space between the 2 sets of numbers. Check to see if your calculator has scientific notation built in.

Exercises 6.2

1. Find the value of n in each statement.

   a. $2500 = 2.5 \times 10^n$

   b. $125 = 1.25 \times 10^n$

   c. $21\ 000 = 2.1 \times 10^n$

   d. $275\ 000 = 2.75 \times 10^n$

   e. $26 = 2.6 \times 10^n$

   f. $6\ 750\ 000 = 6.75 \times 10^n$

   g. $85\ 600 = 8.56 \times 10^n$

   h. $740\ 000\ 000 = 7.4 \times 10^n$

2. Write each of these numbers in scientific notation.

   a. 5400

   b. 520 000

   c. 7 280 000

   d. 6000

   e. 3 020 000

   f. 352 000 000 000

3. Write in standard notation.

   a. 2.4×10^3

   b. 3.6×10^2

   c. 3.7×10^6

   d. 1.72×10^3

   e. 7.35×10^4

   f. 5.0×10^5

4. Express the number in each statement using scientific notation.

   a. The area of Canada is approximately 9 970 000 km^2.

   b. One of Saturn's rings is 26 500 km wide.

   c. The moon is 406 000 km from earth at its farthest point.

   d. The mass of a large elephant is about 4100 kg.

   e. The diameter of the sun is about 1 520 000 000 m.

5. How would a calculator show the following numbers in scientific notation?

   a. 52 000

   b. 125 000 000 000

Turn to the Answer Key at the end of the module to check your work.

Scientific Notation: Small Numbers

Scientists often use very small numbers. For example, the diameter of an atom is about 0.000 000 015 cm.

Another modern-day use of very small numbers is with CDs. The information is permanently encoded on a disc in the form of tiny bits made by a laser beam. On one side of a disc, there are as many as 8 200 000 000 bits, each measuring approximately 0.000 000 5 m across.

We can avoid writing all the zeros in very small numbers by using scientific notation. To write 0.000 000 015 in scientific notation, we move the decimal point as shown.

$$\begin{aligned} &\overset{\text{from here}}{\downarrow}\qquad\quad \overset{\text{to here}}{\downarrow} \\ &0.000\ 000\ 015 = 1.5 \times 0.000\ 000\ 01 \\ & = 1.5 \times 10^{-8} \end{aligned}$$

Notice that the decimal point has been moved 8 places to the right, and the exponent of 10 is –8.

Example

Express in scientific notation:

$$\begin{aligned} 0.000\ 062\ 5 &= 6.25 \times 0.000\ 01 \\ &= 6.25 \times 10^{-5} \end{aligned}$$

Remember these rules when working with very small numbers.

1. **The exponent of 10 is the number of places that the decimal point has to be moved.**
2. **If the original number is less than 1, the exponent is negative.**

Examples

Write the following numbers in scientific notation:

1. 0.035

2. 0.000 256

Answer

1. 3.5×10^{-2} 2. 2.56×10^{-4}

Once again, you may be given numbers written in scientific notation and will be asked to write them in standard form. Work through the following example, before trying the second one on your own.

Express in scientific notation:

1. $1.6 \times 10^{-5} = 1.6 \times 0.000\,01$
   $= 0.000\,016$

2. $5.08 \times 10^{-2} =$ ______________________________

   = ______________________________

Did you get 0.050 8 as your answer?

Small numbers written using scientific notation can also be shown on a calculator. For example, 0.000 21 will be seen as:

$$\boxed{21 \quad -04} \rightarrow 2.1 \times 10^{-4}$$

Exercises 6.3

1. Find the value of n in each statement.

   a. $0.025 = 2.5 \times 10^{n}$

   b. $0.000\ 65 = 6.5 \times 10^{n}$

   c. $0.37 = 3.7 \times 10^{n}$

   d. $0.000\ 042 = 4.2 \times 10^{n}$

   e. $0.001\ 03 = 1.03 \times 10^{n}$

   f. $0.000\ 003\ 75 = 3.75 \times 10^{n}$

2. Write these numbers in scientific notation.

   a. 0.65

   b. 0.125

   c. 0.005 5

   d. 0.000 25

   e. 0.000 000 58

   f. 0.375

   g. 0.000 075

3. Write each of the following in standard notation.

   a. 2.5×10^{-3}

   b. 3.7×10^{-7}

   c. 1.25×10^{-6}

   d. 6.15×10^{-8}

4. Write the number in each statement using scientific notation.

   a. The hummingbird of Cuba has a mass of 0.001 98 kg.

   ____

   b. The mass of a spider is 0.000 102 kg.

   ____

   c. An influenza virus measures 0.000 000 1 m across.

   ____

   d. The diameter of a molecule of water is 0.000 000 028 cm.

   ____

Turn to the Answer Key at the end of the module to check your work.

More Work On Writing Scientific Notation

Exercises 6.4

In these exercises you will be dealing with very large and very small numbers.

1. Write these numbers in scientific notation.

   a. 0.000 000 72 ____________________

   b. 6 000 000 ____________________

   c. 0.012 ____________________

   d. 7100 ____________________

   e. 0.6 ____________________

2. Write these numbers in standard notation.

   a. 7.2×10^{-5} ____________________

   b. 6.1×10^{2} ____________________

   c. 2.6×10^{-1} ____________________

Turn to the Answer Key at the end of the module to check your work.

Exercises 6.5

1. Write each in scientific notation.

Distance	Standard Notation	Scientific Notation
a. Circumference of Earth	39 900 000 m	
b. Diameter of Earth	12 742 000 m	
c. Diameter of Sun	1 392 000 km	
d. Diameter of Milky Way	9 000 000 000 000 000 000 km	
e. Speed of light	1 070 000 000 km/h	

2. Write each in scientific notation.

Radiation Type	Average Wave Length	Scientific Notation
a. Gamma Ray	0.000 000 000 061m	
b. X-ray	0.000 000 000 472 m	
c. Cosmic Rays	0.000 000 000 000 000 000 34 m	
d. Blue light	0.000 004 91 m	
e. Regular light	0.000 000 643 m	

3. Complete the chart.

Distance from Sun	Scientific Notation	Standard Notation
a. Mercury	5.79×10^7 km	
b. Earth	1.5×10^8 km	
c. Jupiter	7.7×10^8 km	
d. Neptune	4.5×10^9 km	
e. Pluto	5.91×10^9 km	

4. Complete the chart.

Mass of Particles	Scientific Notation	Standard Notation
a. Gamma Ray	3.04×10^{-2} km	
b. X-ray	2.1×10^{-7} km	
c. Cosmic Rays	1.35×10^{-8} km	

Turn to the Answer Key at the end of the module to check your work.

Calculations Using Scientific Notation

Earlier in this lesson you reviewed the rules of multiplying and dividing exponents. Let's see if you remember them.

State the value.

1. $10^2 \times 10^3$ ________________________
2. $10^{-1} \times 10^4$ ________________________
3. $10^{-5} \times 10^6$ ________________________
4. $10^7 \div 10^2$ ________________________
5. $10^2 \div 10^5$ ________________________
6. $10^{-1} \div 10^4$ ________________________

Did you remember the rules? If the bases are the same when multiplying you add the exponents, and when dividing, you subtract the exponents.

Check your answers:

1. 10^5
2. 10^3
3. 10^1
4. 10^5
5. 10^{-3}
6. 10^{-5}

Make sure you are comfortable with the ideas we have covered before you move forward in the lesson.

We will use the above rules and scientific notation to simplify calculations with very large numbers and very small numbers.

Study this Example

$42\,000 \times 500$	$= 4.2 \times 10^4 \times 5.0 \times 10^2$	Change each number into scientific notation
	$= 4.2 \times 5.0 \times 10^4 \times 10^2$	Move powers of 10 beside each other
	$= 21 \times 10^4 \times 10^2$	Do arithmetic
	$= 21 \times 10^6$	Add powers **Note:** No longer in scientific notation.
	$= 2.1 \times 10^7$	In scientific notation

Your turn to try. Just take your time and work slowly through step by step. Show work.

a. $12\,000 \times 4300$

b. 0.35×0.002 (You will need negative exponents)

Check your answers with the steps shown below:

a. $12\ 000 \times 4300$
$= 1.2 \times 10^4 \times 4.3 \times 10^3$
$= 1.2 \times 4.3 \times 10^4 \times 10^3$
$= 5.16 \times 10^7$

b. 3.5×0.002
$= 3.5 \times 10^1 \times 2.0 \times 10^{-3}$
$= 3.5 \times 2.0 \times 10^1 \times 10^{-3}$
$= \quad \times 10^{-2}$

Go over the examples carefully. Then do the Exercises.

Exercises 6.6

Find the following answers using scientific notation. Show all work. Remember in your last step to write it in scientific notation.

1. $120\ 000\ 000 \times 27\ 500\ 000$

2. $0.004 \times 0.000\ 023$

3. $5\ 000\ 000\ 000 \times 0.002\ 5$

4. $1\ 875\ 000\ 000\ 000 \times 0.4$

5. $20 \times 0.000\ 000\ 036\ 1$

Turn to the Answer Key at the end of the module to check your work.

Lesson 7
Order of Operations

Learning Outcomes

By the end of this lesson you will be better able to:

- use the **BEDMAS** acronym to solve equations involving more than one operation

Something that tells us what to do with a number or numbers is called an operation.

Addition, subtraction, multiplication, and division are the operations that you know about already. These are called the basic operations.

You have probably done questions about adding and subtracting more than two numbers in other math courses.

Previously, you learned about multiplying and dividing with more than two numbers.

What do you do with a question that involves many different operations?

BEDMAS: More Than Just a Weird Word

$$(-2)(3) \div 6 + 9 - 14 \div (9 - 2)$$

BEDMAS is an acronym that helps you to remember the order of operations.

BEDMAS
First, work out everything that is in brackets.

$$\begin{aligned} &(-2)(3) \div 6 + 9 - 14 \div \mathbf{(9-2)} \\ =\ &(-2)(3) \div 6 + 9 - 14 \div 7 \end{aligned}$$

B**E**DMAS
Next, simplify all of the exponents. (There are no exponents in this question.)

BE**DM**AS

Do all of the division and multiplication in the order they appear from left to right.

$$= \mathbf{(-2)(3)} \div 6 + 9 - 14 \div 7$$
$$= \mathbf{-6 \div 6} + 9 - \mathbf{14 \div 7}$$
$$= -1 + 9 - 2$$

BEDM**AS**

Finally, do the addition and subtraction.

$$= \mathbf{-1 + 9} - 2$$
$$= \mathbf{8 - 2}$$
$$= 6$$

Exercises 7.1

Solve the following.

1. $(-3)(7) =$
2. $4 \times 9 =$
3. $-13 \times 3 =$
4. $42 \div (-6) =$
5. $(8)(-1)(-4) =$
6. $12 + 6 =$
7. $15 \div 5 + 7 =$
8. $2 - 3 \times 4 =$
9. $3 + \frac{4}{2} =$
10. $-16 + (4)(3) =$
11. $-5 - (2)(-1)(-18) \div 4 =$
12. $6 \times 5 \div 3 =$
13. $18 \div 2 + 4 =$
14. $18 \div (2 + 4) =$

15. $6 \times 8 + 12 + 3 \times 9 =$

16. $3 + 11 \times 4 + 12 \div 3 =$

17. $7 - 3 \times 5 =$

18. $(7 - 3) \times 5 =$

19. $36 \div 9 + 2 + 1 \times 9 + 6 - 5 =$

20. $6 \times 7 \div 14 - 3 + 2 \times 4 =$

21. $5 - 1 + 2 - 4 \times 3 \div 6 =$

Turn to the Answer Key at the end of the module to check your work.

Answer Key

Lesson 1: Fractions

Exercises 1.1

a 1. denominator

d 2. improper fraction

c 3. mixed number

e 4. numerator

b 5. proper fraction

a. the 3 in $\frac{2}{3}$

b. $\frac{4}{7}$

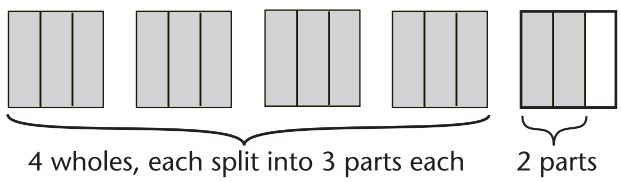
c. $2\frac{1}{5}$

d. $\frac{9}{4}$

e. the 2 in $\frac{2}{5}$

Exercises 1.2

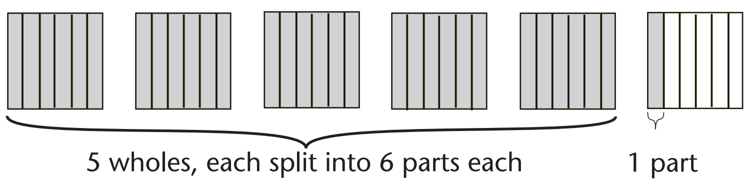
1. a.



$4 \times 3 + 2 = 14$ parts

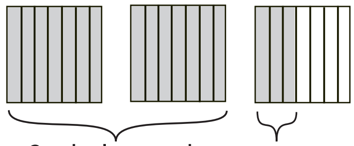
$4\frac{2}{3} = \frac{14}{3}$

b.



$5 \times 6 + 1 = 31$ parts

$5\frac{1}{6} = \frac{31}{6}$

c. 

$2 \times 7 + 3 = 17$ parts

$2\frac{3}{7} = \frac{17}{7}$

2. a. $8\frac{1}{5}$ x +

$8\frac{1}{5} = \frac{41}{5}$

b. $6\frac{2}{9}$ x +

$6\frac{2}{9} = \frac{56}{9}$

c. $3\frac{1}{4}$ x +

$3\frac{1}{4} = \frac{13}{4}$

Exercises 1.3

1. $\begin{array}{r}6\\8\overline{)53}\end{array}$ *remainder* 5 $\frac{53}{8} = 6\frac{5}{8}$

2. $\begin{array}{r}7\\5\overline{)35}\end{array}$ *remainder* 0 $\frac{35}{5} = 7$

3. $\begin{array}{r}3\\12\overline{)42}\end{array}$ *remainder* 6 $\frac{42}{12} = 3\frac{6}{12} = 3\frac{1}{2}$

4. $\begin{array}{r}4\\13\overline{)54}\end{array}$ *remainder* 2 $\frac{54}{13} = 4\frac{2}{13}$

5. $\begin{array}{r}3\\6\overline{)20}\end{array}$ *remainder* 2 $\frac{20}{6} = 3\frac{2}{6} = 3\frac{1}{3}$

Exercises 1.4

1. $\frac{2}{3} \times \frac{4}{9} = \frac{2 \times 4}{3 \times 9} = \frac{8}{27}$

2. $\frac{1}{5} \times \frac{3}{4} = \frac{1 \times 3}{5 \times 4} = \frac{3}{20}$

3. $\frac{4}{5} \times \frac{6}{13} = \frac{4 \times 6}{5 \times 13} = \frac{24}{65}$

4. $\frac{2}{7} \times \frac{1}{5} = \frac{2 \times 1}{7 \times 5} = \frac{2}{35}$

5. $\frac{8}{3} \times \frac{5}{7} = \frac{8 \times 5}{3 \times 7} = \frac{40}{21} = 1\frac{19}{21}$

Exercises 1.5

1. $\frac{3}{7} \times 3 = \frac{3}{7} \times \frac{3}{1} = \frac{3 \times 3}{7 \times 1} = \frac{9}{7} = 1\frac{2}{7}$

2. $\frac{4}{5} \times 3\frac{3}{4} = \frac{4}{5} \times \frac{15}{4} = \frac{4 \times 15}{5 \times 4} = \frac{60}{20} = 3$

3. $\frac{9}{10} \times 8 = \frac{9}{10} \times \frac{8}{1} = \frac{9 \times 8}{10 \times 1} = \frac{72}{10} = 7\frac{2}{10} = 7\frac{1}{5}$

4. $\frac{8}{5} \times 7 = \frac{8}{5} \times \frac{7}{1} = \frac{8 \times 7}{5 \times 1} = \frac{56}{5} = 11\frac{1}{5}$

5. $2\frac{1}{4} \times \frac{6}{11} = \frac{9}{4} \times \frac{6}{11} = \frac{9 \times 6}{4 \times 11} = \frac{54}{44} = \frac{27}{22} = 1\frac{5}{22}$

6. $2\frac{1}{2} \times 1\frac{1}{9} = \frac{5}{2} \times \frac{10}{9} = \frac{5 \times 10}{2 \times 9} = \frac{50}{18} = \frac{25}{9} = 2\frac{7}{9}$

7. $\frac{4}{5} \times 6 = \frac{4}{5} \times \frac{6}{1} = \frac{4 \times 6}{5 \times 1} = \frac{24}{5} = 4\frac{4}{5}$

Exercises 1.6

1. a. $\frac{3}{2}$

   b. $\frac{4}{1}$ *or* 4

   c. $\frac{1}{7}$

   d. $\frac{5}{14}$ Convert the mixed number to an improper fraction before finding the reciprocal.

2. a. $\frac{7}{5} \div 3 = \frac{7}{5} \div \frac{3}{1} = \frac{7}{5} \times \frac{1}{3} = \frac{7 \times 1}{5 \times 3} = \frac{7}{15}$

   b. $\frac{5}{6} \div 4 = \frac{5}{6} \div \frac{4}{1} = \frac{5}{6} \times \frac{1}{4} = \frac{5 \times 1}{6 \times 4} = \frac{5}{24}$

   c. $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{1 \times 3}{2 \times 2} = \frac{3}{4}$

   d. $\frac{1}{5} \div 9 = \frac{1}{5} \div \frac{9}{1} = \frac{1}{5} \times \frac{1}{9} = \frac{1 \times 1}{5 \times 9} = \frac{1}{45}$

   e. $5\frac{1}{5} \div \frac{1}{5} = \frac{26}{5} \div \frac{1}{5} = \frac{26}{5} \times \frac{5}{1} = \frac{26 \times \cancel{5}^{1}}{\cancel{5}_{1} \times 1} = 26$

   f. $\frac{1}{4} \div 2 = \frac{1}{4} \div \frac{2}{1} = \frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$

   g. $\frac{1}{5} \div \frac{1}{4} = \frac{1}{5} \times \frac{4}{1} = \frac{\cancel{1} \times 4}{5 \times \cancel{1}} = \frac{4}{5}$

   h. $\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{\cancel{2} \times 3}{5 \times \cancel{2}} = \frac{3}{5}$

i. $\frac{11}{9} \div \frac{1}{5} = \frac{11}{9} \times \frac{5}{1} = \frac{11 \times 5}{9 \times 1} = \frac{55}{9} = 6\frac{1}{9}$

j. $7 \div 2\frac{3}{4} = \frac{7}{1} \div \frac{11}{4} = \frac{7}{1} \times \frac{4}{11} = \frac{7 \times 4}{1 \times 11} = \frac{28}{11} = 2\frac{6}{11}$

Exercises 1.7

1. terminating
2. repeating
3. terminating
4. repeating

Exercises 1.8

	Fraction	Decimal
1. 4 tenths, 9 hundredths	$\frac{49}{100}$	0.49
2. 7 tenths, 3 hundredths, 5 thousandths	$\frac{735}{1000} = \frac{147}{200}$	0.735
3. 6 tenths, 4 hundredths, 2 thousandths	$\frac{642}{1000} = \frac{321}{500}$	0.642
4. 3 tenths, 2 hundredths, 1 thousandth	$\frac{321}{1000}$	0.321

Exercises 1.9

1. $\frac{4}{10}$, 0.4
2. $\frac{64}{100}$, 0.64
3. $\frac{35}{100}$, 0.35
4. $\frac{246}{1000}$, 0.246

Exercises 1.10

1. a. 0.080808...
   b. 0.353535...
   c. 0.151515...
   d. 0.848484...

2. The two repeating digits are the numerator.
   99 is the denominator.

3. a. $\frac{5}{9}$
   b. $\frac{34}{99}$
   c. $\frac{789}{999} = \frac{263}{333}$
   d. $\frac{2468}{9999}$
   e. $\frac{15}{999} = \frac{5}{333}$

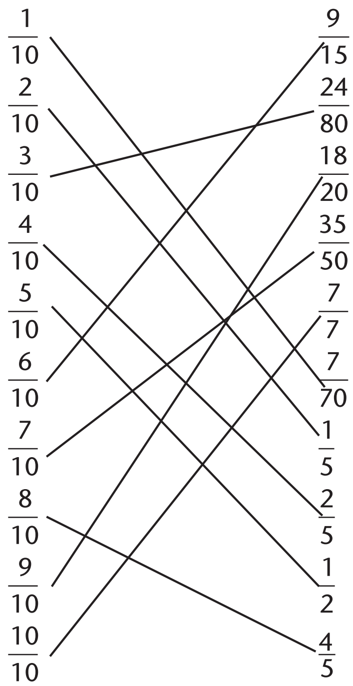
4.

Fraction	Decimal Not Rounded	Rounded
a. $\frac{16}{25}$	0.64	\$0.64
b. $\frac{80}{120}$	0.6666..	\$0.67
c. $\frac{17}{200}$	0.085	\$0.09
d. $\frac{125}{300}$	0.416666...	\$0.42

Exercises 1.11

1. $\frac{1}{2} = \frac{(1 \times 4)}{(2 \times 4)} = \frac{4}{8}$

2.



Exercises 1.12

1. b. $0.78 > \frac{3}{5}$

   c. $0.96 > \frac{19}{20}$

   d. $1.27 < \frac{38}{25}$

   e. $3.45 > \frac{162}{50}$

2. b. $0.2 < \frac{6}{25}$

   c. $0.4 > \frac{7}{20}$

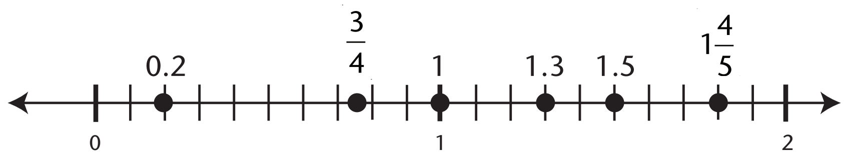
   d. $0.78 > \frac{37}{50}$

   e. $0.93 < \frac{19}{20}$

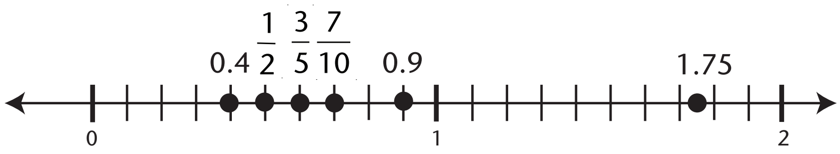
3. student's own answer

Exercises 1.13

1.



2.



Lesson 2: Ratios and Rates

Exercises 2.1

1. a. number of black counters to the number of grey counters
   b. number of black counters to the number of white counters
   c. number of grey counters to the total number of counters
   d. number of black counters to the number of white counters to the number of grey counters
   e. number of white counters to the total number of counters (6:16 can also be written as 3:8, an equivalent ratio)
2. a. part-to-part ratio
   b. part-to-part ratio
   c. part-to-whole ratio
   d. part-to-part ratio
   e. part-to-whole ratio
3. a. 12:15 (can also be written 4:5)
   b. 1:2
   c. 3:4
   d. 1:3:4
4. Answers may vary depending on which part you chose to compare.
   a. number of boys to total number of students 12:27
      number of girls to total number of students 15:27
   b. cups of water to total cups of ingredients 1:3
      cups of pancake mix to total cups of ingredients 2:3

c. days of sunshine to days in the week 3:7
   days of rain to days in the week 4:7
d. number of pants to total number of clothing articles 1:8
   number of shorts to total number of clothing articles 3:8
   number of T-shirts to total number of clothing articles 4:8

5. a. The ratios are proportional. Multiply both terms in 2:4 by 3 to get 6:12.
   b. The ratios are not proportional. There is no factor that you can multiply or divide either of the ratios by to get the other ratio.
   c. The ratios are proportional. Divide both terms in 16:30 by 2 to get 8:15.

Exercises 2.2

1. a. 110 km/2 h
   b. $11.19/3 kg
   c. 30 beats/10 s

2. Answers will vary. Sample answers are given below.
   a. Sean drove 400 kilometres and used 28 litres of fuel.
   b. Cindy drove at a speed of 70 kilometres per hour.
   c. Tahlia earned $72 for 5 hours of work.

3. a. $\frac{110 \text{ km}}{2 \text{ h}} = \frac{?}{1 \text{ h}}$

   $\frac{110 \text{ km}}{2 \text{ h}} = \frac{55 \text{ km}}{1 \text{ h}}$ The unit rate is 55 km/h.

   b. $\frac{\$11.19}{3 \text{ kg}} = \frac{?}{1 \text{ kg}}$

   $\frac{\$11.19}{3 \text{ kg}} = \frac{\$3.73}{1 \text{ kg}}$ The unit rate is $3.73/kg.

   c. $\frac{30 \text{ beats}}{10 \text{ s}} = \frac{?}{1 \text{ s}}$

   $\frac{30 \text{ beats}}{10 \text{ s}} = \frac{3 \text{ beats}}{1 \text{ s}}$ The unit rate is 3 beats/s.

4. 4-pack unit price $= \dfrac{\text{cost}}{\text{quantity}}$

$$= \frac{\$6.68}{4 \text{ batteries}}$$

$$= \$1.67/\text{battery}$$

10-pack unit price $= \dfrac{\text{cost}}{\text{quantity}}$

$$= \frac{\$13.90}{10 \text{ batteries}}$$

$$= \$1.39/\text{battery}$$

The price per battery is lower if you buy the 10-pack.

Exercises 2.3

1. First convert 30 minutes to 0.5 hours.

$$\text{rate of speed} = \frac{\text{distance traveled}}{\text{time}}$$

$$= \frac{8.5 \text{ km}}{0.5 \text{ h}}$$

$$= 17 \text{ km/h}$$

Cassie rides her bike at 17 km/h.

2.

$$\overset{\times 3}{\frac{1 \text{ can}}{9 \text{ m}^2} = \frac{x}{27 \text{ m}^2}} \qquad \textit{OR} \qquad \frac{9 \text{ m}^2}{1 \text{ can}} = \frac{27 \text{ m}^2}{x}$$

$$\times 3$$

$$\frac{1 \text{ can}}{9 \text{ m}^2} = \frac{3 \text{ cans}}{27 \text{ m}^2}$$

It will take 3 cans of paint to paint the room.

3. a. Hardware store job: hourly wage $= \dfrac{\text{amount paid}}{\text{hours worked}}$

$$= \frac{\$440}{40 \text{ h}}$$

$$= \$11/\text{h}$$

Chris would earn $11/h working at the hardware store.

Library job: $\text{hourly wage} = \dfrac{\text{amount paid}}{\text{hours worked}}$

$$= \frac{\$350}{25\text{ h}}$$

$$= \$14/\text{h}$$

Chris would earn \$14/h working at the library.

b. It depends what Chris wants. If she needs money, she should work the first one because she'll make more per week. If she only wants a part time job, the library pays a better rate.

4. Set up a proportion:

$$\frac{\$11.47}{194\text{ kWh}} = \frac{x}{230\text{ kWh}}$$

This solution shows the cross-product method.

$$(\$11.47)(230\text{ kWh}) = (194\text{ kWh})(x)$$

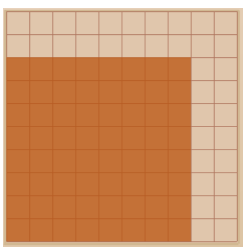
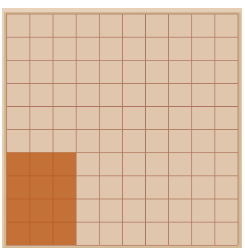
$$\frac{(\$11.47)(230\text{ kWh})}{194\text{ kWh}} = \frac{(194\text{ kWh})(x)}{194\text{ kWh}}$$

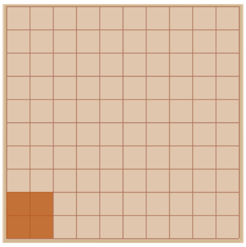
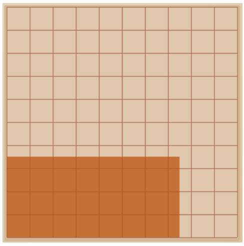
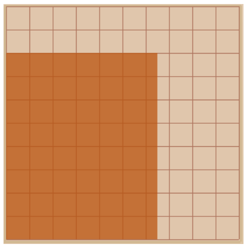
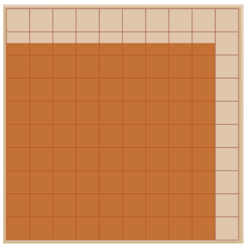
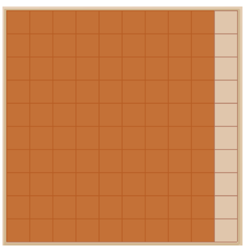
$$\$13.60 = x$$

Marcel's bill will be \$13.60 next month.

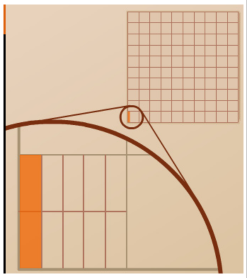
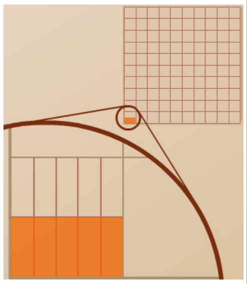
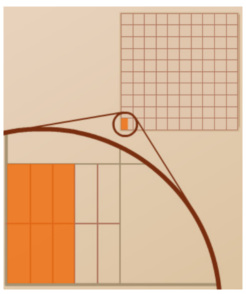
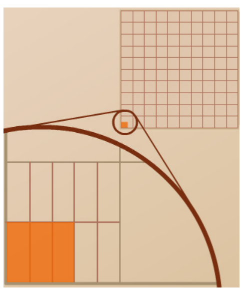
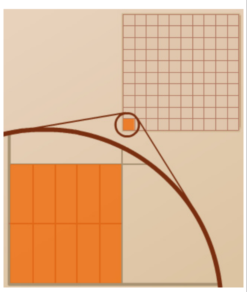
Lesson 3: Percent

Exercises 3.1

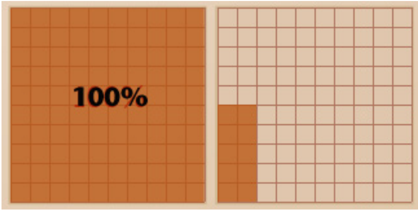
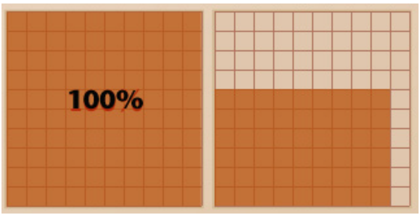
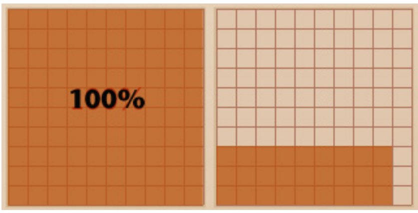
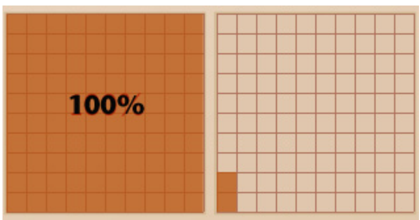
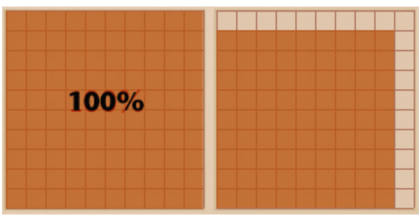
Picture	Percent	Fraction	Ratio	Decimal
	64%	$\frac{64}{100}$	64:100	0.64
	12%	$\frac{12}{100}$	12:100	0.12

	4%	$\frac{4}{100}$	4:100	0.04
	26.25%	$\frac{26.25}{100}$	26.25:100	0.2625
	52%	$\frac{52}{100}$	52:100	0.52
	76.5%	$\frac{76.5}{100}$	76.5:100	0.765
	90%	$\frac{90}{100}$	90:100	0.9

Exercises 3.2

Picture	Percent	Fraction	Ratio	Decimal
	0.2%	$\frac{2}{1000}$	2:1000	0.002
	0.5%	$\frac{5}{1000}$	5:1000	0.005
	0.6%	$\frac{6}{1000}$	6:1000	0.006
	0.3%	$\frac{3}{1000}$	3:1000	0.003
	1%	$\frac{10}{1000} = \frac{1}{100}$	1:100	0.01

Exercises 3.3

Picture	Percent	Fraction	Ratio	Decimal
	110%	$\frac{110}{100}=1\frac{10}{100}$	110:100	1.1
	154%	$\frac{154}{100}=1\frac{54}{100}$	154:100	1.54
	127%	$\frac{127}{100}=1\frac{27}{100}$	127:100	1.27
	102%	$\frac{102}{100}=1\frac{2}{100}$	102:100	1.02
	181%	$\frac{181}{100}=1\frac{81}{100}$	181:100	1.81

Exercises 3.4

1. a. $\frac{171}{300}=\frac{57}{100}=57\%$

   b. $\frac{41}{20}=\frac{205}{100}=205\%$

c. You can use the cross-product method to solve this problem.

$$\frac{3}{125}=\frac{x}{100}$$
$$(3)(100)=(125)(x)$$
$$\frac{(3)(100)}{125}=\frac{(125)(x)}{125}$$
$$2.4=x$$

So, $\frac{3}{125}=2.4\%$

d. You can use the cross product method to solve this problem.

$$1\frac{7}{15}=\frac{x}{100}$$
$$\frac{22}{15}=\frac{x}{100}$$
$$(22)(100)=(15)(x)$$
$$\frac{(22)(100)}{15}=\frac{(15)(x)}{15}$$
$$146.\overline{6}=x$$

So, $1\frac{7}{15}=146.\overline{6}\%=146.7\%$

2. a. 14%
   b. 0.5%
   c. 10%
   d. 123%

3.

	Decimal	Fraction
a. 0.07%	0.0007	$\frac{7}{10\,000}$
b. $23\frac{1}{2}\%$	23.5% = 0.235	$\frac{235}{1000}=\frac{47}{200}$
c. 325%	3.25	$3\frac{25}{100}=3\frac{1}{4}$

4. a. $7:4 = \frac{7}{4}$

$$\frac{7}{4} \xrightarrow{\times 25} \frac{x}{100}$$

$$\frac{7}{4} = \frac{175}{100} = 175\%$$

b. $1:500 = \frac{1}{500}$

$$\frac{1}{500} \xrightarrow{\div 5} \frac{x}{100}$$

$$\frac{1}{500} = \frac{0.2}{100} = 0.2\%$$

Exercises 3.5

1. a. $120.00 × 5%
   = $120.00 × 0.05
   = $6.00

   b. $120.00 × 7%
   = $120.00 × 0.07
   = $8.40

2. GST = 5% of $70.00 = 0.05 × $70.00 = $3.50
   PST = 7% of $70.00 = 0.07 × $70.00 = $ 4.90
   The total amount of taxes = $3.50 + $4.90 = $8.40
   The check out price is: $70.00 + $8.40 = $ 78.40

3. original price × 5% = $3.40
   original price × 0.05 = $3.40
   $$\frac{\text{original price} \times 0.05}{0.05} = \frac{\$3.40}{0.05}$$
   original price = $68.00

4. original price × 7% = $10.00
   original price × 0.07 = $10.00
   $$\frac{\text{original price} \times 0.07}{0.07} = \frac{\$10.00}{0.07}$$
   original price = $142.86

5. original price × 12% = $9.60
   original price × 0.12 = $9.60
   $$\frac{\text{original price} \times 0.12}{0.12} = \frac{\$9.60}{0.12}$$
   original price = $80.00

Exercises 3.6

1. Start by finding the price before taxes (retail):

   Work backwards. The cost of an item including taxes can be found by multiplying the retail price by 112% (this is because you pay 100% of the retail price, plus 7% PST and 5% GST).

   retail price × 112% = \$120.00
   retail price × 1.12 = \$120.00

   $$\frac{\text{retail price} \times 1.12}{1.12} = \frac{\$120.00}{1.12}$$

   retail price = \$107.14

   Find the profit:
   profit = retail price – wholesale price
   profit = \$107.14 – \$90.00
   profit = \$17.14

   The vendor makes a \$17.14 profit on the jeans.

2. sale price = original price – discount

   $$\$300 = x - (20\% \text{ of } x)$$
   $$\$300 = x - 0.20x$$
   $$\$300 = 0.80x$$
   $$\frac{\$300}{0.80} = \frac{0.80x}{0.80}$$
   $$\$375 = x$$

3. price after original 10% discount = \$80 – 10% of \$80
   = \$80 – 0.10(\$80)
   = \$80 – \$8
   = \$72

   price after 15% discount = \$72 – 15% of \$72
   = \$72 – 0.15(\$72)
   = \$72 – \$10.80
   = \$61.20

4. price after 30% discount = \$160 – 30% of \$160
   = \$160 – 0.30(\$160)
   = \$160 – \$48
   = \$112
   PST (7%) = \$112 × 0.07 = \$7.84
   GST (5%) = \$112 × 0.05 = \$5.60

   The total cost = \$112 + \$7.84 + \$5.60 = \$125.44

5. final course grade = class portion + provincial exam portion
= 80% from Class grade + 20% from Prov. Exam Grade
= 0.80 (70%) + 0.20 (73%)
= 56% + 14.6%
= 70.6% overall

6. a. $\frac{2}{3}$ of 2.5%

$\frac{2}{3} \times \frac{2.5}{100} = \frac{5}{300}$

Convert to a percent:

$\frac{5}{300} = 0.0166\overline{6} = 1.67\%$

b. 2.5% – 1.67% = 0.83%

About 0.83% of the world's water is available to be used for drinking water.

Lesson 4: Integers, Part 1

Exercises 4.1

These answers are approximate. Yours should be close.

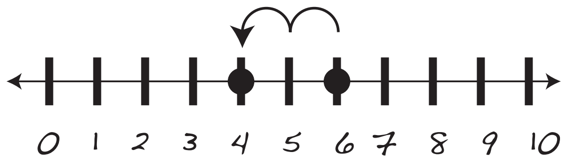
1. a. +60 m
   b. +25 m
   c. –40 m
   d. –50 m
   e. –125 m
2. a. –50
   b. +10
   c. –200
   d. –3
   e. +5
   f. –6

Exercises 4.2

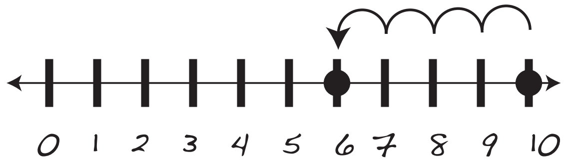
1. a. +5
   b. +9
   c. –2
2. a. +3
   b. –11
   c. –21
3. –8, –2, 0, +5, +12

Exercises 4.3

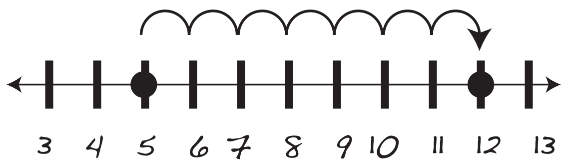
1. b. 6 – 2 = 4



c. 10 – 4 = 6



d. 5 + 7 = 12



2. a. (–6) + (– 2) =
   b. (+7) + (–3) =
   c. (+9) + (–6) =
   d. (–12) + (+3) + (–7) =
   e. (–64) + (+32) +(+11) =

Exercises 4.4

1. a. +2
   b. +1
   c. –1
   d. –3
   e. –1
   f. +2
2. a. (–4) + (+5) = +1
   b. (+2) + (–3) = –1
   c. (+4) + (–2) = +2
   d. (–3) + (–1) = –4

Exercises 4.5

1. a. –5
   b. +8
   c. +5
   d. –3
   e. +3
   f. –5
2. a. –7
   b. +1
3. a. +10
   b. +2
4. Some answers may include:
   (+1) + (–8) = –7
   (–6) + (–1) = –7
   (–3) + (–4) = –7
   (–10) + (+3) = –7

Exercises 4.6

1. a. Negative
   b. Negative
2. a. +8
   b. –8
   c. –40
   d. –25
3. a. +10
   b. +23
4. –17, –6, 0, 2, 16, 22, 27
5. a. +5
   b. 0
   c. –10

6. The 3rd step
7. a. +3
   b. –5
   c. +1
   d. –4
   e. –8
   f. 0
8. a. –2
   b. +4
   c. –2
   d. –6
9. a. –36
   b. –45
10. a. positive
    b. negative
11. a. –25
    b. –5
    c. +20
    d. +73
    e. +93

Exercises 4.7

1. a. +5
   b. –4
   c. –8
   d. –5
2. a. +3
   b. +1
   c. 0
   d. –5

3. (+2000) – (–200) = +2200

   Cullen is right

4. a. –4

   b. +3

   c. +8

   d. –8

5. a. +5

   b. +5

   c. –5

   d. –4

6. (+3) – (–1) = +4

   The eagle and the salmon are 4 metres apart.

7. (+8848) – (–411) = +9259

   These two elevations are 9259 metres apart.

Lesson 4: Integers, Part 2

Exercises 4.8

1. Groups: boxes
   Items: bulbs
   # of groups: Susan packed **370** boxes.
   # of items in one group: There were **24** bulbs in each box.
   370 × **24** = **8880**
   Susan packed **8880** bulbs.

2. Groups: days
   Items: bales of hay
   # of groups: Amir needs hay for **45** days.
   # of items in one group: He needs **4** bales of hay each day.
   45 × **4** = **180**
   Amir needs **180** bales of hay.

3. Groups: packs of trading cards
Items: cards in each pack
\# of groups: **24** packs
\# of items in one group: **15** cards in each pack
24 × 15 = 360
There are **360** trading cards in a box.

4. Groups: pouches of beads
Items: beads
\# of groups: **6** pouches of beads
\# of items in one group: **22** beads in each pouch
6 × 22 = 132
There are **132** beads in the kit.

5. Groups: the hours that Chris worked
Items: the dollars that Chris earned per hour
\# of groups: **21 hours**
\# of items in one group: $**9**
21 × 9 = 189
Chris earned **$189** last week.

6. There are 3726 groups with 5¢ in each group. Multiply to find the total.
3726 × 5¢ = 18 630¢ = $186.30
The band has made $186.30.

Exercises 4.9

1. Groups: boxes
Items: bulbs
\# of items in one group: There were **24** bulbs in each box.
Total items: There were **60 000** bulbs altogether.
60 000 ÷ 24 = 2500
They filled **2500** boxes with bulbs.

2. Groups: calendars
Items: dollars for each calendar that they sell
\# of items in one group: The class earns $**3** for each calendar that they sell.
Total items: The class wants to raise $**465**.
465 ÷ 3 = 155
They need to sell **155** calendars to reach their goal.

3.	Groups:	new houses
	Items:	seedlings
	# of groups:	There are **13** new houses.
	Total items:	There are **585** seedlings.
		585 ÷ 13 = 45
		The landscaper can plant **45** coleus seedlings in each yard.
4.	Groups:	the hours that Chris works
	Items:	dollars ($) that Chris earns
	# of items in one group:	**9**
	Total items:	**648**
		648 ÷ 9 = 72
		Chris needs to work for **72** hours to earn the money to buy the guitar.
5.	Groups:	bracelets
	Items:	beads
	# of items in one group:	**7**
	Total items:	**56**
		56 ÷ 7 = 8
		Alexis can make **8** bracelets.

6. The items in this question are dollars. There are $18 in total.

   The groups are the packs of cards. There are 24 packs of cards.

   Divide to find the number of items in one group (the number of dollars per pack).

   18 ÷ 24 = 0.75

   Nancy pays $0.75 for each pack of cards.

Exercises 4.10

1.	Groups:	The number of days.
		We don't know.
		Divide to find the number of groups.
	Items:	Bales of hay needed for each day — 4.
	Total:	192 bales of hay in total
		192 ÷ 4 = 48
		Amir has enough hay to feed his cows for **48** days.

2. Groups: The calendars — 32.
Items: Dollars for each calendar — \$14.
Total: Total amount of money they have collected.
(We don't know. Multiply to find the total.)
$32 \times 14 = \$448$
The class has collected **\$448**.

3. Groups: The band members — 5
Items: Dollars (\$) each band member gets.
(We don't know. Divide to find the number of items in each group.)
Total: Total amount in dollars (\$) that the band earned. The total is 700.
$700 \div 5 = 140$
Each band member gets **\$140**.

4. Groups: The cedar hedges — 13
Items: Number of plants in each hedge. — 8
Total: Total number of plants needed.
(We don't know. Multiply to find the total.)
$13 \times 8 = 104$
The landscaper needs **104** cedar plants.

Exercises 4.11

1.

a. $4 \times -3 = -12$	b. $-4 \times 3 = -12$
c. $12 \div 1 = 12$	d. $-1 \times 12 = -12$
e. $12 \div -4 = -3$	f. $-12 \div 4 = -3$
g. $2 \times -6 = -12$	h. $-2 \times 6 = -12$
i. $-12 \div 3 = -4$	j. $12 \div -3 = -4$
k. $12 \div 3 = 4$	l. $-12 \times 1 = -12$
m. $12 \times -1 = -12$	n. $12 \div 2 = 6$
o. $12 \div -2 = -6$	p. $12 \div -6 = -2$
q. $-12 \div 6 = -2$	r. $3 \times -4 = -12$
s. $4 \times 3 = 12$	t. $6 \times 2 = 12$
u. $-6 \times 2 = -12$	v. $6 \times -2 = -12$

2. $-3 \times 5 = -15$ It will be 15° C colder on the fifth day than it is today.

3. $-900 \div 3 = -300$ Each person owes \$300.

Exercises 4.12

1. $4 \times 5 = 20$
2. $-4 \times -5 = 20$
3. $4 \times -5 = -20$
4. $20 \div 4 = 5$
5. $-20 \div -5 = 4$
6. $-10 \times -2 = 20$
7. $-20 \div -2 = 10$
8. $20 \div 2 = 10$
9. $-1 \times -20 = 20$
10. $3 \times -8 = -24$
11. $-24 \div 8 = -3$
12. $-4 \times 6 = -24$
13. $24 \times -1 = -24$
14. $24 \div -4 = -6$
15. $2 \times -12 = -24$
16. $-24 \div 3 = -8$
17. $-24 \div 1 = -24$
18. $-7 \times -7 = 49$
19. $7 \times 7 = 49$
20. $49 \div -7 = -7$
21. $49 \div 7 = 7$
22. $-5 \times -5 = 25$
23. $5 \times 5 = 25$
24. $-25 \div -25 = 1$
25. $-1 \times -5 = 5$
26. $-4 \times -4 = 16$
27. $4 \times 4 = 16$
28. $16 \div -4 = -4$
29. $-16 \div 4 = -4$
30. $4 \div 2 = 2$
31. $4 \div -2 = -2$
32. $-4 \div 2 = -2$
33. $-4 \div -2 = 2$
34. $56 \div 8 = 7$

Exercises 4.13

1. $\frac{12}{(3)(-1)(2)} = -2$
2. $\frac{(3)(-3)(-7)}{(-9)} = -7$
3. $(4)(5)(-1) = -20$
4. $\frac{(-16)(25)(-2)}{(10)(-4)} = -20$
5. $(2)(-5)(7)(-2) = 140$
6. $\frac{(6)(-4)(2)}{-12} = 4$
7. $\frac{(24)(-14)}{(-8)(-7)(-1)} = 6$
8. $\frac{(8)(-7)}{4(14)} = -1$
9. $(-1)(2)(-3)(4)(-5) = -120$
10. $\frac{(-15)(6)}{-9} = 10$
11. $(5)(-3)(2) = -30$
12. $\frac{(-21)(9)}{(7)(-3)} = 9$

Lesson 5: Powers

Exercises 5.1

1. a. 4^2
   b. 9^3
   c. 2^5
   d. 7^4
   e. 10^4
   f. $\left(\frac{2}{5}\right)^2$
   g. $\left(\frac{1}{10}\right)^3$
   h. $\left(\frac{3}{4}\right)^2$
2. a. 8
   b. 25
   c. 27
   d. 100
   e. 1000
   f. $\frac{1}{9}$
   g. $\frac{1}{8}$
   h. $\left(\frac{27}{64}\right)$

Exercises 5.2

1. $10^{5+2} = 10^7 = 10\ 000\ 000$
2. $3^{4+2} = 3^6 = 729$
3. $2^{2+3} = 2^5 = 32$
4. $4 \times 9 = 36$ (different bases)
5. $2^{2+4} = 2^6 = 64$
6. $10^{1+2} = 10^3 = 1000$

7. $2^{4+5} = 2^9 = 512$
8. $6^{2+2+2} = 6^6 = 46\ 656$
9. $3^{2+3} = 3^5 = 243$
10. $5^{1+2} = 5^3 = 125$
11. $2^{2+1+2} = 2^5 = 32$
12. $10^{3+1} = 10^4 = 10\ 000$
13. $36 \times 4 = 144$ (bases are different)
14. $9 \times 25 = 225$ (bases are different)
15. $2^{4+4} = 2^8 = 256$
16. $9 \times 64 = 576$ (bases are different)

Exercises 5.3

1. $10^{5-2} = 10^3 = 1000$
2. $3^{4-2} = 3^2 = 9$
3. $8^{7-5} = 8^2 = 64$
4. $10^{6-4} = 10^2 = 100$
5. $9^{5-4} = 9^1 = 9$

Exercises 5.4

1. a. 1

   b. $\frac{1}{11}$

   c. $10^2 \times 10^1 = 10^3 = 1000$

   d. $\frac{3^4 \times 3^1}{3^5} = \frac{3^5}{3^5} = 3^{5-5} = 3^0 = 1$

2. (a) (i) $9^{5-3} = 9^2$ (ii) 81
   (b) (i) $10^{5-2} = 10^3$ (ii) 1000
   (c) (i) $5^{4-4} = 5^0$ (ii) 1
   (d) (i) $10^{4-1} = 10^3$ (ii) 1000
   (e) (i) $8^{10-8} = 8^2$ (ii) 64

3. (a) $n = 9$ (b) $n = 3$ (c) $n = 0$ (d) $n = 6$
   (e) $n = 1$ (f) $n = 4$ (g) $n = 3$ (h) $n = 7$
   (i) $n = 4$ (j) $n = 5$

Exercises 5.5

1. $\dfrac{1}{3\times 3}=\dfrac{1}{9}$
2. $\dfrac{1}{5}$
3. $\dfrac{1}{10}=\dfrac{1}{10\times 10\times 10\times 10\times 10\times 10\times 10\times 10}=0.000\ 000\ 01$
4. $\dfrac{1}{9\times 9}=\dfrac{1}{81}$
5. $\dfrac{1}{10}=\dfrac{1}{10\times 10\times 10\times 10\times 10\times 10\times 10\times 10\times 10}=0.000\ 000\ 001$
6. $2^{(6\ +\ -4)}=2^2=4$
7. $3^{(-7\ +\ 10)}=3^3=27$
8. $10^{(2\ +\ -5)}=10^{-3}=0.001$
9. $10^{(-3\ +\ -6)}=10^{-9}=0.000\ 000\ 001$
10. $10^{(5\ +\ -2)}=10^3=1000$

Lesson 6: Scientific Notation

Exercises 6.1

1. a. 10^9
   b. 10^1
   c. 10^3
   d. 10^6
   e. 10^8

2. a. n = 1
   b. n = –3
   c. n = 5
   d. n = 0
   e. n = 10

3. a. 0.0001
   b. 100
   c. 0.001
   d. 1
   e. 100 000

4. a. 10^{-3}
   b. 10^{4}
   c. 10^{-1}
   d. 10^{2}
   e. 10^{0}

Exercises 6.2

1. a. 3
   b. 2
   c. 4
   d. 5
   e. 1
   f. 6
   g. 4
   h. 8

2. a. 5.4×10^{3}
   b. 5.2×10^{5}
   c. 7.28×10^{6}
   d. 6×10^{3}
   e. 3.02×10^{6}
   f. 3.52×10^{11}

3. a. 2400
   b. 360
   c. 3 700 000
   d. 1720
   e. 73 500
   f. 500 000

4. a. 9.97×10^{6}
   b. 2.65×10^{4}
   c. 4.06×10^{5}
   d. 4.1×10^{3}
   e. 1.52×10^{9}

5. a. $\boxed{5.2\ \ 04}$
   b. $\boxed{1.25\ \ 11}$

Exercises 6.3

1. a. –2
   b. –4
   c. –1
   d. –5
   e. –3
   f. –6

2. a. 6.5×10^{-1} b. 1.25×10^{-1}
   c. 5.5×10^{-3} d. 2.5×10^{-4}
   e. 5.8×10^{-7} f. 3.75×10^{-1}
   g. 7.5×10^{-5}

3. a. 0.0025 b. 0.000 000 37
   c. 0.000 001 25 d. 0.000 000 061 5

4. a. 1.98×10^{-3} b. 1.02×10^{-4}
   c. 1×10^{-7} d. 2.8×10^{-8}

Exercises 6.4

1. a. 7.2×10^{-7}
   b. 6.0×10^{6}
   c. 1.2×10^{-2}
   d. 7.1×10^{3}
   e. 6.0×10^{-1}

2. a. 0.000 072
   b. 610
   c. 0.26

Exercises 6.5

1. a. 3.99×10^{7} b. 1.2742×10^{7}
   b. 1.392×10^{6} d. 9×10^{18}
   e. 1.07×10^{9}

2. a. 6.1×10^{-11} b. 4.72×10^{-10}
   b. 3.4×10^{-19} d. 4.91×10^{-6}
   e. 6.43×10^{-7}

3. a. 57 900 000 b. 150 000 000
   c. 770 000 000 d. 4 500 000 000
   e. 5 910 000 000

4. a. 0.0304 b. 0.000 000 21
   c. 0.000 000 013 5

Exercises 6.6

1. $1.2 \times 10^{8} \times 2.75 \times 10^{7} = 3.3 \times 10^{15}$
2. $4.0 \times 10^{-3} \times 2.3 \times 10^{-5} = 9.2 \times 10^{-8}$
3. $5.0 \times 10^{9} \times 2.5 \times 10^{-3} = 12.5 \times 10^{6} = 1.25 \times 10^{7}$
4. $1.875 \times 10^{12} \times 4.0 \times 10^{-1} = 7.5 \times 10^{11}$
5. $2.0 \times 10^{1} \times 3.61 \times 10^{-8} = 7.22 \times 10^{-7}$

Lesson 7: Order of Operations

Exercises 7.1

1. $(-3)(7) = -21$
2. $4 \times 9 = 36$
3. $-13 \times 3 = -39$
4. $42 \div -6 = -7$
5. $(8)(-1)(-4) = 32$
6. $12 + 6 = 18$
7. $15 \div 5 + 7 = 10$
8. $2 - 3 \times 4 = -10$
9. $3 + \frac{4}{2} = 5$
10. $-16 + (4)(3) = -4$
11. $-5 - (2)(-1)(-18) \div 4 = -14$
12. $6 \times 5 \div 3 = 10$
13. $18 \div 2 + 4 = 9 + 4 = 13$
14. $18 \div (2 + 4) = 18 \div 6 = 3$
15. $6 \times 8 + 12 + 3 \times 9 = 87$
16. $3 + 11 \times 4 + 12 \div 3 = 51$
17. $7 - 3 \times 5 = 7 - 15 = -8$
18. $(7 - 3) \times 5 = 4 \times 5 = 20$
19. $36 \div 9 + 2 + 1 \times 9 + 6 - 5 = 16$
20. $6 \times 7 \div 14 - 3 + 2 \times 4 = 8$
21. $5 - 1 + 2 - 4 \times 3 \div 6 = 4$