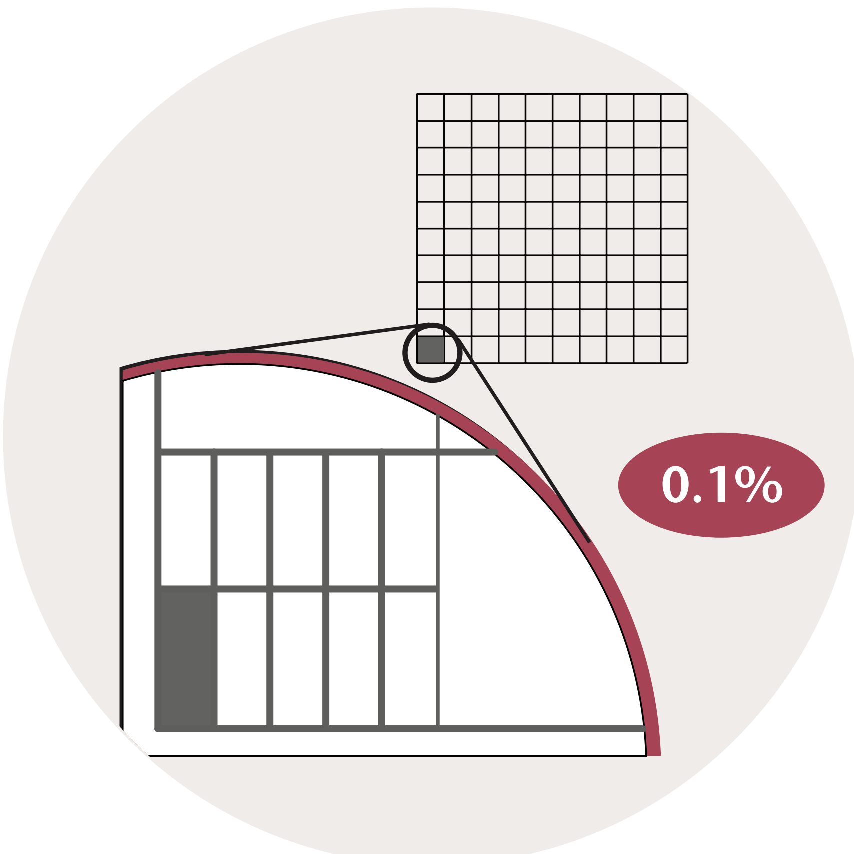
Module 4
Version 03
Section 1–3

Math 8

Module 4
Fractions, Ratios, and Probability



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Course Overview

Welcome to Mathematics 8!

In this course you will continue your exploration of mathematics. You'll have a chance to practice and review the math skills you already have as you learn new concepts and skills. This course will help you to increase your ability to think mathematically.

Organization of the Course

The Mathematics 8 course is made up of four modules. These modules are:

Module 1: Exploring 2–D and 3–D Connections

Module 2: Squares, Integers, and the Pythagorean Theorem

Module 3: Data, Graphing, and Linear Equations

Module 4: Fractions, Ratios, and Probability

Organization of the Modules

Each module has three sections. The sections have the following features:

Pretest

This is for students who feel they already know the concepts in the section. It is divided by lesson, so you can get an idea of where you need to focus your attention within the section.

Lessons

Each section is divided into lessons. Each lesson is made up of the following parts:

Essential Questions

Essential Questions are based on the concepts in each lesson. This activity will help you organize information and reflect on your learning.

Warm–up

This is a brief drill or review to get ready for the lesson.

Explore

This is the main teaching part of the lesson. Here you will explore new concepts and learn new skills.

Try it! Activities

These are activities for you to complete to solidify your new skills. You will mark these using Solutions at the end of each module.

At the end of each module you will find:

Solutions
This contains all of the solutions to the Pretests, Warm–ups and Try it! Activities.

Templates
Templates to pull out, cut, colour, or fold in order to complete specific activities. You will be directed to these as needed.

Glossary
This is a list of key terms and their definitions.

More about the Pretest

There is a pretest at the beginning of each section. This pretest has questions for each lesson in the section. Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in the section. Mark your answers using Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Thinking Space

The column on the outside edge of most pages is called the Thinking Space. You can use this space to

- write questions about things you don't understand
- note things that you want to look at again
- respond to a question in the Thinking Space or the text
- draw pictures that help you understand the math
- identify words that you don't understand
- connect what you are learning to what you already know
- make your own notes or comments

Materials and Resources

There is no textbook required for this course. All of the necessary materials and exercises are found in the modules.

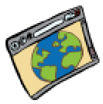
In some cases, you will be referred to templates to pull out, cut, colour, or fold. These templates will always be found near the end of the module, just in front of the answer key.

You will need a scientific calculator for some of the activities. A geometry set would also be helpful, although for many activities you can use a straightedge rather than a ruler.

If you have Internet access, you might want to do some exploring online. The Math 8 Course Website will be a good starting point. Go to **http://www.openschool.bc.ca/courses/math/math8/mod4.html** and find the lesson that you're working on. You'll find relevant links to websites with games, activities, and extra practice.

Icons

You will see a few icons used on the left–hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.



Explore Online

Essential Questions

Solutions

Use a Calculator

Module 4 Overview

You've probably worked with ratios, rates, percentages and probability—and not just in math class. These concepts are all around you.

Maybe you've tried to figure out the distance between two places using the scale on a map. That scale is a ratio. If you then tried to figure out how long it would take you to travel that distance, you probably had to use your speed, which is a rate. Or maybe you've used percents to figure out how much an item will cost after tax. Have your parents ever bought a lottery ticket? What do you think their chances of winning are? Well, you can use probability to figure it out.

What do ratios, rates, percents and probabilities have in common? They all use fractions. We will start this module by working with fractions. Once we we're comfortable with fractions, we'll look at ratios, rates, percents, and probability.

Section Overviews

Section 4.1: Multiplying and Dividing Fractions

In the first section of Module 4, you'll review fractions, including addition and subtraction and changing between mixed numbers and improper fractions. After that you'll move on to multiplying and dividing fractions. You'll start your work with each operation by using fraction strips—found at the back of the module—and move toward multiplying and dividing fractions using rules. After you learn the basics of multiplying and dividing, you'll use benchmarks to help you estimate answers. Finally, you'll use the multiplication and division of fractions to solve problems. Throughout this section, you can watch videos from the Math 8 website http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html to help you understand the processes and different operations.

Section 4.2: Ratios, Rates, and Percents

In the second section, you'll first work with ratios and rates and discover why they're different. Then you'll learn two different ways to solve proportions, including using the cross-products. In the last part of this section, you'll work with percents, including very small percents (0–1%) and larger ones (over 100%). Percents are very important when shopping, so you'll work through lots of word problems that deal with sales tax, discounts and sales, and store markup.

On the Math 8 website http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html you'll find an interactive media tool that allows you to shade in blocks on hundreds grids, and then calculates the equivalent fraction, decimal and percent. You'll also be able to watch two videos that show you how to convert among fractions, decimals, ratios and percents.

Section 4.3: Probability

In the final section of Module 4 (and perhaps the course, if you've already done the previous three modules), you'll work with probability. First you'll review the math terms about probability, and then you'll do some questions about the probability of a single event. Next you'll be introduced to the idea of independent events and must decide if events are independent or dependent. Following this, you'll work through how to calculate the probability of multiple independent events. Finally, you're introduced to tree diagrams, which you can help you organize information and calculate the probabilities of multiple independent events.

The Math 8 website http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0843a1f_probability.html offers help for calculating the probabilities using a coin, die and/or spinner through the use of *Exploring Probability*. If you would like some fun practice with calculating the probability of one event or two independent events, you can play the *Toads and Vines* game.

Course Map

On the following page you'll find a course map. If you colour in the box for each section and lesson as you complete it, you'll easily be able to see how much of the course you've finished, and how much is still left to complete.

Mathematics 8

Module 1

Section 1 Nets and Views
- Lesson A: Visualizing in Three Dimensions
- Lesson B: Prisms and Cylinders and their Nets
- Lesson C: Top View, Side View, and Front View

Section 2 Calculations in 2-D and 3-D
- Lesson A: Total Area of the Net
- Lesson B: More about Area in 2-D and 3-D
- Lesson C: The Amount of Space

Section 3 Shapes and Design
- Lesson A: What is a Tessellation Anyways?
- Lesson B: Transformations in Tessellations
- Lesson C: Creating a Tessellation

Module 2

Section 1 Multiplying and Dividing Integers
- Lesson A: Should I Multiply or Divide?
- Lesson B: Mult. and Div. with Negative Numbers
- Lesson C: Expressions–More than one Operation

Section 2 Squares and Square Roots
- Lesson A: Perfect Squares
- Lesson B: Square Roots of Perfect Squares
- Lesson C: Estimating Square Roots
- Lesson D: Calculating Square Roots

Section 3 The Pythagorean Theorem
- Lesson A: The Pythagorean Theorem
- Lesson B: Pythagorean Triples
- Lesson C: Using the Pythagorean Theorem

Module 3

Section 1 Analyzing statistical graphs
- Lesson A: Types of Graphs and Data
- Lesson B: Adv. and Dis. of Graph Types
- Lesson C: Misleading Graphs

Section 2 Graphing on the Coordinate Plane
- Lesson A: Ordered Pairs and Linear Equations
- Lesson B: Tables of Values
- Lesson C: Relationships and Missing Values

Section 3 Linear Equations
- Lesson A: Algebra Tiles
- Lesson B: Solving Linear Equations Symbolically
- Lesson C: More Linear Equations

Module 4

Section 1 Multiplying and Dividing Fractions
- Lesson A: Proper, Improper Fractions, Mixed Numbers
- Lesson B: Multiplying Fractions
- Lesson C: Dividing Fractions
- Lesson D: Estimating and Solving Problems

Section 2 Ratios, Rates and Percents
- Lesson A: Ratios and Rates in Everyday Life
- Lesson B: Making Sense of Percents
- Lesson C: Solving Problems with Percents

Section 3 Probability
- Lesson A: The Probability of Independent Events
- Lesson B: Problem Solving with Probability

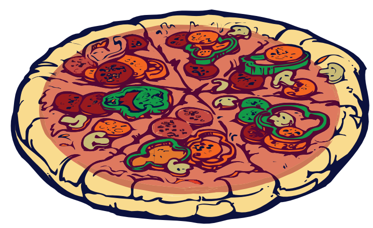
Section 1
Multiplying and Dividing Fractions

In this section you will:

- write a given positive mixed number as an improper fraction
- write a given positive improper fraction as a mixed number
- multiply two given fractions
- divide two given fractions
- solve problems involving multiplication and division of fractions

Where in the World...?

You're inviting twelve people to your birthday party, and you plan to serve pizza and cake. You and your mom make the cake beforehand, using fractions and ratios to scale the recipe so it makes enough cake for all the guests. You also have to plan ahead when you order the pizza so that you have enough pieces for everyone.



When the guests arrive and the pizza is delivered, each person gets a share. Likewise, the cake has to be divided up and split among all the guests (and you, of course! It's your birthday!)

After eating the cake, your friends surprise you with a gift that they pitched together to buy. They divided the cost of the gift by the total number of people buying it, to make the gift affordable.

When we divide a whole object into pieces, this creates fractions. We see fractions in many parts of our lives.

Section 1
Pretest

Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Please complete this pretest **without** using a calculator.

Lesson A: Proper and Improper Fractions and Mixed Numbers

Reduce all answers to lowest terms where necessary.

1. Write the following mixed numbers as improper fractions.

   a. $1\frac{1}{3}$ b. $5\frac{2}{4}$ c. $4\frac{3}{9}$ d. $3\frac{6}{7}$

2. Write the following improper fractions as mixed numbers.

   a. $\frac{38}{6}$ b. $\frac{51}{12}$ c. $\frac{36}{16}$ d. $\frac{13}{6}$ e. $\frac{28}{8}$

Lesson B: Multiplying Fractions

Complete the following multiplication problems. Your answers should be left as either *proper fractions* or *mixed numbers*, and should be *reduced* to lowest terms.

1. a. $\frac{1}{2}\times\frac{1}{3}$ b. $\frac{3}{4}\times\frac{4}{5}$ c. $\frac{5}{6}\times\frac{6}{7}$ d. $\frac{7}{8}\times\frac{8}{9}$ e. $\frac{9}{10}\times\frac{10}{11}$

2. a. $\frac{1}{2}\times 3$ b. $\frac{3}{10}\times 4\frac{1}{6}$ c. $\frac{1}{4}\times 6$ d. $\frac{2}{5}\times 10\frac{1}{3}$ e. $\frac{3}{8}\times 12$

Lesson C: Dividing Fractions

Complete the following division problems. Your answers should be left as either *proper fractions* or *mixed numbers*, and should be *reduced* to lowest terms.

1. a. $\frac{1}{2}\div\frac{1}{3}$ b. $\frac{3}{4}\div\frac{4}{5}$ c. $\frac{5}{6}\div\frac{6}{7}$ d. $\frac{7}{8}\div\frac{8}{9}$ e. $\frac{9}{10}\div\frac{10}{11}$

2. a. $\frac{1}{2} \div 3$ b. $\frac{3}{10} \div 1\frac{1}{6}$ c. $\frac{1}{4} \div 6$ d. $1\frac{2}{5} \div 10$ e. $\frac{3}{8} \div 12$

Lesson D: Estimating and Solving Problems

1. Write a mathematical expression for each of the following sentences. You do not need to solve the expressions.

    a. one third of a half

    b. sharing half a pizza between three people equally

    c. you are doubling a recipe that calls for $1\frac{3}{4}$ cups of flour

2. Can the product of two proper fractions be greater than 1? Explain your answer using an example.

3. A full gas tank in a truck holds 72 litres of fuel. How many litres are in the gas tank if the gauge reads $\frac{3}{4}$ full?

4. A pair of jeans regularly costs $68. If the jeans are advertised at $\frac{1}{4}$ off during a sale, what is the reduced price of the jeans?

Turn to Solutions at the end of the module and mark your work.

Lesson A

Proper and Improper Fractions and Mixed Numbers

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
When might you see a mixed number?		
When can you use improper fractions?		
How do I convert between mixed numbers and improper fractions?		

Thinking Space

Warm–up

1. Complete the following:

   a. $8 + 11 =$

   b. $17 - 13 =$

   c. $5 \times 7 =$

   d. $48 \div 6 =$

   e. $6 \times 5 + 2 =$

   f. $4 \times 12 + 3 =$

2. Find an equivalent fraction by replacing the **?** with the correct number:

   a. $\frac{2}{3} = \frac{?}{15}$

   b. $\frac{3}{4} = \frac{?}{36}$

3. Reduce the following improper fractions to lowest terms.

   a. $\frac{6}{4} = \frac{\quad}{\quad}$

   b. $\frac{20}{8} = \frac{\quad}{\quad}$

Turn to Solutions at the end of the module and mark your work.

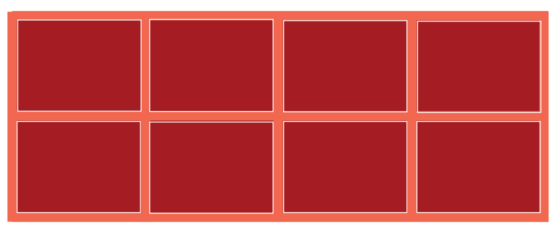
Thinking Space

Explore

Vocabulary of Mixed Numbers and Improper Fractions

Let's begin with a review of fractions. We can think of any object as being "whole." For example, the picture below shows one whole chocolate bar.

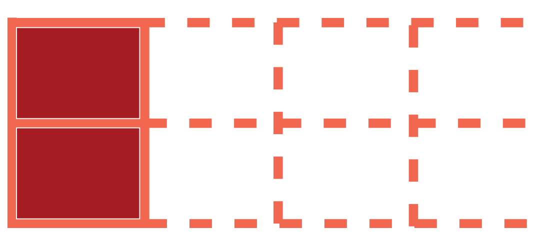
We can also think of it as being one whole object made of any number of equal parts. Think of dividing the chocolate bar into equal pieces to share with your friends. Before you share the chocolate bar, you divide it into 8 pieces. You still have one whole chocolate bar, because you still have 8 out of 8 pieces. In this case the whole object can be viewed as a fraction that equals "1".



$$1 = \frac{8}{8}$$

If you were to share with three of your friends, you might give each person 2 pieces, leaving only 2 pieces for yourself.

$$\frac{2}{8} = \frac{\textbf{numerator}}{\textbf{denominator}} = \frac{\text{number of equal parts I have}}{\text{total number of equal parts}}$$



Thinking Space

In a **proper fraction** the numerator is smaller than the denominator.

Example:

$$\frac{2}{3}$$

In an **improper fraction**, the numerator is bigger than the denominator.

Example:

$$\frac{5}{3}$$

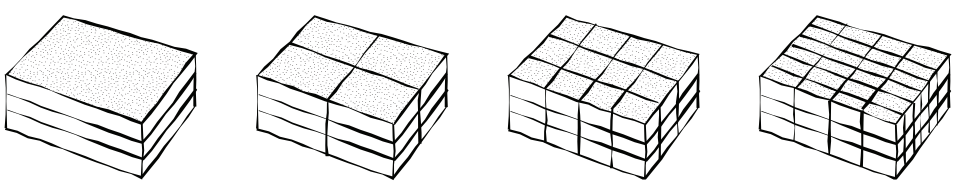
Fractions that have the same value or represent the same ratio are called **equivalent fractions**.

Example:

$$\frac{3}{4} = \frac{9}{12} = \frac{18}{24}$$

x3

x6



A **mixed number** is made of a whole number and a fraction. It is really a short hand notation for the addition of a whole number and a fraction.

Example:

$$2\frac{1}{3} = 2 + \frac{1}{3}$$

Thinking Space

Try It!
Activity 1

Match the term on the left to the example on the right.

______ 1. denominator	a. the 3 in $\frac{2}{3}$
______ 2. improper fraction	b. $\frac{4}{7}$
______ 3. mixed number	c. $2\frac{1}{5}$
______ 4. numerator	d. $\frac{9}{4}$
______ 5. proper fraction	e. the 2 in $\frac{2}{5}$

Turn to Solutions at the end of the module and mark your work.

Thinking Space

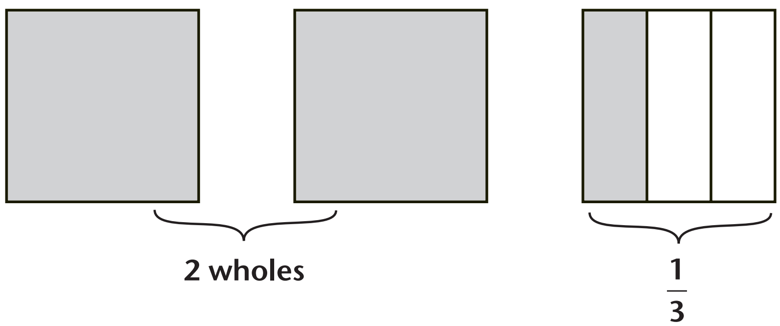
Explore

Proper and Improper Fractions and Mixed Numbers

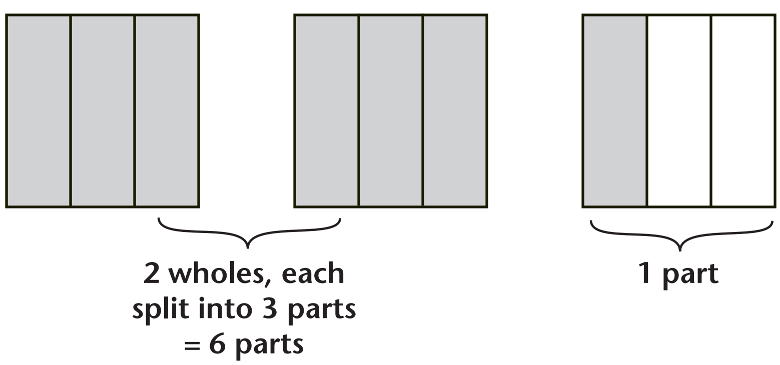
When we're working with fractions, we often see mixed numbers. It can be difficult to perform operations (addition, subtraction, multiplication and division) on mixed numbers. In order to work with fractions, we have to know how to convert from a mixed number to an improper fraction.

Let's work through an example. We'll change $2\frac{1}{3}$ to an improper fraction.

First let's draw a picture of the mixed number.



We can divide up the wholes into the same number of parts as the fraction: 3 parts for each whole.



Thinking Space

Now, we can find the total number of parts from the picture. One way is just to count them. Another way is to do the following operation:

$$(2 \text{ wholes} \times 3 \text{ parts in each}) + 1 \text{ part}$$
$$2 \times 3 + 1$$
$$= 6 + 1$$
$$= 7$$

So, we have seven parts. Remember that the "parts" are actually thirds. So we have seven thirds or $\frac{7}{3}$.

$$2\frac{1}{3} = \frac{7}{3}$$

The Steps

For any mixed number, you can follow these steps to convert to an improper fraction.

Step 1: Multiply the whole number by the denominator of the fraction.

Step 2: Add the numerator of the fraction to this product.

Step 3: Put the sum on top of the denominator.

Let's look at one more example.

Write $6\frac{3}{5}$ as an improper fraction.

Step 1: Multiply the whole number by the denominator of the fraction.

$$6\frac{3}{5}$$

$6 \times 5 = 30$

Step 2: Add the numerator of the fraction to this product.

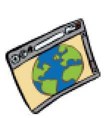
$$6\frac{3}{5}$$

$30 + 3 = 33$

Step 3: Put the sum on top of the denominator.

$$\frac{33}{5}$$

So, $6\frac{3}{5} = \frac{33}{5}$.



Watch the first example in the video *Changing Mixed Numbers and Improper Fractions*. Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html and click on Module 4.

Thinking Space

Try It!
Activity 2

1. Convert the following mixed numbers into improper fractions using pictures for each.

   a. $4\frac{2}{3}$

   b. $5\frac{1}{6}$

   c. $2\frac{3}{7}$

2. Convert the following mixed numbers into improper fractions.

   a. $8\frac{1}{5}$

   b. $6\frac{2}{9}$

Thinking Space

c. $3\frac{1}{4}$

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Converting from an Improper Fraction to a Mixed Number

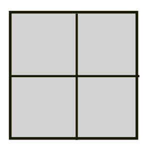
After you perform operations on fractions, you will often be asked to express your answer as a proper fraction or a mixed number. If your answer is an improper fraction, you will have to convert it to a mixed number.

Lets try an example. We'll change the improper fraction $\frac{9}{4}$ to a mixed number.

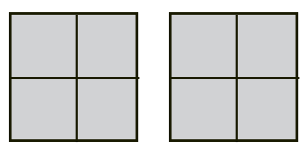
Let's try drawing a picture.

First of all, we know that our "wholes" have been cut into quarters. We'll draw quarters until we have 9 of them.

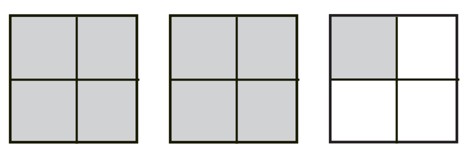
Here's 4 quarters:



Here's 8 quarters:



Now we only need one more quarter to make 9. We'll draw another square, but shade in only one part.



You can see from the drawing that we have two wholes and one quarter, so our mixed number is $2\frac{1}{4}$.

$$\frac{9}{4} = 2\frac{1}{4}$$

Thinking Space

The Steps

For any improper fraction, you can follow these steps to convert to a mixed number.

Step 1: Divide the numerator by the denominator. Make note of the whole number and the remainder.

Step 2: The whole number from the division becomes the whole number in your mixed number.

Step 3: The remainder becomes the numerator of the fraction in your mixed number.

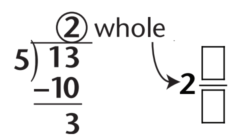
Step 4: The denominator of this fraction is the same as the denominator of the improper fraction you started with.

Let's work through an example. We'll change $\frac{13}{5}$ to a mixed number.

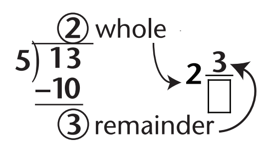
Step 1: Divide the numerator by the denominator. Make note of the whole number and the remainder.

$$\begin{array}{r} 2 \\ 5\overline{)13} \\ \underline{-10} \\ 3 \end{array}$$

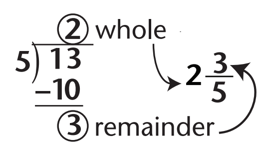
Step 2: The whole number from the division becomes the whole number in your mixed number.



Step 3: The remainder becomes the numerator of the fraction in your mixed number.

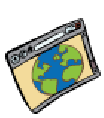


Step 4: The denominator of this fraction is the same as the denominator of the improper fraction you started with.



The improper fraction we started with was $\frac{13}{5}$, so our denominator is 5.

So $\frac{13}{5} = 2\frac{3}{5}$.



Watch the second example in the video *Changing Mixed Numbers and Improper Fractions*. Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html and click on Module 4.

Thinking Space

Try It!
Activity 3

Convert the following improper fractions to mixed numbers and reduce when necessary:

1. $\frac{53}{8}$

2. $\frac{35}{5}$

3. $\frac{42}{12}$

4. $\frac{54}{13}$

5. $\frac{20}{6}$

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B
Multiplying Fractions

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How do I multiply fractions?		

Thinking Space

Warm–up

1. Reduce the following fractions to lowest terms.

   a. $\frac{6}{10}$ b. $\frac{54}{60}$ c. $\frac{32}{72}$

2. Convert the following to improper fractions.

   a. 7   b. $2\frac{1}{5}$ c. 63

3. Convert the following to mixed numbers.

   a. $\frac{11}{3}$ b. $\frac{51}{8}$ c. $\frac{109}{12}$

4. Multiply.

   a. 4×3

   b. 5×4

   c. 6×7

   d. 9×8

   e. 3×11

   f. 7×12

   g. 9×2

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Multiplying Fractions Using Fraction Strips

For this Explore, you will need:

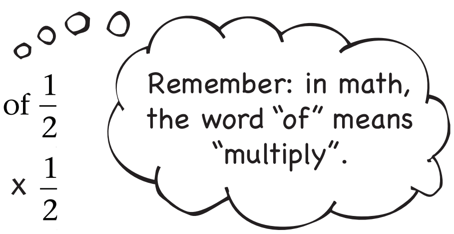
- fraction strips (from Appendix)

You had a whole chocolate bar hidden in your room! Unfortunately, your brother found it and ate half of it, so now you've only got half a chocolate bar. You want to share it equally with your friends Cory and Sarah (so that makes three of you).

When you split something into three equal shares, each person gets $\frac{1}{3}$. So the big question is, what fraction of the whole chocolate bar does each person get? Remember that we don't have a whole chocolate bar to start with, we only have $\frac{1}{2}$ of the chocolate bar.

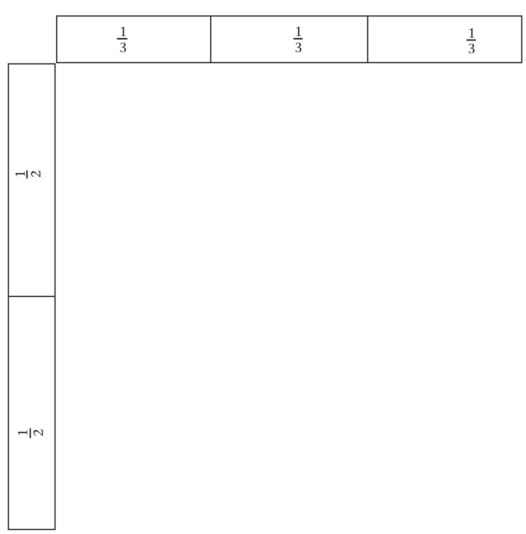
Our expression looks like this: $\frac{1}{3}$ of $\frac{1}{2}$

$\frac{1}{3} \times \frac{1}{2}$



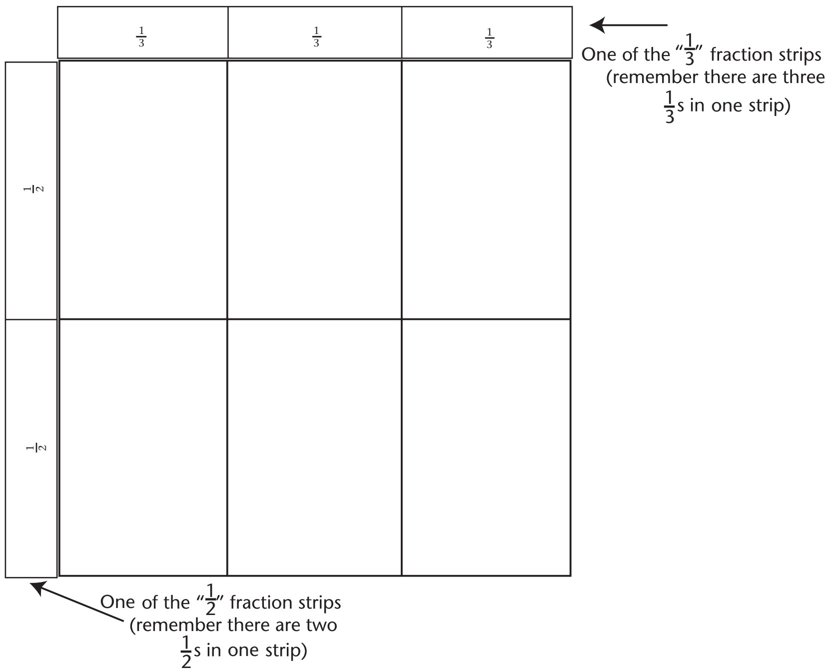
Let's investigate this with the fraction strips.

Get a piece of blank paper and, using your fraction strips as templates, sketch the $\frac{1}{3}$ strip across the top, and the $\frac{1}{2}$ strip down the side.

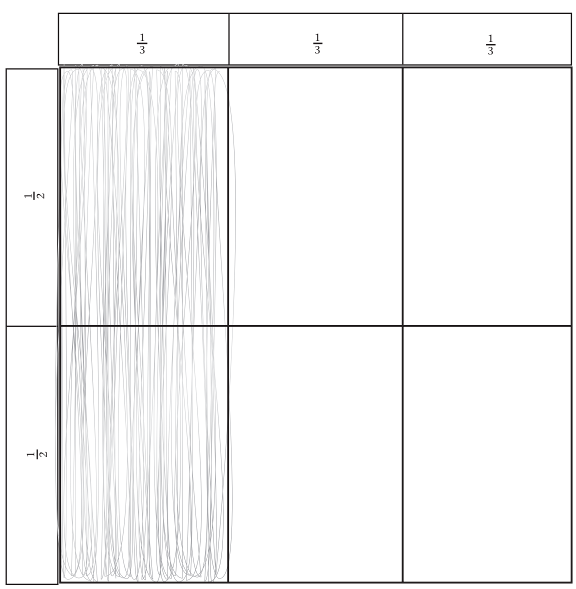


Thinking Space

Now draw the grid that forms by drawing down from the $\frac{1}{3}$ strip and across from the $\frac{1}{2}$ strip, like this:

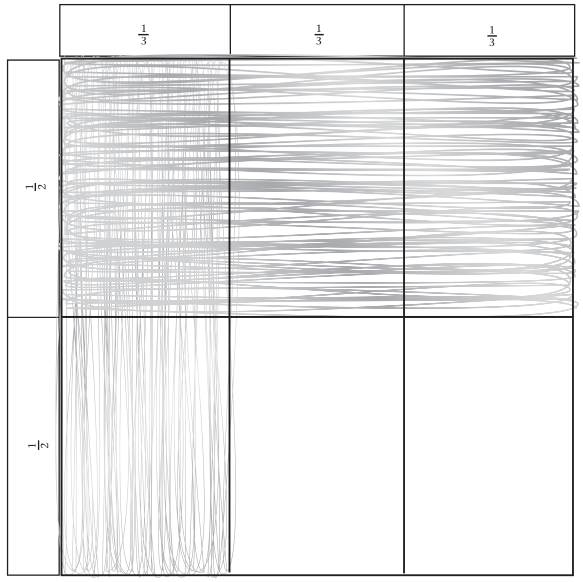


Shade with light vertical lines all the way down to show $\frac{1}{3}$.



Thinking Space

Shade with light horizontal lines all the way across to show $\frac{1}{2}$.

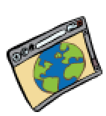


It doesn't make a difference to the answer if you put the $\frac{1}{2}$ fraction strip on top and the $\frac{1}{3}$ fraction strip on the left side. Why is this?

You can now read the answer to the multiplication question from the grid. The double-shaded piece gives the numerator (1) and the number of parts in the grid gives you the denominator (6).

So $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Remember to answer the original question. What fraction of the whole chocolate bar does each person get? Each person gets $\frac{1}{6}$ of the chocolate bar.



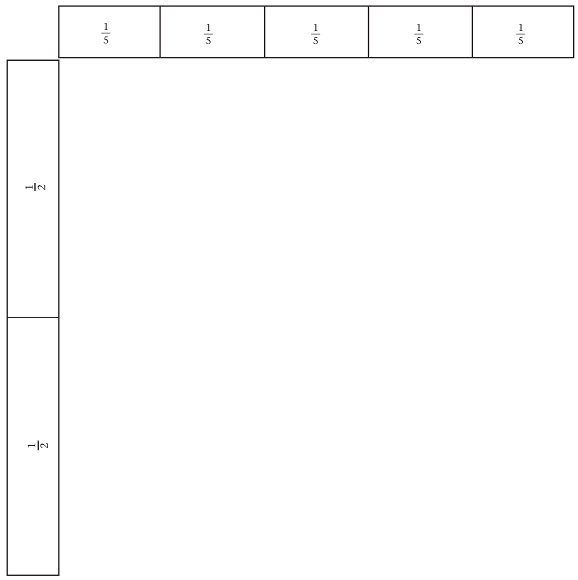
Watch *Multiplying with Fraction Strips* to see some more examples. Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html and click on Module 4.

Let's look at one more example:

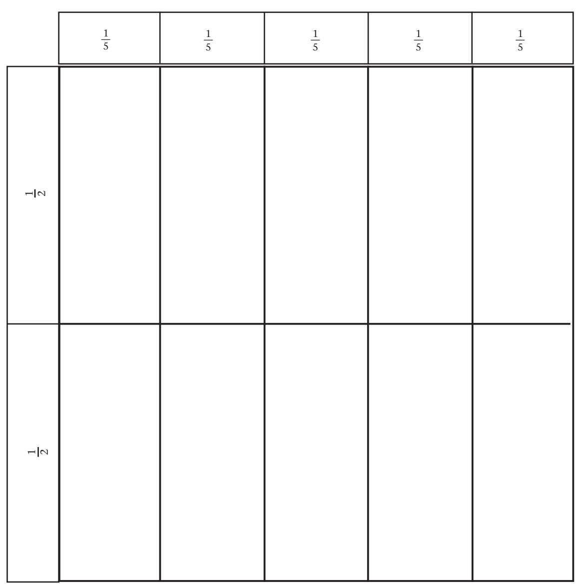
Tammy likes to run on a $\frac{4}{5}$ km track to prepare for racing at track meets. After she finished her training today, she spotted a friend half way around the track. How far away is the friend along the track?

Thinking Space

On a piece of blank paper, and using your fraction strips as templates, sketch the $\frac{1}{5}$ strip across the top, and the $\frac{1}{2}$ strip down the side.

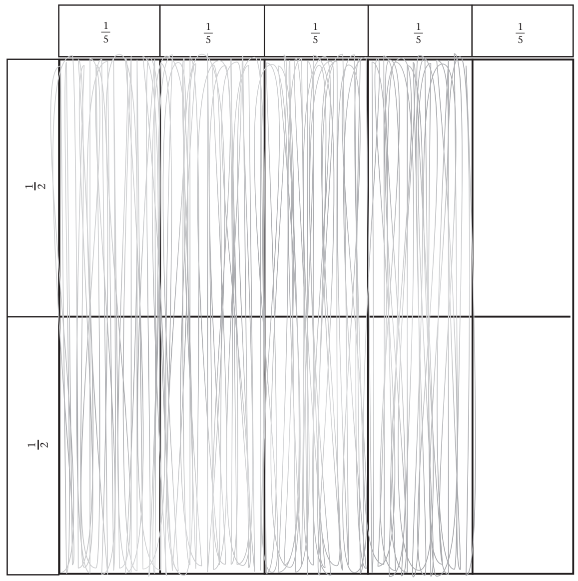


Now draw the grid that forms by drawing down from the $\frac{1}{5}$ strip and across from the $\frac{1}{2}$ strip, like this:

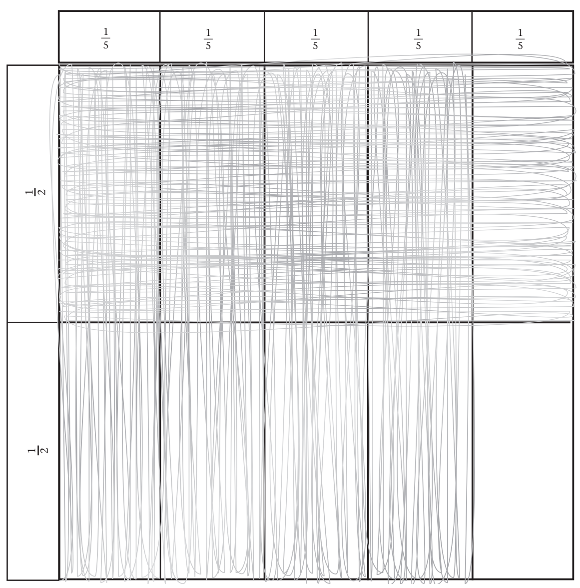


Thinking Space

Shade the first 4 columns with light vertical lines all the way down to show $\frac{4}{5}$.



Shade with light horizontal lines all the way across to show $\frac{1}{2}$.



You can now read the answer to the multiplication question from the grid. The double-shaded piece gives the numerator (4) and the grid gives you the denominator (10).

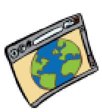
So $\frac{4}{5}$ of $\frac{1}{2} = \frac{4}{5} \times \frac{1}{2} = \frac{4}{10}$

Thinking Space

Let's see if we can find a pattern. Fill in the table below using the two examples above. One of the examples from the video is filled in for you.

Problem	Number of segments across the top	Number of segments down the left side	Total number of segments created from the two fraction strips	Number of segments shaded by both	Fraction that results
$\frac{1}{3} \times \frac{1}{2}$	3	2	6	1	$\frac{1}{6}$
$\frac{2}{3} \times \frac{3}{4}$	3	4	12	6	$\frac{6}{12}$
$\frac{4}{5} \times \frac{1}{2}$	5	2	10	4	$\frac{4}{10}$

Can you spot a pattern here? See if you can come up with a rule that you can use to multiply fractions. Write it here, and then move on to the next Explore to check your work.



If you have Internet access and you would like to do an activity with fraction area models, go to the Math 8 website at **http://www.openschool.bc.ca/courses/math/math8/mod4.html**. Click on the link under *Lesson 4.1B: Multiplying Fractions*.

Thinking Space

Explore
Multiplying Fractions

When we multiply fractions, we can use the following rule:

$$\frac{\text{numerator A}}{\text{denominator A}} \times \frac{\text{numerator B}}{\text{denominator B}} = \frac{\text{numerator A} \times \text{numerator B}}{\text{denominator A} \times \text{denominator B}}$$

Let's see this rule in action. We'll work through three multiplication problems together.

Example 1:

$$\frac{3}{4} \times \frac{1}{3}$$

Multiply the two numerators, and multiply the two denominators.

$$= \frac{3 \times 1}{4 \times 3}$$
$$= \frac{3}{12}$$

Don't forget to reduce your answer to lowest terms.

If we would have simplified before multiplying, we wouldn't have to reduce our answer. See if you can figure out what could have been simplified.

$$= \frac{3 \div 3}{12 \div 3}$$
$$= \frac{1}{4}$$

Example 2:

$$\frac{1}{4} \times \frac{3}{5}$$

Multiply the two numerators, and multiply the two denominators.

$$= \frac{1 \times 3}{4 \times 5}$$
$$= \frac{3}{20}$$

This fraction cannot be reduced any further, so this is your final answer.

Thinking Space

Example 3:

$$\frac{11}{4} \times \frac{2}{5}$$

Multiply the two numerators, and multiply the two denominators.

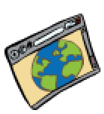
$$= \frac{11 \times \cancel{2}^{1}}{\cancel{4}_{2} \times 5}$$

Simplify by dividing 2 and 4 by a common factor of 2.

$$= \frac{11}{10}$$

This is an improper fraction. You should always leave your answers as proper fractions or mixed numbers. If we convert this to a mixed number we get:

$$1\frac{1}{10}$$



Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html, click on Module 4, and watch the first example in the video *Multiplying Fractions*.

If you want to review simplifying, watch the video *Simplifying* in Module 2.

Now it's your turn to try multiplying fractions.

Thinking Space

Try It!
Activity 4

Complete the following multiplication problems. Your answers should be left as either *proper fractions* or *mixed numbers*, and should be *reduced* to lowest terms.

1. $\frac{2}{3} \times \frac{4}{9}$

2. $\frac{1}{5} \times \frac{3}{4}$

3. $\frac{4}{5} \times \frac{6}{13}$

4. $\frac{2}{7} \times \frac{1}{5}$

5. $\frac{8}{3} \times \frac{5}{7}$

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Multiplying a Fraction by a Whole Number or a Mixed Number

Now that we know how to multiply two proper fractions, what happens when we have a fraction multiplied by a whole number? What about a fraction multiplied by a mixed number? Let's work through some examples.

Multiplying by a Whole Number

Example 1

$$\frac{2}{3} \times 4$$

Remember, we can write any whole number as an improper fraction. 4 is the same as $\frac{4}{1}$. Let's rewrite the expression, and solve the problem.

$$\frac{2}{3} \times 4$$

$$= \frac{2}{3} \times \frac{4}{1}$$

Now, this looks familiar! We can use the multiplication rule for fractions.

$$= \frac{2 \times 4}{3 \times 1}$$

$$= \frac{8}{3}$$

We shouldn't leave the answer as an improper fraction: convert to a mixed number.

$$= 2\frac{2}{3}$$

Thinking Space

Example 2

$$8 \times \frac{3}{5}$$

We can write the whole number as an improper fraction. 8 is the same as $\frac{8}{1}$. Let's rewrite the expression, and solve the problem.

$$\begin{aligned}
&8 \times \frac{3}{5} \\
&= \frac{8}{1} \times \frac{3}{5} \\
&= \frac{8 \times 3}{1 \times 5} \\
&= \frac{24}{5} \\
&= 4\frac{4}{5}
\end{aligned}$$

Multiplying by a Mixed Number

Example 1

$$1\frac{1}{5} \times \frac{3}{4}$$

Remember: we can convert any mixed number to an improper fraction. $1\frac{1}{5}$ is the same as $\frac{6}{5}$. Let's rewrite the expression, and solve the problem.

$$\begin{aligned}
&1\frac{1}{5} \times \frac{3}{4} \\
&= \frac{6}{5} \times \frac{3}{4} \\
&= \frac{\cancel{6}^{3} \times 3}{5 \times \cancel{4}_{2}} \\
&= \frac{9}{10}
\end{aligned}$$

Now, this looks familiar! We can use the multiplication rule for fractions.

Simplifying before multiplying means we don't have to reduce our answer. It also makes the multiplication easier!

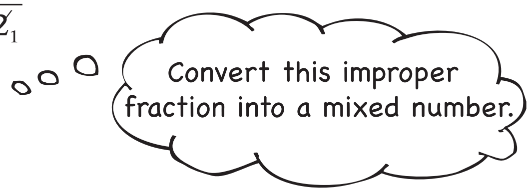
Thinking Space

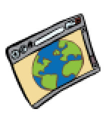
Example 2

$$\frac{6}{11} \times 4\frac{1}{2}$$

We can write the mixed number as an improper fraction. $4\frac{1}{2}$ is the same as $\frac{9}{2}$. Let's rewrite the expression, and solve the problem.

$$\frac{6}{11} \times 4\frac{1}{2}$$
$$= \frac{6}{11} \times \frac{9}{2}$$
$$= \frac{\cancel{6}^{3} \times 9}{11 \times \cancel{2}_{1}}$$
$$= \frac{27}{11}$$
$$= 2\frac{5}{11}$$





Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html, click on Module 4, and watch the second example in the video *Multiplying Fractions*.

Now it's time for you to try. Look back at the examples at any time if you need a hint.

Thinking Space

Try It!
Activity 2

Complete the following multiplication problems. Your answers should be left as either proper fractions or mixed numbers, and should be reduced to lowest terms.

1. $\frac{3}{7} \times 3$

2. $\frac{4}{5} \times 3\frac{3}{4}$

3. $\frac{9}{10} \times 8$

4. $\frac{8}{5} \times 7$

5. $2\frac{1}{4} \times \frac{6}{11}$

Thinking Space

6. $2\frac{1}{2} \times 1\frac{1}{9}$

7. $\frac{4}{5} \times 6$

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson C
Dividing Fractions

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How do I divide fractions?		

Thinking Space

Warm–up

1. Divide the following:

   a. $65 \div 5$

   b. $32 \div 8$

   c. $63 \div 9$

   d. $18 \div 2$

   e. $45 \div 5$

2. Divide the following:

   a. $\frac{18}{3}$

   b. $\frac{35}{7}$

   c. $\frac{54}{6}$

3. Evaluate $12 \div 5$. Write your answer as a fraction. Do this question **without** a calculator.

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Dividing Fractions Using Fraction Strips

For this Explore, you will need:

- fraction strips (from Appendix)

Sam's family had pizza for dinner last night. Sam and his sister get to share the leftovers to take for lunch today. In the morning, Sam opens the fridge to see how much is left. It looked to Sam like there was about a quarter of the pizza left. Sam began splitting the pizza.

Let's try to figure out how much of the pizza Sam and his sister will get if they divide the leftovers equally.

Our expression looks like this:

$\frac{1}{4}$ shared among 2 people

or

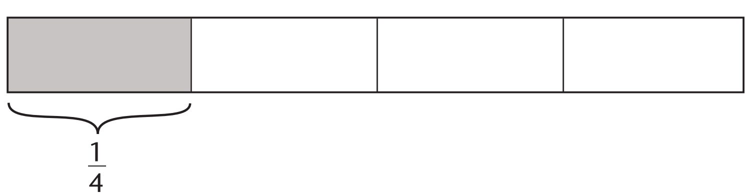
$\frac{1}{4}$ divided by 2

or

$\frac{1}{4} \div 2$

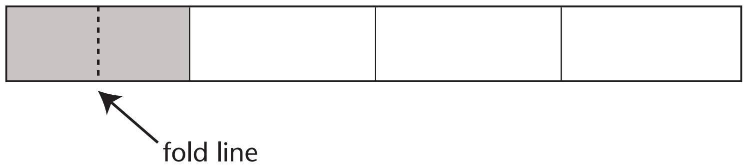
Let's investigate this with the fraction strips.

Get your "quarters" fraction strip. The whole strip represents the whole pizza. There is only a quarter of the pizza left, so we will just work with one section of the "quarters" strip. You might want to shade in $\frac{1}{4}$ so it's easy to see.



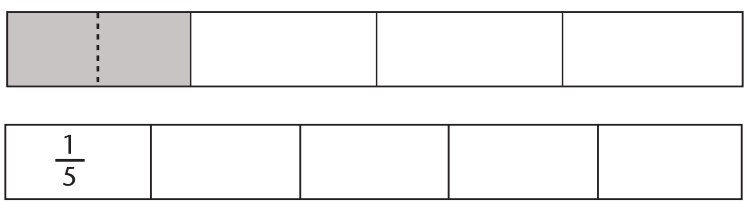
Thinking Space

Now, we need to divide this quarter into two parts. The easiest way to do this is to fold the quarter in half.

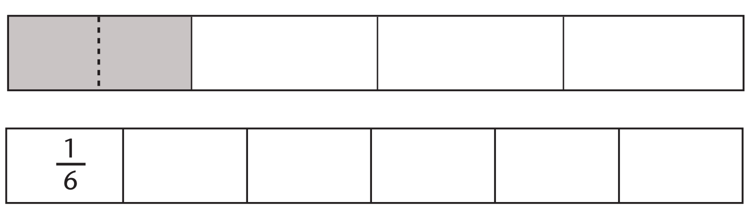


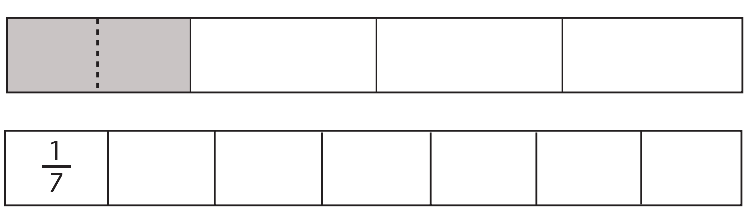
We have divided the quarter into two pieces, but how big is each piece? We can use the other fraction strips to figure it out. Look for a fraction strip that has divisions that are the same size as the ones we've created by folding the quarter. We know that each piece is smaller than a quarter, so we only need to look at the fraction strips with divisions smaller than a quarter.

Start with the "fifths" strip. Line this strip up below the quarters strip as shown below.

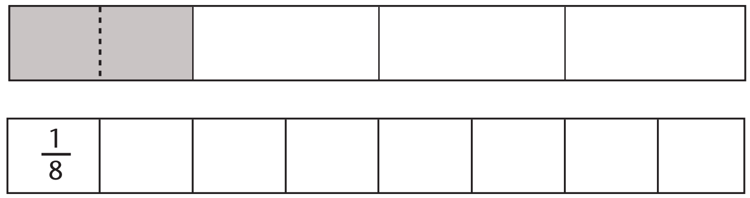


One fifth is bigger than the piece we've created. Using your fraction strips, try the same thing with smaller and smaller fractions until you find the one that fits.





Thinking Space



There! The divisions we created are the same size as the divisions on the "eighths" strip. That means that our pieces are $\frac{1}{8}$ big. Now we can answer the problem.

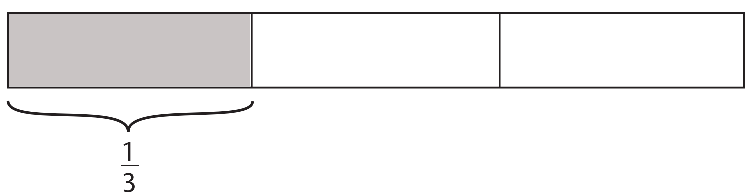
$$\frac{1}{4} \div 2 = \frac{1}{8}$$

Sam and his sister each get $\frac{1}{8}$ of the pizza.

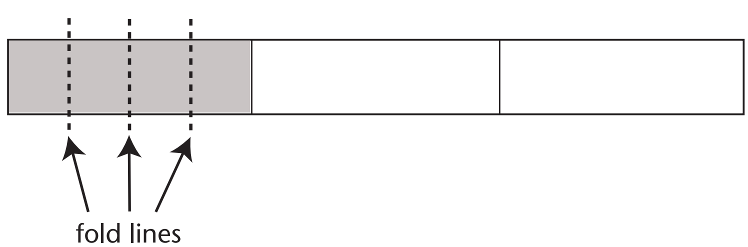
Let's try another example with the fraction strips.

$$\frac{1}{3} \div 4 = ?$$

Get your "thirds" fraction strip out. You may want to shade in $\frac{1}{3}$ so it's easy to see.



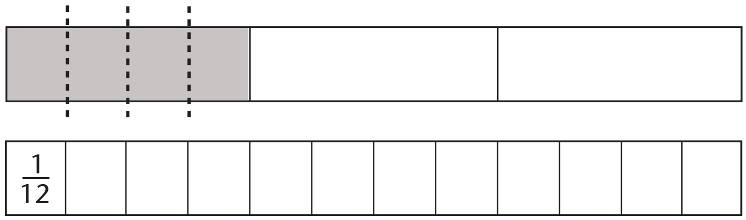
Now, we need to divide the shaded area into four equal pieces. To do this, fold the shaded area in half, and then in half again. When you unfold it, you should see four parts separated by fold lines.



We now have four pieces, but we don't know what fraction of the whole each piece is. Just like in the previous example, you can use your other fraction strips to figure it out. This time you try it on your own, then continue reading.

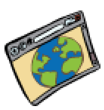
Thinking Space

Hopefully you found that the divisions we created are the same size as the divisions on the "twelfths" strip.



That means that our pieces are $\frac{1}{12}$ big. Now we can answer the problem.

$$\frac{1}{3} \div 4 = \frac{1}{12}$$



Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html, click on Module 4, and watch *Dividing with Fraction Strips* to see more examples.

Let's see if we can find a pattern. Fill in the table below using the two examples above. One of the examples from the video is filled in for you.

Problem	Fraction that results:
$\frac{1}{4} \div 2$	
$\frac{1}{3} \div 4$	
$\frac{3}{4} \div \frac{1}{8}$	6

Can you spot a pattern here? See if you can come up with a rule that you can use to divide fractions. Write it here, then move on to the next Explore to check your work.

Thinking Space

Explore
Dividing Fractions

The rule for dividing fractions is: multiply the first fraction by the **reciprocal** of the second fraction.

You're probably wondering, "What is a reciprocal?" In mathematics, a reciprocal is a number that you multiply a fraction by so that the result equals one. If you start with a whole number, put it over 1 first. The easiest way to find it is to just flip the fraction over.

Here are two examples:

What is the reciprocal of $\frac{4}{5}$?

We flip the fraction to find that the reciprocal is $\frac{5}{4}$

What would be the reciprocal of $1\frac{1}{4}$?

What is the reciprocal of 3?

3 is the same as $\frac{3}{1}$, so we flip and the reciprocal is $\frac{1}{3}$.

In general, the rule for division of fractions looks like this:

$$\frac{a}{A} \div \frac{b}{B}$$
$$= \frac{a}{A} \times \frac{B}{b}$$
$$= \frac{a \times B}{A \times b}$$

Now that we have a rule for division, let's work through an example so you know how to use it.

$$\frac{1}{4} \div 2 = ?$$

Thinking Space

Following the rule, we'll rewrite the first fraction, and then multiply it by the reciprocal of the second.

$$\begin{aligned} &\frac{1}{4} \div 2 \\ &= \frac{1}{4} \div \frac{2}{1} \\ &= \frac{1}{4} \times \frac{1}{2} \\ &= \frac{1 \times 1}{4 \times 2} \\ &= \frac{1}{8} \end{aligned}$$

2 is the same as $\frac{2}{1}$. Flip it to get the reciprocal.

Now you have a multiplication problem—we did lots of these in the last lesson!

It may seem a bit strange to be changing operations and flipping fractions. To help you understand this process, think back to the first example. Sam and his sister were sharing a quarter of a pizza. We can think of this scenario in two ways:

We must split the leftover pizza into two parts.	**OR**	Each person gets half of the leftover pizza.

We can translate these sentences into mathematical language. Remember, there was a quarter of the pizza left in the fridge for Sam and his sister to share equally.

the leftover pizza divided by 2	**OR**	half of the leftover pizza
$\frac{1}{4} \div 2$	**OR**	$\frac{1}{2} \times \frac{1}{4}$

With a little shuffling around, you'll find our division rule!

Remember, you can multiply two numbers in any order. $\frac{1}{4} \times \frac{1}{2}$ is the same as $\frac{1}{2} \times \frac{1}{4}$

$$\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2}$$

Thinking Space

Let's try another example using the division rule.

$$\frac{2}{3} \div \frac{3}{4} = ?$$

Rewrite the first fraction, change the operation to multiplication, and flip the second fraction.

$$\begin{aligned} &\frac{2}{3} \div \frac{3}{4} \\ &= \frac{2}{3} \times \frac{4}{3} \\ &= \frac{2 \times 4}{3 \times 3} \\ &= \frac{8}{9} \end{aligned}$$

The answer is a proper fraction, and can't be reduced, so we can leave it as it is.

Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html, click on Module 4, and watch *Dividing Fractions* to see more examples of division.

Mixed Numbers

In the video you just watched, you saw an example where the first fraction is actually a mixed number. Let's look at an example where the second fraction is a mixed number.

$$\frac{5}{6} \div 1\frac{2}{3} = ?$$

By the division rule, we know we need to find the reciprocal of the second fraction. Before we can do this, we need to convert the mixed number to an improper fraction.

$$\begin{aligned} &\frac{5}{6} \div 1\frac{2}{3} \\ &= \frac{5}{6} \div \frac{5}{3} \\ &= \frac{5}{6} \times \frac{3}{5} \\ &= \frac{\cancel{5}^1 \times \cancel{3}^1}{\cancel{6}_2 \times \cancel{5}_1} \\ &= \frac{1}{2} \end{aligned}$$

Now you can flip the second fraction. The reciprocal of $\frac{5}{3}$ is $\frac{3}{5}$.

Simplify before you multiply.

Thinking Space

Try It!
Activity 1

1. Write the reciprocal of each of the following numbers.

   a. $\frac{2}{3}$

   b. $\frac{1}{4}$

   c. 7

   d. $2\frac{4}{5}$

2. Complete the following division problems. Your final answers should be either proper fractions or mixed numbers, and should be reduced to lowest terms.

   a. $\frac{7}{5} \div 3$

   b. $\frac{5}{6} \div 4$

   c. $\frac{1}{2} \div \frac{2}{3}$

   d. $\frac{1}{5} \div 9$

   e. $5\frac{1}{5} \div \frac{1}{5}$

Thinking Space

f. $\frac{1}{4} \div 2$

g. $\frac{1}{5} \div \frac{1}{4}$

h. $\frac{2}{5} \div \frac{2}{3}$

i. $\frac{11}{9} \div \frac{1}{5}$

j. $7 \div 2\frac{3}{4}$

3. If a student insists that $\frac{12}{35} \div \frac{4}{5} = \frac{7}{3}$, what mistake is the student likely making?

Turn to Solutions at the end of the module and mark your work.

Optional: If you would like to explore another model for dividing fractions, go to the Math 8 website **http://www.openschool.bc.ca/courses/math/math8/mod4.html** and click on the link under *Lesson 4.1C: Dividing Fractions*.

You've finished Lesson C. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson D

Estimating and Solving Problems

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

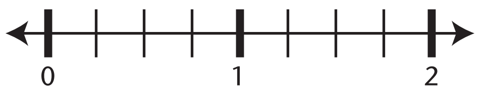
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How will estimating help me multiply and divide fractions?		
How do I know which operation to use when I solve a problem?		
In what mathematical situations would we multiply and divide fractions?		

Thinking Space

Warm–up

1. Place the following fractions on the number line below.

   $1\frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{7}{4}$



2. Use <, >, = to make the following statements true.

   a. $\frac{2}{3} \boxed{} \frac{1}{3}$

   b. $1\frac{1}{2} \boxed{} \frac{3}{2}$

   c. $2 \boxed{} \frac{6}{5}$

   d. $\frac{2}{3} \boxed{} \frac{3}{4}$

   e. $\frac{9}{3} \boxed{} 3$

3. What operations are usually indicated by the following words?

   a. total ____________________

   b. shared ____________________

   c. times ____________________

   d. less than ____________________

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
When to Multiply or Divide

In the previous lessons, you learned how to multiply and divide fractions. Now that you have the skills you need, it's time to think about when you might need to use them. In what situations would you ever need to multiply or divide fractions?

Vocabulary Review

Multiplication is a way of scaling one number by another, or showing repeated additions. For example 3×4 can be thought of as $3 + 3 + 3 + 3$ or as $4 + 4 + 4$. When we multiply, the answer is called the product.

For example:

$3 \times 8 = 24$	24 is the product
$2 \times 9 = 18$	18 is the product

Division looks at how many groupings you can make. The number you're dividing is called the dividend. The number you are dividing by is called the divisor. The answer is made of two parts: the quotient and the remainder.

For example:

$$32 \div 8 = \frac{32}{8} = 4$$

32 is the dividend, 8 is the divisor, 4 is the quotient.

$$47 \div 9 = \frac{47}{9} = 5 \text{ R2}$$

47 is the dividend, 9 is the divisor, 5 is the quotient, 2 is the remainder.

Thinking Space

Multiply or Divide?

When you're solving a problem, how will you know what operation to use?

Well, if it's a word problem, you can look for certain key words that will give you a hint. *Multiply*, *divide*, *product*, and *quotient* are the most obvious ones, but there are others. For example, "of" usually means multiply. Or, if you see the word "share" or "parts" it often means divide. It is important to read the problem carefully and think about what it is asking—then, compare the hints in the problem to what you know about multiplying and dividing.

It won't always be a word problem. You could be in the grocery store, in the kitchen baking cookies, or building something in the workshop. Real world problems won't be written out for you; you'll have to analyze the situation yourself and collect the relevant information.

The problems you see in math class help you practise your problem-solving skills. No matter what kind of a problem you are solving, it's important to analyze carefully and organize your information.

Example 1

Roger is baking cookies to bring to school for a class party. He knows that if he wants enough cookies for his whole class, he'll need to double the recipe. He's worried that he might not have enough chocolate chips. The recipe calls for $\frac{3}{4}$ cup of chocolate chips. How many cups will Roger need if he doubles the recipe?

Think it through:

> If we double the recipe, we need to double the amount of chocolate chips. "Double" means "twice as much", or "multiply by 2". We have to find out how many cups of chocolate chips Roger will need if he doubles the recipe. The original recipe calls for $\frac{3}{4}$ cup of chocolate chips, so we need to multiply this by 2 to double it.

From the thought process above you can write the expression:

$\frac{3}{4}$ cup chocolate chips × 2

$\frac{3}{4} \times 2$

Thinking Space

Then, to solve the problem, simply complete the multiplication.

$$\begin{aligned}
&\frac{3}{4}\times 2\\
&=\frac{3}{4}\times\frac{2}{1}\\
&=\frac{3\times 2}{4\times 1}\\
&=\frac{6}{4}\\
&=\frac{3}{2}\\
&=1\frac{1}{2}
\end{aligned}$$

Roger will need $1\frac{1}{2}$ cups of chocolate chips to make cookies for his class.

Example 2

Jen has half of an orange and she wants to share it equally with her friends Chris and Angela. How much of the orange does each person get?

Sometimes it helps to rephrase the question in your own words:

> Jen has $\frac{1}{2}$ of an orange and she needs to share it equally three ways.

Then, think it through:

> If you take one whole orange and cut it into three equal parts (1 divided by 3 people), each person would get $\frac{1}{3}$ of one orange.
>
> Since Jen only has $\frac{1}{2}$ of an orange, we should divide the $\frac{1}{2}$ orange by 3 people. Then each person can only have $\frac{1}{3}$ of $\frac{1}{2}$ the orange.

The thought process above shows that we can actually solve this problem using division or multiplication.

Thinking Space

We can divide the $\frac{1}{2}$ orange by 3 $\frac{1}{2} \div 3$

OR

We can take $\frac{1}{3}$ of the $\frac{1}{2}$ orange $\frac{1}{3} \times \frac{1}{2}$

You can see that not only is it important to get the operation correct, but the numbers are important too! We can either divide the half-orange by 3 or multiply it by $\frac{1}{3}$.

If you divide by $\frac{1}{3}$, you have to use the rule for dividing fractions. How is this related to the multiplication expression we found?

Finding the expression is usually the tricky part. Once you have the expression, you can use your multiplication and division skills to solve it. Try this one before you move to the next example. Choose to use either multiplication or division to solve it.

Answer:

Each person will get $\frac{1}{6}$ of the orange.

Now try a few questions on your own.

Thinking Space

Try It!
Activity 1

Match the following example questions with the correct expressions.

Note: Each question can have more than one answer, and each answer can be used more than once.

Questions:

______ 1. $\frac{4}{5}$ of a pizza needs to be divided into 7 pieces.

______ 2. $\frac{2}{3}$ of seats in an arena were filled. $\frac{2}{5}$ of the seats were filled with children. What portion of the total arena seats are filled with children?

______ 3. Half of a fish is cut up to share with five people. How much does each person get?

______ 4. Seven friends get together to pick fruit for a big party. Each person picks four fifths of a box of fruit. How much fruit do they pick?

______ 5. A box of beads is half full. Each person needs $\frac{1}{5}$ of a box. How many people can get what they need from this box?

______ 6. Two brothers pool their allowance and buy $\frac{2}{5}$ of a box of booster card packs for a collectable card game. One brother paid $\frac{2}{3}$ of the price and gets $\frac{2}{3}$ of the cards. What portion of booster packs does he get?

Expressions:

a. $\frac{1}{2} \div 5$

b. $\frac{2}{3} \times \frac{2}{5}$

c. $\frac{4}{5} \div 7$

d. $\frac{4}{5} \times 7$

e. $\frac{1}{2} \div \frac{1}{5}$

f. $\frac{4}{5} \times \frac{1}{7}$

g. $\frac{1}{2} \times \frac{1}{5}$

h. $\frac{2}{3} \div \frac{2}{5}$

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Estimating Products and Quotients

When we solve problems, it's helpful to know if the answer we got is reasonable. Estimation can help us decide how reasonable an answer is.

For example, if I am splitting one orange among three people, it doesn't make sense to say that each person will get more than one orange. I only had one to begin with, so an answer greater than one isn't reasonable. On the other hand, if my answer comes out as less than one, I know it's reasonable because I am splitting up an orange into smaller pieces—pieces that are smaller than the whole orange.

How do we decide if an answer is reasonable? Sometimes there are hints in the wording of the question. Other times you can estimate the approximate size of the answer based on the numbers in the question. You can even use rounding to help you estimate.

With fractions, you can get a general idea of how big the answer will be based on the type of question, and the fractions involved.

Multiplication

There are three types of multiplication problems (shown in the table below). Multiplying two proper fractions will always give you a proper fraction—that is, your answer will always be between zero and one. Multiplying two improper fractions will always give you an improper fraction—that is, your answer will always be greater than one. When you multiply a proper fraction and an improper fraction, your answer may be a proper fraction (less than one) or an improper fraction (greater than one)—it depends on the size of the two fractions.

Thinking Space

Type of Multiplication	Answer	Example
proper × proper	between 0 and 1	$\frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$
proper × improper	depends on the fractions, can be less than or greater than 1	$\frac{1}{2}\times\frac{4}{3}=\frac{2}{3}$ $\frac{12}{5}\times\frac{3}{4}=\frac{9}{5}$
improper × improper	greater than 1	$\frac{4}{3}\times\frac{7}{5}=\frac{28}{15}$

Recognizing the type of multiplication problem you have will help you decide if your answer is reasonable. For example, if you multiply two proper fractions and your answer is greater than 1, you know you've made a mistake.

If you are multiplying a proper fraction and an improper fraction, you'll have to do a bit more work to estimate the size of your answer. You can approximate the fractions and then estimate.

For example:

$$\frac{5}{6}\times\frac{5}{3}$$

Here we have a proper fraction multiplied by an improper fraction. We can use approximation to see what number our answer will be close to.

$\frac{5}{6}$ is close to 1, so we will approximate it to 1.

$\frac{5}{3}$ is $1\frac{2}{3}$, which is close to $1\frac{1}{2}$, so we will approximate it to $1\frac{1}{2}$ (or $\frac{3}{2}$).

Our multiplication becomes: $1\times\frac{3}{2}=\frac{3}{2}$ (or $1\frac{1}{2}$)

So, we estimate that our answer will be close to $1\frac{1}{2}$. Let's multiply the original question to see how close our approximation was.

$$\frac{5}{6}\times\frac{5}{3}=\frac{25}{18}=1\frac{7}{18}$$

Thinking Space

The answer $1\frac{7}{18}$ is very close to our approximation of $1\frac{1}{2}$ $(1\frac{9}{18} = 1\frac{1}{2})$, so we know our answer is reasonable.

Even if you know that your answer should be between 0 and 1, you can use approximation to see if your answer will be closer to 0 or closer to 1, or if it's in the middle (close to $\frac{1}{2}$).

For example:

$$\frac{5}{8} \times \frac{7}{9}$$

These fractions are both proper, so we know the answer will be between 0 and 1.

$\frac{5}{8}$ is close to $\frac{1}{2}$, so we will approximate it to $\frac{1}{2}$.

Why is $\frac{5}{8}$ close to $\frac{1}{2}$?

$\frac{7}{9}$ is close to 1, so we will approximate it to 1.

Our multiplication becomes: $\frac{1}{2} \times 1 = \frac{1}{2}$

So, we estimate that our answer will be close to $\frac{1}{2}$. Let's multiply the original question to see how close our approximation was.

$$\frac{5}{8} \times \frac{7}{9} = \frac{35}{72}$$

The answer $\frac{35}{72}$ is very close to our approximation of $\frac{1}{2}$ $(\frac{36}{72} = \frac{1}{2})$, so we know our answer is reasonable.

Thinking Space

We talked about these terms earlier in the lesson. You can look at the definitions again if you need to.

Division

The guidelines for division are a bit simpler. You just need to know which is bigger, the dividend or the divisor. To figure out which fraction is bigger, you can plot them on a number line, or you can create equivalent fractions with a common denominator.

You can use the following points to judge the reasonableness of your answer.

Division Type	Answer	Example
dividend greater than divisor	greater than 1	$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$
dividend less than divisor	between 0 and 1	$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

Even if you know that your answer should be between 0 and 1, you can use approximation to see if your answer will be closer to 0 or closer to 1, or if it's in the middle (close to $\frac{1}{2}$).

Example:

$\frac{3}{5} \div \frac{7}{8}$

$\frac{3}{5}$ can be approximated to $\frac{1}{2}$.

$\frac{7}{8}$ can be approximated to 1.

Our division becomes $\frac{1}{2} \div 1 = \frac{1}{2}$.

So, we estimate that our answer will be close to $\frac{1}{2}$. Let's divide the original question to see how close our approximation was.

$$\frac{3}{5} \div \frac{7}{8} = \frac{3}{5} \times \frac{8}{7} = \frac{24}{35}$$

The answer $\frac{24}{35}$ is not too far off from our approximation of $\frac{1}{2}$, so we know our answer is reasonable.

Thinking Space

Try It!
Activity 2

1. Give an example for each of the following, or explain why no example exists.

   a. The product of two improper fractions that is greater than 1.

   b. The quotient of a proper fraction divided by an improper fraction that is less than 1.

   c. The product of two proper fractions that is greater than 1.

2. Without calculating the actual answers, use < or > to make the following statements true.

   a. $\frac{3}{4} \times \frac{8}{5} \ \square\ 1$

   b. $\frac{13}{8} \times \frac{3}{2} \ \square\ 1$

   c. $\frac{3}{4} \times \frac{5}{8} \ \square\ 1$

   d. $2\frac{1}{5} \div 4\frac{2}{5} \ \square\ 1$

Thinking Space

3. Estimate.

   a. $\frac{3}{5} \times \frac{4}{7}$

   b. $\frac{5}{12} \div \frac{4}{5}$

   c. $3\frac{5}{6} \div 2\frac{1}{5}$

4. A student multiplied $\frac{4}{2}$ and $\frac{4}{3}$, getting a product of $\frac{2}{3}$. Is the student's answer reasonable? Explain why or why not.

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Solving Problems

In this Explore, we're going to put our problem-solving skills and our estimation skills together with our fraction skills, and work on some word problems. The best way to master problem solving is to get lots of practice!

Problem 1

Esther is a track and field athlete, and she often runs on the track at her school. If the track is $\frac{2}{5}$ of a kilometer long and she runs $8\frac{1}{4}$ times around the track, how far does she run?

Think about it:

Esther runs $8\frac{1}{4}$ times around a $\frac{2}{5}$ km track. "Times" makes me think of multiplication.

Write an expression:

$8\frac{1}{4}$ times around a $\frac{2}{5}$ km track OR $8\frac{1}{4} \times \frac{2}{5}$

Estimate:

$8\frac{1}{4}$ is close to 8

$\frac{2}{5}$ is close to $\frac{1}{2}$

Our multiplication becomes $8 \times \frac{1}{2} = 4$

Our answer should be close to 4.

Thinking Space

Solve:

$$8\frac{1}{4} \times \frac{2}{5}$$
$$= \frac{33}{4} \times \frac{2}{5}$$
$$= \frac{33 \times \cancel{2}^{1}}{\cancel{4}_{2} \times 5}$$

Simplify by dividing 2 and 4 by 2.

$$= \frac{33}{10}$$
$$= 3\frac{3}{10}$$

Answer the question:

Esther ran $3\frac{3}{10}$ km.

Is your answer reasonable?

Yes. Our estimate was "close to 4" and our answer was slightly bigger than 3.

Problem 2

A store owner wants to make parking spaces outside her store. If she has 35 m available and the parking spaces have a length of $6\frac{1}{10}$ m, how many parking spaces can she make?

Rephrase the problem:

The store owner needs to split up the 35 m outside her store into $6\frac{1}{10}$ m pieces. How many of these pieces can she make?

Write an expression:

35 m divided by $6\frac{1}{10}$ m

OR

$35 \div 6\frac{1}{10}$

Thinking Space

Estimate:

35 is greater than $6\frac{1}{10}$, so we know our answer will be greater than 1.

$6\frac{1}{10}$ is close to 6, so our answer will be close to $\frac{35}{6}$. She should be able to make around 5 spaces and she might have some space left over.

Solve:

$$\begin{aligned}
&35 \div 6\frac{1}{10} \\
&= 35 \div \frac{61}{10} \\
&= \frac{35}{1} \times \frac{10}{61} \\
&= \frac{35 \times 10}{1 \times 61} \\
&= \frac{350}{61} \\
&= 5\frac{45}{61}
\end{aligned}$$

Answer the question:

The store owner can make 5 complete parking spaces and she will have a bit of space left over.

The solution to our expression was $5\frac{45}{61}$. Why can we only make 5 parking spaces?

Is your answer reasonable?

Yes. Our estimate and our answer were almost the same.

Problem 3

Joshua's regular rate of pay is $14/h. He gets paid time and a half when he works overtime. Last week, Joshua worked 40 hours at regular time, and 3 hours of overtime. How much will he get paid for last week's work?

Rephrase the problem:

When Joshua works overtime, he makes $1\frac{1}{2}$ times his regular rate. Last week, Joshua worked 40 hours at a rate of $14/h and 3 hours at a rate of $1\frac{1}{2}$ times $14/h.

Thinking Space

Write an expression:

40 hours at \$14/h and 3 hours at $1\frac{1}{2}$ times \$14/h

OR

$(40 \times 14) + 3 \times (1\frac{1}{2} \times 14)$

Estimate:

In a question like this where there are several operations, whole numbers, and fractions, you could spend a lot of time rounding off numbers and performing operations to get your estimate. An estimate should be a quick way of generating a "ballpark" answer.

Joshua's regular rate of pay is \$14/h. 14 is pretty much in the middle of 10 and 20. 10 and 20 are both easy numbers to multiply by. We can use these numbers to find a range that our answer will fit within.

Joshua worked 43 hours (don't worry about the overtime rate for now).

$43 \times 10 = 430$

This is not the only way to estimate. Would you use a different strategy?

$43 \times 20 = 860$

Our answer should be somewhere between \$430 and \$860.

Solve:

$$
\begin{aligned}
&(40 \times 14) + 3 \times \ 1\frac{1}{2} \times 14 \\
&= 560 + 3 \times \left(\frac{3}{2} \times \frac{14}{1}\right) \\
&= 560 + 3 \times \left(\frac{3 \times \cancel{14}^{7}}{\cancel{2}_{1} \times 1}\right) \\
&= 560 + 3 \times 21 \\
&= 560 + 63 \\
&= 623
\end{aligned}
$$

There are several operations to complete. Remember to use order of operations (BEDMAS).

Simplify by dividing 14 and 2 by 2.

Thinking Space

Answer the question:

Joshua made $623 for last week's work.

Is your answer reasonable?

Yes. Our estimate gave us a range and our answer fell within that range.

Tips for Solving Problems

There are many ways to solve problems and there is no magic method that will work for all problems. The list of tips below might help you work through the problems you encounter. You may need to use only one or two of these strategies, or you may use them all. Do what works best for you, and remember, practice makes perfect!

- Read carefully (maybe even read it aloud).
- Highlight key information.
- Rephrase the problem in your own words.
- Look for hints that tell you what operation to use.
- Draw a picture.
- Make a table or chart.
- Write an expression.
- Estimate.

Are there any other strategies you use to solve problems? Add them to the list.

Once you know how to solve the problem, solve it! Don't forget to answer the question, and make sure the answer is reasonable.

Thinking Space

Try It!
Activity 3

1. The Greater Vancouver Regional District consists of 21 municipalities including the City of Vancouver. Approximately half of the population of British Columbia lives in the Greater Vancouver region. About a quarter of these people live in the City of Vancouver. What fraction represents the population of the City of Vancouver in relation to the population of BC?

2. A dessert recipe you're following says that it makes 5 servings. It calls for $1\frac{1}{4}$ cups of sugar. How much sugar is in each serving?

Temperature can be measured in degrees Celsius (°C) or degrees Fahrenheit (°F). To convert between them, you can use the following formulas:

$$F = \frac{9}{5}C + 32$$

$$C = (F - 32) \div \frac{9}{5}$$

where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

3. Convert the following temperatures into degrees Fahrenheit.

   a. 30 °C

Thinking Space

b. 10 °C

4. Convert the following temperatures into degrees Celsius.

a. 59 °F

b. 65 °F

5. A hoodie that you want regularly costs $48. The store is advertising a sale where everything is $\frac{1}{3}$ off. What is the reduced price of the hoodie?

a. Write an expression that you can use to solve the problem.

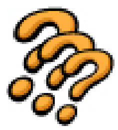
b. Estimate the reduced price of the hoodie.

c. Calculate the actual reduced price of the hoodie.

Thinking Space

d. Answer the question and comment on the reasonableness of your answer.

Turn to Solutions at the end of the module and mark your work.



You've finished Lesson D. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Section Summary

Completing this section has helped you to:

- write a given positive mixed number as an improper fraction
- write a given positive improper fraction as a mixed number
- multiply two given fractions
- divide two given fractions
- solve problems involving multiplication and division of fractions

Section 2
Fractions with a Purpose: Rates, Ratios, and Percents

In this section you will:

- demonstrate an understanding of ratio and rate
- demonstrate an understanding of percents greater than or equal to 0%
- solve problems that involve rates, ratios and proportional reasoning

Where in the World...?

At some point, you've probably bought or rented a movie to watch on your home television. Have you ever wondered what is meant by "full screen" or "wide screen?" Have you ever wished those annoying black bars on the top and bottom of your screen would go away?

Movies are filmed in a format that is wider than our TV's display. A common ratio of length to width for the big screen is 2.35:1. Normal TVs and TV programs have a ratio of 4:3 (or 1.33:1). To make a movie work on your TV, one of two alterations happens.

- If you want the movie to be full screen, the image needs to be cropped from a ratio of 2.35:1 to a ratio of 4:3. It will then fill your whole TV screen, but small portions of the picture will be cut off.
- Instead of cutting off part of the picture, the movie can be made to fit the width of the TV. Then, black bars appear above and below the image where there is nothing to show. This is called wide screen. One common format for wide screen is 16:9 or $\frac{16}{9} = 1.78$.

Movie shown on TV in full screen mode

Movie shown on TV in wide screen mode

Section 2
Pretest

Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Please complete this pretest **without** using a calculator.

Lesson A: Ratios and Rates in Every Day Life

Reduce all answers to lowest terms where necessary.

1. Write a part-to-part ratio for each of the following descriptions. Reduce the ratios to lowest terms where possible.

    a. In a class period you spend 10 minutes watching a video and 30 minutes doing an activity.

    b. On a given day you spend sixteen hours awake and eight hours sleeping.

    c. One Saturday, Jocelyn spends three hours watching television, five hours playing soccer and two hours studying.

    d. You drink six glasses of water and two glasses of milk.

2. For each description in question 1, write a part-to-whole ratio. Reduce the ratios to lowest terms where possible.

   a.

   b.

   c.

   d.

3. Write a unit rate for each of the following descriptions.

   a. Bill travelled 70 m in 7s.

   b. Elsie earned $77.50 for five hours of work.

   c. Richard travelled 512 km and used 72 L of fuel.

4. If one dozen eggs cost $4.50, what is the price

   a. per egg?

   b. of 5 eggs?

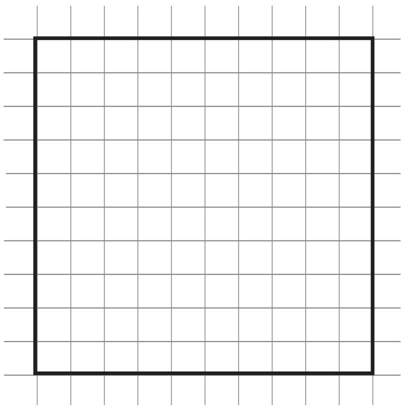
5. A juice mixture is composed of mango juice, apple juice, and orange juice in a 1:2:3 ratio. If there are 18 L of mixed juice, how much apple juice is in the mixture?

6. A quality control inspector examined 300 light bulbs and found 10 of them to be defective. At this rate, how many defective bulbs will there be in a lot of 4500?

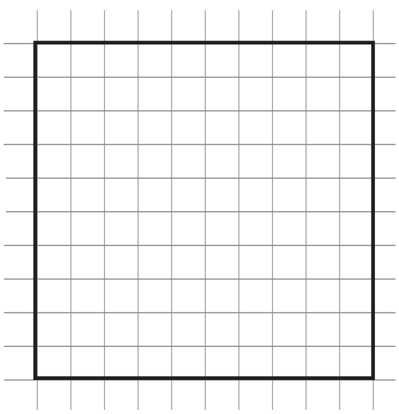
Lesson B: Making Sense of Percents

1. Using the grids provided, draw a picture that represents each of the following percents.

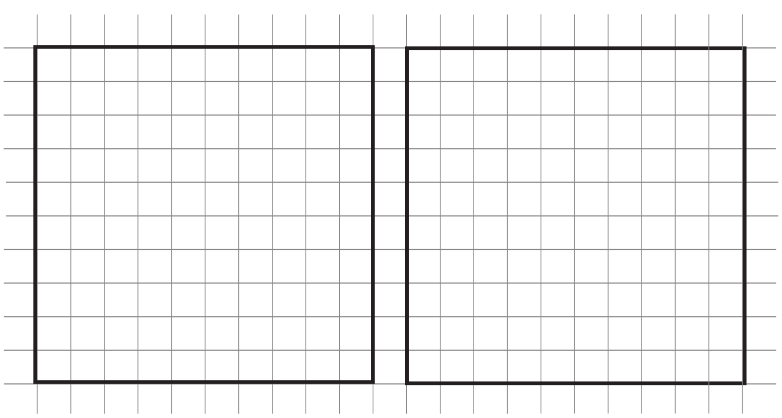
   a. 0.5%



   b. 67%



   c. 197%



2. Fill in the following table by converting between ratios, fractions, decimals, and percents. Please put ratios and fractions in lowest terms. The first row is done for you as an example.

Part-to-Whole Ratio	Fraction	Decimal	Percent
1:2	$\frac{1}{2}$	0.5	50%
1:500			
	$\frac{27}{50}$		
		1.75	
			0.0075%
	$\frac{137}{200}$		
1403:400			

Lesson C: Solving Problems with Percents

1. A video game store offers a buyback program for used console games. The store pays 40% of the current new price. If you have a game that sells for $55.00 new, what will the store buy it back for?

2. A bike is on sale for \$550. What would be the total cost including GST (5%) and PST (7%)?

3. The cost of an item, including PST (7%) and GST (5%), is \$95.20. If the store bought the items wholesale for \$62.00, what is the profit they make on the item?

4. The price of a DVD player was reduced from $240.00 to $192.00. Find the percent discount.

5. Property taxes increased from $1800.00 to $2000.00. What was the percent increase?

Turn to Solutions at the end of the module and mark your work.

Lesson A
Ratios and Rates in Every Day Life

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What is a ratio?		
What is a rate?		
What is the difference between a rate and a ratio?		
How do you use rates in your day-to-day life?		

Thinking Space

Warm–up

Write equivalent fractions for each of the following fractions.

1. $\frac{2}{6}$

2. $\frac{2}{5}$

3. $\frac{11}{15}$

4. $\frac{4}{16}$

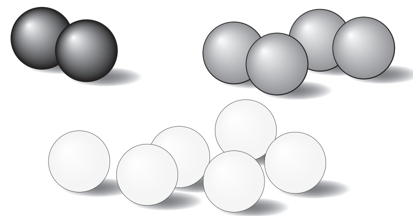
5. $\frac{3}{8}$

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Ratios

Look at the picture below.



We can see from this picture that:

- There are 2 black marbles.
- There are 4 grey marbles.
- There are 6 white marbles.
- There are 12 marbles all together.

Using this information, we can make some comparisons. For example, we can compare:

What other comparisons can you make?

- the number of grey marbles to the number of white marbles
- the number of black marbles to the number of grey marbles to the number of white marbles
- the number of white marbles to the total number of marbles

In math, we can describe these comparisons using ratios. A **ratio** is a comparison of two or more numbers. We can write each comparison listed above as a ratio by separating the numbers with a colon.

the number of grey marbles to the number of white marbles	4:6
the number of black marbles to the number of grey marbles to the number of white marbles	2:4:6
the number of white marbles to the total number of marbles	6:12

Notice that the order in which the numbers appear is very important. Write the numbers in the ratio in the same order that they are listed in the words.

Thinking Space

grey to white

4 : 6

Each number in a ratio is called a *term*. The ratio 4:6 is a *two-term* ratio because it contains two terms, 4 and 6. The ratio 2:4:6 is a *three-term* ratio because it contains three terms, 2, 4, and 6.

Part-to-Part Ratios

A *part-to-part* ratio describes certain parts of a group, or certain parts of a whole. In the marble example above, the ratio of grey marbles to white marbles (4:6) is a part-to-part ratio. It compares different parts of a collection of marbles. The three-term ratio 2:4:6 is also a part-to-part ratio. It describes three parts of the collection.

Part-to-Whole Ratios

A *part-to-whole* ratio describes a part of a group in comparison to the whole group. In the marble example above the ratio of white marbles to the total number of marbles (6:12) is a part-to-whole ratio. It compares a specific part of the group to the whole group.

Part-to-whole ratios can also be written as fractions. For example, we could write the ratio of white marbles to the total number of marbles as 6:12 or $\frac{6}{12}$.

Proportions

Let's look at the ratio of white marbles to the total number of marbles again. This ratio can be written as the fraction $\frac{6}{12}$. Remember that we can create equivalent fractions by multiplying or dividing the numerator and the denominator by a common factor. In this case:

$$\frac{6}{12} \underset{\div 6}{\overset{\div 6}{\longrightarrow}} \frac{1}{2}$$

$\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions.

Thinking Space

We can do the same thing with ratios. 6:12 can be written as 1:2.

$$6:12 \xrightarrow[\div 6]{\div 6} 1:2$$

A pair of equivalent ratios is called a **proportion**. We say that these two ratios are *proportional* to each other.

Thinking Space

Try It!
Activity 1

1. Look at the counters below. The ratios below describe how the coloured counters relate to each other. Explain the relation for each ratio. The first one has been done for you.

   a. 3:7 Number of black counters to the number of grey counters

   b. 3:6 ______________________________

   c. 7:16 ______________________________

   d. 3:6:7 ______________________________

   e. 3:8 ______________________________

2. Classify each ratio in question 1 as either a part-to-part ratio or a part-to-whole ratio.

   a.

   b.

   c.

   d.

   e.

Thinking Space

3. Write a part-to-part ratio for each of the comparisons below.

   a. There are 12 boys and 15 girls in a Grade 8 math class.

   b. To prepare pancakes from a packaged mix, you need 1 cup of water and 2 cups of pancake mix.

   c. This week, the forecast calls for three days of sunshine and four days of rain.

   d. In your dresser drawer you have one pair of pants, three pairs of shorts, and four T-shirts.

4. For each of the comparisons in question 3, write a part-to-whole ratio.

   a.

   b.

   c.

   d.

Thinking Space

5. Which of the following pairs of ratios are proportional? How do you know?

   a. 2:4 and 6:12

   b. 1:3 and 4:15

   c. 16:30 and 8:15

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Rates

Earlier in the lesson, we explored ratios. Ratios compare quantities of the same kind: numbers of girls and boys (people), numbers of rainy days and sunny days (days), numbers of pants, shorts, and shirts in a dresser drawer (articles of clothing). But what if we want to compare different types of things?

A rate is a way of comparing two measurements or quantities. For example, speed is a rate. The rate 50 km/h compares distance and time. The rate means that you can travel 50 kilometres in one hour.

Other examples of rates are:

- the number of litres of water you use in the shower every week
- the amount of rain that falls in a year
- the amount of money paid for every hour you work
- the distance you can travel in a vehicle with a certain amount of fuel

Can you think of any other examples of rates?

When we worked with ratios, we did not include units. When we work with rates, the units are very important. If your friend told you that oranges were on sale for 1.99/1, you would probably ask for more information. You might assume that your friend meant $1.99, since they're talking about price, but you wouldn't know if they meant $1.99 per orange or $1.99 per pound of oranges, or $1.99 per kilogram of oranges. The rate 1.99/1 is not as meaningful as the rate $1.99/kg.

Unit Rates

In rates, as in ratios, equivalent fractions play a very important role. We change ratios and rates into fractions, then use our fraction knowledge to find equivalent forms. For example, if you get paid $36.00 for 4 hours of work on your part time job, what is your hourly wage?

Your rate of pay is $36.00/4 hours. To figure out your hourly wage, create equivalent fractions.

Thinking Space

$$\frac{\$36.00}{4 \text{ hours}} = \frac{?}{1 \text{ hour}}$$

÷ 4

$$\frac{\$36.00}{4 \text{ hours}} = \frac{\$9.00}{1 \text{ hour}}$$

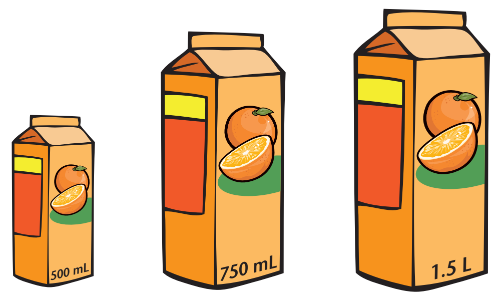
÷ 4

So, your wage is \$9.00/h.

Notice that the second term in this rate is 1. A rate that has 1 as its second term is called a **unit rate**. Unit rates are often used to make comparisons. For example, if you were grocery shopping you might want to compare the prices of two different brands. Or, you might want to compare the prices of the same item at different stores.

Where else might you use unit rates?

Which carton of juice is the best buy?

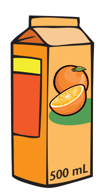


\$1.89 **\$2.45** **\$4.29**

If we can find the price per unit volume of each carton, we can compare them to figure out which is the best deal.

If we buy this carton, we get 500 mL of juice for \$1.89.

$$\begin{aligned} \text{unit price} &= \frac{\text{cost}}{\text{volume}} \\ &= \frac{\$1.89}{500 \text{ mL}} \\ &= \$0.00378/\text{mL} \end{aligned}$$

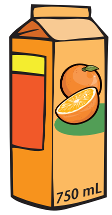


The unit price of this carton is \$0.00378/mL.

If we buy this carton, we get 750 mL of juice for \$2.45.

Thinking Space

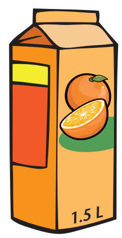
$$\begin{aligned}\text{unit price} &= \frac{\text{cost}}{\text{volume}}\\ &= \frac{\$2.45}{750\text{ mL}}\\ &= \$0.00327/\text{mL}\end{aligned}$$

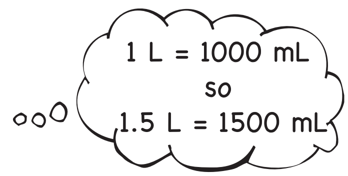


The unit price of this carton is \$0.00327/mL.

If we buy this carton, we get 1.5 L of juice for \$4.29. To compare this price to the others, we need the unit price to be in the same units. To convert litres to millilitres, multiply by 1000.

$$\begin{aligned}\text{unit price} &= \frac{\text{cost}}{\text{volume}}\\ &= \frac{\$4.29}{1500\text{ mL}}\\ &= \$0.00286/\text{mL}\end{aligned}$$





Are there other factors to consider when making your purchase? Do you always buy the product with the cheapest unit price?

The unit price of this carton is \$0.00286/mL.

The largest container has the lowest unit price, so it is the best value.

Thinking Space

Try It!
Activity 2

1. Write a rate for each sentence below.

   a. Adrian travelled 110 kilometres in two hours.

   b. Angelique paid $11.19 for three kilograms of apples.

   c. David took his heart rate after jogging. He counted 30 beats in a 10-second time period.

2. Write a description for each rate below.

   a. $\frac{400 \text{ km}}{28 \text{ L}}$

   b. 70 km/h

   c. $\frac{\$72}{5 \text{ h}}$

Thinking Space

3. Calculate the unit rate for each of the rates described in question 1.

   a.

   b.

   c.

4. You can buy a package of four batteries for $6.67 or a package of 10 for $13.90. Which package is the best buy?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Solving Problems with Ratios and Rates

Let's apply our knowledge of ratios and rates to solve some problems.

Problem 1

At a hockey game, your favourite team out shot their opponent 2 to 1. If your team made 30 shots, how many shots did their opponent make?

The ratio of shots made by your team to the number of shots made by their opponent is 2:1. That means that for every two shots your team made, their opponent only made one. We can use proportions to solve this problem.

$$2:1 = 30:\square$$

×15

$$2:1 = 30:\square$$

×15

$$2:1 = 30:15$$

We could also set up this proportion using fractions.

×15

$$\frac{2}{1} = \frac{30}{\square}$$

×15

$$\frac{2}{1} = \frac{30}{15}$$

Your favourite team's opponent made 15 shots on goal.

Problem 2

Jillian works at a coffee shop. Last week she worked 25 hours and earned $225.

a. What is her hourly rate of pay?

b. She is scheduled to work 31 hours next week. How much money will she earn?

Thinking Space

To answer part (a), we need to find out how much Jillian makes in one hour.

$$\begin{aligned}\text{unit rate} &= \frac{\text{amount earned}}{\text{hours worked}}\\ &= \frac{\$225}{25\text{ h}}\\ &= \$9/\text{h}\end{aligned}$$

Jillian's rate of pay is \$9 per hour.

To answer part (b), we need to figure out how much she'd earn if she worked 31 hours. We can use the unit rate we found in part (a) and multiply it by the number of hours she is scheduled to work next week.

$$\overset{\times 31}{\frac{\$9}{1\text{ h}} = \frac{\square}{31\text{ h}}}$$

$$\underset{\times 31}{}$$

$$\frac{\$9}{1\text{ h}} = \frac{\$274}{31\text{ h}}$$

Jillian will earn \$274 next week if she works all her scheduled hours.

Problem 3

Stephen consulted his map to find the distance between Nanaimo and Courtenay. He used a ruler to measure the distance on the map: 8.25 cm. "Great!" he thought, "Now I'll just look at the scale."

Unfortunately, the bottom of the map was ripped, and the scale was missing. Stephen was discouraged for a moment, but then he had an idea. "I know that it's about 20 km from Nanaimo to Ladysmith. I'll measure that distance on the map and make my own scale!" Stephen found the distance between Nanaimo and Ladysmith to be 1.5 cm on the map.

Set up a proportion and find the distance between Nanaimo and Courtenay using Stephen's scale.

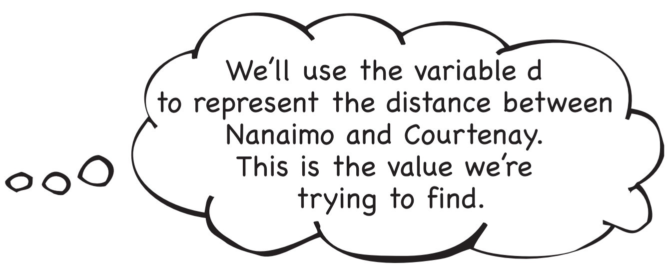
Thinking Space

Maps are created using a scale. This means that any distance shown on a map is proportional to the actual distance.

$$\frac{\text{distance on map}}{\text{actual distance}}$$

We can use this ratio to set up a proportion with the information in the problem.

$$\frac{1.5\text{ cm}}{20\text{ km}} = \frac{8.25\text{ cm}}{d}$$



Thinking Space

$$\frac{1.5\text{ cm}}{20\text{ km}} = \frac{8.25\text{ cm}}{d} \quad (\times \square)$$

What can we multiply 1.5 by to get 8.25?

$$\frac{1.5\text{ cm}}{20\text{ km}} = \frac{8.25\text{ cm}}{d} \quad (\times 5.5)$$

Divide to find what number multiplied by 1.5 gives 8.25.
8.25 ÷ 1.5 = 5.5

From the proportion you can see that

$$d = 20 \times 5.5$$
$$d = 110$$

So the distance from Nanaimo to Courtenay is 110 km.

The Cross-Product Method

In some proportions, it's easy to determine what factor you should multiply or divide by to find the missing number. As you saw in Problem 3 (above) it's not always so easy. We'll try solving Problem 3 again, using a different method. But first, let's look at a simple proportion.

$$\frac{1}{2} = \frac{2}{4}$$

Try this:

Multiply the numerator of the first fraction by the denominator of the second fraction.

$$\frac{1}{2} = \frac{2}{4} \qquad 1 \times 4 = 4$$

Multiply the denominator of the first fraction by the numerator of the second fraction.

$$\frac{1}{2} = \frac{2}{4} \qquad 2 \times 2 = 4$$

Thinking Space

How is this method different from the way we solved the problem before? How is it similar?

Notice that you get the same answer for both. These are called *cross-products*. In any proportion, the cross-products are equal.

If $\frac{a}{A} = \frac{b}{B}$ then $aB = Ab$

We can use this to help us solve proportion problems.

Let's go back to Problem 3 (the map problem). Here's our proportion:

$$\frac{1.5 \text{ cm}}{20 \text{ km}} = \frac{8.25 \text{ cm}}{d} \qquad \frac{1.5 \text{ cm}}{20 \text{ km}} = \frac{8.25 \text{ cm}}{d}$$

We can write the cross products as an equation.

Remember, side-by-side brackets mean multiply.

$$(1.5 \text{ cm})(d) = (20 \text{ km})(8.25 \text{ cm})$$

Now, solve the equation.

To undo multiplying by 1.5 cm, divide by 1.5 cm.

What you do to one side of the equation, you must also do to the other side.

$$\frac{(1.5 \text{ cm})(d)}{1.5 \text{ cm}} = \frac{(20 \text{ km})(8.25 \text{ cm})}{1.5 \text{ cm}}$$

$$d = \frac{(20 \text{ km})(8.25 \text{ cm})}{1.5 \text{ cm}}$$

$$d = 110 \text{ km}$$

We got the same answer as we did before; the actual distance from Nanaimo to Courtenay is 110 km.

When you're solving problems, you can use whichever method works best for you.

Thinking Space

Try It!
Activity 3

1. Cassie rides her bike to school. The school is 8.5 km away from her house, and it usually takes her 30 minutes to get there. What is Cassie's rate of speed on her bike (in km/h)?

2. If a can of paint covers 9 square metres, how many cans of paint does it take to paint a room which has 27 square metres of wall area?

3. Beverly is offered two different jobs.
   - The first job is working in a hardware store. The manager says he will pay her $440/week if she works 40 hours a week. He would pay her the same hourly wage if she wants fewer hours.
   - The second job is at the library. The librarian says she will pay Beverly $350/week for 25 hours of work per week. The librarian is not flexible about the number of hours Beverly can work.

   a. Calculate the hourly rate of pay for each job.

Thinking Space

b. Which job should Beverly take? Explain your answer.

4. Marcel's BC Hydro bill arrived in the mail. The bill showed that he used 194 kWh of electricity for which he was charged $11.47. If he uses 230 kWh next month, how much will his bill be?

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B
Making Sense of Percents

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How can I use a ten-by-ten grid to show percents?		
How do I convert among decimals, fractions, ratios, and percents?		

Thinking Space

Warm–up

1. Write each fraction as a percent.

   a. $\frac{70}{100}$

   b. $\frac{6}{10}$

   c. $\frac{3}{4}$

2. Write each percent as a decimal.

   a. 85%

   b. 27%

   c. 3%

3. Write each percent as a fraction in lowest terms.

   a. 50%

   b. 60%

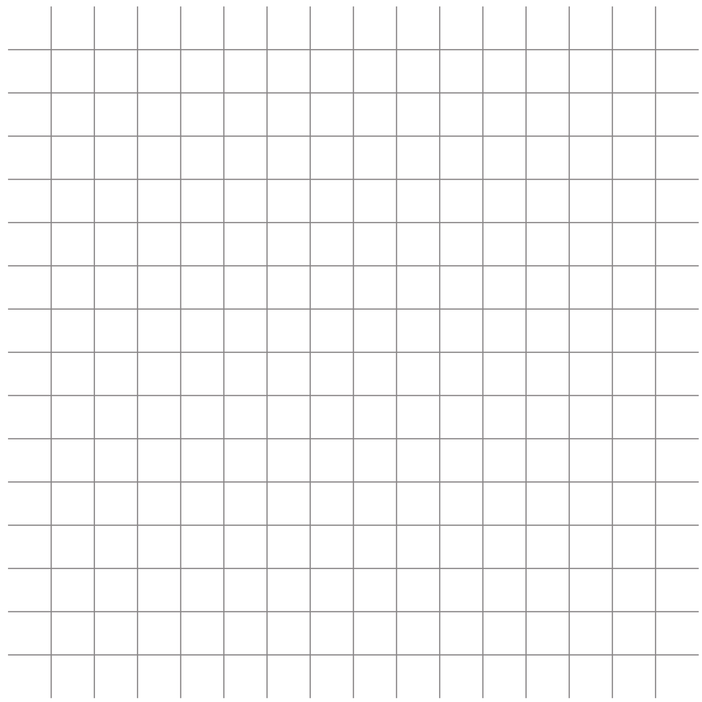
   c. 43%

Turn to Solutions at the end of the module and mark your work.

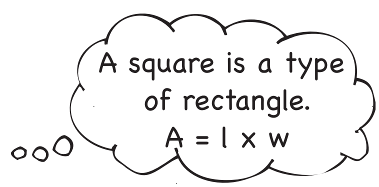
Thinking Space

Explore
Drawing Percentages

Draw a ten by ten square on the grid below. Shade it in lightly.



We can calculate the area of this square as follows:



A = *length* × *width*

A = 10 × 10

A = 100 square units

Next draw a five by five square, starting in one of the corners of the ten by ten square. Shade this square a different colour. Calculate the area of this square as follows:

A = *length* × *width*

A = 5 × 5

A = 25 square units

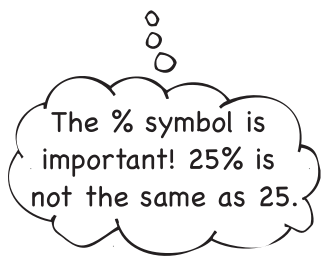
Thinking Space

Let's compare the ratio of the area of the smaller square to the area of the bigger square. We can think of the big square as a whole, and the smaller square as a part of the whole. The ratio of their areas, then, will be a part-to-whole ratio. We can write this in three ways:

25 to 100 $\qquad$ 25 : 100 $\qquad$ $\frac{25}{100}$

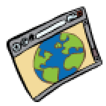
You may remember that a fraction of a whole, expressed as a fraction of 100, is called a percent (or a percentage). So, we can actually write our area ratio above in two more ways: as a percent and as a decimal.

25 to 100 $\qquad$ 25 : 100 $\qquad$ $\frac{25}{100}$ $\qquad$ 25% $\qquad$ 0.25



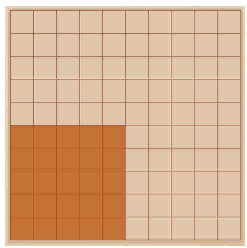
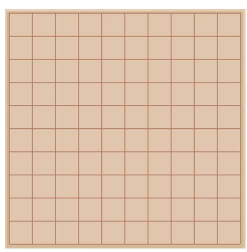
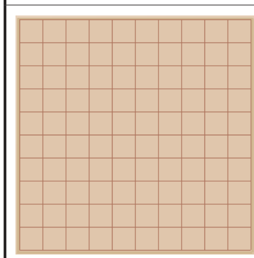
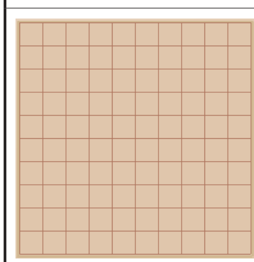
Thinking Space

Try It!
Activity 1

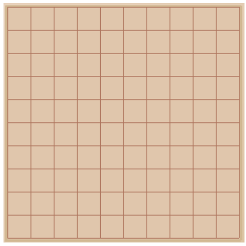
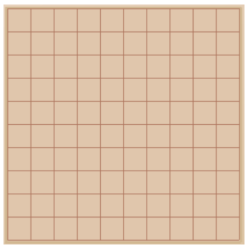
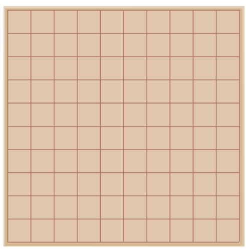
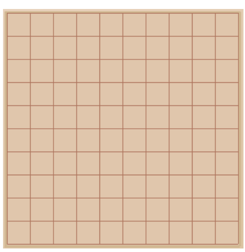


Go to **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0842b1f_percent.html** and open *Perfecting Percents*. Click on “1 to 100%” and follow the instructions.

Use *Perfecting Percents* to complete the following table. The first line is done for you.

Picture	Percent	Fraction	Ratio	Decimal
	25%	$\frac{25}{100}$	25:100	0.25
	64%		64:100	
		$\frac{12}{100}$		
				0.04

Thinking Space

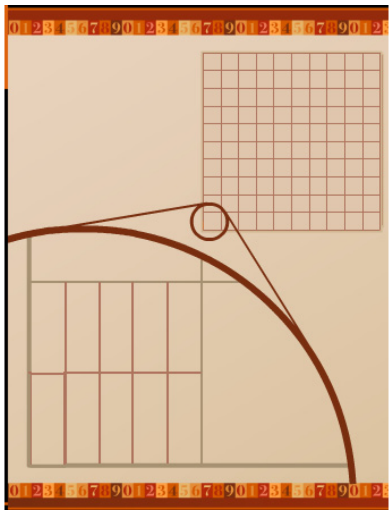
				0.2625
	52%		52:100	
				0.765
		$\frac{90}{100}$		

Turn to Solutions at the end of the module and mark your work.

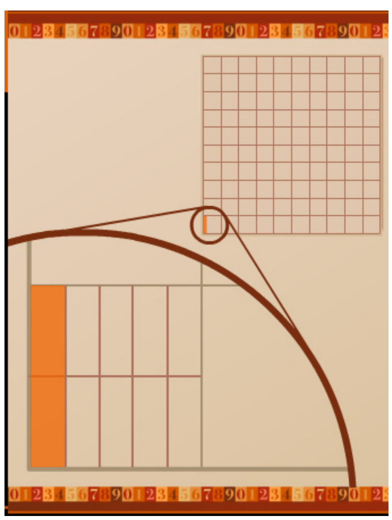
Thinking Space

Explore
Drawing Percentages

In the ten-by-ten grid we've been working with, each square represents 1%. What happens if we divide that 1% square into smaller pieces?



Here you can see that the 1% square is divided into ten pieces (or tenths). If you shade in two of those pieces, what percent do you have?



You have:

two tenths of one percent

$$\frac{2}{10} \times \frac{1}{100}$$

This is the same as $\frac{2}{1000}$.

To write this as a percent, we need to convert it to a fraction with a denominator of 100.

$$\overset{\div 10}{\frac{2}{1000} = \frac{?}{100}}_{\div 10}$$

$$\frac{2}{1000} = \frac{0.2}{100} = 0.2\%$$

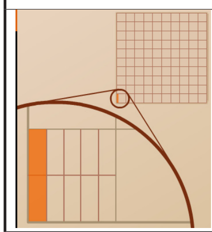
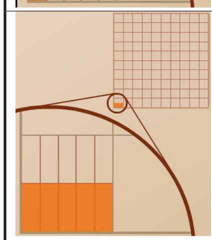
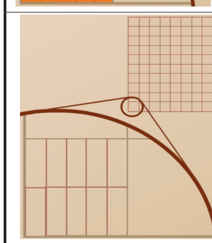
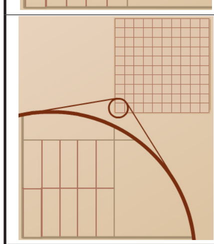
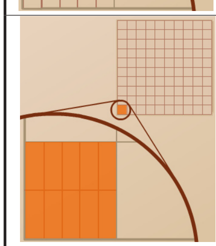
We can write fractional percents as percents, fractions, ratios, and decimals. Give it a try in the next activity.

Thinking Space

Try It!
Activity 2

Go to **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0842b1f_percent.html** and open *Perfecting Percents*. Click on "0 to 1%" and follow the instructions.

Use *Perfecting Percents* to complete the following table. The first line is completed for you.

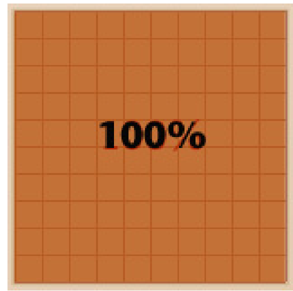
Picture	Percent	Fraction	Ratio	Decimal
	0.2%	$\frac{2}{1000}$	2:1000	0.002
		$\frac{5}{1000}$		
	0.6%			
				0.003
				

Turn to Solutions at the end of the module and mark your work.

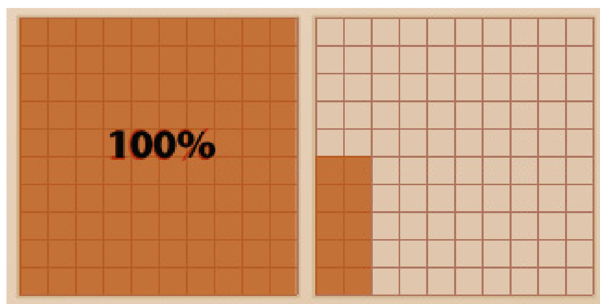
Thinking Space

Explore
Percents Greater Than 100%

One whole ten by ten grid represents 100%.



We can use multiple ten-by-ten grids to represent percentages that are greater than 100%.



1st grid has 100 shaded squares = 100%

2nd grid has 10 shaded squares = 10%

Total = 110 shaded squares = 110%

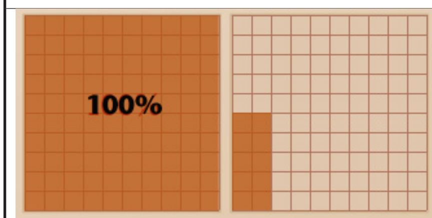
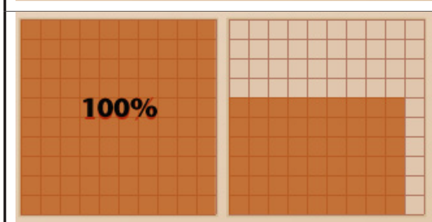
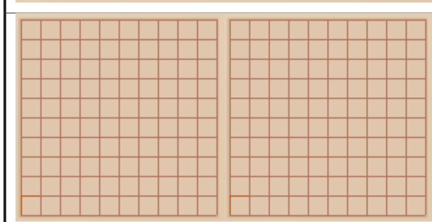
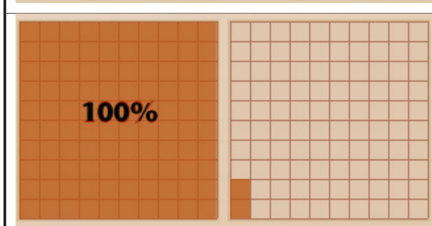
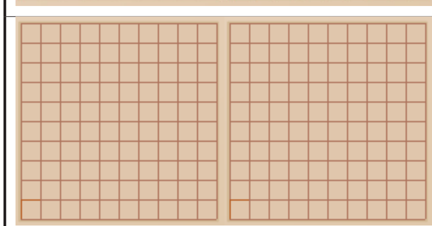
We can write percentages that are greater than 100% as percents, fractions, ratios, and decimals. Give it a try in the next activity.

Thinking Space

Try It!
Activity 3

Go to **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0842b1f_percent.html** and open *Perfecting Percents*. Click on "100% to 200%" and follow the instructions.

Use *Perfecting Percents* to complete the following table. The first line is completed for you.

Picture	Percent	Fraction	Ratio	Decimal
	110%	$\frac{110}{1000} = 1\frac{10}{100}$	110:100	1.1
			154:100	
	127%			1.27
				
				1.81

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Calculations With Percents

You have seen that percents can range from 0% to more than 100%. In the next lesson we'll explore a number of situations in which you might find percents, and we'll solve several problems. For now, we'll work on our calculation skills, and get some practice converting between percents, fractions, ratios, and decimals.

Let's work through several examples together.

Example 1

Write the following fractions as percents.

a. $\frac{14}{20}$

b. $\frac{3}{250}$

c. $1\frac{3}{5}$

To write the fractions as percents, create equivalent fractions out of 100. This is a proportion. You can solve proportions by figuring out what factor to multiply the numerator and denominator by to create the equivalent fraction, or you can use the cross product method that we used in Lesson A.

a.
$$\overset{\times 5}{\underset{\times 5}{\frac{14}{20} = \frac{x}{100}}}$$

$$\frac{14}{20} = \frac{70}{100} = 70\%$$

b.
$$\frac{3}{250} = \frac{x}{100}$$

$$(3)(100) = (250)(x)$$

$$\frac{(3)(100)}{250} = \frac{(250)x}{250}$$

$$1.2 = x$$

So,

$$\frac{3}{250} = \frac{1.2}{100} = 1.2\%$$

If you're not sure what to multiply or divide by, use the cross-product method.

Thinking Space

c. $1\frac{3}{5} = \frac{x}{100}$

First change this to an improper fraction.

$$\frac{8}{5} \overset{\times 20}{=} \frac{x}{100}$$

$$\frac{8}{5} = \frac{160}{100}$$

$$\frac{8}{5} = 160\%$$

Example 2

Write each decimal as a percent.

a. 0.35

b. 0.0001

c. 1.67

To convert from decimals to percents, simply multiply by 100 and add the percent symbol. A quick way to do this is to move the decimal point two places to the right.

a. $0.35 \times 100 = 35$

So, $0.35 = 35\%$

Don't forget the percent symbol!

b. $0.0001 \times 100 = 0.01$

So, $0.0001 = 0.01\%$

c. $1.67 \times 100 = 167$

So, $1.67 = 167\%$

Part-to-whole ratios can be written as percents. Can rates be written as percents? Why or why not?

Example 3

Write each part-to-whole ratio as a percent.

a. 1:200

b. 2:10

c. 8:3

Remember that part-to-whole ratios can be written as fractions. We can set up proportions to find an equivalent fraction with a denominator of 100.

Thinking Space

a. $1:200 = \dfrac{1}{200}$

$\div 2$

$$\frac{1}{200} = \frac{x}{100}$$

$\div 2$

$$\frac{1}{200} = \frac{0.5}{100} = 0.5\%$$

b. $2:10 = \dfrac{2}{10}$

$\times 10$

$$\frac{2}{10} = \frac{x}{100}$$

$\times 10$

$$\frac{2}{10} = \frac{20}{100} = 20\%$$

c. $8:3 = \dfrac{8}{3}$

$$\frac{8}{3} = \frac{x}{100}$$

Use the cross-product method to solve this one.

$$(8)(100) = (3)(x)$$

$$\frac{(8)(100)}{3} = \frac{(3)(x)}{3}$$

$$266.\overline{6} = x$$

So,

$$8:3 = 266.\overline{6} \approx 266.7\%$$

Example 4

Write each of the following percents as a decimal and as a fraction.

a. 41.5%

b. 140%

c. $\frac{3}{4}\%$

To convert a percent to a decimal, simply divide by 100 and remove the percent sign. A quick way to do this is to move the decimal point two places to the left.

Once you have a decimal, you can easily convert to a fraction. Don't forget to reduce to lowest terms.

Thinking Space

a. $41.5 \div 100 = 0.415$

so $41.5\% = 0.415$

$41.5\cancel{\%} = 0.415$

To convert the decimal to a fraction, start by checking the place value of the last digit. The 5 is in the thousandths place, so

$$0.415 = \frac{415}{1000}$$

Now reduce the fraction to lowest terms.

$$\frac{415}{1000} \overset{\div 5}{=} \frac{83}{200}$$

So, $41.5\% = \frac{83}{200}$.

b. $140 \div 100 = 1.4$

so $140\% = 1.4$

Now convert the decimal to a fraction. We have one whole, and four tenths so

$$1.4 = 1\frac{4}{10}$$

Now reduce the fraction to lowest terms.

$$1\frac{4}{10} \overset{\div 2}{=} 1\frac{2}{5}$$

So, $140\% = 1\frac{2}{5}$.

How might you convert a percent to a fraction without converting to a decimal first?

Thinking Space

c. Start by writing $\frac{3}{4}\%$ as a decimal percentage.

$\frac{3}{4}\% = 0.75\%$

$3 \div 4 = 0.75$

Now convert the percent to a decimal.

$0.75 \div 100 = 0.0075$

So, $0.75\% = 0.0075$

We can also move the decimal point two places to the left.
$0.75\cancel{\%} = 0.0075$

The 5 is in the ten thousandths place, so

$$0.0075 = \frac{75}{10\,000}$$

Reduce the fraction to the lowest terms.

$$\frac{75}{10\,000} \underset{\div 25}{\overset{\div 25}{=}} \frac{3}{400}$$

So, $0.75\% = \frac{3}{400}$.

Now it's your turn to practice converting between percents, decimals, fractions, and ratios.

Thinking Space

Try It!
Activity 4

1. Write the following fractions as percents.

   a. $\frac{171}{300}$

   b. $\frac{41}{20}$

   c. $\frac{3}{125}$

   d. $1\frac{7}{15}$

2. Write the following decimals as percents.

   a. 0.14

   b. 0.005

   c. 0.1

   d. 1.23

3. Write each of the following percents as a decimal and as a fraction.

	Decimal	Fraction
a. 0.07%		
b. $23\frac{1}{2}\%$		
c. 325%		

Thinking Space

4. Write the following part-to-whole ratios as percents.

   a. 7:4

   b. 1:500

5. Part-to whole ratios can be written as percents. Can rates be represented by percents? Why or why not?

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson C

Solving Problems with Percents

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How do I combine percents?		
How do I find the percent of a percent?		

Thinking Space

Warm–up

1. What is 20% of 10?

2. What 15% of 110?

3. Convert 70% to a decimal.

4. Convert 7% to a decimal.

5. What is 0.05% of $10,000.00?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Taxes (GST and PST)

You were so excited to buy that new hoodie! But when you went to pay, the total that came up on the cash register was more than the price on the tag. What happened?

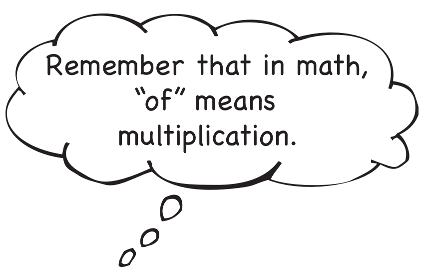
In some provinces two taxes are added to purchases: the Goods and Services Tax (GST) and the Provincial Sales Tax (PST). Other provinces have just one tax, the Harmonized Sales Tax (HST). BC may soon adopt the HST, but in this lesson we'll explore how to work with two taxes. The current rates are:

- GST 5%
- PST 7%

Example 1

A skateboard has a displayed price of \$140.00. What is the total amount of taxes on this item? What is the check out price?

$5\% = \frac{5}{100} = 0.05$ $\qquad$ $7\% = \frac{7}{100} = 0.07$



GST = 5% of \$140.00 = 0.05 × 140.00 = \$7.00

PST = 7% of \$140.00 = 0.07 × 140.00 = \$ 9.80

The total amount of taxes = \$7.00 + \$9.80 = \$16.80

The check out price is: \$140.00 + \$16.80 = \$ 156.80

Note: The GST and PST are each calculated separately—the calculation of one does not affect the calculation of the other. We could perform these calculations in any order. In fact, we could combine the GST and PST into one total percentage (5% + 7% = 12%).

Thinking Space

Working Backward

Sometimes we might want to find the price of an item before taxes. To do this we can work backward from the cost of the taxes.

Example 2

For example, if we know the GST on an item is $9.00, we can find the original price.

We know the percentage and the cost of the tax, but we don't know the original amount. In the previous example, we used the following expression to find the tax cost.

original price × percentage tax = tax cost

We can use this same expression, substituting what we know.

original price × 5% = $9.00

OR

original price × 0.05 = $9.00

Solve for the original price:

$$\frac{\text{original price} \times 0.05}{0.05} = \frac{\$9.00}{0.05}$$

Divide both sides by 0.05 to get "original price" by itself on the left side.

original price = $180.00

How big do you think the original price will be compared to its taxes? Try estimating before we solve it.

Example 3

You pay $3.00 in taxes on a T-shirt. How much did the T-shirt cost before taxes?

Total taxes include 5% GST + 7% PST = 12%

original price × 12% = $3.00

original price × 0.12 = $3.00

$$\frac{\text{original price} \times 0.12}{0.12} = \frac{\$3.00}{0.12}$$

original price = $25.00

The T-shirt cost $25.00 before taxes.

Thinking Space

Try It!
Activity 1

For the questions below, assume that the GST is 5% and the PST is 7%.

1. If the display price of an item is $120.00, what is the
   a. GST paid on the item?

   b. PST paid on the item?

2. If the price tag on a pair of jeans reads $70.00, what is the total amount of taxes? What is the check out price?

Thinking Space

3. If the GST paid on a new collector's edition of a video game is $3.40, what was the original price?

4. If the PST paid on an item is $10.00, what was the original price of the item?

5. If the total tax on an item is $9.60, what was the original price of the item?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Profits, Taxes, and Discounts

Stores mark up the price on their merchandise to cover expenses, wages, and make a profit. Sometimes a store will decide to mark up an item based on a specific dollar amount. This is called simple markup. Other times, a store will decide on a percentage markup, and apply the same percent markup to similar items. The price that a store sells items at is called the retail price.

Stores often offer discounts in order to get rid of old merchandise or to encourage people to buy more items. There are many ways to offer a discount. One of the most common is a percent discount.

Let's work through some examples.

Example 1 (Simple Markup)

A store buys a box of Wii® accessories for \$200.00. The store plans to sell the accessories and wishes to mark it up by \$25.00. If you were to purchase the accessories from this store, how much would you pay including taxes?

Solution:

Since the store wants to markup the Wii® accessories by \$25.00, the price tag will read \$225.00 (\$200.00 cost + \$25.00 markup = \$225.00).

When you purchase the accessories, you also have to pay GST (5%) and PST (7%).

$$\text{GST} = 5\% \text{ of } \$225.00 = 0.05 \times \$225.00 = \$\ 11.25$$

$$\text{PST} = 7\% \text{ of } \$225.00 = 0.07 \times \$225.00 = \$15.75$$

So the total cost for you = \$225.00 + \$11.25 + \$15.75 = \$252.00.

Note: The GST and PST are each calculated separately—the calculation of one does not affect the calculation of the other. We could perform these calculations in any order. In fact, we could combine the GST and PST into one total percentage (5% + 7% = 12%).

Thinking Space

To make sure this works, check that you get the same answer.

12% of \$225.00 = 0.12 × 225.00 = \$27.00

Previously we calculated that the GST was \$11.25 and the PST was \$15.75. Combining these, you can see that we get \$27.00, which is the same answer.

In what other situations might you be able to combine percents? Are there any situations where this might not work?

Example 2 (Percent Markup)

A store buys hoodies at a wholesale price of \$60.00 each. They usually mark up the price of a clothing item by 35%. What is the retail price for the hoodies?

Solution:

The markup is 35% of the wholesale cost.

35% × \$60.00 = 0.35 × \$60.00 = \$21

retail price = cost of item + markup

= \$60.00 + \$21

= \$82.00

The retail price of the hoodies is \$82 each.

Example 3 (Percent Discount)

A CD regularly sells for \$16. You can buy it on sale for 15% off. What is the sale price?

Solution:

sale price = original price – discount amount

= \$16 – (15% of \$16)

= \$16 – (0.15 × \$16)

= \$16 – \$2.40

= \$13.60

The sale price is \$13.60.

Thinking Space

Another way to approach this problem is to consider how much of the original retail price you will be paying. If you are getting a 15% discount, then you are paying 85% of the original price.

100% - 15% = 85%

Then,

$$\begin{aligned} \text{sale price} &= 85\% \text{ of the original price} \\ &= 85\% \times \$16 \\ &= 0.85 \times \$16 \\ &= \$13.60 \end{aligned}$$

You get the same answer; the discounted price is $13.60.

Example 4 (Percent Discount: Working Backwards)

A pair of jeans is marked down by 20%, and a sale tag now advertises the sale price is $46.40. What was the original price?

Solution:

Sale price = Original Price – (20% of the original price)

Let x represent the original price, and substitute the values we know.

$$\begin{aligned} \$46.80 &= x - (20\% \text{ of } x) \\ \$46.80 &= x - 0.20x \\ \$46.80 &= 0.80x \\ \frac{\$46.40}{0.80} &= \frac{0.80x}{0.80} \\ \$58 &= x \end{aligned}$$

Simplify like terms.
x = 1x and
1x – 0.20x = 0.80x.

The original price of the pair of jeans was $58.00.

We can approach this problem another way.

The original price for the item can be considered 100%. If 20% is taken off, that would leave 80% of the price (100% – 20% = 80%).

Can you think of another way to solve the problem? Try setting up a proportion.

So, the expression we can use is:

80% of the original price is $46.40.

Try using this expression to solve the problem. You should get the same answer.

Thinking Space

Example 5 (Combined Discounts)

A portable DVD player usually sells for $150.00 at a local store. The weekend flyer had an advertisement for a 10% discount. You go to check it out and find out that the store is giving a further discount of 20% off any discounted price! What will be the new ticket price?

Solution:

Price after 10% discount = \$150.00 – 10% of \$150.00

= \$150.00 – 0.10 × 150.00

= \$150.00 – \$15.00

= \$135.00

Price after a further 20% discount = \$135.00 – 20% of \$135.00

= \$135.00 – 0.20 × \$135.00

= \$135.00 – \$27.00

= \$108.00

So the overall discounted price is $108.00.

Note: In Example 1 we found that we could combine the two taxes before figuring out the cost of the taxes. It didn't matter which order we calculated the taxes, or if we combined them first.

Example 5 is different. In Example 5, there is a discount on a discount. This is called a compounding percent. The order that you calculate these percents is very important, and you cannot simply add the percents together.

Check it out! Try applying the discounts in a different order, or combining them first before calculating the sale price. You should find that you get a different answer for each.

Other Problems

We have solved several percent problems related to shopping: profits, taxes, and discounts. There are many other applications of percents. The next activity will ask you to solve a number of problems. Some will be similar to the ones we solved in this Explore, but some will be a bit different. Think through the problems carefully, and use what you know about percents, ratios, and proportions to help you.

Thinking Space

Try It!
Activity 2

For the questions below, assume that the GST is 5% and the PST is 7%.

1. A street vendor buys a pair of jeans wholesale for $90.00 and sells it for $120.00 including taxes. What is the profit amount for the vendor? (GST is 5% and PST is 7%)

2. Cole bought a Blackberry for $300.00 after a 20% discount. What was the original listed price?

3. A classic video game discounted by 10% has been advertised for a further 15% discount. If the original price was $80.00, what was the price of the game after both discounts?

Thinking Space

4. A winter jacket has a listed price of \$160.00. If the store advertises a discount of 30%, how much does it cost after the discount and the taxes are added? (GST is 5% and PST is 7%)

5. In Grade 10, students face their first provincial exams. The provincial exam is worth 20% of their final mark, the remainder comes from their class grade. Alex is a student in 10th grade. If he has 70% in his class mark and 73% on his provincial exam, what mark does he get for a final grade in the course?

6. Most of the water on Earth is saltwater. Only approximately 2.5% of the water on Earth is freshwater. Two thirds of that freshwater is frozen in icecaps and glaciers. Our drinking water comes from freshwater sources such as groundwater, rivers, and lakes.

    a. What percent of the Earth's freshwater is frozen? (Express your answer to the nearest hundredth.)

Thinking Space

b. What percent of the Earth's water is available to us for use? (Express your answer to the nearest hundredth.)

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson C. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Section Summary

Completing this section has helped you to:

- demonstrate an understanding of ratio and rates
- demonstrate an understanding of percents
- solve problems involving percents ratios and rates

Section 3
Probability

In this section you will:

- list possible outcomes in random experiments
- decide if events are independent or dependent
- determine the probability of two independent events and verify the probability using a different strategy
- solve problems involving probability of independent events

For this section you will need:

- calculator

Where in the World...?

You're going on a camping trip next weekend. How can you find out if it's going to rain? Meteorologists provide their predictions about the weather for weather reports which list the chance for rain as a probability or percent. If a weather report says there is a 40% chance of rain, will it rain? Probability looks at the outcome you want—sunny days—out of the possible outcomes—sunny and rainy days.

Section 3
Pretest

Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Please complete this pretest **without** using a calculator.

Lesson A: The Probability of Independent Events

1. Which of the following are independent events?

    a. eating candy and cavities in teeth ________________

    b. tossing a coin and rolling a die ________________

    c. draw a card from a deck of cards.
    Leave it out of the deck and then draw a second card. ________________

    d. driving to work and eating a sandwich ________________

2. What is the sample space for spinning a spinner with four equal sections numbered from 1 to 4 and tossing a coin?

3. A six-sided die is rolled once. What is the probability that the result is:

   a. an odd number

   b. a number less than 5

   c. at least a 2

Lesson B: Problem Solving with Probability

4. Two fair, six-sided dice are rolled at once. What is the probability that the sum of the sides, (your result) is:

   a. 3

   b. at most 3

   c. at least 3

d. a multiple of 6

e. more than 10

5. A single card is drawn from a well-shuffled 52-card deck of playing cards. What is the probability that it is:

   a. a red card?

   b. an Ace ?

   c. a face card (J, Q, K)?

   d. a King or a Queen?

6. To start a game, Esther has to throw a six with a fair, six-sided die. Find the probability that Esther starts the game on:

   a. her first throw

   b. her second throw

7. A letter is randomly selected from the 26 letters of the alphabet, and two fair coins are tossed. (Assume that the letter ‘y’ is a consonant).

   a. What is the probability that a vowel is selected and at least one head turns up on the coins?

   b. What is the probability that consonant is selected and two tails turn up on the coins?

   c. What is the probability that a letter ‘y’ is selected and exactly one tail turns up on the coins?

   d. What is the probability that a letter ‘w’ is selected and that two heads turn up on the coins?

8. Two cards are drawn from standard deck of 52 playing cards; the first card is replaced before the second card is drawn. Find the probability that the cards will be:

   a. a heart followed by club

   b. a Queen followed by a red card

   c. a black card followed by a diamond in any order.

Turn to Solutions at the end of the module and mark your work.

Lesson A

The Probability of Independent Events

For this lesson you will need:

- calculator

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What are independent events?		
How do **I** determine the probability of two independent events?		

Thinking Space

Warm-up

1. a. $\frac{1}{4} \times \frac{1}{2} =$

   b. $\frac{1}{5} \times \frac{2}{7} =$

   c. $\frac{3}{4} \times \frac{5}{8} =$

2. If you toss a coin, what results could you get?

3. If you look at a standard deck of cards, what values of cards are possible? What suits are possible?

4. If a fair die is rolled, what possible numbers could be rolled?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
What is Probability? A Brief Review

Review the following terms with their definitions:

probability	the likelihood of a specific event occurring
outcome	a particular result
random experiment	an experiment that produces equally likely outcomes
favourable outcome	an outcome or result that we are hoping for
sample space	all possible outcomes of a probability experiment
biased	an experiment that produces specific outcomes more often than others

Thinking Space

Try It!
Activity 1

Match the correct term to each of the real life examples.

_____	1. a pair of dice consistently rolls a sum of seven	a.	Probability
_____	2. rolling a two on a six-sided die	b.	Outcome
_____	3. 40% chance of getting a concert ticket	c.	Random Experiment
_____	4. rolling a fair six-sided die	d.	Favourable Outcome
_____	5. 3 ways of winning a ticket in a contest	e.	Sample Space
_____	6. rolling a die and getting 1, 2, 3, 4, 5, or 6	f.	Biased

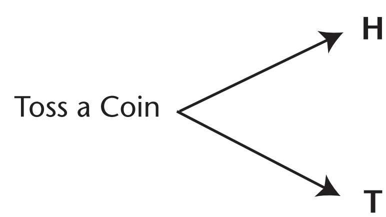
Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Possible Outcomes

When you conduct an experiment where there is an equal probability or chance of any outcome, the experiment is considered random. There are a number of examples we can use to show random experiments, such as tossing a coin and rolling a six-sided die. There are a number of ways to show possible outcomes for a random experiment. You could use a tree diagram, outcome notation, or a table.

Tree Diagrams



Outcome Notation

When you put information into curly brackets { }, the information inside represents the possible outcomes or the specific favourable outcomes of an experiment. This form is known as outcome notation.

When we toss a coin, we can list the possible outcomes as {H, T}. If we are interested in just heads, we can write it as {H}.

When we roll a six-sided die, the possible outcomes are {1, 2, 3, 4, 5, 6}

If we're interested in only the even rolls, the favourable outcomes are {2, 4, 6}

Equation for Probability of a Given Outcome

When we think about probability, we are interested in knowing what the chances are of getting "what we want" or a "favourable" outcome compared to all the possible outcomes. To make this simpler, we use a formula:

$$P(\text{outcome}) = \frac{\text{number of favourable outcomes}}{\text{all possible outcomes}}$$

Thinking Space

For example, if we want to toss heads on a coin:

$$P(\text{heads}) = \frac{\text{number of favourable outcomes}}{\text{all possible outcomes}}$$

heads

$$P(\text{heads}) = \frac{1}{2} = 0.5$$

heads or tails

For example, if we want to roll a 5 on a fair six-sided die:

$$P(5) = \frac{\text{number of favourable outcomes}}{\text{all possible outcomes}}$$

rolling a 5

$$= \frac{1}{6} \approx 0.167$$

rolling a 1, 2, 3, 4, 5, or 6

As shown above, the **fraction** created by this formula can also be turned into a decimal. Remember, in probability the results will always form a decimal from 0 to 1.

- 0 represents 0% or no chance of the outcome occurring.
- 1 represents 100% chance of the outcome occurring.

We converted among fractions, decimals, and percents in Section 2.

We can also look at the possibility of "not getting" a specific outcome. For example, above we looked at rolling a 5. Sometimes we might want to roll anything but a 5, which can be shown as "not a 5".

We can write this in a formula:

$$P(\text{not a } 5) = \frac{\text{number of outcomes that are not a 5}}{\text{all possible outcomes}}$$

Thinking Space

A shorter way to write this is to use the "not event" notation. We do this by putting a line over the event we do not want.

not a 5

$$P(\bar{5}) = \frac{\text{number of outcomes that are not a 5}}{\text{all possible outcomes}}$$

On a six sided die, there are 5 ways to NOT get a 5: {1, 2, 3, 4, 6}.
Therefore, $P(\bar{5}) = \frac{5}{6}$.

The combination of these two events $P(5) = \frac{5}{6}$ and $P(\bar{5}) = \frac{5}{6}$ gives us ALL the possible outcomes. Any time we do probability, the sum of the "favourable outcomes" and the "not favourable outcomes" adds up to the full sample set, or 100% of the possible outcomes (a probability of 1). These events are called complimentary events. Using the example above:

$$P(\text{favourable}) + P(\overline{\text{favourable}}) = \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1$$

Thinking Space

Try It!
Activity 2

Use your calculator to express the answers as both fractions and decimals (for example: $\frac{3}{8} = 0.375$).

1. A number is picked at random from 1 to 20. What is the probability that the number is divisible by 5?

2. A drawer contains 9 pairs of socks: 3 white, 2 black, and 4 green.

   a. What is the probability that a randomly picked pair of socks is black or white?

   b. What is the probability of picking a pair of socks that is not green?

   c. What do you notice about these two answers?

3. A card is picked from a well-shuffled deck of standard playing cards. What is the probability that the card is red?

Turn to Solutions at the end of the module and mark your work.

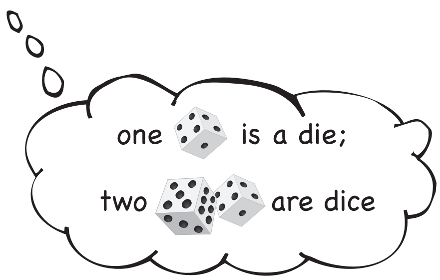
Thinking Space

Explore
Independent Events

Would you go swimming in a lake on a cold and rainy day?

For most people, the probability would be zero. If it's a hot and sunny day, the probability is closer to one (or 100%). The event "swimming in a lake" depends on the weather. This is an example of dependent events, where one event depends on the result of another event.

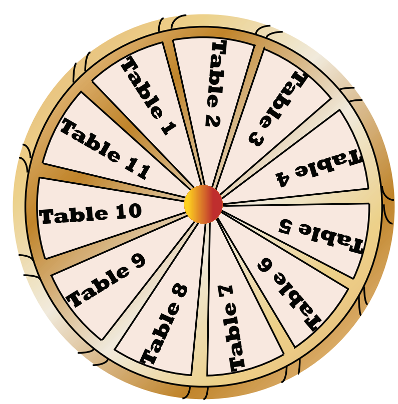
In a different example, we look at tossing a coin and rolling a six-sided die.



When you toss a coin and roll a die at the same time, the coin toss does not change or affect the die roll. The die roll does not affect the coin toss. When one event does not change another we call them independent events.

Here's another example of independent events. At a party there is a bag of numbered tiles that correspond to the seats in the room, and a large spinner with many sections on it.

The Master of Ceremonies at the party draws a numbered tile out of the bag. The person sitting in the seat with that number gets to spin the spinner to choose which table gets to go to the buffet next. Another tile is drawn and that person gets to spin the spinner to decide the next table to go to the buffet. The possibility of your numbered tile being drawn is not affected by the spinner. The result of the spinner does not affect the numbered tile drawn.



In this course, we focus on independent events.

Thinking Space

Winning Prizes at the School Fair

At the entrance to the school spring fair, there is a bag which contains thirty game tiles. In the bag are two game tiles that show an mp3 player and five tiles that show free bottled water. The remaining tiles read, “Good luck—try again!” The first event involves drawing a tile out of the bag.

What is the probability of winning an mp3 player? What is the probability of winning a bottle of water?

$$P(\text{mp3 player}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}} = \frac{\text{tiles showing mp3}}{\text{tiles in the bag}} = \frac{2}{30}$$

$$P(\text{bottled water}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}} = \frac{\text{tiles showing water}}{\text{tiles in the bag}} = \frac{5}{30}$$

Someone has been lucky and won an mp3 player! The tile is placed back into the bag. What is the probability that the second tile drawn is also for an mp3 player?

$$P(\text{mp3 player}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}} = \frac{\text{tiles showing mp3}}{\text{tiles in the bag}} = \frac{2}{30}$$

What is the probability that the second tile drawn is for bottled water?

$$P(\text{bottled water}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}} = \frac{\text{tiles showing water}}{\text{tiles in the bag}} = \frac{5}{30}$$

Notice how the probability of winning a mp3 player or a bottle of water in the second event has not changed from the probabilities for the first event. By replacing the tile, you have the same opportunities each event. Both the first event and the second event are independent. If we didn’t replace the tile, the second event would depend on the first event.

Another way to determine if events are independent is to check if they are physically different. For example, if a fair coin is tossed and a fair die is rolled, the events of turning up a head on the coin and a six on the die are independent, because the coin and die are physically different pieces of equipment.

Thinking Space

Try It!
Activity 3

Determine if the following pairs of events are independent or not and explain your choice.

1. Drawing a card from a normal 52-card deck. The card is a heart. Without replacing the card, drawing again. The second event also is a heart.

   Independent or dependent? ______________________

   Explanation:

   ____

   ____

2. Turning up a six on the roll of a fair die and drawing a black card from a regular deck of cards.

   Independent or dependent? ______________________

   Explanation:

   ____

   ____

3. Picking a white pair of socks (from a drawer containing two white pairs and three green pairs) and turning up a head on the flip of a fair coin.

   Independent or dependent? ______________________

   Explanation:

   ____

   ____

Thinking Space

4. Buying a single Lotto 6/49 ticket and a single Lotto Super 7 ticket.

   Independent or dependent? ______________________

   Explanation:

   ____

   ____

5. Having a sundae and playing computer games.

   Independent or dependent? ______________________

   Explanation:

   ____

   ____

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Determining the Sample Space of Independent Events

What methods can you use to show all the possible outcomes or sample space?

For a fair coin, we have two possible outcomes: heads and tails.

For a fair six-sided die, there are six possible outcomes: 1, 2, 3, 4, 5, 6.

If our experiment involves rolling a die and tossing a coin together, we have to look at all the combinations that can be made from these possibilities. There are 12 unique combinations. Can you find them all?

One way to find all the possibilities is to use a table. To make the table, put all the possible numbers from the die across the top, and all the possible outcomes for the coin (heads or tails) down the side.

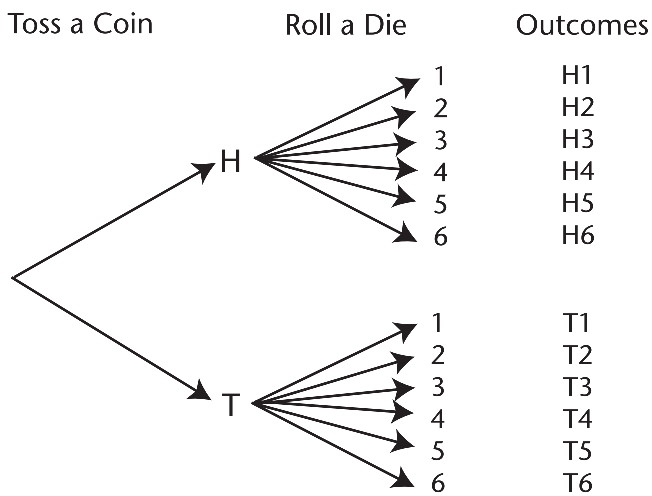
	1	**2**	**3**	**4**	**5**	**6**
Heads (H)	H1	H2	H3	H4	H5	H6
Tails (T)	T1	T2	T3	T4	T5	T6

From the table you can see the 12 possible outcomes. This represents the sample space.

Another useful way to show this is to use tree diagrams. How could you show tossing a coin and rolling a die using a tree diagram?

Thinking Space

One possible way to do this:



In many cases, we want to look at a portion of the sample space. For example, on a coin toss, the sample space is heads and tails. However, there are times we wish to consider only one of these as the result we want.

For rolling a six-sided die, we have the sample space {1, 2, 3, 4, 5, 6}.

Some parts of the sample space are:

- the even rolls {2, 4, 6}
- the odd rolls {1, 3, 5}
- rolls greater than 2 {3, 4, 5, 6}

When we roll a die and toss a coin, there are 12 outcomes in the sample space. What outcomes from this sample space result in tails and an odd number?

To find the answer to this and other probability experiments, we'll use one of our media tools.

Go to **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0843a1f_probability.html** and open *Exploring Probability*.

Thinking Space

On the left side, for Event One, choose 1 Die. For Event Two, choose 1 Coin. Move your mouse pointer over the outcomes in the list of Possible Outcomes at the bottom right to see how each outcome was formed.

In the Show Me box at the bottom left, click on Tails and Odd Number. You'll see the Favourable Outcomes, Possible Outcomes and Event Probability listed at the bottom right.

For Tails and Odd Number, they are:

FAVOURABLE OUTCOMES: 3	POSSIBLE OUTCOMES: 12	EVENT PROBABILITY: $\frac{3}{12} = \frac{1}{4}$

Thinking Space

Try It!
Activity 4

Go to **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0843a1f_probability.html** and open *Exploring Probability*. Use the media to answer the following questions.

1. You and a friend each toss a coin. What is the sample space for the two coin tosses together? List the sample space in outcome notation.

2. Friends get together to play a board game. The board game uses two fair six-sided dice for determining moves. How many possible outcomes are there when rolling two dice?

3. If you use a spinner that is divided into three equal sections coloured red, white, and blue, and toss a fair coin, what is the sample space?

4. You toss a coin and roll a die. What outcomes from the sample space have heads and a roll less than or equal to 4?

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B

Problem Solving with Probability

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What are the strategies that I can use to help me calculate probability of multiple independent events and to solve problems about probability?		

Thinking Space

Warm-up

1. If you roll two dice, are the results on each die independent?

2. You have a bag with a red ball, a green ball, and a blue ball. If you draw out one ball, and then draw out a second without putting the first one back, are the two draws independent?

3. If a fair coin is tossed and a fair die is rolled, what is the number of all possible outcomes? How many of those outcomes contain an even number and heads?

4. If a card is picked from a regular deck and a fair die is rolled, what are the outcomes from the sample space that show a spade face card and an even number?

Thinking Space

5. A coin is tossed and die is rolled. List the favourable outcomes, if you want to get heads and a number less than 5.

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Determining the Probability of Independent Events

In the review we looked at independent events. In the last lesson, we looked at possible outcomes or sample space. Now we'll calculate the probabilities of multiple independent events.

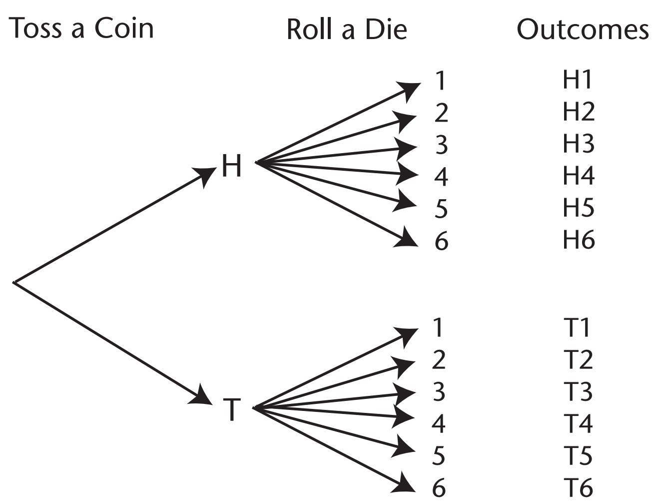
Example 1: Coin and Die

If a fair coin is tossed and a fair die is rolled, what is the probability of tossing heads and rolling a six?

Let's start by listing the sample space. There are 12 unique combinations that we'll show in a table.

	1	**2**	**3**	**4**	**5**	**6**
Heads (H)	H1	H2	H3	H4	H5	H6
Tails (T)	T1	T2	T3	T4	T5	T6

We can show them in a tree diagram.



How many possibilities out of the 12 show (heads and a 6)?

Thinking Space

There is *one* outcome which is (H, 6). There are 12 possible outcomes. This means that there is 1 outcome we want out of 12 possibilities. So the probability of turning up a head on the coin and a six on the die is:

$$P(\text{head and a 6}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}} = \frac{1}{12}$$

So,

The probability of tossing heads with a fair coin is $\frac{1}{2}$.

The probability of rolling a 6 on a fair six-sided die is $\frac{1}{6}$.

The probability of tossing heads and rolling a 6 is $\frac{1}{12}$.

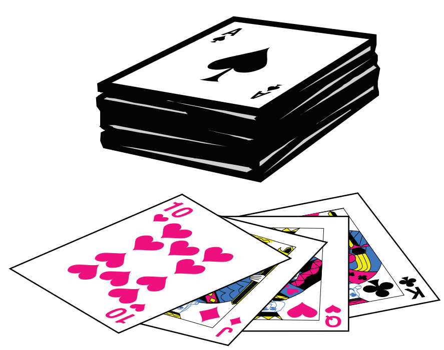
What mathematical operation could we use with the first two probabilities in the chart that will result in a probability of $\frac{1}{12}$?

We can summarize these probabilities in a chart.

P(heads)	P(6)	P(heads and 6) from tree diagram
$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$

Example 2: Spinner and Deck of Cards

Use a spinner with four equal sections to randomly pick a number between 1 and 4. Next, pick a card from a regular deck of playing cards. What is the probability of spinning an even number and drawing a black (B) card?



Since these are "physically" different events, they are independent.

Thinking Space

If there are four sections in the spinner, the even results are 2 and 4. In a standard deck of cards, half the deck is black and half the deck is red. The results that are of interest to us are (spinning 2, drawing black) and (spinning 4 and drawing black).

Make a table to show all the possible results:

		Spinner Results			
		1	2	3	4
Card Drawn	black	1, black	2, black	3, black	4, black
	red	1, red	2, red	3, red	4, red

The sample space is {(1,B), (2,B), (3,B), (4,B),(1,R), (2,R), (3,R), (4,R)}.

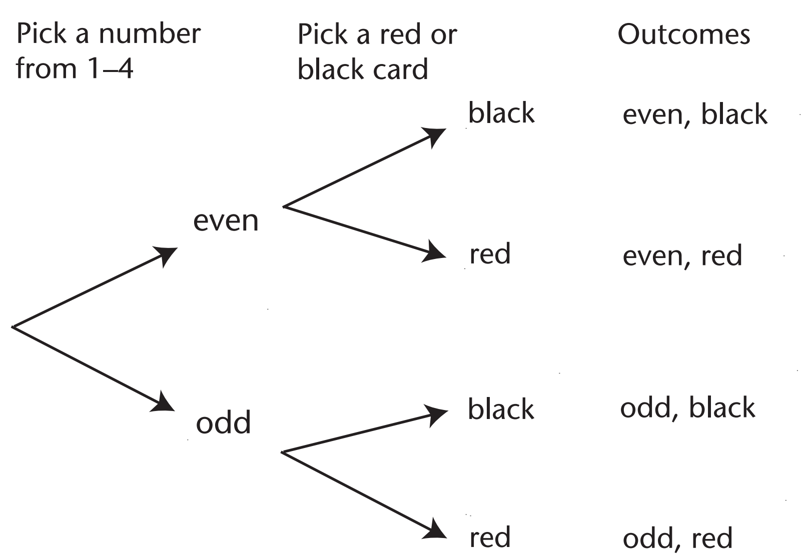
Out of the 8 results shown in the table, 2 of them are favourable outcomes.

$$P(\text{even, black}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}} = \frac{2}{8} = \frac{1}{4}$$

We can show this with a tree diagram. One way to show this is to show ALL the possibilities individually. However, with a deck of cards, that means 52 options for that alone. That would be a lot of work!

Can you think of more than one way to do this?

Another way to do this is to show it as only what you want and what you don't want. On the spinner, half of the numbers (2 and 4) are even and half (1 and 3) are odd. With the deck of cards, half are red and half are black.



Thinking Space

So,

The probability of spinning an even number on a spinner with numbers 1–4 is $\frac{1}{2}$.

The probability of drawing a black card from a deck of cards is $\frac{1}{2}$.

The probability of spinning an even number and drawing a black card is $\frac{1}{4}$.

Write these probabilities into the chart below.

P(even)	P(B)	P(even and B) from table

When we know the probability of spinning an even number—P(even)—and the probability of picking a black card—P(B)—we can calculate the probability of getting them both—P(even and B). Do you know which operation to use? (Keep reading for more information—all will be revealed below!)

Do you see a mathematical relationship between the probabilities for the two independent events, and the probability for the events both happening together?

You may have noticed from the first example that $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$, and from the second example that $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

The second example can be written mathematically as:

$$\text{P(even and black)} = \text{P(even)} \times \text{P(black)} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Thinking Space

It turns out that this is not coincidence: If event A and event B are independent then the probability of event A and event B both occurring is given by:

$$P(A \text{ and } B) = P(A) \times P(B)$$

In fact we can test if two events are independent by checking if this equality holds.

Example 3: Two Coins

If two fair coins are tossed what is the probability that two heads will turn up?

Using the rule above we have:

$$P(H) = \frac{1}{2}$$

So $P(H \text{ and } H) = P(H) \times P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Using the basic definition we have:

Event (head and head) = {(H,H)}

Sample Space = {(H,H), (H,T), (T,H), (T,T)}

$$\text{So } P(HH) = \frac{\text{number of ways to get (heads and heads)}}{\text{total number of possibilities}}$$

$$= \frac{1}{4} \text{ (as expected!)}$$

Thinking Space

Try It!
Activity 1

Go to **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0843a1f_probability.html** and open *Exploring Probability*. Use the media to answer the following questions.

1. You roll a fair six-sided die.

   a. What are the possible outcomes?

   What is the probability of rolling:

   b. a 3?

   c. an even number?

   d. a number less than 3?

   e. a number greater than or equal to 4?

Thinking Space

2. If a fair coin is tossed and a fair die is rolled, find the probability of tossing a head and rolling 2.

3. If a fair coin is tossed and a fair die is rolled:

   a. Find the probability of obtaining a tail and a number less than 3.

   b. Find the probability of obtaining a head and at least a 3.

   c. Find the probability of obtaining a tail and an odd number less than 5.

Thinking Space

4. A bag contains four balls numbered 1 to 4. If a ball is picked at random and a card is drawn from a regular deck of 52 playing cards,

   a. Find the probability of picking an even number and a face card (Jack, Queen, King)

   b. Find the probability of an odd number and an Ace.

   c. Find the probability of a 4 and a heart.

   d. Find the probability of picking a number less than one and a diamond.

Thinking Space

5. Two fair dice are rolled. What is the probability that the sum of the numbers that turn up is 12? Hint: the number of possible outcomes is 36 {(1,1), (1,2), (1,3), …, (6,4), (6,5), (6,6)}.

6. If two fair coins are tossed, what is the probability of turning up:

   a. a head or a tail?

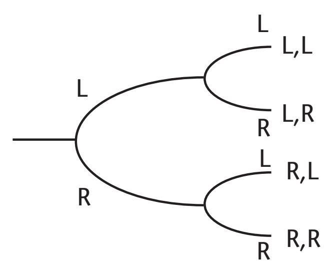
   b. a head and a tail?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Pathways

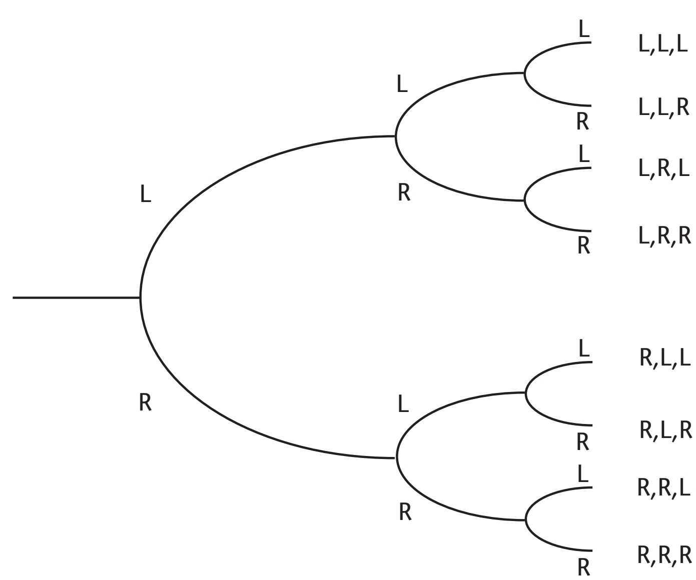
When looking at problems with probability, there are many examples we can explore. One type is called a pathway problem. Pathway problems involve having different possible options for routes to complete an activity. Some examples of this could be a race along streets and avenues or different paths in a board game or computer game. For example, in a computer game, players can work their way along a path. The path branches into two paths. Each new path has a branch point resulting in two new paths.



What is the probability of taking a left path and a right path to finish the race?

There are four possible routes players could take. Two routes have a left and a right. One route has two lefts. One route has two rights. The probability that players took a left and a right at some point in their journey is $\frac{2}{4}$.

Try this again if you add one more branch to the tree.



Thinking Space

How many possibilities include two lefts and one right?

Your tree will include 8 paths now. Using L = left and R = right, the possible paths you have are: LLL, LLR, LRL, LRR, RLL, RLR, RRL, and RRR. Out of the eight possible paths, three paths involve two lefts and one right (LLR, LRL, and RLL). So,

$$P(\text{two lefts and one right}) = \frac{3}{8}$$

You may notice we just included more than two events. How could you use your knowledge of independent events to calculate the probability without drawing out the tree?

Determining probabilities in these cases simply involves multiplying individual probabilities.

$$\begin{aligned}&P(\text{left}) \times p(\text{left}) \times p(\text{right})\\&= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\\&= \frac{1}{8}\end{aligned}$$

You might notice this is not the same answer as the one we found using a tree diagram. Why do you think this is the case?

Notice that in the second case, we listed the choices as left, left, right. Does this give you any ideas about why there is a difference?

The reason is that in the first case, we did not specify an order. Since there are three choices for when we make the right turn (LLR, LRL, or RLL), each with a probability of $\frac{1}{8}$, we can multiply by 3 to find the answer:

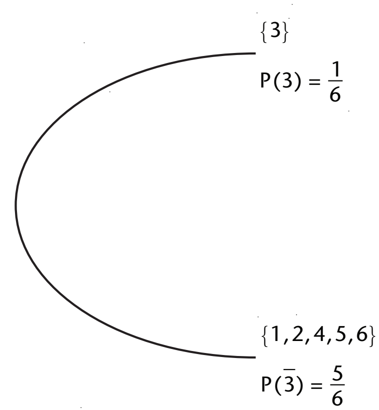
$$P(\text{two lefts and one right}) = 3 \times \frac{1}{8} = \frac{3}{8}$$

Another common type of problem involves the idea of "not" a possibility. For example, when you roll a die, you might want to roll anything but a 6. In other words, you want to find the probability of rolling a 1, 2, 3, 4, or 5, which can also be written as rolling 5 or less. If you use the "not" version, you would simply say the probability of "not a 6".

Thinking Space

This is also useful for simplifying tree diagrams. For example, we can look at rolling a 3 on a six-sided die. You can draw all six options in a tree and find the answer. However, it's far more efficient to make a branched tree with two options—one branch being what you want (a 3), and the other being everything else (not a 3). On each branch, we could simply put the probabilities of each. On the "3" branch, we would list $P(3) = \frac{1}{6}$. On the "not a 3" branch, we would list $P(\text{not a }3) = \frac{5}{6}$

Roll a Die



Now let's look at the pathway problem and the "not" problem together.

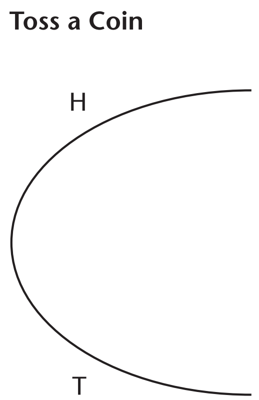
We can look at other multiple events. For example, if you roll a die, toss a coin, and spin a spinner (4 numbered sections), how many possible outcomes are there?

We know coins have two possibilities—heads and tails. A die can be 1, 2, 3, 4, 5, 6. The spinner can give 1, 2, 3, 4. This will make quite an elaborate tree. However, with specific outcomes in mind, this becomes a much simpler branched tree.

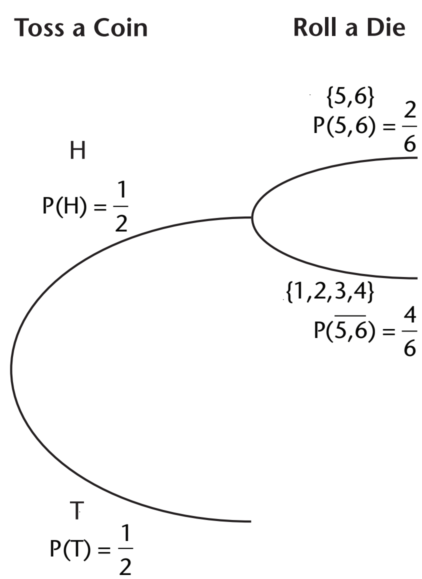
What is the probability of tossing a head, rolling 5 or greater, and spinning a 2?

We can begin our tree with two branches of heads or tails.

Thinking Space

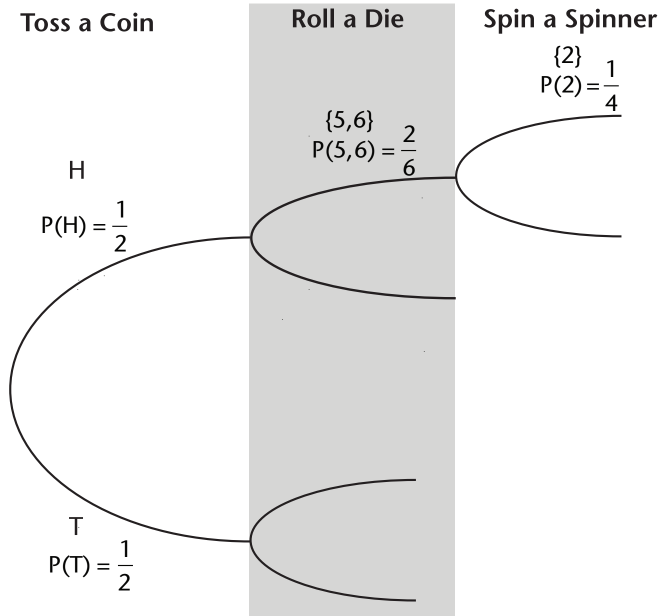


Rather than writing out all the possible die rolls, make the branch from the heads path show two paths. Label one path as {5, 6} and list its probability as $P(5 \text{ or } 6) = \frac{2}{6}$. Label the other path "not 5 or 6" $(\overline{5 \text{ or } 6})$ (which means 1, 2, 3, 4) with a $P(\overline{5 \text{ or } 6}) = \frac{4}{6}$. We could copy this branch onto the tails path as well if we were interested in those outcomes, but for this example we'll just focus on the outcomes that we want.



We can do something similar with the spinner options.

Thinking Space



Using our tree, we can calculate the probability of tossing a head, rolling 5 or greater, and spinning a 2 by multiplying the individual probabilities together.

$$\begin{aligned}
&P(\text{heads}) \times P(5 \text{ or greater}) \times P(2) \\
&= \frac{1}{2} \times \frac{2}{6} \times \frac{1}{4} \\
&= \frac{2}{48} \\
&= \frac{1}{24} \\
&\approx 0.042
\end{aligned}$$

Thinking Space

Try It!
Activity 2

1. You toss a coin and roll a die. What is the probability of tossing heads and not getting a number less than three?

2. You are in a race. On each path you could take, there are three branch points. Each branch point has two directions to choose from—left and right. What is the probability of choosing a right, left, right in that order?

3. Look again at the race from question 2. What is the probability of choosing a route that includes two rights and a left? How is this question different from question 2?

Thinking Space

4. A new game mixes cards from a regular 52-card deck, dice, and coins. What is the probability of drawing a face card, rolling 5 or 6, and tossing tails?

Turn to Solutions at the end of the module and mark your work.

Have fun calculating probabilities for one event or two independent events by playing *Toads and Vines*, found at **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0843b1f_toadvine.html.**

You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Section Summary

Completing this section has helped you to:

- List possible outcomes in random experiments.
- Decide if events are independent or dependent.
- Determine the probability of two independent events, and verify the probability using a different strategy.
- Solve problems involving probability of independent events.

Appendix

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Solutions

Section 1

Pretest

Lesson A: Proper and Improper Fractions and Mixed Numbers

1. a. $\frac{4}{3}$ b. $\frac{22}{4}$ c. $\frac{39}{9}$ d. $\frac{27}{7}$

2. a. $6\frac{2}{6} = 6\frac{1}{3}$ b. $4\frac{3}{12} = 4\frac{1}{4}$ c. $2\frac{4}{16} = 2\frac{1}{4}$ d. $2\frac{1}{6}$ e. $3\frac{4}{8} = 3\frac{1}{2}$

Lesson B: Multiplying Fractions

1. a. $\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$

   b. $\frac{3}{4} \times \frac{4}{5} = \frac{3 \times 4}{4 \times 5} = \frac{12}{20} = \frac{3}{5}$

   c. $\frac{5}{6} \times \frac{6}{7} = \frac{5 \times 6}{6 \times 7} = \frac{30}{42} = \frac{5}{7}$

   d. $\frac{7}{8} \times \frac{8}{9} = \frac{7 \times 8}{8 \times 9} = \frac{56}{72} = \frac{7}{9}$

   e. $\frac{9}{10} \times \frac{10}{11} = \frac{9 \times 10}{10 \times 11} = \frac{90}{110} = \frac{9}{11}$

2. a. $\frac{1}{2} \times 3 = \frac{1}{2} \times \frac{3}{1} = \frac{1 \times 3}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$

   b. $\frac{3}{10} \times 4\frac{1}{6} = \frac{3}{10} \times \frac{25}{6} = \frac{3 \times 25}{10 \times 6} = \frac{75}{60} = \frac{5}{4} = 1\frac{1}{4}$

   c. $\frac{1}{4} \times 6 = \frac{1}{4} \times \frac{6}{1} = \frac{1 \times 6}{4 \times 1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$

   d. $\frac{2}{5} \times 10\frac{1}{3} = \frac{2}{5} \times \frac{31}{3} = \frac{2 \times 31}{5 \times 3} = \frac{62}{15} = 4\frac{2}{15}$

   e. $\frac{3}{8} \times 12 = \frac{3}{8} \times \frac{12}{1} = \frac{3 \times 12}{8 \times 1} = \frac{36}{8} = \frac{9}{2} = 4\frac{1}{2}$

Lesson C: Dividing Fractions

1\. a. $\frac{1}{2} \div \frac{1}{3}$
$= \frac{1}{2} \times \frac{3}{1}$
$= \frac{1 \times 3}{2 \times 1}$
$= \frac{3}{2}$
$= 1\frac{1}{2}$

b. $\frac{3}{4} \div \frac{4}{5}$
$= \frac{3}{4} \times \frac{5}{4}$
$= \frac{3 \times 5}{4 \times 4}$
$= \frac{15}{16}$

c. $\frac{5}{6} \div \frac{6}{7}$
$= \frac{5}{6} \times \frac{7}{6}$
$= \frac{5 \times 7}{6 \times 6}$
$= \frac{35}{36}$

d. $\frac{7}{8} \div \frac{8}{9}$
$= \frac{7}{8} \times \frac{9}{8}$
$= \frac{7 \times 9}{8 \times 8}$
$= \frac{63}{64}$

e. $\frac{9}{10} \div \frac{10}{11}$
$= \frac{9}{10} \times \frac{11}{10}$
$= \frac{9 \times 11}{10 \times 10}$
$= \frac{99}{100}$

2\. a. $\frac{1}{2} \div 3$
$= \frac{1}{2} \div \frac{3}{1}$
$= \frac{1}{2} \times \frac{1}{3}$
$= \frac{1 \times 1}{2 \times 3}$
$= \frac{1}{6}$

b. $\frac{3}{10} \div 1\frac{1}{6}$
$= \frac{3}{10} \div \frac{7}{6}$
$= \frac{3}{10} \times \frac{6}{7}$
$= \frac{3 \times 6}{10 \times 7}$
$= \frac{18}{70}$
$= \frac{9}{35}$

c. $\frac{1}{4} \div 6$
$= \frac{1}{4} \div \frac{6}{1}$
$= \frac{1}{4} \times \frac{1}{6}$
$= \frac{1 \times 1}{4 \times 6}$
$= \frac{1}{24}$

d. $1\frac{2}{5} \div 10$
$= \frac{7}{5} \div \frac{10}{1}$
$= \frac{7}{5} \times \frac{1}{10}$
$= \frac{7 \times 1}{5 \times 10}$
$= \frac{7}{50}$

e. $\frac{3}{8} \div 12$
$= \frac{3}{8} \div \frac{12}{1}$
$= \frac{3}{8} \times \frac{1}{12}$
$= \frac{3 \times 1}{8 \times 12}$
$= \frac{3}{96}$
$= \frac{1}{32}$

Lesson D: Estimating and Solving Problems

1\. a. $\frac{1}{3} \times \frac{1}{2}$

b. $\frac{1}{2} \div 3$ OR $\frac{1}{2} \times \frac{1}{3}$

c. $2 \times 1\frac{3}{4}$

2\. The product of two proper fractions will never be greater than 1. In each fraction, the numerator is smaller than the denominator. Take $\frac{1}{2} \times \frac{3}{4}$ for example. 1 is smaller than 2, and 3 is smaller than 4. The product of the numerators will be smaller than the product of the denominators. In our example, the answer would be $\frac{3}{8}$, which is a proper fraction. (Examples will vary.)

3. There are 54 L of gas in the tank.

4. The reduced price of the jeans is $51.

Lesson A: Proper and Improper Fractions and Mixed Numbers

Warm-up

1. a. 19

   b. 4

   c. 35

   d. 8

   e. 32

   f. 51

2. a. $\frac{2}{3} = \frac{10}{15}$

   b. $\frac{3}{4} = \frac{27}{36}$

3. a. $\frac{6}{4} = \frac{3}{2}$

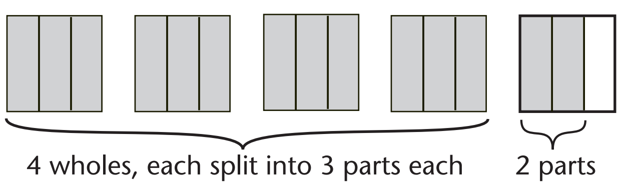
   b. $\frac{20}{8} = \frac{10}{4} = \frac{5}{2}$

Try It! Activity 1

a	6. denominator	a. the 3 in $\frac{2}{3}$
d	7. improper fraction	b. $\frac{4}{7}$
c	8. mixed number	c. $2\frac{1}{5}$
e	9. numerator	d. $\frac{9}{4}$
b	10. proper fraction	e. the 2 in $\frac{2}{5}$

Try It! Activity 2

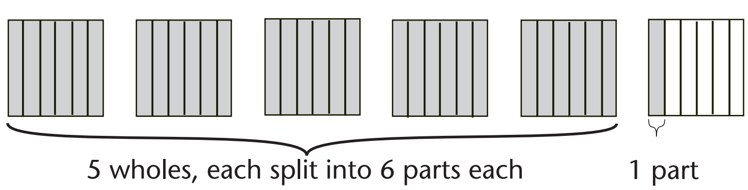
1. a.



$$4 \times 3 + 2 = 14 \text{ parts}$$

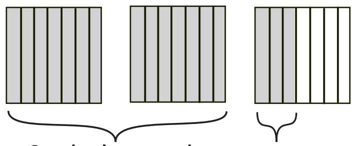
$$4\frac{2}{3} = \frac{14}{3}$$

b.



$$5 \times 6 + 1 = 31 \text{ parts}$$

$$5\frac{1}{6} = \frac{31}{6}$$

c. 

$$2 \times 7 + 1 = 15 \text{ parts}$$

$$2\frac{3}{7} = \frac{17}{7}$$

2. a. $8\frac{1}{5}$

$$8\frac{1}{5} = \frac{41}{5}$$

b. $6\frac{2}{9}$

$$6\frac{2}{9} = \frac{56}{9}$$

c. $3\frac{1}{4}$

$$3\frac{1}{4} = \frac{13}{4}$$

Try It! Activity 3

1. $\begin{array}{r} 6 \\ 8\overline{)53} \end{array}$ *remainder* 5 $\quad \frac{53}{8} = 6\frac{5}{8}$
2. $\begin{array}{r} 7 \\ 5\overline{)35} \end{array}$ *remainder* 0 $\quad \frac{35}{5} = 7$
3. $\begin{array}{r} 3 \\ 12\overline{)42} \end{array}$ *remainder* 6 $\quad \frac{42}{12} = 3\frac{6}{12} = 3\frac{1}{2}$
4. $\begin{array}{r} 4 \\ 13\overline{)54} \end{array}$ *remainder* 1 $\quad \frac{54}{13} = 4\frac{1}{13}$
5. $\begin{array}{r} 3 \\ 6\overline{)20} \end{array}$ *remainder* 2 $\quad \frac{20}{6} = 3\frac{2}{6} = 3\frac{1}{3}$

Lesson B: Multiplying Fractions

Warm-up

1. a. $\frac{3}{5}$ b. $\frac{9}{10}$ c. $\frac{4}{9}$

2. a. $\frac{7}{1}$ b. $\frac{11}{5}$ c. $\frac{63}{1}$

3. a. $3\frac{2}{3}$ b. $6\frac{3}{8}$ c. $9\frac{1}{12}$

4. a. 12
   b. 20
   c. 42
   d. 72
   e. 33
   f. 84
   g. 18

Try It! Activity 1

1. $\frac{2}{3} \times \frac{4}{9} = \frac{2 \times 4}{3 \times 9} = \frac{8}{27}$

2. $\frac{1}{5} \times \frac{3}{4} = \frac{1 \times 3}{5 \times 4} = \frac{3}{20}$

3. $\frac{4}{5} \times \frac{6}{13} = \frac{4 \times 6}{5 \times 13} = \frac{24}{65}$

4. $\frac{2}{7} \times \frac{1}{5} = \frac{2 \times 1}{7 \times 5} = \frac{2}{35}$

5. $\frac{8}{3} \times \frac{5}{7} = \frac{8 \times 5}{3 \times 7} = \frac{40}{21} = 1\frac{19}{21}$

Try It! Activity 2

1. $\frac{3}{7} \times 3 = \frac{3}{7} \times \frac{3}{1} = \frac{3 \times 3}{7 \times 1} = \frac{9}{7} = 1\frac{2}{7}$

2. $\frac{4}{5} \times 3\frac{3}{4} = \frac{4}{5} \times \frac{15}{4} = \frac{4 \times 15}{5 \times 4} = \frac{60}{20} = 3$

3. $\frac{9}{10} \times 8 = \frac{9}{10} \times \frac{8}{1} = \frac{9 \times 8}{10 \times 1} = \frac{72}{10} = 7\frac{2}{10} = 7\frac{1}{5}$

4. $\frac{8}{5} \times 7 = \frac{8}{5} \times \frac{7}{1} = \frac{8 \times 7}{5 \times 1} = \frac{56}{5} = 11\frac{1}{5}$

5. $2\frac{1}{4} \times \frac{6}{11} = \frac{9}{4} \times \frac{6}{11} = \frac{9 \times 6}{4 \times 11} = \frac{54}{44} = \frac{27}{22} = 1\frac{5}{22}$

6. $2\frac{1}{2} \times 1\frac{1}{9} = \frac{5}{2} \times \frac{10}{9} = \frac{5 \times 10}{2 \times 9} = \frac{50}{18} = \frac{25}{9} = 2\frac{7}{9}$

7. $\frac{4}{5} \times 6 = \frac{4}{5} \times \frac{6}{1} = \frac{4 \times 6}{5 \times 1} = \frac{24}{5} = 4\frac{4}{5}$

Lesson C: Dividing Fractions

Warm-up

1. a. 13
   b. 4
   c. 7
   d. 9
   e. 9

2. a. 6
   b. 5
   c. 9

3. $\frac{12}{5}$ or $2\frac{2}{5}$

Try It! Activity 1

1. a. $\frac{3}{2}$

   b. $\frac{4}{1}$ *or* 4

   c. $\frac{1}{7}$

   d. $\frac{5}{14}$ Convert the mixed number to an improper fraction before finding the reciprocal.

2. a. $\frac{7}{5} \div 3 = \frac{7}{5} \div \frac{3}{1} = \frac{7}{5} \times \frac{1}{3} = \frac{7 \times 1}{5 \times 3} = \frac{7}{15}$

   b. $\frac{5}{6} \div 4 = \frac{5}{6} \div \frac{4}{1} = \frac{5}{6} \times \frac{1}{4} = \frac{5 \times 1}{6 \times 4} = \frac{5}{24}$

   c. $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{1 \times 3}{2 \times 2} = \frac{3}{4}$

   d. $\frac{1}{5} \div 9 = \frac{1}{5} \div \frac{9}{1} = \frac{1}{5} \times \frac{1}{9} = \frac{1 \times 1}{5 \times 9} = \frac{1}{45}$

   e. $5\frac{1}{5} \div \frac{1}{5} = \frac{26}{5} \div \frac{1}{5} = \frac{26}{5} \times \frac{5}{1} = \frac{26 \times \cancel{5}^{1}}{\cancel{5}_{1} \times 1} = 26$

   f. $\frac{1}{4} \div 2 = \frac{1}{4} \div \frac{2}{1} = \frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 1} = \frac{1}{8}$

   g. $\frac{1}{5} \div \frac{1}{4} = \frac{1}{5} \times \frac{4}{1} = \frac{\cancel{1} \times 4}{5 \times \cancel{1}} = \frac{4}{5}$

   h. $\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{\cancel{2} \times 3}{5 \times \cancel{2}} = \frac{3}{5}$

   i. $\frac{11}{9} \div \frac{1}{5} = \frac{11}{9} \times \frac{5}{1} = \frac{11 \times 5}{9 \times 1} = \frac{55}{9} = 6\frac{1}{9}$

3. The student probably flipped the wrong fraction. The student's work probably looked like this:

   $$\frac{12}{35} \div \frac{4}{5} = \frac{35}{12} \times \frac{4}{5} = \frac{7}{3}$$

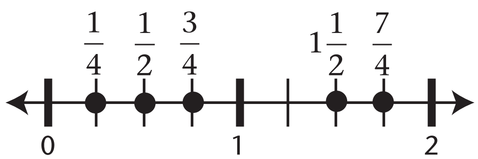
   To solve the problem properly, the work should look like this:

   $$\frac{12}{35} \div \frac{4}{5} = \frac{12}{35} \times \frac{5}{4} = \frac{3}{7}$$

Lesson D: Estimating and Solving Problems

Warm-up

1.



2. a. $\frac{2}{3} \boxed{>} \frac{1}{3}$

   b. $1\frac{1}{2} \boxed{=} \frac{3}{2}$

   c. $2 \boxed{>} \frac{6}{5}$

   d. $\frac{2}{3} \boxed{<} \frac{3}{4}$

   e. $\frac{9}{3} \boxed{=} 3$

3. a. addition

   b. division

   c. multiplication

   d. subtraction

Try It! Activity 1

	Questions:	Expressions:
c, f	1. $\frac{4}{5}$ of a pizza needs to be divided into 7 pieces.	a. $\frac{1}{2} \div 5$
b	2. $\frac{2}{3}$ of seats in an arena were filled. $\frac{2}{5}$ of the seats were filled with children. What portion of the total arena seats are filled with children?	b. $\frac{2}{3} \times \frac{2}{5}$
a, g	3. Half of a fish is cut up to share with five people. How much does each person get?	c. $\frac{4}{5} \div 7$
d	4. Seven friends get together to pick fruit for a big party. Each person picks four fifths of a box of fruit. How much fruit do they pick?	d. $\frac{4}{5} \times 7$
e	5. A box of beads is half full. Each person needs $\frac{1}{5}$ of a box. How many people can get what they need from this box?	e. $\frac{1}{2} \div \frac{1}{5}$
b	6. Two brothers pool their allowance and buy $\frac{2}{5}$ of a box of booster card packs for a collectable card game. One brother paid $\frac{2}{3}$ of the price and gets $\frac{2}{3}$ of the cards. What portion of booster packs does he get?	f. $\frac{4}{5} \times \frac{1}{7}$
		g. $\frac{1}{2} \times \frac{1}{5}$
		h. $\frac{2}{3} \div \frac{2}{5}$

Try It! Activity 2

1. (Answers will vary. Examples are given below.)

   a. $\frac{4}{3} \times \frac{2}{1} = \frac{8}{3} > 1$

   b. $\frac{3}{4} \div \frac{12}{5} = \frac{5}{16} < 1$

   c. No example exists. The product of two proper fractions is always less than 1.

2. a. $\frac{3}{4} \times \frac{8}{5} \boxed{>} 1$

   b. $\frac{13}{8} \times \frac{3}{2} \boxed{>} 1$

   c. $\frac{3}{4} \times \frac{5}{8} \boxed{<} 1$

   d. $2\frac{1}{5} \div 4\frac{2}{5} \boxed{<} 1$

3. Your estimation might be a bit different from the solutions given below. There is more than one way to approximate an answer.

   a. $\frac{3}{5} \approx \frac{1}{2}$
   $\frac{4}{7} \approx \frac{1}{2}$
   $\frac{3}{5} \times \frac{4}{7} \approx \frac{1}{2} \times \frac{1}{2} \approx \frac{1}{4}$

   b. $\frac{5}{12} \approx \frac{1}{2}$
   $\frac{4}{5} \approx 1$
   $\frac{5}{12} \div \frac{4}{5} \approx \frac{1}{2} \div 1 \approx \frac{1}{2}$

   c. $3\frac{5}{6} \approx 4$
   $2\frac{1}{5} \approx 2$
   $3\frac{5}{6} \div 2\frac{1}{5} \approx 4 \div 2 \approx 2$

4. The student's answer is not reasonable. When you multiply two improper fractions, you should get an answer that is greater than 1. The student's answer is less than 1.

Try It! Activity 3

1. $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ One eighth of the population of BC lives in the City of Vancouver.

2. $1\frac{1}{4} \div 5$

   $= \frac{5}{4} \div \frac{5}{1}$

   $= \frac{\cancel{5}^{1} \times 1}{4 \times \cancel{5}_{1}}$

   $= \frac{1}{4}$ There is a quarter cup of sugar in each serving of dessert.

3. Use the formula $F = \frac{9}{5}C + 32$

   a. $F = \frac{9}{5}C + 32$

   $= \frac{9}{5}(30) + 32$

   $= \frac{9 \times \cancel{30}^{6}}{\cancel{5}_{1}} + 32$

   $= 54 + 32$

   $= 86°F$

   b. $F = \frac{9}{5}C + 32$

   $= \frac{9}{5}(10) + 32$

   $= \frac{9 \times \cancel{10}^{2}}{\cancel{5}_{1}} + 32$

   $= 18 + 32$

   $= 50°F$

4. Use the formula $C = (F - 32) \div \frac{9}{5}$

   a. $C = (F - 32) \div \frac{9}{5}$

   $= (59 - 32) \div \frac{9}{5}$

   $= 27 \div \frac{9}{5}$

   $= \cancel{27}^{3} \times \frac{5}{\cancel{9}_{1}}$

   $= 15°C$

   b. $C = (F - 32) \div \frac{9}{5}$

   $= (65 - 32) \div \frac{9}{5}$

   $= 33 \div \frac{9}{5}$

   $= 33 \times \frac{5}{9}$

   $= 18\frac{1}{3}°C$

5. a. $\$48 \times \frac{2}{3}$ *or* $\$48 - (48 \times \frac{1}{3})$

   b. $48 \approx 50$

   $\frac{2}{3}$ is a bit more than $\frac{1}{2}$

   $50 \times \frac{1}{2} = 25$

   The reduced price of the hoodie is a bit more than $25.

   c. $\$48 \times \frac{2}{3} = \$32$

   d. Answers will vary.

Section 2

Pretest

1. a. 10:30 = 1:3 (video : activity)

   b. 16:8 = 2:1 (awake : sleeping)

   c. 3:5:2 (TV : soccer : studying)

   d. 6:2 = 3:1 (water : milk)

2. Answers may vary. Sample responses are given below.

   a. 10:40 = 1:4 (video : total minutes in class)

   b. 16:24 = 2:3 (sleeping : total hours in a day)

   c. 3:10 (TV : total time spent on Saturday activities)

   d. 6:8 = 3:4 (water : total # of glasses of liquid)

3. a. 70m/7s = 10m/s

   b. $77.50/5hr = $15.50/hr

   c. 512km/72L = 7.11 km/L

4. a. $4.50/12 eggs = $0.38/egg

   b. $\dfrac{\$0.38}{1\text{ egg}} = \dfrac{x}{5\text{ egg}}$

   $x = \$1.88$ Five eggs would cost $1.88.

5. 1 part mango : 2 parts apple : 3 parts orange juice makes a total of 6 parts.

   Let x be the amount of apple juice in litres.

   $$\frac{2\text{ parts apple}}{6\text{ parts total}} = \frac{x}{18\text{ L}}$$

   $$\frac{2}{6} = \frac{6}{18}$$

   $$x = 6$$

   There are 6 L of apple juice in the mixture.

6.

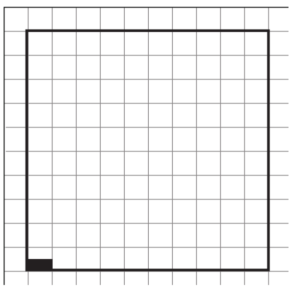
$$\frac{10}{300} = \frac{x}{4500}$$
$$(10)(4500) = (300)(x)$$
$$\frac{(10)(4500)}{300} = \frac{(300)(x)}{300}$$
$$150 = x$$

There will be 150 defective bulbs in a lot of 4500.

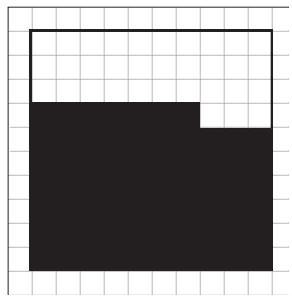
Lesson B: Making Sense of Percents

1.

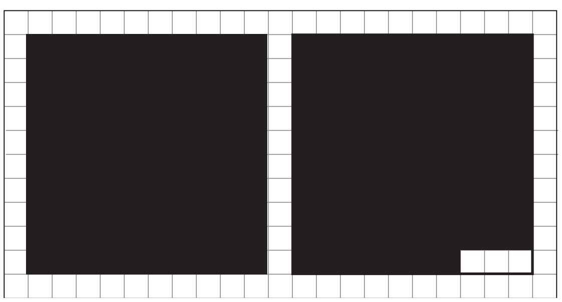
a. 0.5%



b. 67%



c. 197%



2.

Part-to-Whole Ratio	Fraction	Decimal	Percent
1:2	$\frac{1}{2}$	0.5	50%
1:500	$\mathbf{\frac{1}{500}}$	**0.002**	**0.2%**
27:50	$\frac{27}{50}$	**0.54**	**54%**
7:4	$\mathbf{1\frac{3}{4}}$ **or** $\mathbf{\frac{7}{4}}$	1.75	**175%**
3:40 000	$\mathbf{\frac{3}{40\,000}}$	**0.000075**	0.0075%
137:200	$\frac{137}{200}$	**0.685**	**68.5%**
1403:400	$\mathbf{3\frac{203}{400}}$ **or** $\mathbf{\frac{1403}{400}}$	**3.5075**	**350.75%**

Lesson C: Solving Problems with Percents

1. 40% of \$55.00
   = 0.40 × \$55.00
   = \$22.00

   The store will pay \$22.00 to buy back the game.

2. GST = 5% of \$550.00 = 0.05 × \$550.00 = \$27.50

   PST = 7% of \$550.00 = 0.07 × \$550.00 = \$38.50

   total amount of taxes = \$38.50 + \$27.50 = \$66.00

   total cost = \$550.00 + \$66.00 = \$616.00

   The total cost of the bike is \$616.00.

3. Solution: <u>Find the price before taxes (retail price):</u>

   Work backwards. The cost of an item including taxes can be found by multiplying the retail price by 112% (this is because you pay 100% of the retail price, plus 7% PST and 5% GST).

   retail price × 112% = \$95.20

retail price × 1.12 = \$95.20

$$\frac{\text{retail price} \times 1.12}{1.12} = \frac{\$120.00}{1.12}$$

retail price = \$85.00

<u>Find the profit:</u>

profit = retail price – wholesale price

profit = \$85.00 – \$62.00

profit = \$23.00

The store makes a \$23.00 profit on the item.

4. discount = original price – sale price

   discount = \$240.00 – \$192.00

   discount = \$48.00

   <u>percent discount:</u>

$$\frac{\$48.00}{\$240.00} = \frac{x}{100}$$
$$(48)(100) = (240)(x)$$
$$\frac{(48)(100)}{240} = \frac{(240)(x)}{240}$$
$$20 = x$$

The price of the DVD player was reduced by 20%.

5. increase in cost = \$2000 – \$1800

   increase in cost = \$200

   percent increase:

$$\frac{\$200}{\$1800} = \frac{x}{100}$$
$$(200)(100) = (1800)(x)$$
$$\frac{(200)(100)}{1800} = \frac{(1800)(x)}{1800}$$
$$11.\overline{1} = x$$

So,

$$\frac{\$200}{\$1800} = \frac{11.\overline{1}}{100}$$

Property taxes increased by 11.1%.

Lesson A: Ratios and Rates in Every Day Life

Warm-up

There is an infinite number of possible answers.

1. $\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$ *or* $\frac{3}{9}$ *or* $\frac{4}{12}$
2. $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$ *or* $\frac{6}{15}$ *or* $\frac{8}{20}$
3. $\frac{11}{15} = \frac{11 \times 3}{15 \times 3} = \frac{33}{45}$ *or* $\frac{22}{30}$ *or* $\frac{44}{60}$
4. $\frac{4}{16} = \frac{4 \div 4}{16 \div 4} = \frac{1}{4}$ *or* $\frac{2}{8}$ *or* $\frac{8}{32}$
5. $\frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16}$ *or* $\frac{9}{24}$ *or* $\frac{12}{32}$

Try It! Activity 1

1. a. number of black counters to the number of grey counters

   b. number of black counters to the number of white counters

   c. number of grey counters to the total number of counters

   d. number of black counters to the number of white counters to the number of grey counters

   e. number of white counters to the total number of counters (6:16 can also be written as 3:8, an equivalent ratio)

2. a. part-to-part ratio

   b. part-to-part ratio

   c. part-to-whole ratio

   d. part-to-part ratio

   e. part-to-whole ratio

3. a. 12:15 (can also be written 4:5)

   b. 1:2

   c. 3:4

   d. 1:3:4

4. Answers may vary depending on which part you chose to compare.
   a. number of boys to total number of students 12:27
      number of girls to total number of students 15:27
   b. cups of water to total cups of ingredients 1:3
      cups of pancake mix to total cups of ingredients 2:3
   c. days of sunshine to days in the week 3:7
      days of rain to days in the week 4:7
   d. number of pants to total number of clothing articles 1:8
      number of shorts to total number of clothing articles 3:8
      number of T-shirts to total number of clothing articles 4:8

5. a. The ratios are proportional. Multiply both terms in 2:4 by 3 to get 6:12.
   b. The ratios are not proportional. There is no factor that you can multiply or divide either of the ratios by to get the other ratio.
   c. The ratios are proportional. Divide both terms in 16:30 by 2 to get 8:15.

Try It! Activity 2

1. a. 110 km/2 h
   b. $11.19/3 kg
   c. 30 beats/10 s

2. Answers will vary. Sample answers are given below.
   a. Sean drove 400 kilometres and used 28 litres of fuel.
   b. Cindy drove at a speed of 70 kilometres per hour.
   c. Tahlia earned $72 for 5 hours of work.

3. a. $\frac{110\text{ km}}{2\text{ h}} = \frac{?}{1\text{ h}}$

   $\frac{110\text{ km}}{2\text{ h}} = \frac{55\text{ km}}{1\text{ h}}$ The unit rate is 55 km/h.

b. $\dfrac{\$11.19}{3\text{ kg}} = \dfrac{?}{1\text{ kg}}$

$\dfrac{\$11.19}{3\text{ kg}} = \dfrac{\$3.73}{1\text{ kg}}$ The unit rate is \$3.73/kg.

c. $\dfrac{30\text{ beats}}{10\text{ s}} = \dfrac{?}{1\text{ s}}$

$\dfrac{30\text{ beats}}{10\text{ s}} = \dfrac{3\text{ beats}}{1\text{ s}}$ The unit rate is 3 beats/s.

4. 4-pack

$$\begin{aligned}\text{unit price} &= \frac{\text{cost}}{\text{quantity}}\\ &= \frac{\$6.68}{4\text{ batteries}}\\ &= \$1.67/\text{battery}\end{aligned}$$

10-pack

$$\begin{aligned}\text{unit price} &= \frac{\text{cost}}{\text{quantity}}\\ &= \frac{\$13.90}{10\text{ batteries}}\\ &= \$1.39/\text{battery}\end{aligned}$$

The price per battery is lower if you buy the 10-pack.

Try It! Activity 3

1. First convert 30 minutes to 0.5 hours.

$$\begin{aligned}\text{rate of speed} &= \frac{\text{distance traveled}}{\text{time}}\\ &= \frac{8.5\text{ km}}{0.5\text{ h}}\\ &= 17\text{ km/h}\end{aligned}$$

Cassie rides her bike at 17 km/h.

2.

$$\frac{1 \text{ can}}{9 \text{ m}^2} \overset{\times 3}{=} \frac{x}{27 \text{ m}^2} \qquad OR \qquad \frac{9 \text{ m}^2}{1 \text{ can}} = \frac{27 \text{ m}^2}{x}$$

$$\frac{1 \text{ can}}{9 \text{ m}^2} = \frac{3 \text{ cans}}{27 \text{ m}^2}$$

It will take 3 cans of paint to paint the room.

3. a. Hardware store job:

$$\begin{aligned} \text{hourly wage} &= \frac{\text{amount paid}}{\text{hours worked}} \\ &= \frac{\$440}{40 \text{ h}} \\ &= \$11/\text{h} \end{aligned}$$

Beverly would earn \$11/h working at the hardware store.

Library job:

$$\begin{aligned} \text{hourly wage} &= \frac{\text{amount paid}}{\text{hours worked}} \\ &= \frac{\$350}{25 \text{ h}} \\ &= \$14/\text{h} \end{aligned}$$

Beverly would earn \$14/h working at the library.

b. It depends what Beverly wants. If she needs money, she should work the first one because she'll make more per week. If she only wants a part time job, the library pays a better rate.

4. Set up a proportion:

$$\frac{\$11.47}{194 \text{ kWh}} = \frac{x}{230 \text{ kWh}}$$

This solution shows the cross-product method.

$$\begin{aligned} (\$11.47)(230 \text{ kWh}) &= (194 \text{ kWh})(x) \\ \frac{(\$11.47)(230 \text{ kWh})}{194 \text{ kWh}} &= \frac{(194 \text{ kWh})(x)}{194 \text{ kWh}} \\ \$13.60 &= x \end{aligned}$$

Marcel's bill will be \$13.60 next month.

Lesson B: Making Sense of Percents

Warm-up

1. a. 70%

   b. 60%

   c. 75%

2. a. 0.85

   b. 0.27

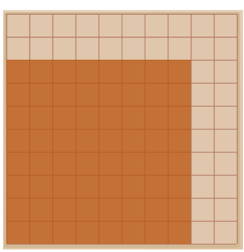
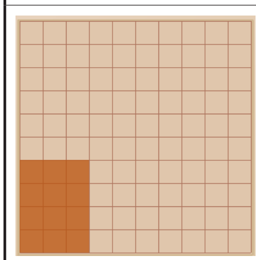
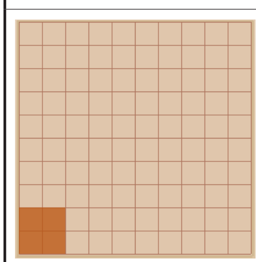
   c. 0.03

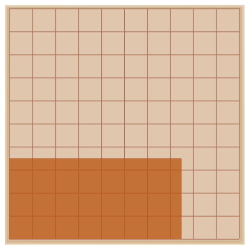
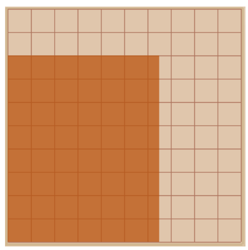
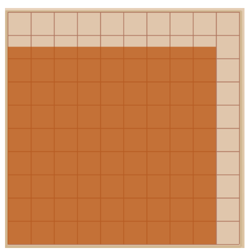
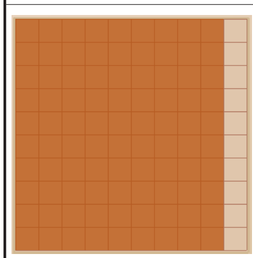
3. a. $\frac{1}{2}$

   b. $\frac{3}{5}$

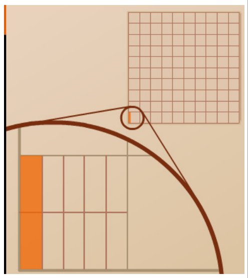
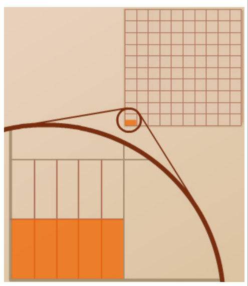
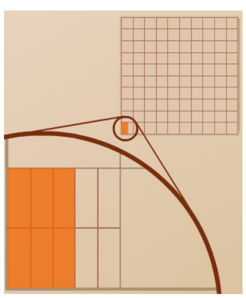
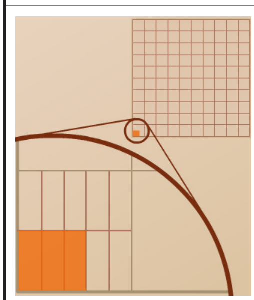
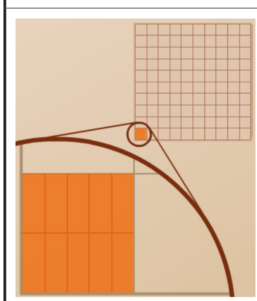
   c. $\frac{43}{100}$

Try It! Activity 1

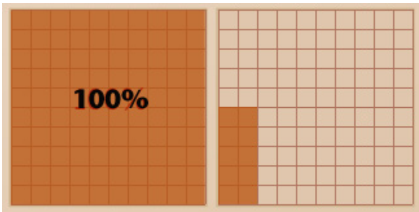
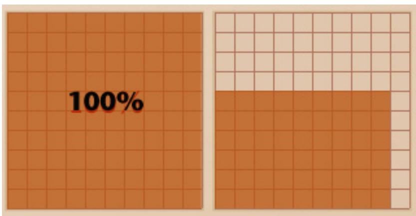
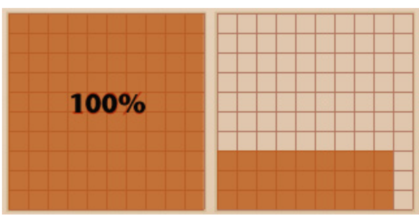
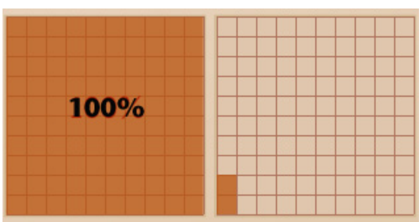
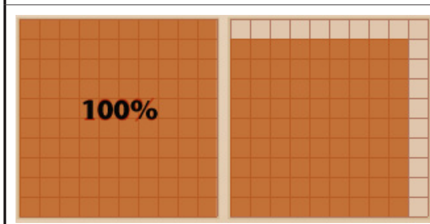
Picture	Percent	Fraction	Ratio	Decimal
	64%	$\frac{64}{100}$	64:100	0.64
	12%	$\frac{12}{100}$	12:100	0.12
	4%	$\frac{4}{100}$	4:100	0.04

	26.25%	$\frac{26.25}{100}$	26.25:100	0.2625
	52%	$\frac{52}{100}$	52:100	0.52
	76.5%	$\frac{76.5}{100}$	76.5:100	0.765
	90%	$\frac{90}{100}$	90:100	0.9

Try It! Activity 2

Picture	Percent	Fraction	Ratio	Decimal
	0.2%	$\frac{2}{1000}$	2:1000	0.002
	0.5%	$\frac{5}{1000}$	5:1000	0.005
	0.6%	$\frac{6}{1000}$	6:1000	0.006
	0.3%	$\frac{3}{1000}$	3:1000	0.003
	1%	$\frac{10}{1000} = \frac{1}{100}$	1:100	0.01

Try It! Activity 3

Picture	Percent	Fraction	Ratio	Decimal
	110%	$\frac{110}{1000}=1\frac{10}{100}$	110:100	1.1
	154%	$\frac{154}{1000}=1\frac{54}{100}$	154:100	1.54
	127%	$\frac{127}{1000}=1\frac{27}{100}$	127:100	1.27
	102%	$\frac{102}{1000}=1\frac{2}{100}$	102:100	1.02
	181%	$\frac{181}{1000}=1\frac{81}{100}$	181:100	1.81

Try It! Activity 4

1. a. $\frac{171}{300}=\frac{57}{100}=57\%$

   b. $\frac{41}{20}=\frac{205}{100}=205\%$

c. You can use the cross-product method to solve this problem.

$$\frac{3}{125} = \frac{x}{100}$$
$$(3)(100) = (125)(x)$$
$$\frac{(3)(100)}{125} = \frac{(125)(x)}{125}$$
$$2.4 = x$$

So, $\frac{3}{125} = 2.4\%$

d. You can use the cross product method to solve this problem.

$$1\frac{7}{15} = \frac{x}{100}$$
$$\frac{22}{15} = \frac{x}{100}$$
$$(22)(100) = (15)(x)$$
$$\frac{(22)(100)}{15} = \frac{(15)(x)}{15}$$
$$146.\overline{6} = x$$

So, $1\frac{7}{15} = 146.\overline{6}\% = 146.7\%$

2. a. 14%
   b. 0.5%
   c. 10%
   d. 123%

3.

	Decimal	**Fraction**
a. 0.07%	0.0007	$\frac{7}{10\,000}$
b. $23\frac{1}{2}\%$	23.5% = 0.235	$\frac{235}{1000} = \frac{47}{200}$
c. 325%	3.25	$3\frac{25}{100} = 3\frac{1}{4}$

4. a. $7:4 = \frac{7}{4}$

$\times 25$

$$\frac{7}{4} = \frac{x}{100}$$

$\times 25$

$$\frac{7}{4} = \frac{175}{100} = 175\%$$

b. $1:500 = \frac{1}{500}$

$\div 5$

$$\frac{1}{500} = \frac{x}{100}$$

$\div 5$

$$\frac{1}{500} = \frac{0.2}{100} = 0.2\%$$

5. Rates cannot be represented as percents. A rate is a ratio that compares two quantities that are measured in different units. A rate is not a part-to-whole ratio. A percent compares similar things, and it represents a part-to-whole relationship.

Lesson C: Solving Problems with Percents

Warm-up

1. $10 \times 20\% = 10 \times 0.20 = 2$

2. $110 \times 15\% = 110 \times 0.15 = 16.5$

3. $70\% = \frac{70}{100} = 0.70$

4. $7\% = \frac{7}{100} = 0.07$

5. $\$10,000.00 \times 0.05\% = \$10,000.00 \times 0.0005 = \$5.00$

Try It! Activity 1

1. a. $\$120.00 \times 5\%$

   $= \$120.00 \times 0.05$

   $= \$6.00$

   b. $\$120.00 \times 7\%$

   $= \$120.00 \times 0.07$

   $= \$8.40$

2. GST = 5% of \$70.00 = 0.05 × \$70.00 = \$3.50

   PST = 7% of \$70.00 = 0.07 × \$70.00 = \$ 4.90

   The total amount of taxes = \$3.50 + \$4.90 = \$8.40

   The check out price is: \$70.00 + \$8.40 = \$ 78.40

3. original price × 5% = \$3.40

   original price × 0.05 = \$3.40

   $$\frac{\text{original price} \times 0.05}{0.05} = \frac{\$3.40}{0.05}$$

   original price = \$68.00

4. original price × 7% = \$10.00

   original price × 0.07 = \$10.00

   $$\frac{\text{original price} \times 0.07}{0.07} = \frac{\$10.00}{0.07}$$

   original price = \$142.86

5. original price × 12% = \$9.60

   original price × 0.12 = \$9.60

   $$\frac{\text{original price} \times 0.12}{0.12} = \frac{\$9.60}{0.12}$$

   original price = \$80.00

Try It! Activity 2

1. Start by finding the price before taxes (retail):

   Work backwards. The cost of an item including taxes can be found by multiplying the retail price by 112% (this is because you pay 100% of the retail price, plus 7% PST and 5% GST).

   retail price × 112% = $120.00

   retail price × 1.12 = $120.00

   $$\frac{\text{retail price} \times 1.12}{1.12} = \frac{\$120.00}{1.12}$$

   retail price = $107.14

   Find the profit:

   profit = retail price – wholesale price

   profit = $107.14 – $90.00

   profit = $17.14

   The vendor makes a $17.14 profit on the jeans.

2. sale price = original price – discount

   $$\begin{aligned}\$300 &= x - (20\% \text{ of } x)\\ \$300 &= x - 0.20x\\ \$300 &= 0.80x\\ \frac{\$300}{0.80} &= \frac{0.80x}{0.80}\\ \$375 &= x\end{aligned}$$

3. price after original 10% discount = $80 – 10% of $80

   = $80 – 0.10($80)

   = $80 – $8

   = $72

   price after 15% discount = $72 – 15% of $72

   = $72 – 0.15($72)

   = $72 – $10.80

   = $61.20

4. price after 30% discount = \$160 – 30% of \$160

= \$160 – 0.30(\$160)

= \$160 – \$48

= \$112

PST (7%) = \$112 × 0.07 = \$7.84

GST (5%) = \$112 × 0.05 = \$5.60

The total cost = \$112 + \$7.84 + \$5.60 = \$125.44

5. final course grade = class portion + provincial exam portion

= 80% from Class grade + 20% from Prov. Exam Grade

= 0.80 (70%) + 0.20 (73%)

= 56% + 14.6%

= 70.6% overall

6. a. $\frac{2}{3}$ of 2.5%

$$\frac{2}{3} \times \frac{2.5}{100} = \frac{5}{300}$$

Convert to a percent:

$$\frac{5}{300} = 0.0166\overline{6} = 1.67\%$$

b. 2.5% – 1.67% = 0.83%

About 0.83% of the world's water is available to be used for drinking water.

Section 3

Pretest

Lesson A: The Probability of Independent Events

1. a. dependent as there is evidence cavities occur more if you eat candy than not

   b. independent—they do not influence each other

   c. dependent—if the first card is not returned to the deck, the possible options for the second card change.

   d. independent—they do not influence each other

2. {(H,1), (H,2), (H,3), (H,4), (T,1), (T,2), (T,3), (T,4)}

Lesson B: Problem Solving With Probability

3. a. $P = \dfrac{\text{number of odds}}{\text{total number of possibilities}} = \dfrac{3}{6} = \dfrac{1}{2}$

   b. $P = \dfrac{\text{number less than 5}}{\text{total number of possibilities}} = \dfrac{4}{6} = \dfrac{2}{3}$

   c. $P = \dfrac{\text{number of at least 2}}{\text{total number of possibilities}} = \dfrac{5}{6}$

4. a. $P = \dfrac{\text{number of combos with the sum of 3}}{\text{total number of sums}} = \dfrac{2}{36} = \dfrac{1}{18}$

   $\{(1,2),(2,1)\}$

   b. $P = \dfrac{\text{number of combos with the sum of at most 3}}{\text{total number of sums}} = \dfrac{3}{36} = \dfrac{1}{12}$

   $\{(1,1),(1,2),(2,1)\}$

   c. $P = 1 - P(\text{sum of at most 3}) \quad P = 1 - \dfrac{3}{36}(\text{from question 4b})$

   $P = \dfrac{36}{36} - \dfrac{3}{36} = \dfrac{33}{36} = \dfrac{11}{12}$

   d. $P = \dfrac{\text{number of combos with the sum that is a multiple of 6}}{\text{total number of sums}} = \dfrac{6}{36} = \dfrac{1}{6}$

   (a value of 6 $\{(1,5),(5,1),(2,4),(4,2),(3,3)\}$ *or* a value of 12 $\{(6,6)\}$)

   e. $P = \dfrac{\text{number of combos with the sum more than 10}}{\text{total number of sums}} = \dfrac{3}{36} = \dfrac{1}{12}$

   (a value of 11 $\{(5,6),(6,5)\}$ *or* a value of 12 $\{(6,6)\}$)

5. a. $P = \dfrac{\text{number of red cards}}{\text{total number of cards}} = \dfrac{26}{52} = \dfrac{1}{2}$

(there are 13 hearts and 13 diamonds)

b. $P = \dfrac{\text{number of aces}}{\text{total number of cards}} = \dfrac{4}{52} = \dfrac{1}{13}$

(there are 4 aces, one in each suit)

c. $P = \dfrac{\text{number of face cards}}{\text{total number of cards}} = \dfrac{12}{52} = \dfrac{1}{13}$

(there are 4 suits, each with $\text{JQK} = 12$)

d. $P = \dfrac{\text{number of Kings and 4 Queens}}{\text{total number of cards}} = \dfrac{8}{52} = \dfrac{2}{13}$

(there are 4 Kings and 4 Queens)

6. a. $P = \dfrac{\text{number of ways to get a 6}}{\text{total of possibile outcomess}} = \dfrac{1}{6}$

b. $P = \underbrace{\dfrac{\text{number of ways to NOT to get a 6}}{\text{total of possibile outcomes}}}_{\text{First Toss}} \times \underbrace{\dfrac{\text{number of ways to get a 6}}{\text{total of possibile outcomes}}}_{\text{Second Toss}}$

$= \dfrac{5}{6} \times \dfrac{1}{6} = \dfrac{5}{36}$

7. a. $P = \dfrac{\text{number of vowels}}{\text{number of letters}} \times \dfrac{\text{number of ways to get at least 1 head}}{\text{number of possible tosses}}$

$= \dfrac{5}{26} \times \dfrac{3}{4} = \dfrac{15}{26}$

b. $P = \dfrac{\text{number of consanants}}{\text{number of letters}} \times \dfrac{\text{number of ways to get 2 tails}}{\text{number of possible tosses}}$

$= \dfrac{21}{26} \times \dfrac{1}{4} = \dfrac{21}{104}$

c. $P = \dfrac{\text{number of '}y\text{'}}{\text{number of letters}} \times \dfrac{\text{number of ways to get exactly 1 tail}}{\text{number of possible tosses}}$

$= \dfrac{1}{26} \times \dfrac{2}{4} = \dfrac{1}{52}$

d. $P = \dfrac{\text{number of '}w\text{'}}{\text{number of letters}} \times \dfrac{\text{number of ways to get 2 heads}}{\text{number of possible tosses}}$

$= \dfrac{1}{26} \times \dfrac{1}{4} = \dfrac{1}{104}$

8. a. $P = \frac{\text{number of hearts}}{\text{total number of cards}} \times \frac{\text{number of clubs}}{\text{total number of cards}}$

$= \frac{13}{52} \times \frac{13}{52} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

(there are 13 hearts and 13 clubs)

b. $P = \frac{\text{number of Queens}}{\text{total number of cards}} \times \frac{\text{number of red cards}}{\text{total number of cards}}$

$= \frac{4}{52} \times \frac{26}{52} = \frac{1}{13} \times \frac{1}{2} = \frac{1}{16}$

(there are 4 Queens... 13 hearts and 13 diamonds = 26)

c. $P = \frac{\text{number of black cards}}{\text{total number of cards}} \times \frac{\text{number of diamonds}}{\text{total number of cards}}$

$= \frac{26}{52} \times \frac{13}{52} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

(13 clubs and 13 spades = 26 and 13 diamonds)

Lesson A: The Probability of Independent Events

Warm-up

1. a. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

b. $\frac{1}{5} \times \frac{2}{7} = \frac{2}{35}$

c. $\frac{3}{4} \times \frac{5}{8} = \frac{15}{32}$

2. heads or tails

3. Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King

Diamonds, Spades, Hearts, Clubs

4. 1, 2, 3, 4, 5, 6

Try It! Activity 1

f	1. A pair of dice consistently rolls a sum of seven	a. Probability
b	2. Rolling a two on a six-sided die	b. Outcome
a	3. 40% chance of getting a concert ticket	c. Random Experiment
c	4. Rolling a fair six-sided die	d. Favourable Outcome
d	5. 3 ways of winning a ticket in a contest	e. Sample Space
e	6. Rolling a die and getting 1, 2, 3, 4, 5, or 6	f. Biased

Try It! Activity 2

1. $$\text{P(divisible by 5)} = \frac{\text{4 numbers divisible by 5 }\{5, 10, 15, 20\}}{20\ \{\text{all the numbers from 1 to 20}\}} = \frac{4}{20} = \frac{1}{5} = 0.20$$

2. a. $$\text{P(B or W)} = \frac{\text{5 pairs }\{\text{3 white, 2 black}\}}{\text{9 pairs }\{\text{3 white, 2 black, 4 green}\}} = \frac{5}{9} = 0.56$$

   b. $$\text{P}(\bar{\text{G}}) = \frac{\text{5 pairs }\{\text{3 white, 2 black}\}}{\text{9 pairs }\{\text{3 white, 2 black, 4 green}\}} = \frac{5}{9} = 0.56$$

   c. Both answers are the same. They ask for the same information in different ways.

3. $$\text{P(red)} = \frac{\text{26 red cards }\{\text{13 diamonds, 13 hearts}\}}{\text{52 cards}} = \frac{26}{52} = \frac{1}{2} = 0.50$$

Try It! Activity 3

1. **Dependent**

   By not replacing the first card, the opportunities for drawing a red card for the second card have changed.

2. **Independent**

   The events are physically different, so one event can't affect the other.

3. **Independent**

   The events are physically different, so one event can't affect the other.

4. **Independent**

   The events are physically different, so one event can't affect the other.

5. **Independent**

   The events are physically different, so one event can't affect the other.

Try It! Activity 4

1. {(H,H), (H,T), (T,H), (T,T)}
2. 36
3. {(red,H), (white,H), (blue,H), (red,T), (white,T), (blue,T)}
4. {(H,1), (H,2), (H,3), (H,4)}

Lesson B: Problem Solving With Probability

Warm-up

1. Yes. The outcome of one die does not affect the outcome on the other.
2. No. If you don't put the first ball back, the results of the second draw depend on what was drawn first.
3. 12 is the total number of possible outcomes.

   There are 3 outcomes that contain a head and an even number {(H,2), (H,4), (H,6)}

4. There are 13 spades in a deck of cards. Three are face spade face cards J, Q, K. On a fair six-sided die, there are three even numbers {2, 4, 6} out of the six. {(J,2), (J,4), (J,6), (Q,2), (Q,4), (Q,6), (K,2), (K,4), (K,6)}
5. On a die, the possible rolls include 1, 2, 3, 4, 5, 6. Out of these rolls, only 1, 2, 3 and 4 are less than 5. So the possible outcomes would be {(H,1), (H,2), (H,3), (H,4)}

Try It! Activity 1

1. You roll a fair six-sided die.

   a. Total sample space is {1, 2, 3, 4, 5, 6}

   b. {3} $P(3) = \frac{1}{6}$

   c. { 2, 4, 6} $P(\text{even}) = \frac{3}{6}$

   d. { 1, 2} $P(\text{less than } 3) = \frac{2}{6}$

   e. {4, 5, 6} $P(\text{greater than or equal to } 4) = \frac{3}{6}$

2. $P(\text{head}) = \frac{1}{2}$

   $P(2) = \frac{1}{6}$

   $P(\text{head and } 2) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

3. a. $P(T) = \frac{1}{2}$

   $P(<3) = \frac{2}{6} = \frac{1}{3}$

   $P(T \text{ and } <3) = P(T) \times P(<3) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

   b. $P(H) = \frac{1}{2}$

   $P(\text{at least a } 3) = P(3, 4, 5, \text{ or } 6) = \frac{4}{6} = \frac{2}{3}$

   $P(T \text{ and at least a } 3) = P(T) \times P(\text{at least a } 3) = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

   c. $P(T) = \frac{1}{2}$

   $P(\text{odd } <5) = \frac{2}{6} = \frac{1}{3}$

   $P(T \text{ and odd} < 5) = P(T) \times P(\text{odd} < 5) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

4. a. $P(\text{even}) = \frac{2}{4} = \frac{1}{2}$ $P(\text{Face card}) = \frac{12}{52} = \frac{3}{13}$

$P(\text{even and Face card}) = P(\text{even}) \times P(\text{face card}) = \frac{1}{2} \times \frac{3}{13} = \frac{3}{26}$

b. $P(\text{odd}) = \frac{2}{4} = \frac{1}{2}$ $P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$

$P(\text{odd and Ace}) = P(\text{odd}) \times P(\text{Ace}) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$

c. $P(4) = \frac{1}{4}$ $P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$

$P(4 \text{ and heart}) = P(4) \times P(\text{heart}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

d. $P(<1) = \frac{0}{4}$ $P(\text{diamond}) = \frac{13}{52} = \frac{1}{4}$

$P(<1 \text{ and diamond}) = P(<1) \times P(\text{diamond}) = 1 \times \frac{1}{4} = 0$

5.

$$P(\text{sum of dice is 12}) = \frac{\text{1 combo that sums to 12 } \{(6,6)\}}{\text{36 combinations } \{(1,1),(1,2),\ldots(6,6),(6,6)\}} = \frac{1}{36} = 0.028$$

6. a. $$P(\text{H or T}) = \frac{\text{4 possibilities with a head or a tail } \{(H,H),(H,T),(T,H),(T,T)\}}{\text{4 possibilities } \{(H,H),(H,T),(T,H),(T,T)\}} = \frac{4}{4} = 1.0$$

b. $$P(\text{H and T}) = \frac{\text{2 possibilities with a head and a tail } \{(H,T),(T,H)\}}{\text{4 possibilities } \{(H,H),(H,T),(T,H),(T,T)\}} = \frac{2}{4} = 0.5$$

Try It! Activity 2

1.

$P(\text{head and not less than a 3}) = P(\text{head}) \times P(\text{not less than 3})$

$= \frac{1}{2} \times \frac{4}{6}$

$= \frac{2}{6} = \frac{1}{3}$

2.

$$
\begin{aligned}
&\text{P(right, left, right)} = \text{P(right)} \times \text{P(left)} \times \text{P(right)} \\
&= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\
&= \frac{1}{8}
\end{aligned}
$$

3.

$$
\begin{aligned}
&\text{P(right, right, left)} \times \text{number of arrangements} = \text{P(right)} \times \text{P(right)} \times \text{P(right)} \times 3 \\
&= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 3 \\
&= \frac{3}{8}
\end{aligned}
$$

(RRL, RLR, LRR)

4.

$$
\begin{aligned}
&\text{P(face card, 5 or 6, tails)} = \text{P(face card)} \times \text{P(5 or 6)} \times \text{P(tails)} \\
&= \frac{12}{52} \times \frac{2}{6} \times \frac{1}{2} \\
&= \frac{24}{624} = \frac{1}{26}
\end{aligned}
$$

Glossary

area
Area is the number of square units that fit inside a 2-D shape.

axis (axes)
The axes are the lines that show the number scale on a graph. The x-axis is horizontal, and the y-axis is vertical. Axis is singular and axes is plural.

bar graph
A bar graph is a graph that uses vertical or horizontal rectangular bars to show the quantity being measured. The longer (or higher) the bar, the higher value it represents.

basic operations
Basic operations include addition, subtraction, multiplication, and division.

bias
Bias occurs when a particular outcome is favoured over another.

circle graph/pie chart
A circle graph or pie chart are visual representations of data amounts that together form a total amount or a single quantity.

circumference
Circumference is the distance around a circle.

coefficient
A coefficient is a number that multiplies a variable in a mathematical expression.

> For example, in the expression $3x - 7$, the number 3 is a coefficient. In the expression $\frac{x}{5} + 8$, the coefficient is $\frac{1}{5}$.

constant/constant term
A constant or constant term is a number in a mathematical expression that has no variable attached to it. The number can't be changed.

> For example, in the expression $3x - 7$, the constant is 7. In the expression $\frac{x}{5} + 8$, the constant is 8.

continuous data
Continuous data is data that is part of a set of numbers that can be infinitely divided into smaller and smaller fractions.

For example, time or distance information can be thought of as continuous because they exist in units smaller than we can measure.

coordinates
Coordinates are a set of numbers that can be used to describe a location of a point on a coordinate plane.

coordinate plane or Cartesian plane
A coordinate plane or Cartesian plane is a rectangular area with one or more axes. The plane is designed to show data in a visual way. It is named after its inventor, René Descartes.

congruent
Congruent means “equal to.”

cross section
A cross section is a section cut from a prism or a cylinder. The cut is made parallel to the base.

cylinder
A cylinder is a three-dimensional or 3-D shape which has two circular bases that are parallel to each other and the same distance apart.

data
Data are numbers that represent measurements. Data may represent money, time, distances, or any other amounts.

degrees
Degrees are the measurement of the size of an angle or part of a circle. A full circle is 360 degrees, also written as 360°.

denominator
The denominator is the bottom number in a fraction. It represents the total number of equal parts.

For example, in the fraction $\frac{3}{4}$ where 4 is the denominator, an object or group has been divided into 4 equal parts. (See also **numerator**.)

diameter
In a circle, the diameter is a straight line from one edge of the circle to the other, which passes through the centre of the circle.

discrete data
Discrete data is data that is grouped into separate categories, with no information existing between the categories.

distributive property
The distributive property states that if you add two numbers and then multiply the sum by another number, you'll get the same result as if you multiply each of the two numbers by the other number and then add the products.

For example, $4(2 + 5) = (4)(2) + (4)(5)$.

equation
An equation is a pair of mathematical expressions that are joined by an equals sign (=), and so they represent the same amounts. An equation is a mathematical "complete sentence".

equilateral triangle
An equilateral triangle is a triangle with three equal sides. In an equilateral triangle, all of the interior angles are 60°.

equivalent
When two things are equivalent they have the same value.

For example $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent expressions

event
An event is a specific outcome from the sample set of all possible outcomes.

For example, drawing a five of hearts from a normal deck of 52 cards is an event.

expression
An expression is a mathematical phrase. An expression is made of terms. Terms are joined by the mathematical operators plus or minus (+ or –) into expressions.

For example, $5x - 7$ is a two-term expression.

extrapolate
To extrapolate means to estimate quantities or data beyond the last amounts measured; to extend a graph line beyond the last data point. (See also **interpolate**.)

favourable outome
A favourable outcome means achieving a desired result in a probability experiment.

fraction
A fraction is a number that represents part of a whole.

For example, $\frac{1}{2}$ represents one part out of a total of two parts.

graph
A graph is a visual representation of data using lines, bars, symbols, or areas.

heptagon
A heptagon is a seven-sided closed figure.

hexagon
A hexagon is a six-sided closed figure.

histogram
A histogram is a vertical bar graph.

hypotenuse
1. the side of a right triangle that is not a leg.
2. the longest side of a right triangle.
3. the side of a right triangle that is opposite the right angle.

icon
An icon is a small symbol that represents a quantity of items for a pictograph or in a graph legend. Usually a picture or line drawing of the item is used as an icon.

improper fraction
An improper fraction is a fraction where the numerator is larger than the denominator.

For example, $\frac{7}{5}$ is an improper fraction.

independent event
In a probability experiment, an independent event is when the outcome of one event does not influence or change the possible outcome of another event.

intercept
The intercept is the location where a line graph intersects an axis.

interior angles
Interior angles are angles that are inside a figure. For polygons, interior angles are at each vertex.

interpolate
To interpolate means to estimate the data amounts between data points that were measured. (See also **extrapolate**.)

interval
An interval is the amount between two values; their difference.

irregular polygon
An irregular polygon is a closed figure where all the sides are not equal and all the angles are not equal.

isosceles triangle
An isosceles triangle is one with two equal sides.

legs
Legs refer to:
1. the sides of a right triangle that form the right angle.
2. the parts of the body that the feet are attached to.

line graph
A line graph is a graph using a straight, bent, or curved line to show continuous data.

linear equation/linear relation
A linear equation or linear relation is an equation, table, description or graph that shows the relationship between two variables and forms a straight-line graph.

misinterpret
To misinterpret means to misunderstand or to gain a false impression from a conversation, picture, data or text.

misleading information
Misleading information is information (such as a graph) that is technically correct but would give most viewers an inaccurate impression.

misrepresent
To misrepresent is to present information falsely, visually or in words.

mixed number
A mixed number is a number composed of a whole number and a fraction.

For example, $2\frac{1}{3}$ is a mixed number.

model
1. To model is to create a representation of real-life data.
2. A model is the graph, map, computer program or another item that represents data.

net
A net is a two-dimensional or 2-D construction of a three-dimensional or 3-D object.

numerator
The numerator is the top number in a fraction. It represents the number of equal parts you are working with.

For example, in the fraction $\frac{3}{4}$ where 3 is the numerator, you are working with only 3 of the parts out of 4 total. (See also **denominator**.)

octagon
An octagon is an eight-sided closed figure.

operations
When we do something with a number or numbers, it is called an operation. Addition, subtraction, multiplication, and division are basic operations.

ordered pair
An ordered pair is a pair of numbers (x, y) that represent the values that satisfy a relation and also represent a location on the graph of the relation.

origin
The origin is the point (0,0) on a two-dimensional graph at which the axes intersect.

outcome
The outcome is the result of a single trial or experiment.

pentagon
A pentagon is a five-sided closed figure.

percent
A percent is a fraction of a whole, expressed as a fraction out of 100.

perfect square
A perfect square is a number that represents the area of a square whose sides are whole numbers.

For example, if a square has sides of length 3, its area is 9, and 9 is a perfect square.

It is also the result when a whole number is multiplied by itself.

For example, $5 \times 5 = 25$, and 25 is a perfect square.

perspective
Perspective is the viewer's perception, visually or psychologically.

pictograph
A pictograph is a graph that uses icons or symbols to represent the amount measured in each category, instead of using an axis to show the measurements.

pie chart
See **circle graph**.

plane
A plane is a two-dimensional or 2-D surface.

point
A point is a location on a coordinate plane which can be represented by an ordered pair (x, y).

polygon
A polygon is a closed geometric shape made of 3 or more line segments.

prism
A prism has three-dimensional or 3-D shapes that have the same cross section along a length.

proper fraction
A proper fraction is a fraction whose numerator is less than its denominator.

For example, $\frac{2}{3}$ is a proper fraction.

probability
Probability is the chance or likelihood that a particular event will occur. Probabilities are often listed as ratios (e.g. 1:2 or 2 to 5), fractions (e.g. $\frac{3}{5}$) or percents (e.g. 15%)

proportion
A proportion is a pair of equal ratios.

Pythagorean Theorem
The Pythagorean Theorem describes the relationship among the lengths of the three sides of a right triangle: $a^2 + b^2 = c^2$

Pythagorean Triple
A Pythagorean Triple is a set of three whole numbers that satisfy the Pythagorean Theorem.

> For example, the numbers 3, 4, and 5 form one Pythagorean Triple. The first two numbers in a Pythagorean Triple are the measurements of the legs, and the third (the largest number) is the measurement of the hypotenuse.

quadrilateral
A quadrilateral is a four-sided closed figure.

radius
In a circle, the radius is the distance from the center to the edge of the circle.

random experiment
A random experiment is a a process leading to at least two outcomes with some uncertainty about which will occur.

rate
A rate is a comparison of two quantities in which each quantity is measured in different units. For example \$8 per dozen roses (or \$8.00/12 roses) is a rate. (See also **unit rate**.)

ratio
A ratio is a comparison of two or more numbers. Ratios are written with a ":" (e.g. 2:3), using words (e.g. 2 to 3), or as a fraction (e.g. $\frac{2}{3}$).

reciprocal
A reciprocal is a number that you multiply a fraction by so that the result equals one. If you start with a whole number, put it over 1 first. The easiest way to find it is to just flip the fraction over.

> For example, the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.

rectangular prism
A rectangular prism is a six-sided three-dimensional or 3-D shape made up of rectangles.

regular polygon

A regular polygon is a closed figure with all sides equal and all angles equal.

right angle

A right angle is an angle that measures 90°.

right triangle

A right triangle has one right angle.

round/round off

To round or round off is to remove unwanted place values at the right end of a number, adjusting the first remaining place value if necessary. (See also **truncate**.)

sample space

A sample space includes all the possible outcomes resulting from a probability experiment.

satisfy

To satisfy means to replace variables with values that make an equation into a true statement.

For example, $y = 3x$ can be satisfied with the ordered pair (2, 6), but cannot be satisfied with (4, 9).

square root

The square root symbol tells us to take the square root of the number that's inside.

For example, $5^2 = 25$. The square root of 25 is 5.

square root symbol

$\sqrt{}$ This symbol tells us to take the square root of the number that's inside.

For example: $\sqrt{4} = 2$

surface area

Surface area refers to the total area of the net of a three-dimensional or 3-D object. The units are squared, for example, cm^2, m^2.

term

A term is an item in an expression that is a constant, or variable, or coefficient-and-variable combination. (See also **expression**.)

tessellation
A tessellation is a tiling pattern that covers an entire plane without overlapping or leaving gaps.

three-dimensional (3-D)
Three-dimensional refers to an object that has length, width and depth, or a representation of an object that has the appearance of depth.

triangular prism
A triangular prism is a five-sided three-dimensional or 3-D shape with two triangles that are parallel and equal to each other and joined by rectangles.

truncate
To truncate means to remove unwanted place values at the right end of a number without adjusting the remaining place value. (See also **round/round off.**)

two-dimensional (2-D)
Two-dimensional refers to an object that has length and width, but no depth.

unit rate
A unit rate is a rate where the second term is 1.

For example, wages are often given as a unit rate.

$10.00/hr represents $10.00 earned for every 1 hour worked.

unknown
An unknown is the value(s) that provide the solution to an equation. (See also **variable.**)

variable
A variable is a value that is unknown or that could change. It is often represented in an expression by a letter such as x, but could be represented by a word or other symbol. (See also **unknown**.)

vertex (vertices)
In a closed figure, the vertex refers to the point where two sides meet. Vertex is singular and vertices is plural.

view
The view refers to a two-dimensional or 2-D drawing of a three-dimensional or 3-D object from one particular position—front view, side view, top view, bottom view, etc.

volume
The volume is the amount of space an object takes up. The units are cubed, for example, cm^3, m^3.

x-axis
The x-axis is the horizontal axis of a coordinate plane. (See also **coordinate plane** and **axis**.)

y-axis
The y-axis is the vertical axis of a coordinate. See also **coordinate plane** and **axis**.)

Templates

Section 1 | Lesson B and C: Fraction Strips

1											
$\frac{1}{2}$	$\frac{1}{2}$										
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$									
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$								
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$							
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$						
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$				
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$			
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$		
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

1											
$\frac{1}{2}$	$\frac{1}{2}$										
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$									
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$								
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$							
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$						
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$				
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$			
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$		
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$