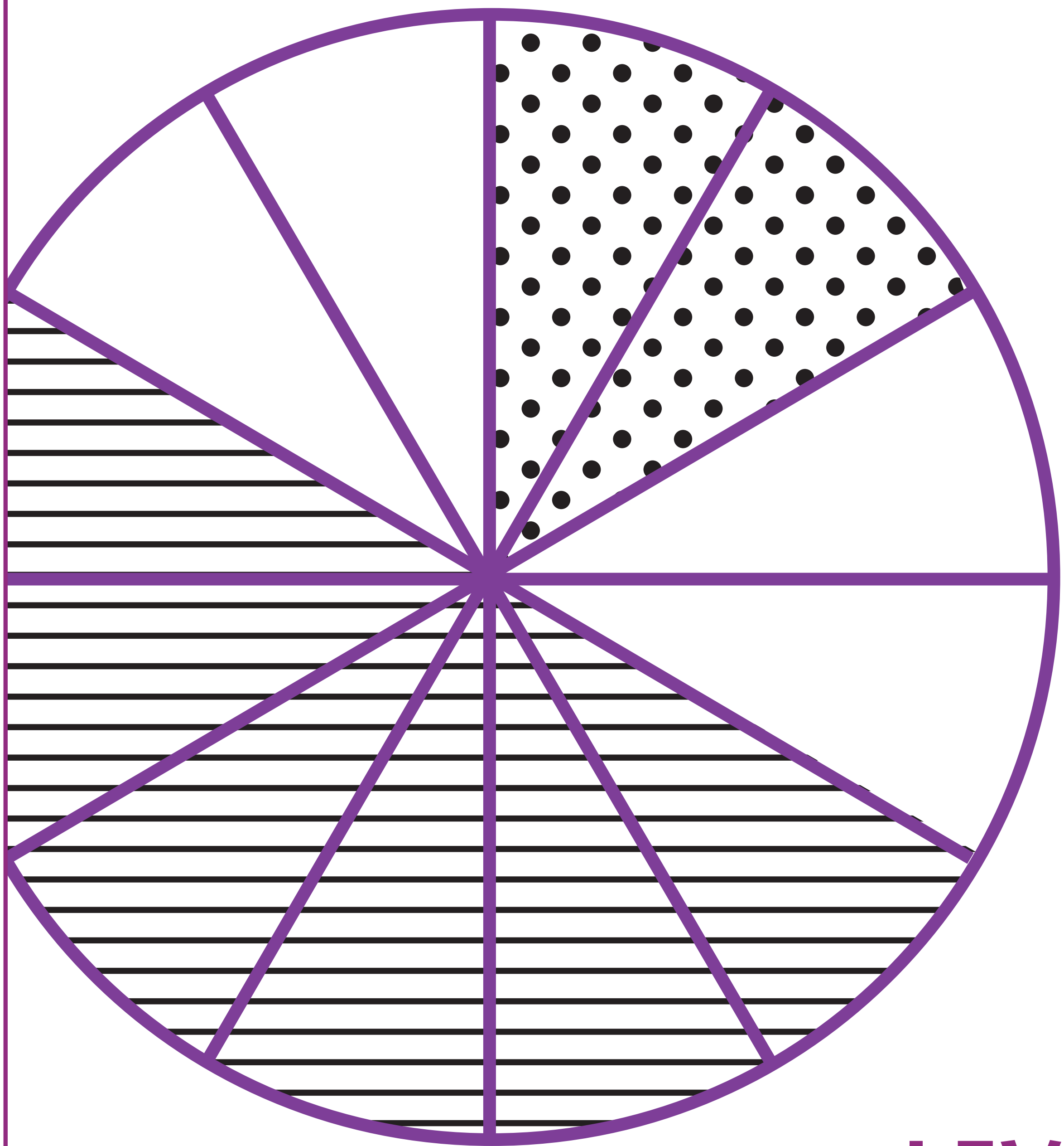
LITERACY FOUNDATIONS MATH



LEVEL 4
NUMBER SENSE

To the Student

This resource covers topics from the British Columbia Ministry of Education's Literacy Foundations Math Level 4. You may find this resource useful if you're a Literacy Foundations Math student, or a K–12 student in grades 7 – 9.

We have provided learning material, exercises, and answers for the exercises, which are located at the back of each set of related lessons. We hope you find it helpful.

Literacy Foundations Math Prescribed Learning Outcomes

The Literacy Foundations Math Prescribed Learning Outcomes (PLOs) are grouped into four areas: Number (A), Patterns and Relations (B), Shape and Space (C), and Statistics and Probability (D). For a complete list of the PLOs in Level 5, search for Literacy Foundations Math curriculum on the BC Ministry of Education's website.

PLOs Represented in This Resource

The PLOs represented in this Level 4 resource are as follows:

Number

A6, A7, A9, A11 – A18

Patterns and Relations

All topics, B1 – B3

Shape and Space

C1 – C5, C7
*C3 topics are represented with the exception of angle construction

Statistics and Probability

D2

PLOs Not Represented in This Resource

The PLOs for which no material is included in this resource are as follows:

Number

There is no material for A1 – A5, read and write numbers, place value, and patterns for multiplying by 10, etc.; A8, compare decimal numbers; nor A10, patterns for multiplying and dividing by 1/10, etc.

Shape and Space

There is no material for C3, construct angles.

Statistics and Probability

There is no material for D1, graph data to solve problems.

Acknowledgements and Copyright

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Lesson 1
Fractions

Learning Outcomes

By the end of this section you will be better able to:

- Describe a quantity using fractions
- Identify and calculate equivalent fractions
- Reduce fractions to lower terms

A fraction is a number that describes a piece of something. In this lesson, all of the fractions describe an amount less than one. $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{13}{16}$ are examples of fractions. The number above the bar is called the numerator of the fraction; the number below the bar is called the denominator.

Consider the square shown in Figure 1 below. This square is divided into 4 equal parts; 1 of these 4 parts is shaded. The fraction $\frac{1}{4}$ tells us what portion of the square is shaded. Similarly, the fraction $\frac{3}{4}$ tells us what portion is not shaded.

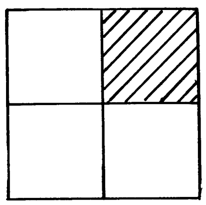


Figure 1

Now consider Figure 2. Here there are 12 small squares; 4 of them are shaded. We can say that $\frac{4}{12}$ of the rectangle is shaded. If, however, we consider the larger squares, we find that 1 of the larger squares is shaded, so we can also say that $\frac{1}{3}$ of the rectangle is shaded.

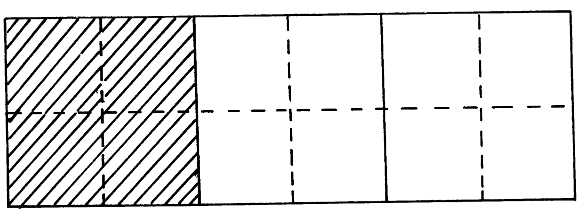
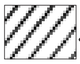
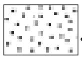


Figure 2

Self Test

1. Give three fractions that tell what part of the square in Figure 3 is marked .

2. Give two fractions that tell what part of the square is marked .

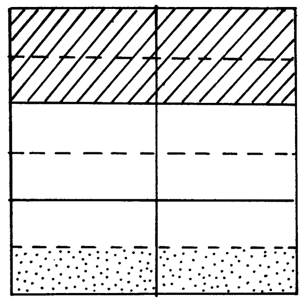


Figure 3

Answers

1. $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}$
2. $\frac{2}{12}, \frac{1}{6}$

Now consider the set of circles and triangles shown in Figure 4.

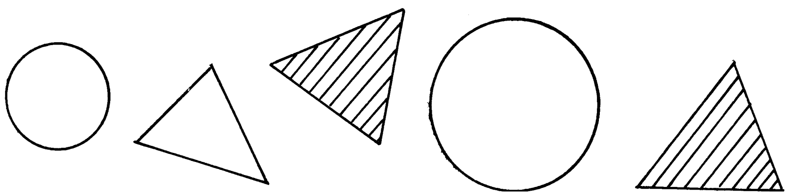


Figure 4

Of the 5 figures, 2 are circles; the fraction $\frac{2}{5}$ tells us what part of the set of figures is circles. Similarly, the fraction $\frac{3}{5}$ tells us what part of the set is triangles.

Self Test

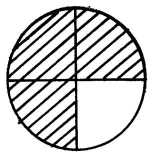
1. In Figure 4, what fractional part of the set of figures is shaded?

2. In Figure 4, what fraction of the set of triangles is shaded?

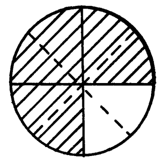
Answers

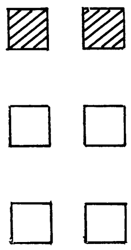
1. $\frac{2}{5}$
2. $\frac{2}{3}$

Equivalent Fractions

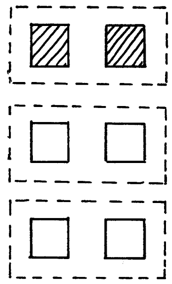


$\frac{3}{4}$ is equivalent to $\frac{6}{8}$





$\frac{2}{6}$ is equivalent to $\frac{1}{3}$

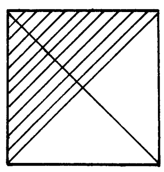


The fractions $\frac{3}{4}$ and $\frac{6}{8}$ represent equivalent portions of the circles; these fractions are called **equivalent fractions**. The fractions $\frac{2}{6}$ and $\frac{1}{3}$ are also equivalent; they represent equivalent parts of the sets of squares.

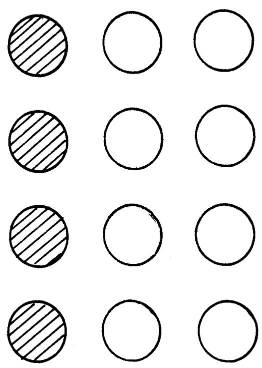
Self Test

Give the pair of equivalent fractions suggested in each case.

1.



2.



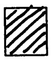
Answers

1. $\frac{2}{4}, \frac{1}{2}$

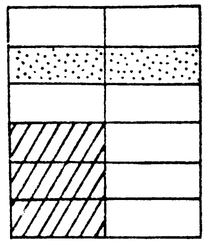
2. $\frac{4}{12}, \frac{1}{3}$ $\left(\text{or } \frac{8}{12}, \frac{2}{3}\right)$

Exercises 1.1

1. In each of (a) and (b) give two fractions that tell what part of the rectangle is marked as indicated.

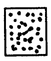
(a)  ____________

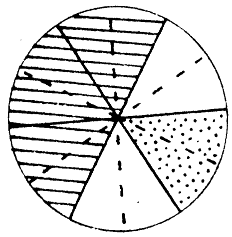
(b)  ____________



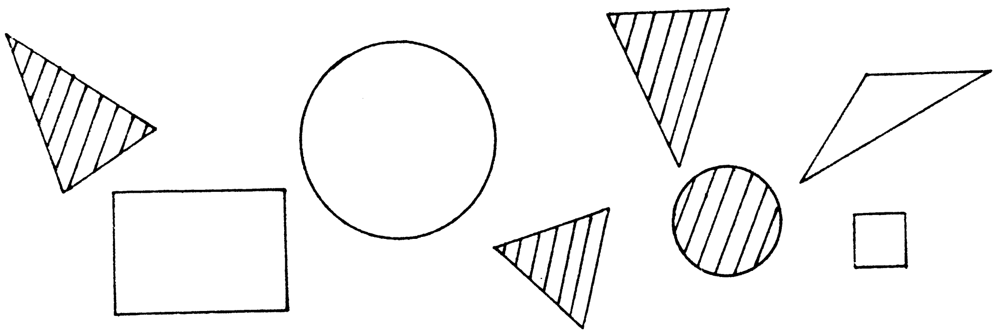
2. In each of (a) and (b), give two fractions that tell what part of the circle is marked as indicated.

(a)  ____________

(b)  ____________



3. This question refers to the set of figures shown below. (Express your answer in lowest terms.)



(a) What fraction of the set of figures is triangles?

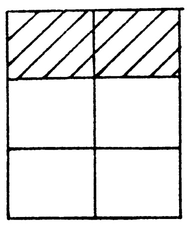
(b) What fraction of the set of figures is circles?

(c) What fraction of the triangles is shaded?

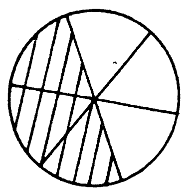
(d) What fraction of the circles is shaded?

4. Give two equivalent fractions suggested in each case.

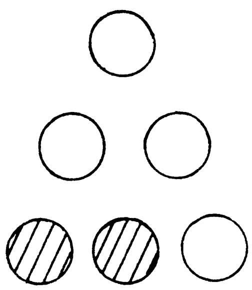
(a)



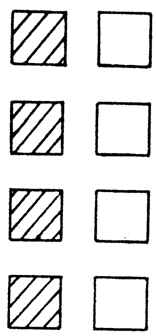
(b)



(c)



(d)



Turn to the Answer Key at the end of the module to check your work.

The Fraction "1"

We will now consider fractions in which the two numbers are the same, that is, in which the numerator is the same as the denominator. These fractions are rather special in that every one of them is equivalent to 1.

$\frac{2}{2}$ is equivalent to 1.

$\frac{3}{3}$ is equivalent to 1.

$\frac{4}{4}$ is equivalent to 1.

....

....

$\frac{29}{29}$ is equivalent to 1.

....

....

To see that this is, in fact, the case, consider the shaded portion of the rectangle in Figure 6(a).

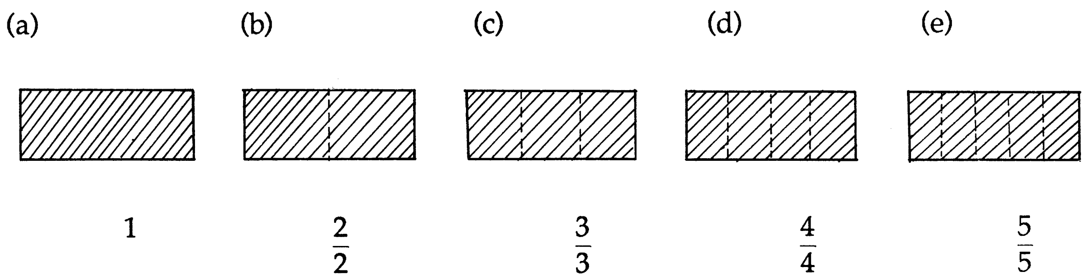


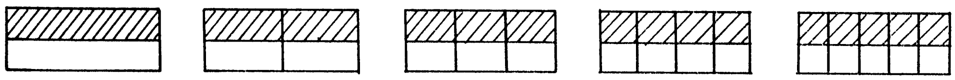
Figure 6

The entire rectangle is shaded, so we can therefore represent the shaded portion by the number 1. (There is 1 rectangle, and all of it is shaded.) Suppose now that we divide this same rectangle into 2 equal regions, Figure 6 (b). Now the fraction $\frac{2}{2}$ represents the shaded portion. (There are 2 regions and both are shaded.) Hence, we see that the fraction $\frac{2}{2}$ is equivalent to 1. If we divide the rectangle into 3 regions, we find that the fraction $\frac{3}{3}$ is also equivalent to 1.

Continuing in this way, we find that any fraction in which the numerator and denominator are the same is equal to 1.

More Equivalent Fractions

Consider these equivalent fractions.



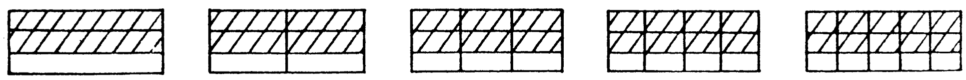
$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ $\frac{5}{10}$

Each of the fractions $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ and so on, are equivalent to the fraction $\frac{1}{2}$.

We can say ***all of these fractions are equivalent*** to $\frac{1}{2}$.

Self Test

What fraction are all the illustrations equivalent to?



$\frac{2}{3}$ $\frac{4}{6}$ $\frac{6}{9}$ $\frac{8}{12}$ $\frac{10}{15}$

Answer

$\frac{2}{3}$

Notice that each of the numerators has the numerator, 2, of $\frac{2}{3}$ as a factor and that each of the denominators has the denominator, 3, of $\frac{2}{3}$ as a factor; that is, 2 will divide into each of the numerators, and 3 will divide into each of the denominators.

$$\frac{2}{3} \qquad \frac{4}{6} \qquad \frac{6}{9} \qquad \frac{8}{12} \qquad \frac{10}{15}$$

$$\frac{1\times 2}{1\times 3} \qquad \frac{2\times 2}{2\times 3} \qquad \frac{3\times 2}{3\times 3} \qquad \frac{4\times 2}{4\times 3} \qquad \frac{5\times 2}{5\times 3}$$

Note the pattern. The first fraction has 1 as factors in numerator and denominator, the second has 2 as factors, the third, 3 as factors, and so on. Do you see that the sixth fraction (although not shown) will have 6 as factors and will be

$$\frac{6\times 2}{6\times 3}=\frac{12}{18}$$

and that the seventh fraction will have 7 as factors and will be

$$\frac{7\times 2}{7\times 3}=\frac{14}{21}?$$

Now consider the fractions equivalent to $\frac{1}{2}$.

$$\frac{1}{2} \qquad \frac{2}{4} \qquad \frac{3}{6} \qquad \frac{4}{8} \qquad \frac{5}{10}\cdots$$

$$\frac{1\times 1}{1\times 2} \qquad \frac{2\times 1}{2\times 2} \qquad \frac{3\times 1}{3\times 2} \qquad \frac{4\times 1}{4\times 2} \qquad \frac{5\times 1}{5\times 2}\cdots$$

The sixth fraction will be $\frac{6\times 1}{6\times 2}=\frac{6}{12}$ the ninth will be $\frac{9\times 1}{9\times 2}$ and so on.

Self Test

1. Write the 12th fraction in the series of fractions equivalent to $\frac{1}{2}$.

2. Write the 8th fraction in the series of fractions equivalent to $\frac{1}{3}$.

Answers

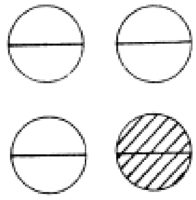
1. $\frac{12\times 1}{12\times 2}=\frac{12}{24}$
2. $\frac{8\times 1}{8\times 3}=\frac{8}{24}$

Exercises 1.2

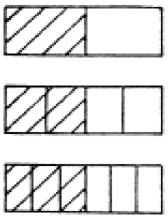
1. Each figure suggests two equivalent fractions. Give these fractions.

(a)

(b)



2. Give the three equivalent fractions that the figures suggests.



3. The fractions $\frac{3}{3}$, $\frac{17}{17}$, and $\frac{31}{31}$ are equivalent to ____________________.

4. Give the missing fractions, following the given pattern below.

$\frac{1\times4}{1\times5}$	$\frac{2\times4}{2\times5}$	a. ________	$\frac{4\times4}{4\times5}$	b. ________	$\frac{6\times4}{6\times5}$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\frac{4}{5}$	$\frac{8}{10}$	$\frac{12}{15}$	c. ______	$\frac{20}{25}$	d. ______

5. Give the three indicated fractions for each group of equivalent fractions.

(a) $\frac{5}{6}$, $\frac{10}{12}$, $\frac{15}{18}$, ______ , ______ , ______ , ...

(b) $\frac{3}{10}$, $\frac{6}{20}$, ______ , $\frac{12}{40}$, ______ , ______ , ...

(c) $\frac{5}{8}$, $\frac{10}{16}$, $\frac{15}{24}$, ______ , ______ , $\frac{30}{48}$, ______ , ...

(d) $\frac{1}{8}$, $\frac{2}{16}$, ______ , ______ , $\frac{5}{40}$, ______ , ...

Turn to the Answer Key at the end of the module to check your work.

Lowest Terms

A fraction is in lowest terms if the numerator and the denominator have no common factor other than 1. For example, the fraction $\frac{8}{9}$ is in lowest terms since 1 is the only common factor of 8 and 9.

$$\frac{8\ \{1,2,4,8\}}{9\ \{1,3,9\}}$$ No common factor other than 1.

The fraction $\frac{12}{15}$ is **not** in lowest terms since 1 and 3 are common factors of 12 and 15.

$$\frac{12\ \{1,2,3,4,6,12\}}{15\ \{1,3,5,15\}}$$ Common factors 1 and 3.

As further example, consider the fractions $\frac{16}{18}$ and $\frac{14}{15}$.

$$\frac{16\ \{1,2,4,8,16\}}{18\ \{1,2,3,6,9,18\}}$$ Common factors 1 and 2.
This fraction is not in lowest terms.

$$\frac{14\ \{1,2,7,14\}}{15\ \{1,3,5,15\}}$$ No common factors other than 1.
This fraction is not in lowest terms.

Self Test

1. List the factors of 15.

 ____

2. List the factors of 20.

 ____

3. Is the fraction $\frac{15}{20}$ in lowest terms? Explain.

 ____

 ____

Answers

1. 1, 3, 5, 15
2. 1, 2, 4, 5, 10, 20
3. No, because common factors of 15 and 20 are 1 and 5.

We can build a series of equivalent fractions from a given lowest- terms fraction. For example, given the fraction $\frac{1}{3}$, we can determine a series of six equivalent fractions.

$$\frac{1\times1}{1\times3},\quad \frac{2\times1}{2\times3},\quad \frac{3\times1}{3\times3},\quad \frac{4\times1}{4\times3},\quad \frac{5\times1}{5\times3},\quad \frac{6\times1}{6\times3}$$

$$= \quad \frac{1}{3},\quad \frac{2}{6},\quad \frac{3}{9},\quad \frac{4}{12},\quad \frac{5}{15},\quad \frac{6}{18}$$

Note that we multiplied the numerator and the denominator of $\frac{1}{3}$ by 1, by 2, by 3, and so on. Similarly, given the fraction $\frac{1}{10}$, we can determine a series of four equivalent fractions.

$$\frac{1\times3}{1\times10},\ \frac{2\times3}{2\times10},\ \frac{3\times3}{3\times10},\ \frac{4\times3}{4\times10} \longrightarrow \frac{3}{10},\ \frac{6}{20},\ \frac{9}{30},\ \frac{12}{40}$$

If we are given a series of equivalent fractions, we can determine the equivalent lowest-terms fraction. For example, in the series

$—, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \cdots$ the lowest-term fraction is not given.

Given the series $$—, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \cdots$$
$$\rightarrow —, \frac{2\times1}{2\times6}, \frac{3\times1}{3\times6}, \frac{4\times1}{4\times6}, \cdots$$

and we see that the lowest-terms fraction of this is $\frac{1}{6}$.

Similarly, since $$—, \frac{4}{18}, \frac{6}{27}, \frac{8}{36}, \cdots$$
$$\rightarrow —, \frac{2\times2}{2\times9}, \frac{3\times2}{3\times9}, \frac{4\times2}{4\times9}, \cdots$$

we see that the lowest-terms fraction of this is $\frac{2}{9}$.

Self Test

1. From the fraction $\frac{1}{4}$ build a series of five equivalent fractions.

2. Find the lowest-terms fraction of this series. $\frac{}{}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}$

Answers

1. $\frac{1\times 1}{1\times 4}, \frac{2\times 1}{2\times 4}, \frac{3\times 1}{3\times 4}, \frac{4\times 1}{4\times 4}, \frac{5\times 1}{5\times 4} \rightarrow \frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}$

2. $\frac{}{}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \rightarrow \frac{}{}, \frac{2\times 2}{2\times 5}, \frac{3\times 2}{3\times 5}, \frac{4\times 2}{4\times 5}$

   The lowest-terms fraction is $\frac{2}{5}$.

Reducing Fractions to Lowest Terms

If we are given a fraction, we can find an equivalent lowest-terms fraction. This is called reducing the fraction to lowest terms.

We can reduce a fraction to lowest terms by ***dividing out*** common factors. For example, suppose that we wish to reduce $\frac{60}{84}$ to lowest terms. We can do this in two ways.

1. Divide out common factors until the fraction is in lowest terms.

$$\frac{60}{84} = \frac{60 \div 2}{84 \div 2} = \frac{30}{42} = \frac{30 \div 2}{42 \div 2} = \frac{15}{21} = \frac{15 \div 3}{21 \div 3} = \frac{5}{7}$$

OR

2. (a) Find the greatest common factor (GCF) of the numerator and the denominator.

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Factors of 84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

GCF of 60 and 84 is 12.

(b) Divide out the GCF.

$$\frac{60}{84} = \frac{60 \div 12}{84 \div 12} = \frac{5}{7}$$

If we know the GCF, all we need do is divide it out, as in 2 (b). If we do not know the GCF, the first method is perhaps the easier.

The following examples further illustrate the reducing of fractions to lowest terms.

$$\frac{20}{48} = \frac{20 \div 2}{48 \div 2} = \frac{10}{24} = \frac{10 \div 2}{48 \div 2} = \frac{5}{12}$$

$$\frac{9}{36} = \frac{9 \div 3}{36 \div 3} = \frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

Note that in each step, we must divide the numerator and the denominator by the same number.

Thus, $\frac{10}{24}=\frac{10\div 2}{24}=\frac{5}{24}$ is incorrect, because we divided only the numerator by 2.

Also, $\frac{10}{24}=\frac{10\div 2}{24\div 3}=\frac{5}{8}$ is incorrect, because we divided the numerator and the denominator by different numbers.

Be very careful not to make these mistakes.

Exercises 1.3

1. (a) List the factors of 8.

   (b) List the factors of 12.

   (c) Is the fraction $\frac{8}{12}$ in lowest terms? Explain.

2. (a) List the factors of 15.

   (b) List the factors of 28.

   (c) Is the fraction $\frac{15}{28}$ in lowest terms? Explain.

3. Explain why each fraction is not in lowest terms.

   (a) $\frac{6}{10}$

   (b) $\frac{25}{30}$

4. Starting with lowest-term fraction, build a group of five equivalent fractions.

(a) $\frac{1}{4}$

(b) $\frac{5}{8}$

5. Reduce each fraction to lowest terms.

(a) $\frac{6}{10}$ (b) $\frac{3}{9}$ (c) $\frac{18}{20}$

(d) $\frac{5}{35}$ (e) $\frac{10}{35}$ (f) $\frac{18}{24}$

(g) $\frac{9}{21}$ (h) $\frac{15}{70}$ (i) $\frac{14}{28}$

(j) $\frac{50}{75}$ (k) $\frac{24}{30}$ (l) $\frac{16}{30}$

(m) $\frac{15}{36}$ (n) $\frac{30}{48}$ (o) $\frac{6}{52}$

Turn to the Answer Key at the end of the module to check your work.

Lesson 2

Adding and Subtracting Fractions

Learning Outcomes

By the end of this section you will be better able to:

- Add and subtract fractions with the same denominator
- Add and subtract fractions with different denominators

Now we shall consider the addition and subtraction of fractions. First, however, let's briefly review some of the main points of Part 1 in which introduced these numbers.

1. Equivalent fractions represent the same amount. For example $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ each represent the same amount.

$$\frac{4}{8}=\frac{1}{2} \qquad \frac{3}{6}=\frac{1}{2} \qquad \frac{2}{4}=\frac{1}{2}$$

2. A fraction may be reduced to lowest terms by dividing out common factors in the numerator and denominator. For example, $\frac{8}{10}$ may be reduced to lowest terms by dividing out the common factor 2.

$$\frac{8}{10}=\frac{8 \div 2}{10 \div 2}=\frac{4}{5}$$

Write the following in lowest terms:

(a) $\frac{20}{35}$

(b) $\frac{28}{42}$

(c) $\frac{40}{50}$

Did you get the following answers?

(a) $\dfrac{20 \div 5}{35 \div 5} = \dfrac{4}{7}$

(b) $\dfrac{28 \div 14}{42 \div 14} = \dfrac{2}{3}$

(c) $\dfrac{40 \div 10}{50 \div 10} = \dfrac{4}{5}$

3. A fraction may be changed to an equivalent fraction by multiplying the numerator and denominator by the same number. For example, $\frac{2}{3}$ may be changed to the equivalent fraction $\frac{8}{12}$ by multiplying the numerator and denominator by 4.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Now it is time to learn the operations of addition and subtraction with fractions.

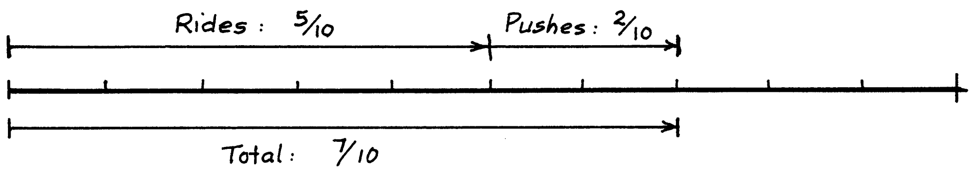
Addition of Fractions

Let us now see what we can discover about the addition of fractions. Consider the following.

Example 1

A boy rides a bicycle $\frac{5}{10}$ of a kilometre.

He then pushes the bicycle $\frac{2}{10}$ of a kilometre.

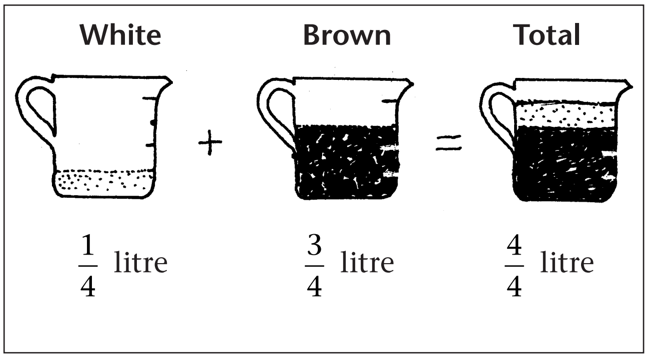


Total distance is $\frac{5}{10}$ km plus $\frac{2}{10}$ or $\frac{7}{10}$ km.

$$\frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

Example 2

A recipe calls for $\frac{1}{4}$ litre of white sugar and $\frac{3}{4}$ litre of brown sugar.



Total amount of sugar is $\frac{1}{4}$ litre plus $\frac{3}{4}$ litre or $\frac{4}{4}$ litre.

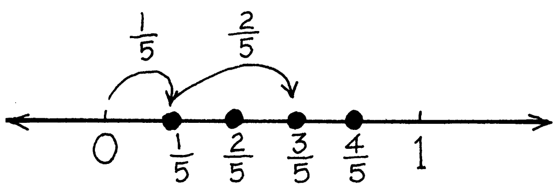
$$\frac{1}{4}+\frac{3}{4}=\frac{4}{4}$$

You will notice in each of the above examples, the denominators of the fractions are the same. We refer to this as like denominators.

Like Denominators

It is easy to add two or more fractions that have the same denominator. We simply add the numerators and keep the same denominator.

For example, let us add $\frac{1}{5}$ and $\frac{2}{5}$.



$$\frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}=\frac{3}{5}$$

Let's work through a few examples together.

Examples

Add the numerators.

(a) $\frac{2}{9}+\frac{5}{9}=\frac{2+5}{9}=\frac{7}{9}$

Keep the same denominator.

(b) $\frac{3}{7}+\frac{4}{7}=\frac{3+4}{7}=\frac{7}{7}=1$ $\left(\text{Reduce } \frac{7}{7} \text{ to } 1\right)$

(c) $\frac{5}{11}+\frac{4}{11}=\frac{5+4}{11}=\frac{9}{11}$

UnLike Denominators

This is when the denominators of at least two of the fractions are different.

Examples

(a) $\frac{1}{2}+\frac{3}{8}=?$

We need the least common multiple (LCM) of the two denominators, 2 and 8. This is called the least common denominator (LCD).

The multiples of 2 are 0, 2, 4, 6, 8, 10, ...

The multiples of 8 are 0, 8, 16, 24, 32, ...

The LCD of 2 and 8 is 8.

The least common denominator (LCD) of two numbers is the smallest nonzero common multiple. Zero is not an LCM.

From what we observed above, the common denominator is 8. We need a fraction that is equivalent to $\frac{1}{2}$ that has a denominator of 8. $\frac{1}{2}=\frac{4\times1}{4\times2}=\frac{4}{8}$

$\frac{1}{2}+\frac{3}{8}=\frac{4}{8}+\frac{3}{8}=\frac{7}{8}$

(b) $\frac{3}{7}+\frac{2}{5}=?$

First, the multiples of 7 are 0, 7, 14, 21, 28, 35, ...

The multiples of 5 are 0, 5, 10, 15, 20, 25, 30, 35, ...

The LCD of 7 and 5 is 35.

$\frac{3}{7}=\frac{5\times 3}{5\times 5}=\frac{15}{35}$ and $\frac{2}{5}=\frac{7\times 2}{7\times 5}=\frac{14}{35}$

$\frac{3}{7}+\frac{2}{5}=\frac{15}{35}+\frac{14}{35}=\frac{29}{35}$

We need equivalent fractions that have a denominator of 35. If you want to review equivalent fractions, look at the previous lesson before continuing.

Self Test

Add the following fractions. Show your work.

a. $\frac{1}{4}+\frac{2}{4}$

b. $\frac{1}{3}+\frac{1}{2}$

c. $\frac{3}{8}+\frac{1}{3}$

Answers

a. $\frac{1}{4}+\frac{2}{4}$

$=\frac{1+2}{4}$

$=\frac{3}{4}$

b. $\frac{1}{3}+\frac{1}{2}$

$=\frac{2}{6}+\frac{3}{6}$

$=\frac{5}{6}$

c. $\frac{3}{8}+\frac{1}{3}$

$=\frac{9}{24}+\frac{8}{24}$

$=\frac{17}{24}$

Subtraction of Fractions

Like Denominators

This is very similar to the procedure used in addition.

Examples

Subtract the numerators.

a. $\frac{7}{9}-\frac{3}{9}=\frac{7-3}{9}=\frac{4}{9}$

Keep the same denominator.

b. $\frac{9}{10}-\frac{3}{10}=\frac{9-3}{10}=\frac{6}{10}=\frac{3}{5}$ $\left(\text{Reduce } \frac{6}{10} \text{ to } \frac{3}{5}\right)$

c. $\frac{7}{16}-\frac{3}{16}=\frac{7-3}{16}=\frac{4}{16}=\frac{1}{4}$ $\left(\text{Reduce } \frac{4}{16} \text{ to } \frac{1}{4}\right)$

UnLike Denominators

Once again, it is necessary to find a common denominator first.

Examples

a. $\frac{9}{12}-\frac{2}{3}=?$

We need the LCM of the two denominators, 12 and 3.

This is called the least common denominator (LCD).

The multiples of 12 are 0, 12, 24, 36, 48, ...

The multiples of 3 are 0, 3, 6, 9, 12, 15, ...

The LCD of 12 and 3 is 12.

$\frac{2}{3}=\frac{4\times 2}{4\times 3}=\frac{8}{12}$

$\frac{9}{12}-\frac{2}{3}=\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$

b. $\frac{4}{5} - \frac{1}{2} = ?$ The LCD of 5 and 2 is 10.

$$\frac{4}{5} = \frac{2 \times 4}{2 \times 5} = \frac{8}{10} \text{ and } \frac{1}{2} = \frac{5 \times 1}{5 \times 2} = \frac{5}{10}$$

$$\frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$$

Self Test

Subtract the following fractions. Show your work.

a. $\frac{5}{6} - \frac{1}{6}$

b. $\frac{7}{12} - \frac{1}{4}$

c. $\frac{11}{15} - \frac{2}{3}$

Answers

a. $\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

b. $\frac{7}{12} - \frac{1}{4} = \frac{7}{12} - \frac{3}{12} = \frac{4}{12} = \frac{1}{3}$

c. $\frac{11}{15} - \frac{2}{3} = \frac{11}{15} - \frac{10}{15} = \frac{1}{15}$

Did you remember to reduce your fractions?

Exercises 2.1

1. Add. Reduce your answers to simplest terms.

a. $\frac{2}{7}+\frac{4}{7}=$

b. $\frac{3}{10}+\frac{2}{10}=$

c. $\frac{7}{9}+\frac{2}{9}=$

d. $\frac{1}{6}+\frac{3}{6}=$

e. $\frac{2}{3}+\frac{1}{6}=$

f. $\frac{1}{4}+\frac{3}{8}=$

g. $\frac{1}{5}+\frac{3}{10}=$

h. $\frac{1}{6}+\frac{1}{2}=$

i. $\frac{2}{5}+\frac{1}{10}=$

2. Subtract. Reduce your answer to simplest terms.

a. $\frac{7}{9}-\frac{2}{9}=$

b. $\frac{8}{10}-\frac{3}{10}=$

c. $\frac{7}{12}-\frac{3}{12}=$

d. $\frac{5}{6}-\frac{5}{6}=$

e. $\frac{5}{6}-\frac{3}{6}=$

f. $\frac{7}{9}-\frac{1}{3}=$

g. $\frac{8}{12}-\frac{1}{4}=$

h. $\frac{5}{10}-\frac{1}{2}=$

i. $\frac{9}{10}-\frac{2}{5}=$

j. $\frac{5}{8}-\frac{1}{4}=$

Turn to the Answer Key at the end of the module to check your work.

Summary

1. To find the sum (or difference) of two fractions that have the same denominator, we find the sum (or difference) of the numerators and leave the denominators unchanged.

$$\frac{1}{7} + \frac{2}{7} = \frac{3}{7}$$

$$\frac{5}{9} - \frac{1}{9} = \frac{4}{9}$$

2. To find the sum (or difference) of two fractions that have different denominators, we must consider two equivalent fractions that have common denominators. The smallest of these common denominators is called the least common denominator of the two fractions.

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

3. The least common denominator of two fractions is the LCM of the two denominators.

Lesson 3

Decimals

Learning Outcomes

By the end of this section you will be better able to:

- describe a fractional quantity using decimal numbers
- compare decimal numbers
- put a list of decimal numbers in order
- add and subtract with decimal numbers
- multiply and divide with decimal numbers

A decimal fraction is part of a whole part of a whole number just like a fraction is. A decimal fraction is another way of writing a common fraction when the denominator (the bottom number) in a fraction is a multiple of 10. The multiples of ten you will be working with are **tenths** and **hundredths**. Later on you will work in thousandths.

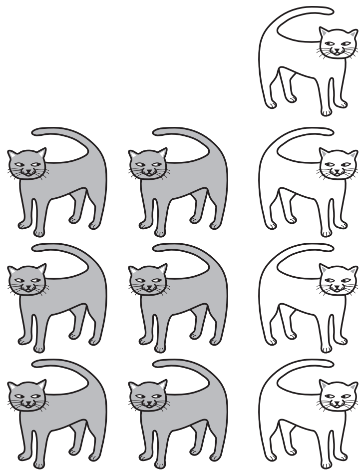
When you use money you are working with decimals.

For example: $7.63

The decimal point separates whole objects, from parts of whole objects. Whole dollars (7 dollars) are separated from tenths of a dollar (6 dimes) and hundreds of a dollar (3 pennies).

6 dimes are $\frac{6}{10}$ of a dollar and is written 0.6.

How do you think 8/10 of a dollar would be written? (0.8)



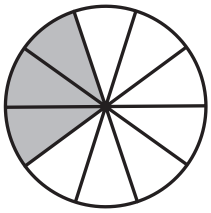
- How many cats are there in this illustration? (10)
- How many of the 10 cats are black? (6)
- What is the fraction of cats out of ten that are black? $\left(\frac{6}{10}\right)$
- If you were to write that fraction in the decimal form, how would you write it? (0.6)
- So 0.6 is the decimal form and $\frac{6}{10}$ is the fraction form.
- How many of the cats are white? (4)
- What is the fraction of cats out of ten that are white? $\left(\frac{4}{10}\right)$
- How would you write that fraction in the decimal form? (0.4)
- What does the zero (0) mean? (It means there are no whole numbers.)

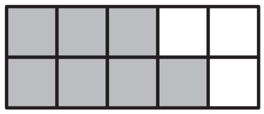
It is important to remember that the decimal point follows the ones' place. If you have a number in front of the decimal, it would tell you how many whole objects you have, and the number after the decimal would tell you how many parts of the whole object you have.

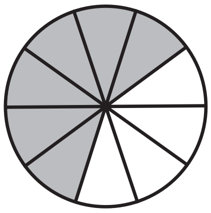
For example, 1.4 would tell you that there was one whole object and 4 parts of the whole object.

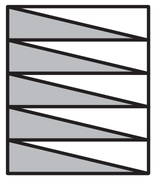
Self Test

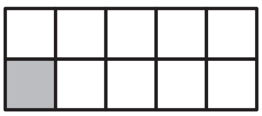
Write a fraction and a decimal for each of the shaded parts.

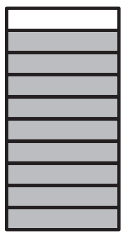
1. 

2. 

3. 

4. 

5. 

6. 

Answers

1. $\frac{3}{10} = 0.3$
2. $\frac{7}{10} = 0.6$
3. $\frac{6}{10} = 0.6$
4. $\frac{5}{10} = 0.5$
5. $\frac{1}{10} = 0.1$
6. $\frac{9}{10} = 0.9$

Exercises 3.1

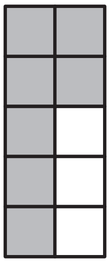
Write the number for each fraction.

1. three fifths ________________
2. one half ________________
3. nine tenths ________________
4. one eighth ________________
5. five tenths ________________
6. one fifth ________________
7. two thirds ________________
8. two sixths ________________
9. seventy hundredths ________________
10. seven eighths ________________

Exercises 3.2

A. Write a common fraction and a decimal fraction for the shaded parts of each diagram.

1.



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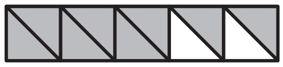
2.



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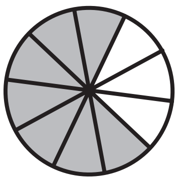
3.



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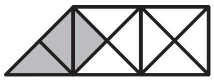
4.



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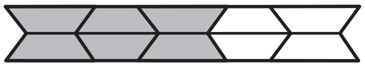
5.



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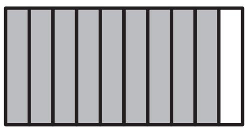
6.



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B. Write each of the decimal fractions below as a common fraction.

Example: $0.5 = \frac{5}{10}$

three tenths = 0.3

1. one tenth ______________________
2. 0.1 ______________________
3. 0.7 ______________________
4. five tenths ______________________
5. nine tenths ______________________
6. 0.8 ______________________

C. Write each common fraction below as a decimal fraction.

Example: $\frac{1}{10} = 0.1$

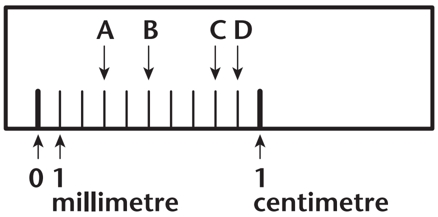
seven tenths = 0.7

1. eight tenths ______________________
2. two tenths ______________________
3. $\frac{3}{10}$ ______________________
4. $\frac{9}{10}$ ______________________
5. six tenths ______________________
6. five tenths ______________________

Exercises 3.3

Measuring a Tenth of a Centimetre

Metric units are always expressed (written) as a decimal.
There are 10 millimetres in 1 centimetre.
Therefore, 1 millimeter = 1/10 cm.



1 millimetre = 0.1 centimetre

Write the following millimeters as centimeters.

1. 3 millimetres = ___________ centimetres

2. 5 millimetres = ___________ centimetres

3. 8 millimetres = ___________ centimetres

4. 9 millimetres = ___________ centimetres

5. What would 10 millimetres be equal to? ________________

Turn to the Answer Key at the end of the module to check your work.

More About Decimals

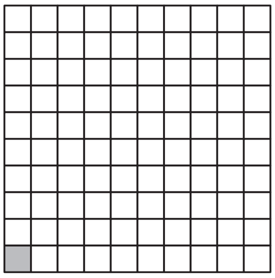
A decimal fraction is another way of writing a common fraction when the denominator—the bottom number in a fraction—is a multiple of 10. Multiples of 10 include numbers like 10, 100, 1000.

Look at these decimal fractions:

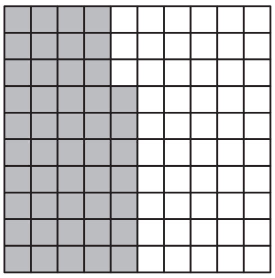
0.6 0.12 0.059

Written as common fractions they would read:

$\frac{6}{10}$ $\frac{12}{100}$ $\frac{59}{1000}$

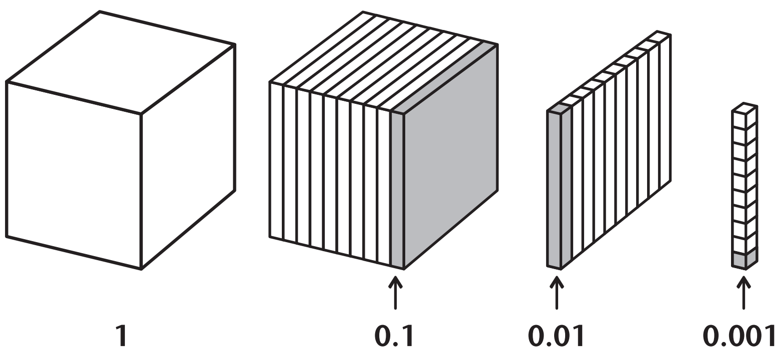


In this diagram one hundredth of the whole or $\frac{1}{100}$ is written as 0.01.

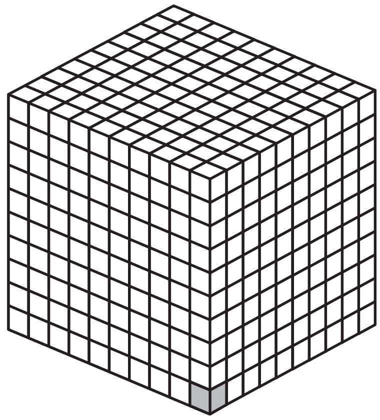


In this diagram 47 hundredths or $\frac{47}{100}$ is written as 0.47.

To understand $\frac{1}{1000}$, let's look at this cube.

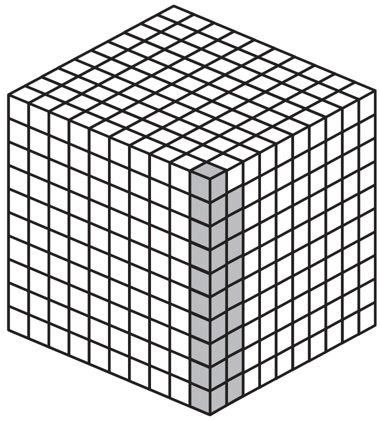


Here is what one thousandth of a whole looks like.



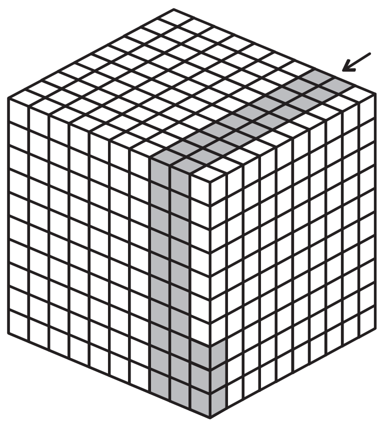
$\frac{1}{1000}$ is written as 0.001.

The next diagram shows you $\frac{19}{1000}$.



How would you write it as a decimal? (0.019)

This last diagram shows you $\frac{203}{1000}$.

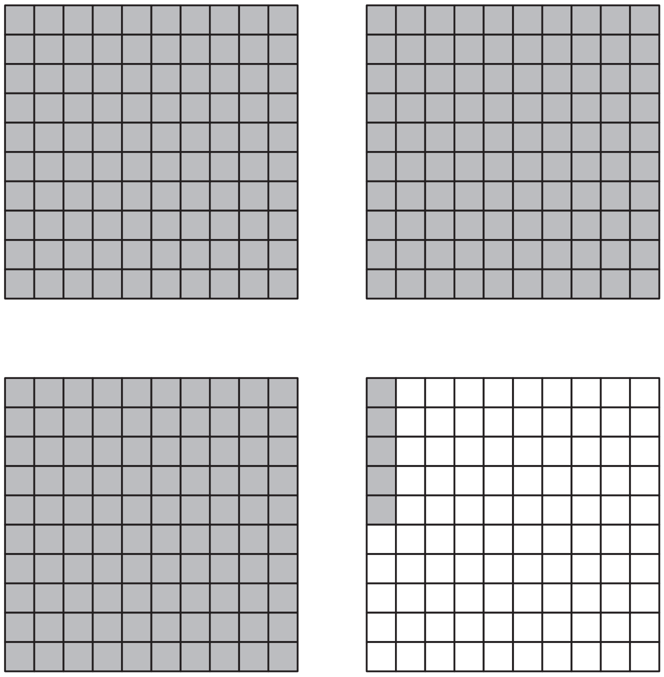


You have to use your imagination here. This shows the whole layer with 100 little cubes in each layer that has been darkened.

The decimal fraction is written as 0.203

You have learned:

- tenths—The number of parts is written on the first place to the right of the decimal point.
- hundredths—The number of parts is written on the second place to the right of the decimal point.
- thousandths—The number of parts is written on the third place to the right of the decimal point.
- whole numbers—Complete items that are not broken up into parts are recorded on the left side of the decimal point.



In the diagram above there are 3 whole and 5 hundredths of a whole which have been darkened.

You write it as: 3.05.

If you wrote 3.50, it would show that 3 whole and 50 hundredths had been darkened.

In this chart you can see whole numbers to the left of the decimal point and decimal parts to the right side.

whole numbers			•	**decimal parts**		
hundreds	tens	ones	•	tenths	hundredths	thousandths

Exercises 3.4

A. Write the following as numerals.

1. seven thousand six hundred five ____________________
2. forty thousand two hundred ____________________
3. nine hundred twenty-three ____________________
4. eight thousand one ____________________
5. sixty-eight thousand ____________________

B. Write in word form.

4200 ____

69 000 ____

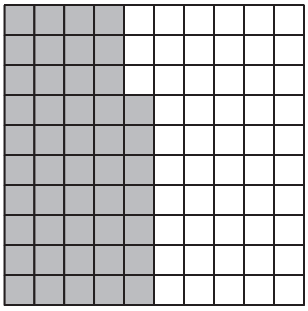
507 ____

30 000 ____

406 000 ____

Exercises 3.5

A. Write the decimal fraction that represents the shaded part of each of the following diagrams.

1. 

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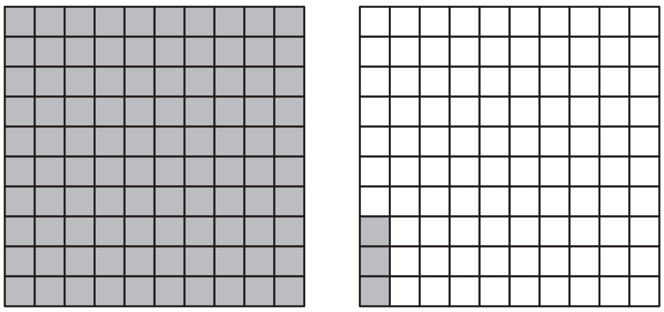
2. 

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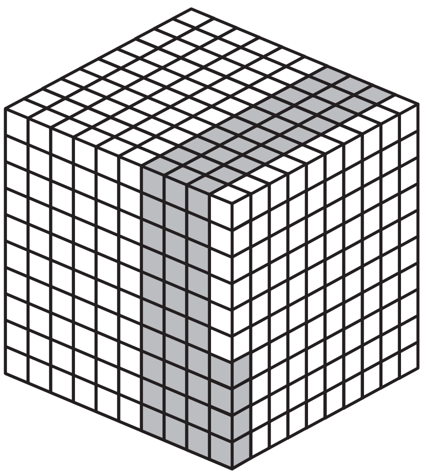
3. 

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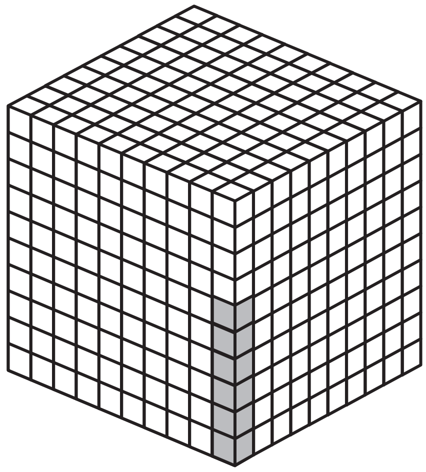

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B. Write the rule that determines which place on the right side of the decimal point a number will be written for the following:

1. tenths ____

____

____

2. hundredths ____

____

____

3. thousandths ____

____

____

C. Write the following as decimal fractions.

1. eight tenths ______________________________

2. six hundred and nine hundredths ________________

3. two thousandths ___________________________

4. seven and fifty-one hundredths _________________

5. forty-four thousandths _______________________

6. five hundred and seven thousandths______________

D. Write the following decimal fractions as words.

1. 0.8 ___________________________________

2. 0.03 __________________________________

3. 0.004 _________________________________

4. 5.36 __________________________________

5. 2000.150________________________________

____

Turn to the Answer Key at the end of the module to check your work.

Making Equivalent Decimals

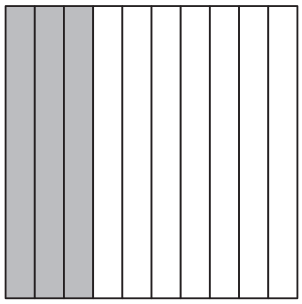
You know a decimal fraction is a part of a whole that is broken up into groups of 10, 100, 1000, and so on.

The first place after the decimal point means tenths, the second place means hundredths, and the third place means thousandths.

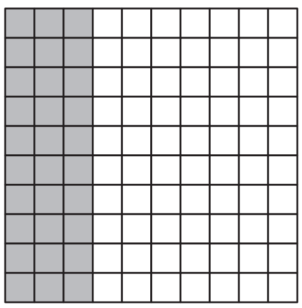
You also know that zero is used as a place holder to indicate how the number is read.

The following illustrations will help you picture equivalent decimal fractions.

In this diagram the square is divided up into tenths. Three of the tenths are darkened. In other words, 0.3 of the whole diagram is darkened.



In this next diagram the same size square is divided up into hundredths. Thirty of the hundredths or 0.30 of the whole diagram are darkened.



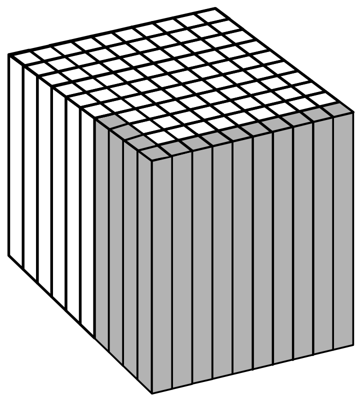
You can see from the two diagrams that the squares are the same size. Therefore 0.3 and 0.30 must show the same part of the whole.

This means they have equivalent value.

The zero at the end is written as a place holder in 0.30 to show that the whole is divided into hundredths.

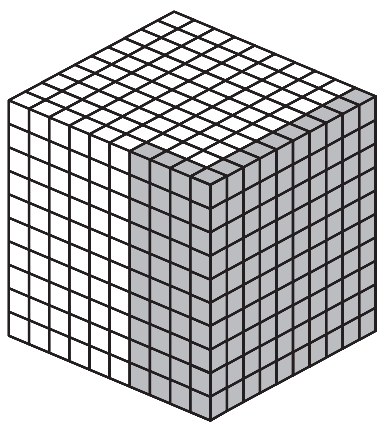
Now let's look at the following cube diagrams. The cubes are divided into hundredths and thousandths and you can see that the decimal fractions have equivalent value.

This diagram shows one whole divided into hundredths. That's 10 rods in each layer × 10 layers = 100.



Thirteen of these hundredths or 0.13 of the diagram is darkened.

Now look at the second diagram. It shows the same cube divided into thousandths. That's 10 layers × 10 rods in each layer x 10 little cubes in each rod.



One hundred thirty of these thousands or 0.130 are darkened. You can see that 0.13 and 0.130 equal the same amount and are equivalent.

You can see how the final number in the decimal fractions shows how many equal pieces the whole is divided into—tenths, hundredths, or thousandths.

Exercises 3.6

Write the following numerals using zeros as place holders.

1. 3 tens ______________________
2. 700 hundreds ______________________
3. 90 thousands ______________________
4. 800 tens ______________________
5. 610 hundreds ______________________
6. 20 thousands ______________________
7. 191 thousands ______________________
8. 3910 tens ______________________

Exercises 3.7

Write equivalent decimal numbers for each of the following decimals.

A. Show as tenths:

1. 0.30 ______________________
2. 0.200 ______________________
3. 5.600 ______________________
4. 0.900 ______________________
5. 3498.00 ______________________
6. 679 ______________________

B. Show as hundredths:

1. 0.3 ____________________
2. 0.9 ____________________
3. 0.890 ____________________
4. 67.8 ____________________
5. 19.2 ____________________
6. 3891 ____________________

C. Show as thousandths:

1. 0.03 ____________________
2. 0.9 ____________________
3. 0.07 ____________________
4. 43.1 ____________________
5. 391 ____________________
6. 4.20 ____________________

Exercises 3.8

A. Circle the greater decimal in each pair.

1. 0.46 or 0.37
2. 3.06 or 4.02
3. 0.18 or 0.72
4. 2.70 or 2.07
5. 0.22 or 0.06
6. 7.60 or 7.51

B. John said that 1.40 is greater than 1.4 because 40 is greater than 4. Is he correct? Use pictures to support your answer.

Turn to the Answer Key at the end of the module to check your work.

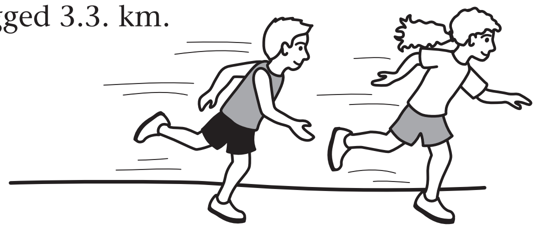
Comparing Decimal Fractions Using Place Value

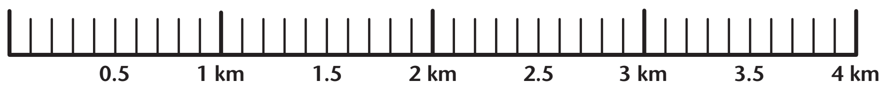
Look at these two decimal fractions: **2.1** and **1.7**.

Which one is the greater? Why?

Bob jogged 2.9 km (kilometers) while Elsa jogged 3.3. km.

Who jogged the greatest distance?





A place value chart is a handy tool to use when you are working with equivalent numbers. This is what it would look like if we placed 24.57 on the place chart below.

Tens		Ones		Tenths		Hundredths
2		4	.	5		7
↓		↓		↓		↓
value		value		value		value
20	+	4	+	0.5	+	0.07
			OR			
20	+	4	+	$\frac{5}{10}$	+	$\frac{7}{100}$
			24.57			

Let's see what 6.09 looks like on a place value chart.

Tens		Ones		Tenths		Hundredths
		6	.	0		9
↓		↓		↓		↓
value		value		value		value
0		6	+	0.0	+	0.09
			OR			
		6	+	$\frac{0}{10}$	+	$\frac{9}{100}$
			6.09			

Now we are going to find which is greater—1.6 or 1.60. To do this we will place each number on the place value chart.

Place 1.6 on this chart.

Tens	Ones	Tenths	Hundredths

1 . 6

↓ ↓

value value

1 + .6

1.6

Place 1.60 on this chart.

Tens	Ones	Tenths	Hundredths

1 . 6 0

↓ ↓ ↓

value value value

1 + .6 + 0

1.60

Zeros on the end of the decimal do not increase value. They just rename a decimal. Look at these examples:

0.4 → 0.40

4 tenths 40 hundredths

0.4 = 0.40

7.8 is seven and eight tenths.

7.80 is seven and eighty hundredths.

7.8 = 7.80

By using this rule you can compare numbers up to the thousandths place or even higher.

Compare these two numbers.

61.090   61.009

Look at the number on the left first (the greatest place value) and then compare the remaining numbers. This is just the same as comparing whole numbers.

The numbers on the left are the same.

61.090   61.009

Look at the next place to the right of the first number and compare.

Again the numbers are the same.

61.090   61.009

Look to the next place to the right and compare.

Once more the numbers are the same.

61.090   61.009

Look to the next place to the right and compare.

The numbers are different. 9 hundredths are more than 0 hundredths. Therefore 61.090 is greater than 61.009.

To sum up: **The first digits compared from the left to the right that are found to be different tell which number, as a whole, has the greater or lesser value.**

Use < (less than), > (greater than) or = to describe the relationship between the following pairs of numbers.

a.	1.70	1.71	b.	9.8	9.08
c.	6.6	6.60	d.	0.8	0.80
e.	6.50	6.45	f.	0.22	0.220

(Answers: a. <, b. >, c. =, d. =, e. >, f. =)

Exercises 3.9

Write the value of each of the underlined digits.

Example: 41$\underline{7}$ 623 $\underline{7}$000

1. 14 2$\underline{3}$5 ______________________
2. 86 $\underline{3}$01 ______________________
3. 11$\underline{4}$ 265 ______________________
4. $\underline{7}$065 ______________________
5. $\underline{1}$78 290 ______________________
6. 4$\underline{5}$4 154 ______________________
7. 1 $\underline{0}$01 321 ______________________
8. $\underline{2}$ 751 345 ______________________

Exercises 3.10

A. Write each number as a decimal in standard form.

Example: two and sixty hundredths $\underline{\mathbf{2.60}}$

1. four and seven tenths ____________________
2. eleven and forty-two hundredths ____________________
3. seven hundredths ____________________
4. nine tenths ____________________

B. Write the value of the underlined digit as a decimal and as a fraction.

Example: $\mathbf{7.\underline{1}5}$

$\mathbf{0.1 = \frac{1}{10}}$

1. $12.1\underline{6}$ ______________________
2. $8.\underline{2}7$ ______________________
3. $19.7\underline{5}$ ______________________
4. $61.\underline{0}5$ ______________________

C. Use the place value chart for each number and put each number into its correct position on the chart.

Example: 16.45

Tens	Ones	Tenths	Hundredths
1	**6** •	**4**	**5**

↑

place the decimal point on the chart

1. 8.45

Tens	Ones	Tenths	Hundredths

2. 12.07

Tens	Ones	Tenths	Hundredths

D. Write the value of each underlined digit.

1. $1\underline{7}.42$ ________________________
2. $11.\underline{7}5$ ________________________
3. $6.0\underline{9}$ ________________________
4. $1\underline{4}.06$ ________________________
5. $20.7\underline{0}$ ________________________
6. $\underline{4}6.41$ ________________________

E. Write < (less than) or > (greater than) to complete each number statement below.

1. 0.21 ____ 0.12
2. 3.16 ____ 4.99
3. 17.21 ____ 7.89
4. 13.01 ____ 13.00
5. 619.444 ____ 691.444
6. 99.002 ____ 98.763
7. 11.310 ____ 11.301
8. 5.005 ____ 5.050
9. 783.90 ____ 784.90
10. 20.016 ____ 20.106

Exercises 3.11

A. Write an equivalent decimal for each of the given decimals below.

Example: 1.70 = 1.7

1. 7.70 __________ 2. 3.4 __________

3. 17.8 __________ 4. 2.3 __________

5. 19.90 __________ 6. 25.60 __________

B. Order from least to greatest.

1. 0.034 0.043 0.039 0.304 0.344

____

2. 392.01 391.02 390.99 392.21 391.22

____

C. Order from greatest to least.

1. 79.41 178.41 77.04 79.14 79.07

____

2. 0.002 0.012 0.200 0.120 0.001

____

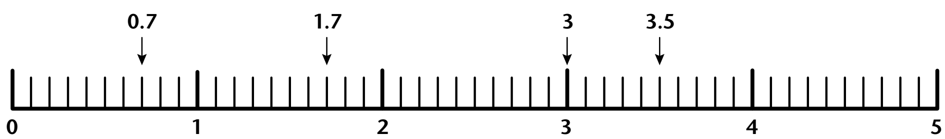
Turn to the Answer Key at the end of the module to check your work.

Ordering Decimals

In the last lesson we compared decimals. In this lesson you will put the decimals in order. You will order them from the greatest to the least, and from the least to the greatest.

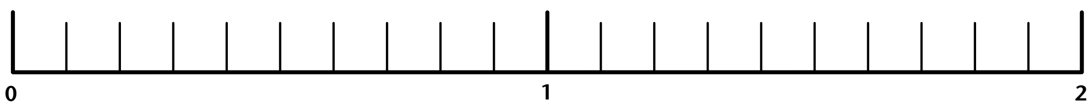
Look at this number line.

The numbers at the bottom of the line are the whole numbers.



The spaces between the whole numbers are divided into ten equal pieces. Each of those pieces is a tenth.

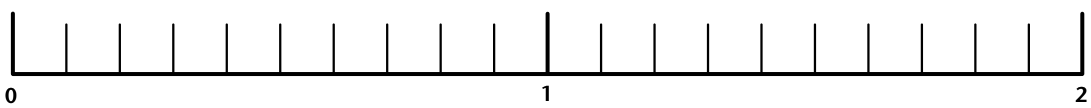
The number 1.7 describes the spot that is seven tenths past the number 1.



Self Test

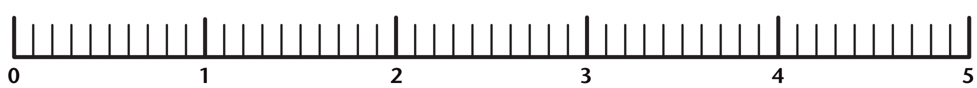
1. Place each of the following decimals on the number line below. Write the decimal numbers in order from the least to the greatest.

   0.5, 1.9, 0.4, 1.6



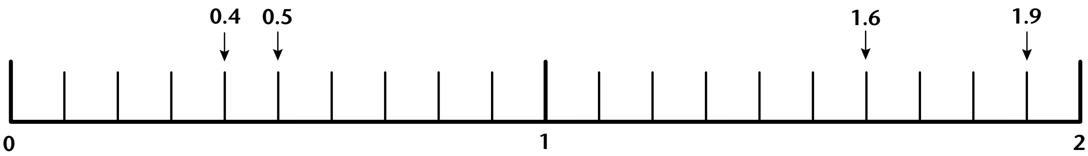
2. Use this number line below to help you write the list of decimals in order from the greatest to the least.

   4.1, 0.8, 2.7, 4.6, 1.5, 3.8, and 3.0

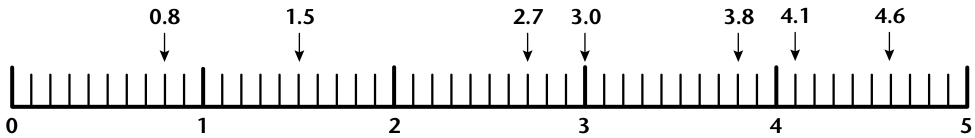


Answers

1.



2.



Exercises 3.12

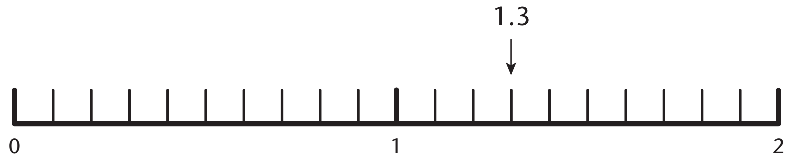
A. Complete the chart below by filling in the common fraction and decimal fraction for each of the numbers given.

Number	Common Fraction	Decimal Fraction
1. $\frac{12}{10}$	__________	__________
2. $\frac{23}{10}$	__________	__________
3. $\frac{38}{10}$	__________	__________
4. $\frac{26}{10}$	__________	__________
5. $\frac{17}{10}$	__________	__________
6. $\frac{10}{10}$	__________	__________

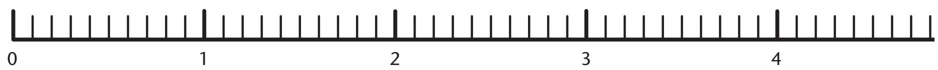
Exercises 3.13

A. Place each of the following decimals on the number line.

Example: 1.3



1. 3.9
2. 0.6
3. 4.7
4. 1.4
5. 2.8
6. 4.2



B. Order each group of numbers from least to greatest.

1. 4.2, 0.4, 2.1 ____________________

2. 7.9, 7.1, 8.2 ____________________

3. 5.3, 6, 5.5, 6.1 ____________________

4. 6.8, 7, 7.3, 7.1 ____________________

5. 2.3, 1, 0.7, 1.6, 1.9 ____________________

6. 3.0, 4.1, 3.9, 4.0 ____________________

Exercises 3.14

Order each group of numbers from the greatest to the least.

1. 7.2, 2.7. 0.7, 7.8, 7.1

   ____

2. 0.9, 9.0, 9.9. 9.1, 9.6

   ____

3. 10.1, 10.6, 10, 10.9, 11.2

   ____

4. 5.1, 7.3, 8.3, 0.9, 4.8, 3.7

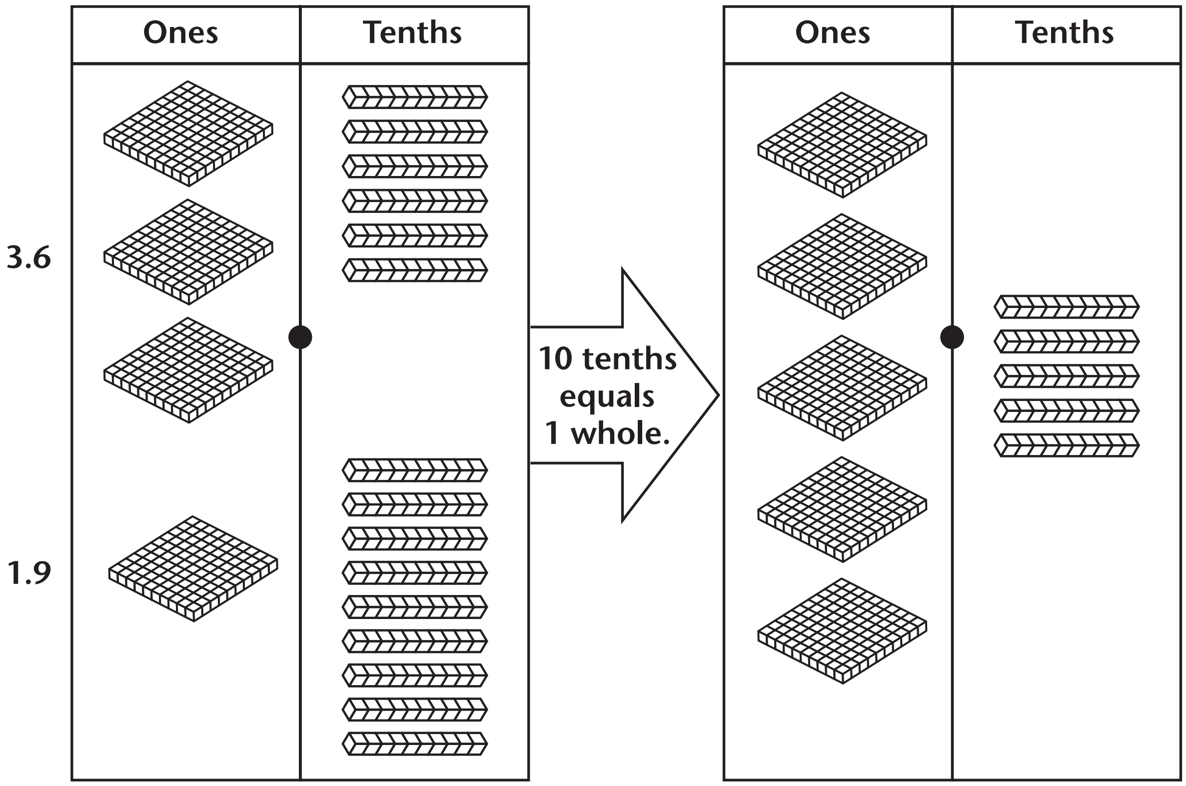
   ____

Turn to the Answer Key at the end of the module to check your work.

Adding Decimal Fractions

You know how to add whole numbers. You can use the same strategies to add decimals.

Look at the pictures of Base 10 blocks. They show you what happens when you add 3.6 + 1.9

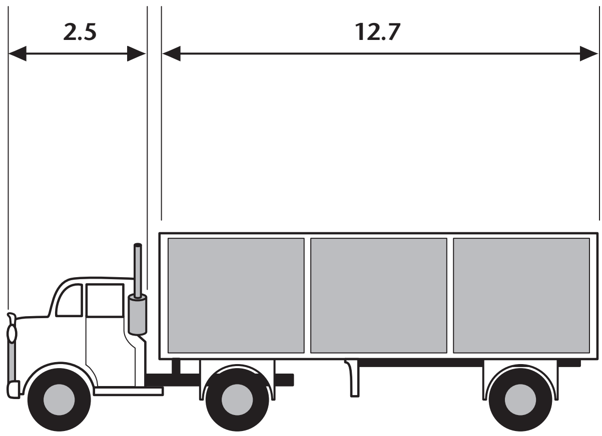


The sum of 3.6 + 1.9 = 5.5

To add without pictures, use place value. Look at this example.

Add the tenths: 12 tenths 2.**8** + 3.**4**	10 tenths equals 1 whole. That's 1 and 2 tenths. **1** 2.8 + 3.4 **2**	Add the ones: 1 **2**.8 + **3**.4 **6.2**

The length of the tractor (or cab) is 2.5 m (metres) and the length of the trailer is 12.7 m. How long is the entire tractor trailer?



Think: 12.7 m + 2.5 m = ?

Write:

	tens	ones	tenths
	1	2^{1}	7
	+	2	5
Answer	1	5	2

The tenths have been regrouped into 1 ones and 2 tenths.

The tractor trailer is 15.2 long.

Mrs. Jones bought 0.5 kg of peaches. The peaches went in a fruit salad with 1.2 kg of pears. How much fruit was in the fruit salad?

Think: "0.5 kg + 1.2 kg = ?"

Write:

	ones	tenths
	0	5
	1	2
Answer	1	7

Mrs. Jones has 1.7 kg of fruit in the salad.

Self Test

Try these addition questions.

$$\begin{array}{r} 4.2 \\ +\ 2.3 \\ \hline \end{array} \qquad \begin{array}{r} 1.7 \\ +\ 5.6 \\ \hline \end{array} \qquad \begin{array}{r} 5.7 \\ +\ 6.7 \\ \hline \end{array} \qquad \begin{array}{r} 3.3 \\ +\ 9.8 \\ \hline \end{array}$$

Answers

6.5 7.3 12.4 13.1

You know that decimal fractions must be lined up correctly before you can add. Study the following problem.

Jake was training for the city cross-country run. He ran 6.1 km on Friday, 5 km on Saturday, 5.2 km on Sunday and 6 km on Monday. How many kilometers did Jake run altogether during those 4 days?

Let's review what we already know about adding decimals.

To add decimal numbers you must follow two simple steps:

- Align (line up) the decimal points, which will line up the place values
- Add the decimal numbers exactly as you would add whole numbers.

To answer this problem, you would write the decimal numbers vertically and then add. It looks like this:

$$\begin{array}{r} 6.1 \\ 5.0 \\ 5.2 \\ +\ 6.0 \\ \hline 22.3 \end{array}$$

Mr. Beaumont mailed three parcels at the post office. The first parcel weighed 0.823 kg, the second 1.3 kg, and the third 0.68 kg. What was the total weight of Mr. Beaumont's three parcels?

You line up the decimal points:

$$\begin{array}{r@{}l} & .823 \\ 1 & .3 \\ & .68 \\ \hline \end{array}$$

Then you add using regrouping as you would with whole numbers.

$$\begin{array}{r@{}l} {}^{1} & {}^{1} \\ & .823 \\ 1 & .3 \\ & .68 \\ \hline 2 & .803 \end{array}$$

Exercises 3.15

Line up the following sets of decimal fractions so the decimal points are in a vertical (straight up and down) line. Use zeros as place holders if necessary.

1. 34.15 600 0.051 6.18 9.136

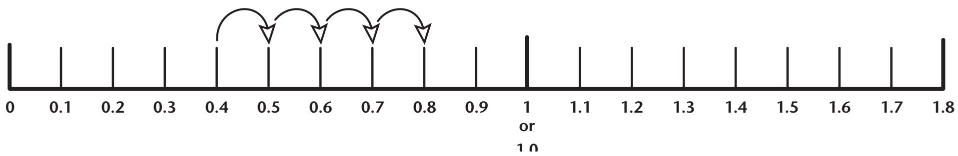
2. 481.2 13 619.51 0.002 1732

3. 14.900 6.84 0.05 182.13 72

Exercises 3.16

A. Use the number line to help you find the sums.

Example: 0.4 + 0.4 = 0.8



1. 0.2 + 0.4 = __________ 2. 0.7 + 0.3 = __________

3. 0.6 + 0.5 = __________ 4. 0.8 + 0.9 = __________

B. Add these questions.

1.	2.	3.	4.
0.2	2.9	27.2	21.6
+0.7	+3.5	+47.9	75.2
			+49.2

C. Line up (align) the decimal points and then add each of the following questions.

1. 14.5 + 5.3 2. \$12.38 + \$1.89 + \$43.98

3. 6.2 + 14.6

4. 8.403 + 12 + 3.98

5. 33.9 + 41.2

D. Solve each problem. Show all your work and then write a statement to answer each question.

1. Jenny jogged 6.2 km (kilometers) on Saturday and 4.8 km on Sunday. How far did she jog on the weekend?

Statement: ______________________________________

2. In a jumping contest the judges combined the best three jumps of each contestant. If Laurie had jumps of 1.3m, 1.6m, 1.7m, and 1.6m, what was her combined score?

Statement: ____

____

Exercises 3.17

Take out your calculator. Here are some activities to help you discover how decimal fractions are recorded on it.

When you enter numbers on a calculator, it is not necessary to press the 0 to the left of the decimal unless it is a whole number place holder (such as 40). The calculator does it for you.

- Press **.96** on your calculator. Remember, you don't need to press 0.96, just .96.
- Now add **one hundredth (0.01)** to .96. Press the equal sign.
- Your calculator should read **0.97**. Continue adding 0.01. Don't forget to press = and then + each time.

What do you notice when you reach **1.00**?

You will notice that your calculator probably reads 1. with no zeros.

Many calculators do not display end zeros in a decimal answer.

Try another number:

Press 23.248 on your calculator.

Now add one thousandth (.001)

Your calculator should show 23.249.

Continue adding .001.

What happens when you reach 23.250?

Your calculator should read 23.25 instead of 23.250.

Now use your calculator to add the following sets of decimal numbers.

1. 19.35 + 18.645 = ____________________

2. 84 + 0.03 + 16.81 = ____________________

3. 93.186 + 4 + 3.027 + 91.5 = ____________________

4. 86.3 + 19.85 + 4.27 + 5.007 = ____________________

Solve this problem.

Karli and her brother Barry entered their pumpkins in the Fall Fair. Karli's pumpkin weighed 13.512 kg. Barry's pumpkin weighed 9.7 kg. What were the combined weights of the 2 pumpkins?

Statement: ____

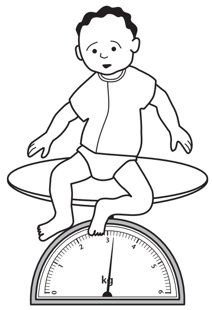
____

Turn to the Answer Key at the end of the module to check your work.

Subtracting Decimal Fractions

Subtracting decimal numbers is very similar to the process you just learned for adding decimal numbers. Line up the decimal points, then subtract just like you would for whole numbers.

When Baby Jamie was born his mass was 3.2 kg.
When he was weighed the next day he had lost 0.4 kg.
What was his mass on the second day?



Think: 3.2 kg – 0.4 kg

Write:

ones	tenths
2	12
~~3~~ .	~~2~~
– 0 .	4
2 .	8 kg

The baby's mass was 2.8 kg on the second day.

Jim is 151.8 cm tall. Cheryl is 3.91 cm shorter than Jim. How tall is Cheryl?

You know the first step is to align the numbers by the decimal points. Because Jim's height is written to the tenth decimal place, and Cheryl's is written to the hundredth place, it is easier to find the difference if you use a zero as a place holder. Look at this example.

$$\begin{array}{r} 151.8\mathbf{0} \\ \underline{-3.91} \end{array}$$

The next step is to subtract the same way you would with whole numbers. It's a good idea to write down the decimal point on the answer line before you calculate the answer.

Look at this example.

$$\begin{array}{r} 151.80 \\ \underline{-3.91} \\ . \end{array}$$

The final step is to complete the subtraction, borrowing (regrouping) if it's necessary. You'll see in this question, you need to regroup.

$$\begin{array}{r} {\scriptstyle 4\ 10\ 17\ 1} \\ 1\cancel{5}\cancel{1}.\cancel{8}0 \\ -\ 3.91 \\ \hline 147.89 \end{array}$$

Cheryl is 147.89 cm tall.

Check: You can add the answer to the piece you subtracted. If you get the number you started with, you know you did the work correctly. *Align the decimal points.*

$$\begin{array}{r} 147.89 \\ +3.91 \\ \hline 151.80 \end{array}$$

Self Test

Try these subtraction questions. Copy the problem onto a piece of paper and line up the decimal points. You should check your work by adding.

1. 8.7 – 7.2 =
2. 100.2 – 84.7 =
3. 7.509 – 0.29 =

Answers

1. 1.5
2. 15.5
3. 7.219

Exercises 3.18

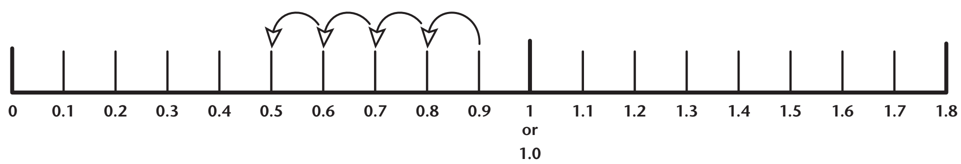
Review your regrouping skills by subtracting these whole numbers.

1. 459 –389	2. 3964 –1892	3. 9733 –4819
4. 6051 –3944	5. 9205 –3856	

Exercises 3.19

A. Use the number line to help you find the differences.

Example: 0.9 – 0.4 = 0.5



1. 0.5 – 0.2 = ________
2. 0.8 – 0.6 = ________
3. 1.0 – 0.3 = ________
4. 1.4 – 0.9 = ________
5. 1.7 – 1.2 = ________
6. 1.3 – 1.2 = ________

B. Align these numbers and find the differences.

1. $284.16 – $103.79

2. 423.1 – 16.5

3. 18 – 9.37

C. Round each decimal to the nearest one and estimate the differences. Find the exact differences to check your estimates. Show your estimation.

1. $\begin{array}{r} 34.5 \\ \underline{-23.6} \end{array}$

2. $\begin{array}{r} 17.4 \\ \underline{-10.7} \end{array}$

3. $\begin{array}{r} 23.50 \\ \underline{-19.78} \end{array}$

4. $\begin{array}{r} 431.25 \\ \underline{-330.55} \end{array}$

Exercises 3.20

A. Complete these questions using your calculator.

1. 6.902 – 4.500 = __________
2. 5.120 – 5.034 = __________
3. $954.89 – $132.45 = __________
4. 98.756 – 0.510 = __________

B. Use your calculator to help solve these problems.

1. Al completed a 10 km run in 45.61 min. His brother Chris completed the same run in 43.19 min. Which brother won the race and by how much?

Statement: ____

____

2. Janice bought a sweater priced at $37.89. She received a sale discount of $6.78 and then was charged $1.87 tax. What did Janice pay for the sweater?

Statement: ______________________________________

____

Turn to the Answer Key at the end of the module to check your work.

Lesson 4

Percent

Learning Outcomes

By the end of this section you will be better able to:

- demonstrate the meaning of percent
- solve problems involving percent

Take out a newspaper and have a look through it. Do you see any fractions? How about decimals? Can you find any percents? You will probably find all three in the sports section. The batting averages in baseball are decimals, basketball free throw averages are usually in percents, and fractions are found when comparing games won and total games played. You will build on your understanding how percents, decimals and fractions are all related in this lesson. After we review some skills, we'll focus on using those skills to solve problems.

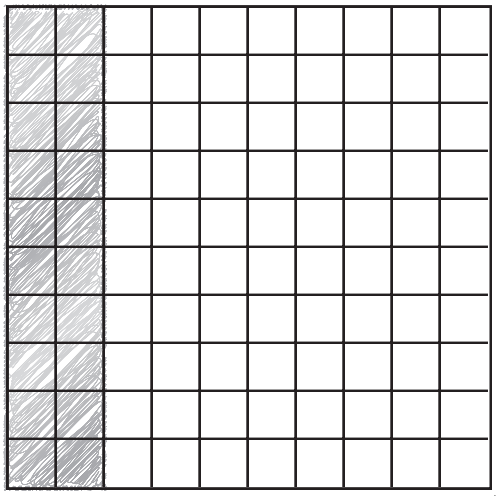
Do you know what percent means?

Let's separate percent into two words: "per" and "cent."

When somebody says that they are driving at 50 kilometres per hour, we can write this as 50 km/h. We can see that "per" is written as a fraction, so whenever you see the word "per" you can think of a fraction.

Now "cent" is French for 100. So whenever you see percent think of "out of 100." This is a fraction with a denominator of 100.

For example, 20% is = $\frac{20}{100}$



We can also write percents as fractions. For instance, 72% is $\frac{72}{100}$.

Remember, we need to simplify fractions.

$$\frac{72}{100} = \frac{72 \div 4}{100 \div 4} = \frac{18}{25}$$

Let's review one more thing. Percents can also be written as decimals. Since a percent can be written as a fraction over 100, we can easily write it as a decimal. Let's take a look at an example.

25% = $\frac{25}{100}$ = 0.25

33% = $\frac{33}{100}$ = 0.33

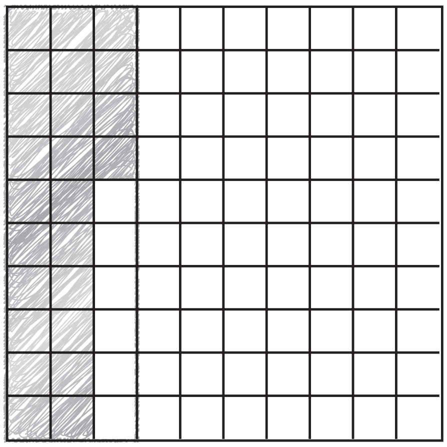
Here are a couple of helpful hints!

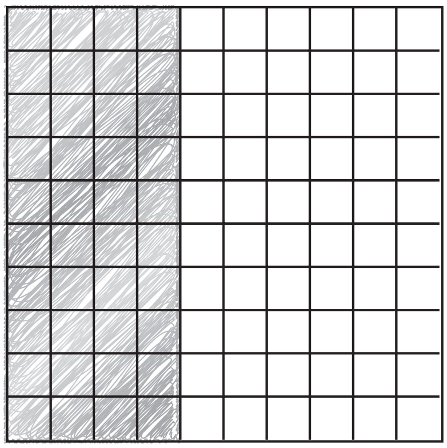
The two 00s in the 100 remind us that there are two decimal points when changing a % to a decimal. For instance 20% = 0.20.

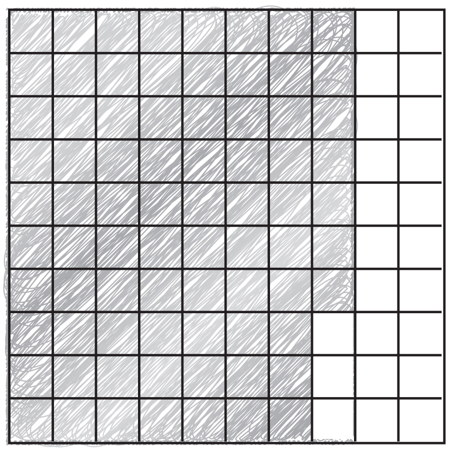
The two 00s in the % symbol also can remind us that % is "**o**ut **o**f" 100. For example, 15% is 15 "out of 100."

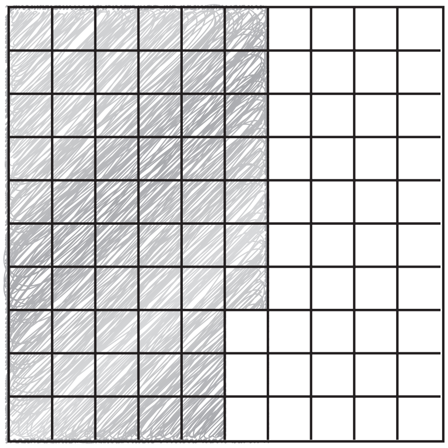
Exercises 4.1

1. Here are some 10 by 10 grids. Find the percent that is shaded in each.

a.  = ____ %

b.  = ____ %

c.  = ____ %

d.  = ____ %

2. Write the following percents as reduced fractions.

   a. 30% =

   b. 85% =

   c. 46% =

   d. 55% =

   e. 28% =

3. Now change these percents to decimals.
   a. 15%

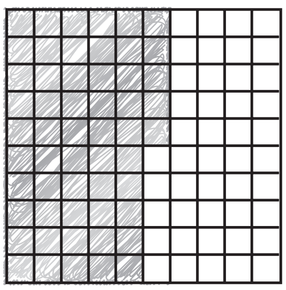
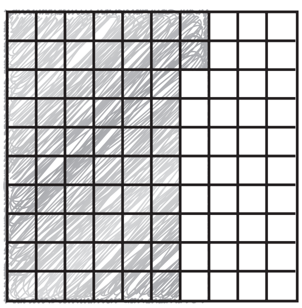
   b. 65%

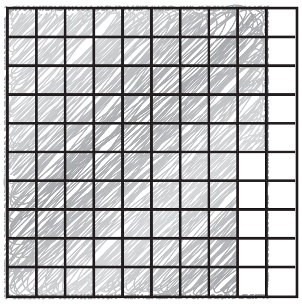
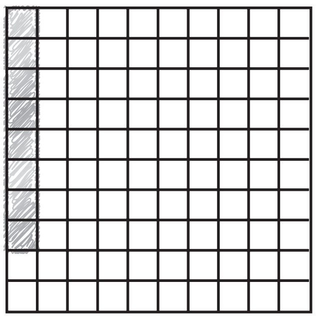
   c. 87%

   d. 59%

   e. 49%

4. Write the amount shaded in the 10 by 10 grid as a fraction (in lowest terms), percent, and decimal.

10 by 10 grid Tenths and hundredths	Fraction (in lowest terms)	Percent	Decimal
			
			

5. Samuel received 75% on his last test. What fraction of the test did he get right?

6. Norio received a quiz back. He received a mark of 3/5. What percent did he get on the quiz?

7. Jessica chose a page out of the newspaper and saw that ads took up one quarter of the page. What percent of the page is covered in ads?

8. Pierre chose a page out of the newspaper and saw that ads took up two thirds of the page. How can this be written as a decimal?

9. Fill in the chart below. The first one is done for you.

Fraction (in lowest terms)	Decimal	Percent
$\frac{1}{4}$	0.25	25%
a. $\frac{19}{20}$		
b. $\frac{43}{50}$		
c.	0.32	
d.	0.74	
e.		92%
f.		8%

Turn to the Answer Key at the end of the module to check your work.

Problem Solving with Percent

All of the word problems we are going to work with use the following equation:

$$\begin{array}{ccccc} (\text{percent}) & \times & (\text{original number}) & = & (\text{amount}) \\ \text{P} & \times & \mathbf{O} & = & \text{A} \end{array}$$

Here's what you need to know to use this equation:

Percent (P) = the percent in DECIMAL form

Original number (O) = the number after the word "of"

Amount (A) = percent of a number

50% of 20 is 10, so let's see what this will look like in the equation:

$$\begin{array}{ccccc} (\text{percent}) & \times & (\text{original number}) & = & (\text{amount}) \\ (0.5) & \times & (20) & = & (10) \end{array}$$

We will be looking at problems where the percent, original number (of number), or the amount is missing.

Let's take a closer look at the equation: **P × O = A**

This equation is solved for A. But what if we want to find P?

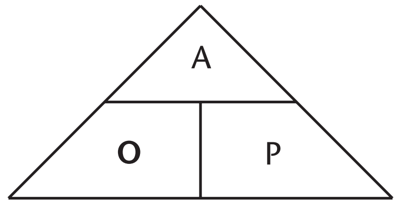
You will learn more about ways to solve equations like this in a future Math course. For now, here is a short explanation.

To have P on its own, we need to move the **O**. Since the P and **O** are being multiplied we need to "un-multiply" them. Un-multiplying is the same dividing. We "un-multiply" by dividing both sides by **O.**

$$\frac{P \times \mathbf{O}}{\mathbf{O}} = \frac{A}{\mathbf{O}} \rightarrow P = \frac{A}{\mathbf{O}}$$

We can use the same "un-multiply" method to solve for **O**. We can isolate **O** by dividing both sides by P. $O = \frac{A}{P}$.

It is important to understand how to isolate variables. But in case you have trouble understanding this right away, you can use a handy trick. Rewrite the equation in a triangle.



A = amount

O = original number (of number)

P = percent in decimal form

Now we will see how to use this triangle by going through examples.

Example 1

20% of 40 is what number?

We know:

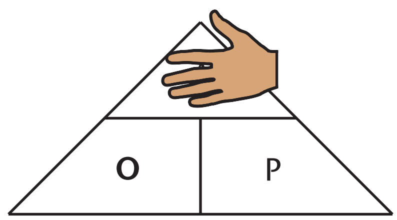
The percent is 20% = 0.20 and the number after "of" is 40 so this is our original number (of number).

P = 0.20

O = 40

A = ?

To find the amount, we cover the A in the triangle and see what we have:



Since the **O** and the P are next to each other, we multiply them.

This is the same as our original equation:

(percent) × (original number) = (amount)

A = O × P

A = 0.20 × 40 = 8

So we know the amount is 8.

Example 2

What percent of 45 is 27?

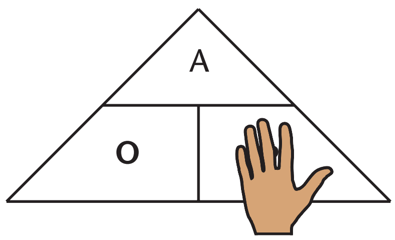
We do not know the percent. The number after "of" is 45, so this is our original number (of number) and our amount is 27.

P = ?

O = 45

A = 27

We do not know P, so let's cover the P in our triangle to see what we need to do:



Since the A is over the **O**, we know $P=\frac{A}{O}$. This is the same equation we found earlier.

$$P = \frac{A}{O}$$

$$P = \frac{27}{45} = 0.6 = 60\%$$

Example 3

80% of what number is 29.6?

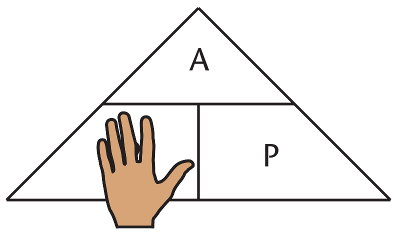
The percent is 80% = 0.80, we do not know the original number (of number), and our amount is 29.6.

P = 0.80

O = ?

A = 29.6

We do not know *O*, so let's cover the *O* in our triangle to see what we need to do:



A is over the P so we need to divide.

$$O = \frac{A}{P}$$

$$O = \frac{29.6}{0.80} = 37$$

Our original number is 37.

Some simple steps can make problem solving easier. In this lesson we'll focus on the following:

1. What is the question asking for?	Read through the problem and identify what you are trying to find. Knowing what sort of answer you're looking for can help you solve the problem.
2. Estimate an answer.	If you have an estimate, then you'll know if your answer is reasonable.
3. Find an answer.	Do the appropriate calculations, and come up with an answer.
4. Make sure the answer is reasonable.	This is a good way to check your work and can help you identify if you've made an error. Compare your answer with your estimate. Is it close? Why or why not? Remember that just because your answer isn't the same as your estimate, doesn't make it wrong. Look carefully at how you rounded the numbers to make your estimate. Stop to think about how your answer compares to your estimate, and decide if your solution is reasonable.

Here's an example problem. We'll use the steps outlined above to solve it.

Scott just ate dinner at a restaurant. His bill came to $23.54. He wants to leave his waiter a 15% tip. How much money should Scott leave for the tip?

Let's follow the four steps:

1. Write what this question is asking (e.g., find 15% of $23.54).
2. Estimate an answer.
3. Find an answer.
4. Make sure the answer is reasonable.

1. What is the question asking for?	What is 15% of $23.54?
2. Estimate an answer.	The bill is about $24. 10% of $24 is $2.40. 15% is 1½ times as much as a 10%. So, $2.40 + $1.20 = $3.60.
3. Find an answer.	The percent is 15% = 0.15, the original number (of number) is 23.54, and we want to find the amount. P = 0.15 **O** = 23.54 A = ? A = **O** × P A = 23.54 × 0.15 = 3.531 = $3.53 The tip should be $3.53.
4. Make sure the answer is reasonable. If not then check over your work.	$3.53 is very close to our estimate, so our answer is reasonable.

Don't forget to answer the question in a sentence.

Scott should leave $3.53 for a tip.

Here are some definitions you may need to review:

Discount = how much money is taken off the original price?

Example: A jacket is originally $100. It is on sale at 25% off.

The discount = 100 × 0.25 = $25

Sales Price = original price – discount.

Example: A jacket is originally $100 and the discount is $25.

The sale price = $100 – $25 = $75

Exercises 4.2

Answer the following questions by following four steps.

1. Write what this question is asking.
2. Estimate an answer. (To estimate, round off the numbers. Then use these rounded numbers to find an estimated answer.)
3. Find an answer.
4. Make sure the answer is reasonable. (Make sure your answer in step 3 is close to your estimate in step 2.)

A chart is provided to guide you for the first few questions. Remember to round to 2 decimal places when working with money.

1. A shirt is regularly priced at $37.49. It is discounted at 30% off. How much is the discount?

What is the question asking for?	
Estimate an answer.	
Find an answer.	
Make sure the answer is reasonable. If not then check over your work.	

2. Christopher just received a mark of 59 out of 70 on his math test. What percent score did he get on the test?

What is the question asking for?	
Estimate an answer.	
Find an answer.	
Make sure the answer is reasonable. If not then check over your work.	

3. Jamie bought a shirt 30% off. It was discounted at $11.27 less than the original price. What was the original price?

What is the question asking for?	
Estimate an answer.	
Find an answer.	
Make sure the answer is reasonable. If not then check over your work.	

4. Two years ago Ben was 120 cm tall. He has grown 20% since then. What is Ben's current height?

Turn to the Answer Key at the end of the module to check your work.

Lesson 5

Ratios and Proportion

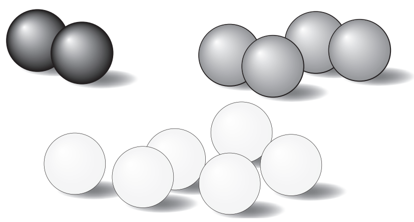
Learning Outcomes

By the end of this section you will be better able to:

- identify and create part-to-part and part-to-whole ratios
- identify and create a proportional statement using ratios

Ratios

Look at the picture below.



We can see from this picture that:

- There are 2 black marbles.
- There are 4 grey marbles.
- There are 6 white marbles.
- There are 12 marbles all together.

Using this information, we can make some comparisons. For example, we can compare:

- the number of grey marbles to the number of white marbles
- the number of black marbles to the number of grey marbles to the number of white marbles
- the number of white marbles to the total number of marbles

In math, we can describe these comparisons using ratios. A **ratio** is a comparison of two or more numbers. We can write each comparison listed above as a ratio by separating the numbers with a colon.

the number of grey marbles to the number of white marbles	4:6
the number of black marbles to the number of grey marbles to the number of white marbles	2:4:6
the number of white marbles to the total number of marbles	6:12

Notice that the order in which the numbers appear is very important. Write the numbers in the ratio in the same order that they are listed in the words.

grey to white

4 : 6

Each number in a ratio is called a *term*. The ratio 4:6 is a *two-term* ratio because it contains two terms, 4 and 6. The ratio 2:4:6 is a *three-term* ratio because it contains three terms, 2, 4, and 6.

Part-to-Part Ratios

A *part-to-part* ratio describes certain parts of a group, or certain parts of a whole. In the marble example above, the ratio of grey marbles to white marbles (4:6) is a part-to-part ratio. It compares different parts of a collection of marbles. The three-term ratio 2:4:6 is also a part-to-part ratio. It describes three parts of the collection.

Part-to-Whole Ratios

A *part-to-whole* ratio describes a part of a group in comparison to the whole group. In the marble example above the ratio of white marbles to the total number of marbles is a part-to-whole ratio. It compares a specific part of the group to the whole group.

Part-to-whole ratios can also be written as fractions. For example, we could write the ratio of white marbles to the total number of marbles as 6:12 or $\frac{6}{12}$.

Ratios can be easier to understand if they are in lowest terms.

Both 6 and 12 are divisible by 6.

$6 \div 6 = 1$

$12 \div 6 = 2$

So $\frac{6}{12}$ and $\frac{1}{2}$.

We could say that $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions.

We could also say that $\frac{6}{12}$ in lowest terms is $\frac{1}{2}$, because the fraction $\frac{1}{2}$ cannot be reduced.

Six of the twelve marbles are white. One out of every two marbles is white. Both of these sentences express the same relationship.

6:12 = 1:2

Proportions

Think about a different pile of marbles. This one is bigger - there are 50 marbles al together.

10 of them are black

15 are grey

25 are white.

How does this pile compare the the smaller collection of marbles we were looking at before?

To answer that question, we will describe some of the relationships using ratios and then write those ratios in lowest terms.

Ratios are easier to understand and compare if they are in lowest terms.

First, we'll look at the ratio of while marbles to the total number of marbles.

Small pile of marbles	Large pile of marbles
white : total	white : total
= 6:12	= 25:50
= 1:2	= 1:2

When we write these ratios in lowest terms, we can see that they are equal.

$$6:12 = 25:50$$

A pair of equivalent ratios is called a **proportion**. We say that these two ratios are proportional to each other.

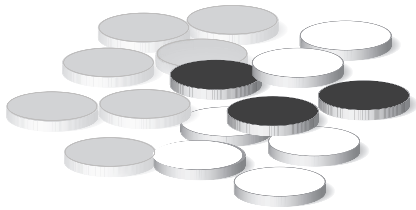
Let's look at another comparison. Are the ratios of black to grey marbles proportional?

Small pile of marbles	Large pile of marbles
black : grey	black : grey
= 2:4	= 10:15
= 1:2	= 2:3

When we write these ratios in lowest terms, we can see that they are NOT proportional.

Exercises 5.1

1. Look at the counters below. The ratios below describe how the coloured counters relate to each other. Explain the relation for each ratio. The first one has been done for you.



   a. 3:7 Number of black counters to the number of grey counters

   b. 3:6 ____

   ____

   c. 7:16 ____

   ____

   d. 3:6:7 ____

   ____

   e. 3:8 ____

   ____

2. Classify each ratio in question 1 as either a part-to-part ratio or a part-to-whole ratio.

   a.

   b.

   c.

   d.

   e.

3. Write a part-to-part ratio for each of the comparisons below.

   a. There are 12 boys and 15 girls in a Grade 8 math class.

   b. To prepare pancakes from a packaged mix, you need 1 cup of water and 2 cups of pancake mix.

   c. This week, the forecast calls for three days of sunshine and four days of rain.

   d. In your dresser drawer you have one pair of pants, three pairs of shorts, and four T-shirts.

4. For each of the comparisons in question 3, write a part-to-whole ratio.

   a.

   b.

   c.

   d.

5. Which of the following pairs of ratios are proportional? How do you know?

   a. 2:4 and 6:12

   b. 1:3 and 4:15

   c. 16:30 and 8:15

Turn to the Answer Key at the end of the module to check your work.

Answer Key

Lesson 1: Fractions

Exercises 1.1

1. (a) $\frac{3}{12}$ $\frac{1}{4}$

   (b) $\frac{2}{12}$ $\frac{1}{6}$

2. (a) $\frac{6}{12}$ $\frac{3}{6}$ $\frac{1}{2}$ (any two)

   (b) $\frac{2}{12}$ $\frac{1}{6}$

3. (a) $\frac{1}{2}$

   (b) $\frac{1}{4}$

   (c) $\frac{3}{4}$

   (d) $\frac{1}{2}$

4. (a) $\frac{1}{3}$ $\frac{2}{6}$

   (b) $\frac{3}{6}$ $\frac{1}{2}$

   (c) $\frac{2}{6}$ $\frac{1}{3}$

   (d) $\frac{4}{8}$ $\frac{1}{2}$ or $\frac{2}{4}$ (any two)

Exercises 1.2

1. (a) 1, 2, 4, 8
   (b) 1, 2, 3, 4, 6, 12
   (c) No, because 2 or 4 can divide into the top and bottom (numerator and denominator).

2. (a) 1, 3, 5, 15
   (b) 1, 2, 4, 7, 14, 28
   (c) Yes, because 1 is the only common factor.

3. (a) Because 2 will divide into the numerator and denominator.
   (b) Because 5 will divide into the numerator and denominator.

4. (a) $\frac{1}{4}$ $\frac{2}{8}$ $\frac{3}{12}$ $\frac{4}{16}$ $\frac{5}{20}$

   (b) $\frac{5}{8}$ $\frac{10}{16}$ $\frac{15}{24}$ $\frac{20}{32}$ $\frac{25}{40}$

Exercises 1.3

1. (a) $\frac{3}{4}$ $\frac{6}{8}$
   (b) $\frac{1}{4}$ $\frac{2}{8}$

2. $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$

3. one (1)

4. (a) $\frac{3 \times 4}{3 \times 5}$ (b) $\frac{5 \times 4}{5 \times 5}$ (c) $\frac{16}{20}$ (d) $\frac{24}{30}$

5. (a) $\frac{20}{24}, \frac{25}{30}, \frac{30}{36}, \ldots$

   (b) $\frac{9}{30}, \frac{15}{50}, \frac{18}{60}, \ldots$

   (c) $\frac{20}{32}, \frac{25}{40}, \frac{35}{56}, \ldots$

   (d) $\frac{3}{24}, \frac{4}{32}, \frac{6}{48}, \ldots$

5. (a) $\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{9}{10}$

(d) $\frac{1}{7}$ (e) $\frac{2}{7}$ (f) $\frac{3}{4}$

(g) $\frac{3}{7}$ (h) $\frac{3}{14}$ (i) $\frac{1}{2}$

(j) $\frac{2}{3}$ (k) $\frac{4}{5}$ (l) $\frac{8}{15}$

(m) $\frac{5}{12}$ (n) $\frac{5}{8}$ (o) $\frac{3}{26}$

Lesson 2: Adding and Subtracting Fractions

Exercises 2.1

1. (a) $\frac{6}{7}$

(b) $\frac{5}{10} = \frac{1}{2}$

(c) $\frac{9}{9} = 1$

(d) $\frac{4}{6} = \frac{2}{3}$

(e)
$$\begin{array}{rl} & \frac{2}{3} = \frac{4}{6} \\ + & \frac{1}{6} = \frac{1}{6} \\ \hline & \frac{5}{6} \end{array}$$

(f)
$$\begin{array}{rl} & \frac{1}{4} = \frac{2}{8} \\ + & \frac{3}{8} = \frac{3}{8} \\ \hline & \frac{5}{8} \end{array}$$

(g) $\frac{1}{5} = \frac{2}{10}$
$+\frac{3}{10} = \frac{3}{10}$
$\frac{5}{10} = \frac{1}{2}$

(h) $\frac{1}{6} = \frac{1}{6}$
$+\frac{1}{2} = \frac{3}{6}$
$\frac{4}{6} = \frac{2}{3}$

(i) $\frac{2}{5} = \frac{4}{10}$
$+\frac{1}{10} = \frac{1}{10}$
$\frac{5}{10} = \frac{1}{2}$

2. (a) $\frac{5}{9}$

(b) $\frac{5}{10} = \frac{1}{2}$

(c) $\frac{4}{12} = \frac{1}{3}$

(d) $\frac{0}{6} = 0$

(e) $\frac{2}{6} = \frac{1}{3}$

(f) $\frac{7}{9} = \frac{7}{9}$
$-\frac{1}{3} = \frac{3}{9}$
$\frac{4}{9}$

(g) $\frac{8}{12} = \frac{8}{12}$
$-\frac{1}{4} = \frac{3}{12}$
$\frac{5}{12}$

(h) $\frac{5}{10} = \frac{5}{10}$
$-\frac{1}{2} = \frac{5}{10}$
$\frac{0}{10} = 0$

(i) $\frac{9}{10} = \frac{9}{10}$
$-\frac{2}{5} = \frac{4}{10}$
$\frac{5}{10} = \frac{1}{2}$

(j) $\frac{5}{8} = \frac{5}{8}$
$-\frac{1}{4} = \frac{2}{8}$
$\frac{3}{8}$

Lesson 3: Decimals

Exercises 3.1

1. $\frac{3}{5}$
2. $\frac{1}{2}$
3. $\frac{9}{10}$
4. $\frac{1}{8}$
5. $\frac{5}{10}$
6. $\frac{1}{5}$
7. $\frac{2}{3}$
8. $\frac{2}{6}$
9. $\frac{70}{100}$
10. $\frac{7}{8}$

Exercises 3.2

A.

1. $\frac{7}{10}$, 0.7
2. $\frac{8}{10}$, 0.8
3. $\frac{7}{10}$, 0.7
4. $\frac{3}{10}$, 0.3
5. $\frac{6}{10}$, 0.6
6. $\frac{9}{10}$, 0.9

B.

1. $\frac{1}{10}$
2. $\frac{1}{10}$
3. $\frac{7}{10}$
4. $\frac{5}{10}$
5. $\frac{9}{10}$
6. $\frac{8}{10}$

C.

1. 0.8
2. 0.2
3. 0.3
4. 0.9
5. 0.6
6. 0.5

Exercises 3.3

1. 0.3 cm
2. 0.5 cm
3. 0.8 cm
4. 0.9 cm
5. 1.0 cm or $\frac{10}{10}$ cm

Exercises 3.4

A. 1. 7605
2. 40 200
3. 923
4. 8001
5. 68 000

B. 1. forty-two hundred or four thousand two hundred
2. sixty-nine thousand
3. five hundred seven
4. thirty thousand
5. four hundred six thousand

Exercises 3.5

A. 1. 0.47
2. 1.03
3. 0.304
4. 0.006

B. 1. **tenths**—The number of parts is written on the ***first*** place to the right of the decimal point.
2. **hundredths**—The number of parts is written on the ***second*** place to the right of the decimal point.
3. **thousandths**—The number of parts is written on the ***third*** place to the right of the decimal point.

C. 1. 0.8
   2. 600.09
   3. 0.002
   4. 7.51
   5. 0.044
   6. 500.007

D. 1. eight tenths
   2. three hundredths
   3. four thousandths
   4. five and thirty-six hundredths
   5. two thousands and one hundred and fifty thousandths or fifteen hundredths

Exercises 3.6

1.	30	2.	70 000
3.	90 000	4.	8000
5.	61 000	6.	20 000
7.	191 000	8.	39 100

Exercises 3.7

A. 1. 0.3
   2. 0.2
   3. 5.6
   4. 0.9
   5. 3498.0
   6. 679.0

B. 1. 0.30
   2. 0.90
   3. 0.89
   4. 67.80
   5. 19.20
   6. 3891.00

C. 1. 0.030
2. 0.900
3. 0.070
4. 43.100
5. 391.000
6. 4.200

Exercises 3.8

A. 1. 0.46
2. 4.02
3. 0.72
4. 2.70
5. 0.22
6. 7.60

B. No – appropriate picture to show they are the same.

Exercises 3.9

1. 30
2. 300
3. 4000
4. 7000
5. 100 000
6. 50 000
7. 0
8. 2 000 000

Exercises 3.10

A. 1. 4.7
2. 11.42
3. 0.07
4. 0.9

B. 1. 0.06 and $\frac{6}{100}$
2. 0.2 and $\frac{2}{10}$
3. 0.05 and $\frac{5}{100}$
4. 0

C. 1.

Tens	Ones	Tenths	Hundredths

8 . 4 5

2.

Tens	Ones	Tenths	Hundredths

1 2 . 0 7

D. 1. 7

2. $\frac{7}{10}$ or 0.7

3. $\frac{9}{100}$ or 0.09

4. 4

5. 0

6. 40

E.
1. 0.21 > 0.12
2. 3.16 < 4.99
3. 17.21 > 7.89
4. 13.01 > 13.00
5. 619.444 < 691.444
6. 99.002 > 98.763
7. 11.310 > 11.301
8. 5.005 < 5.050
9. 783.90 < 784.90
10. 20.016 < 20.106

Exercises 3.11

A.
1. 7.7
2. 3.40
3. 17.80
4. 2.30
5. 19.9
6. 25.6

B.
1. 0.034 0.039 0.043 0.304 0.344
2. 390.99 391.02 391.22 392.01 392.21

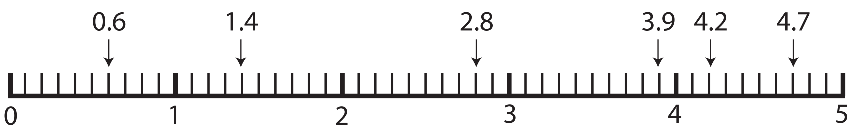
C.
1. 178.41 79.41 79.14 79.07 77.04
2. 0.200 0.120 0.012 0.002 0.001

Exercises 3.12

	Number	Common Fraction	Decimal Fraction
1.	$\frac{12}{10}$	$1\frac{2}{10}$	1.2
2.	$\frac{23}{10}$	$2\frac{3}{10}$	2.3
3.	$\frac{38}{10}$	$3\frac{8}{10}$	3.8
4.	$\frac{26}{10}$	$2\frac{6}{10}$	2.6
5.	$\frac{17}{10}$	$1\frac{7}{10}$	1.7
6.	$\frac{10}{10}$	1	1.0

Exercises 3.13

A.



B. 1. 0.4, 2.1, 4.2
2. 7.1, 7.9, 8.2
3. 5.3, 5.5, 6, 6.1
4. 6.8, 7, 7.1, 7.3
5. 0.7, 1, 1.6, 1.9, 2.3
6. 3, 3.9, 4, 4.1

Exercises 3.14

1. 0.7, 2.7, 7.1, 7.2, 7.8
2. 0.9 9.0, 9.1, 9.6, 9.9
3. 10, 10.1, 10.6,10.9, 11.2
4. 0.9, 3.7, 4.8, 5.1, 7.3, 8.3

Exercises 3.15

1.		2.		3.	
	34.150		481.200		14.900
	600.000		13.000		6.840
	0.051		691.510		0.050
	6.180		0.002		182.130
	9.136		1732.000		72.000

Exercises 3.16

A.

1. 0.6
2. 1.0
3. 1.1
4. 1.7

B.

1.	2.	3.	4.
0.2	2.9	27.2	21.6
+0.7	+3.5	+47.9	75.2
0.9	**6.4**	**75.1**	+49.2
			146.0

C.

1.	2.	3.
14.5	$12.38	6.2
+5.3	1.89	14.6
19.8	43.98	20.8
	$58.25	

4.	5.
8.403	33.9
12.000	41.2
3.980	75.1
24.383	

D.

1.

6.2
+ 4.8
11.0

Jenny jogged 11.0 km.

2.

1.7
1.3
1.6
1.7
+1.6
6.2

Laurie's combined score was 6.2 m.

Exercises 3.17

1. 37.995
2. 100.84
3. 191.713
4. 115.427

$$\begin{array}{r} 13.512 \\ +9.7 \\ \hline 23.212 \text{ kg} \end{array}$$

Exercises 3.18

1. $\begin{array}{r} {\scriptstyle 3\,1} \\ \cancel{4}59 \\ -389 \\ \hline 70 \end{array}$
2. $\begin{array}{r} {\scriptstyle 8\,1} \\ 3\cancel{9}64 \\ -1892 \\ \hline 2072 \end{array}$
3. $\begin{array}{r} {\scriptstyle 8\,1\,2\,1} \\ \cancel{9}7\cancel{3}3 \\ -4819 \\ \hline 4914 \end{array}$
4. $\begin{array}{r} {\scriptstyle 5\,1\,4\,1} \\ \cancel{6}0\cancel{5}1 \\ -3944 \\ \hline 2107 \end{array}$
5. $\begin{array}{r} {\scriptstyle 8\,1\,9\,1} \\ \cancel{9}2\cancel{0}5 \\ -3856 \\ \hline 5349 \end{array}$

Exercises 3.19

A.

1.	0.3	2.	0.2
3.	0.7	4.	0.5
5.	0.5	6.	0.1

B.

1. $\begin{array}{r} \$284.16 \\ -\$103.79 \\ \hline \$180.37 \end{array}$
2. $\begin{array}{r} 423.1 \\ -16.5 \\ \hline 406.6 \end{array}$
3. $\begin{array}{r} 18.00 \\ -9.37 \\ \hline 8.63 \end{array}$

C.

1. $\begin{array}{r} 34.5 \\ -23.6 \\ \hline 10.9 \end{array}$ $\begin{array}{r} 35 \\ -24 \\ \hline 11 \end{array}$
2. $\begin{array}{r} 17.4 \\ -10.7 \\ \hline 6.7 \end{array}$ $\begin{array}{r} 17 \\ -11 \\ \hline 6 \end{array}$
3. $\begin{array}{r} 23.50 \\ -19.78 \\ \hline 3.72 \end{array}$ $\begin{array}{r} 24 \\ -20 \\ \hline 4 \end{array}$
4. $\begin{array}{r} 431.25 \\ -330.55 \\ \hline 100.70 \end{array}$ $\begin{array}{r} 431 \\ -331 \\ \hline 100 \end{array}$

Exercises 3.20

A. 1. 2.402

2. 0.086

3. $822.44

4. 98.246

B. 1.

$$\begin{array}{r} 45.61 \\ -43.19 \\ \hline 2.42 \end{array}$$

Chris won the race by 2.42 minutes.

2.

$$\begin{array}{r} \$37.89 \\ -6.78 \\ \hline \$31.11 \end{array} \qquad \begin{array}{r} \$31.11 \\ +1.87 \\ \hline \$32.98 \end{array}$$

She paid $32.98.

Lesson 4: Percent

Exercises 4.1

1. a. 24
   b. 40
   c. 77
   d. 57

2. a. $30\% = \frac{30}{100} = \frac{3}{10}$
   b. $85\% = \frac{85}{100} = \frac{17}{20}$
   c. $46\% = \frac{46}{100} = \frac{23}{50}$
   d. $55\% = \frac{55}{100} = \frac{11}{20}$
   e. $28\% = \frac{28}{100} = \frac{7}{25}$

3. a. 0.15
   b. 0.65
   c. 0.87
   d. 0.59
   e. 0.49

4. a. $\frac{55}{100} = \frac{11}{20}$, 55%, 0.55
   b. $\frac{62}{100} = \frac{31}{50}$, 62%, 0.62
   c. $\frac{85}{100} = \frac{17}{20}$, 85%, 0.85
   d. $\frac{8}{100} = \frac{2}{25}$, 8%, 0.08

5. $75\% = \frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$

6. $\frac{3}{5} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60\%$

7. 25%

8. 0.67

9.

Fraction (in lowest terms)	Decimal	Percent
$\frac{1}{4}$	0.25	25%
a. $\frac{19}{20}$	**0.95**	**95%**
b. $\frac{43}{50}$	**0.86**	**86%**
c. $\frac{32}{100} = \frac{8}{25}$	0.32	**32%**
d. $\frac{74}{100} = \frac{37}{50}$	0.74	**74%**
e. $\frac{92}{100} = \frac{23}{25}$	**0.92**	92%
f. $\frac{8}{100} = \frac{2}{25}$	**0.08**	8%

Exercises 4.2

1.

1. What is the question asking for?	What is 30% of $37.49?
2. Estimate an answer.	Estimate 30% of $40.00. 10% of $40.00 is $4.00. So, 30% of $40.00 = $12.00.
3. Find an answer.	P = 0.30 A = **O** × P **O** = $37.49 A = $37.49 × 0.30 A = ? A = $11.25 The discount is $11.25.
4. Make sure the answer is reasonable. If not then check over your work.	$11.25 is close to $12.00. The answer is reasonable.

2.

1. What is the question asking for?	Write $\frac{59}{70}$ as a percent.
2. Estimate an answer.	59 is close to 60. 70 is close to 75. $\frac{60}{75} = \frac{20}{25} = \frac{80}{100} = 80\%$
3. Find an answer.	$\frac{59}{70} = 0.8428.... = 84\%$
4. Make sure the answer is reasonable. If not then check over your work.	The answer is close to estimate.

3.

1. What is the question asking for?	\$11.27 is 30% of what?
2. Estimate an answer.	P = 0.30 **O** = ? A = \$12.00 $O = \frac{A}{P} = \frac{12.00}{0.30} = \frac{120}{3} = 40$
3. Find an answer.	P = 0.30 **O** = ? A = \$11.27 $O = \frac{A}{P} = \frac{11.27}{0.30} = 37.57$ The original price is \$37.57.
4. Make sure the answer is reasonable. If not then check over your work.	The answer is close to estimate.

4. 20% of 120 cm

$0.20 \times 120 = 24$ cm

Bert grew 24 cm. His current height is 144 cm

Lesson 5: Ratios and Proportion

Exercises 5.1

1. a. number of black counters to the number of grey counters

   b. number of black counters to the number of white counters

   c. number of grey counters to the total number of counters

   d. number of black counters to the number of white counters to the number of grey counters

   e. number of white counters to the total number of counters (6:16 can also be written as 3:8, an equivalent ratio)

2. a. part-to-part ratio

   b. part-to-part ratio

   c. part-to-whole ratio

   d. part-to-part ratio

   e. part-to-whole ratio

3. a. 12:15 (can also be written 4:5)

   b. 1:2

   c. 3:4

   d. 1:3:4

4. Answers may vary depending on which part you chose to compare.

   a. number of boys to total number of students 12:27
   number of girls to total number of students 15:27

   b. cups of water to total cups of ingredients 1:3
   cups of pancake mix to total cups of ingredients 2:3

   c. days of sunshine to days in the week 3:7
   days of rain to days in the week 4:7

   d. number of pants to total number of clothing articles 1:8
   number of shorts to total number of clothing articles 3:8
   number of T-shirts to total number of clothing articles 4:8

5. a. The ratios are proportional. Multiply both terms in 2:4 by 3 to get 6:12.

   b. The ratios are not proportional. There is no factor that you can multiply or divide either of the ratios by to get the other ratio.

   c. The ratios are proportional. Divide both terms in 16:30 by 2 to get 8:15.