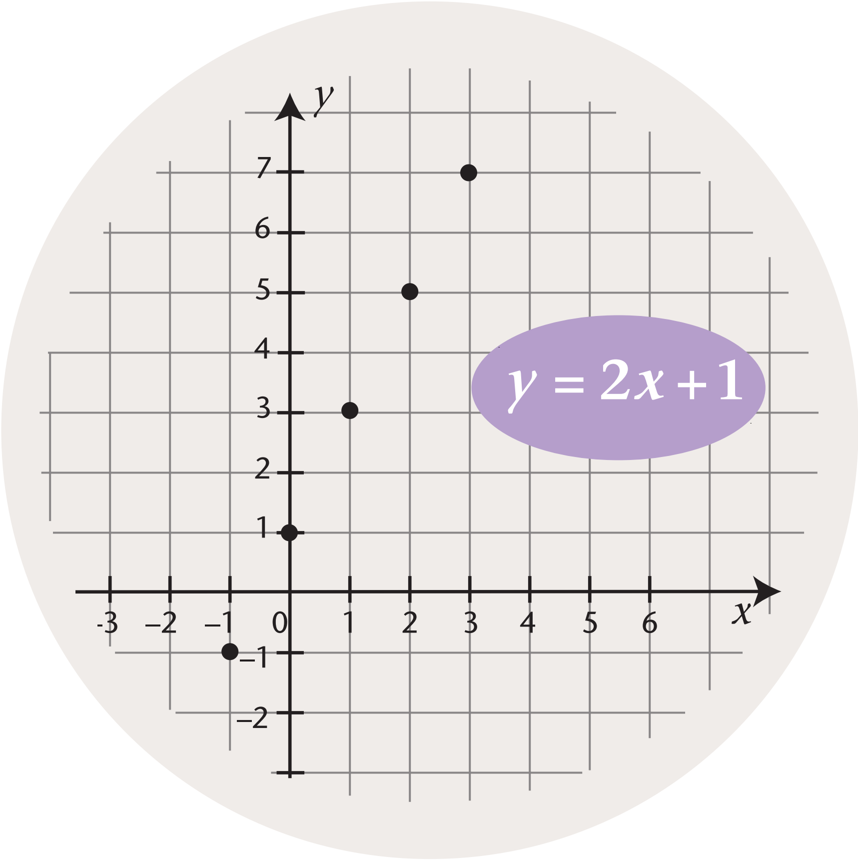
Math 8

Module 3

Data, Graphing, and Linear Equations



BRITISH COLUMBIA

OPEN SCHOOL BC

Project Manager: Christina Teskey, Jennifer Riddel

Planning Team: Dane Allison (School District 8), Sonya Fern (School District 62), Duncan McDougall (Tutor Find Learning Centre), Shelley Moore (School District 38), Jennifer Riddel (Open School BC), Christina Teskey (Open School BC), Angela Voll

Writers: Dan Laidlaw (School District 73), Esther Moreno (School District 39), Angela Voll, Rusé Kampunzi, Clint Surry (School District 63), Christina Teskey (Open School BC)

Course Reviewers: Susan Robinson (School District 64), Clint Surry (School District 63)

Editor: Shannon Mitchell (Paper Hat Editing Services), Angela Voll

Production Technician: Beverly Carstensen

Art: Beverly Carstensen, Cal Jones, Max Licht, Sean Owen, Christine Ramkeesoon

Multimedia
Media Coordinator: Janet Bartz, Christine Ramkeesoon
Media Design: Janet Bartz
Flash Programming: Chris Manuel
Video Scripting: Christina Teskey
Video Production: Caitlin Flanders, Sean Owen, Chris Manuel
Video Talent: Jennifer Riddel, Jeffrey Chan

Print History

Corrected: October 2009
New: August 2009

Contents

Course Overview

Welcome to Mathematics 8!

In this course you will continue your exploration of mathematics. You'll have a chance to practice and review the math skills you already have as you learn new concepts and skills. This course will help you to increase your ability to think mathematically.

Organization of the Course

The Mathematics 8 course is made up of four modules. These modules are:

Module 1: Exploring 2–D and 3–D Connections

Module 2: Squares, Integers, and the Pythagorean Theorem

Module 3: Data, Graphing, and Linear Equations

Module 4: Fractions, Ratios, and Probability

Organization of the Modules

Each module has three sections. The sections have the following features:

Pretest

This is for students who feel they already know the concepts in the section. It is divided by lesson, so you can get an idea of where you need to focus your attention within the section.

Lessons

Each section is divided into lessons. Each lesson is made up of the following parts:

Essential Questions

Essential Questions are based on the concepts in each lesson. This activity will help you organize information and reflect on your learning.

Warm–up

This is a brief drill or review to get ready for the lesson.

Explore

This is the main teaching part of the lesson. Here you will explore new concepts and learn new skills.

Try it! Activities

These are activities for you to complete to solidify your new skills. You will mark these using Solutions at the end of each module.

At the end of each module you will find:

Solutions

This contains all of the solutions to the Pretests, Warm–ups and Try it! Activities.

Templates

Templates to pull out, cut, colour, or fold in order to complete specific activities. You will be directed to these as needed.

Glossary

This is a list of key terms and their definitions.

More about the Pretest

There is a pretest at the beginning of each section. This pretest has questions for each lesson in the section. Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in the section. Mark your answers using Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Thinking Space

The column on the outside edge of most pages is called the Thinking Space. You can use this space to

- write questions about things you don't understand
- note things that you want to look at again
- respond to a question in the Thinking Space or the text
- draw pictures that help you understand the math
- identify words that you don't understand
- connect what you are learning to what you already know
- make your own notes or comments

Materials and Resources

There is no textbook required for this course. All of the necessary materials and exercises are found in the modules.

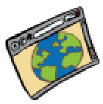
In some cases, you will be referred to templates to pull out, cut, colour, or fold. These templates will always be found near the end of the module, just in front of the answer key.

You will need a scientific calculator for some of the activities. A geometry set would also be helpful, although for many activities you can use a straightedge rather than a ruler.

If you have Internet access, you might want to do some exploring online. The Math 8 Course Website will be a good starting point. Go to **http://www.openschool.bc.ca/courses/math/math8/** and find the lesson that you're working on. You'll find relevant links to websites with games, activities, and extra practice.

Icons

You will see a few icons used on the left–hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.



Explore Online

Essential Questions

Solutions

Use a Calculator

Module 3 Overview

Module 3 consists of three sections on statistical data and graphing, and graphing and solving linear equations

Section Overviews

Section 3.1: Analyzing Statistical Graphs

In the first section of Module 3, you'll review types of statistical graphs and learn some advantages and disadvantages of each. You'll also learn to decide which type of graph is best for representing different mathematical situations. Finally, you'll learn how graphs can be manipulated by changing the formatting, scale or other attributes.

Section 3.2: Graphing on the Coordinate Plane

In the second section, you'll review how to plot points on the coordinate plane and learn how a linear equation is different from other equations that you might work with. You'll be able to identify the parts of a linear equation and create ordered pairs to complete a table of values. When given a table of values or a graph, you'll be able to find missing points. All throughout the section, you'll be analyzing and creating your own graphs on the coordinate plane, and working with multimedia called *Linear Equations* at **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0832c1f_linear.html**.

Section 3.3: Linear Equations

It's time for some hands-on math! In the last section of Module 3, you'll work with algebra tiles to model and solve linear equations. Next, you'll build on the ideas from modeling equations to solve them symbolically using a reverse-BEDMAS method. You'll also learn two different methods of checking your work. In the final part of the section, you'll tackle some harder equations by grouping like terms and working with the distributive property.

Course Map

On the following page you'll find a course map. If you colour in the box for each section and lesson as you complete it, you'll easily be able to see how much of the course you've finished, and how much is still left to complete.

Mathematics 8

Module 1

Section 1: Nets and Views
- Lesson A: Visualizing in Three Dimensions
- Lesson B: Prisms and Cylinders and their Nets
- Lesson C: Top View, Side View, and Front View

Section 2: Calculations in 2-D and 3-D
- Lesson A: Total Area of the Net
- Lesson B: More about Area in 2-D and 3-D
- Lesson C: The Amount of Space

Section 3: Shapes and Design
- Lesson A: What is a Tessellation Anyways?
- Lesson B: Transformations in Tessellations
- Lesson C: Creating a Tessellation

Module 2

Section 1: Multiplying and Dividing Integers
- Lesson A: Should I Multiply or Divide?
- Lesson B: Mult. and Div. with Negative Numbers
- Lesson C: Expressions–More than one Operation

Section 2: Squares and Square Roots
- Lesson A: Perfect Squares
- Lesson B: Square Roots of Perfect Squares
- Lesson C: Estimating Square Roots
- Lesson D: Calculating Square Roots

Section 3: The Pythagorean Theorem
- Lesson A: The Pythagorean Theorem
- Lesson B: Pythagorean Triples
- Lesson C: Using the Pythagorean Theorem

Module 3

Section 1: Analyzing statistical graphs
- Lesson A: Types of Graphs and Data
- Lesson B: Adv. and Dis. of Graph Types
- Lesson C: Misleading Graphs

Section 2: Graphing on the Coordinate Plane
- Lesson A: Ordered Pairs and Linear Equations
- Lesson B: Tables of Values
- Lesson C: Relationships and Missing Values

Section 3: Linear Equations
- Lesson A: Algebra Tiles
- Lesson B: Solving Linear Equations Symbolically
- Lesson C: More Linear Equations

Module 4

Section 1: Multiplying and Dividing Fractions
- Lesson A: Proper, Improper Fractions, Mixed Numbers
- Lesson B: Multiplying Fractions
- Lesson C: Dividing Fractions
- Lesson D: Estimating and Solving Problems

Section 2: Ratios, Rates and Percents
- Lesson A: Ratios and Rates in Everyday Life
- Lesson B: Making Sense of Percents
- Lesson C: Solving Problems with Percents

Section 3: Probability
- Lesson A: The Probability of Independent Events
- Lesson B: Problem Solving with Probability

Section 1

Analyzing Statistical Graphs

In this section you will:

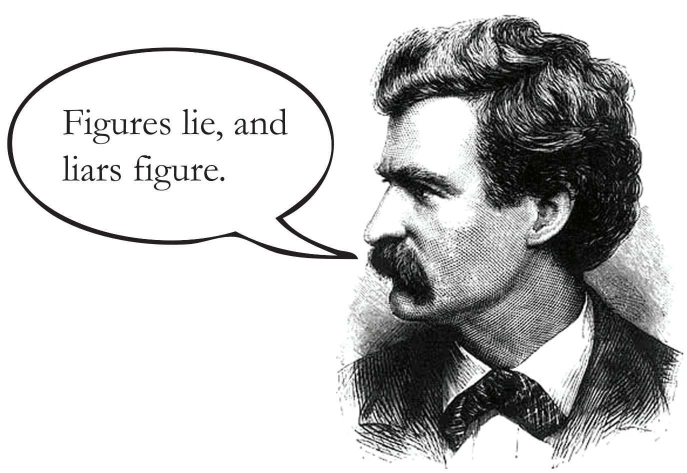
- understand the advantages and disadvantages of each graph type
- identify data sets as continuous or discrete, and know which type(s) of graph to use for a given data set
- identify graphs that show information accurately, and identify those that are misleading
- identify common ways graphs are used to misrepresent information

For this section you will need:

- ruler

Where in the World...?

"Figures lie, and liars figure," said Mark Twain. He wrote stories about life in the southern United States in the 1800s, but he was also a respected politician and humourist, well known for his clever quips and wry observations. He made this quote to show how numbers can be misused to convince people of what isn't necessarily true. But numbers can also illustrate and enhance the truth. It's all in how you present them.



Section 1
Pretest

Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using the Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson A: Types of Graphs and Data

1. Match each graph type on the left with a statement on the right. Use each statement only once. Write the letter of the correct response in the blank.

   ______ Bar graph

   ______ Line graph

   ______ Circle graph

   ______ Pictograph

   a. not very accurate, but easy to understand, shows data as portions of a whole

   b. can be very accurate, but can only be used with continuous data

   c. shows data for categories, easy to read, can show two or more data sets on one graph

   d. shows data in categories, easy to read, uses icons instead of numbers on a scale

2. Angela is studying the salaries earned by several professionals. She learns the average salaries of doctors, lawyers, accountants, engineers, teachers, nurses, and police officers.

   a. Is this data continuous or discrete?

   ____

b. Explain why you chose the answer in part a.

____

____

c. What would be the graph type that would best describe the data?

____

Lesson B: Advantages and Disadvantages of Graph Types

3. A language teacher wants to produce a graph that shows the portions of the school's population that speak Spanish, Italian, French, Hindi, Chinese, and Japanese at home. The graph will include a category for those that speak an "other" language not listed. Which statement below reflects this teacher's best choice of graph? Place a check mark beside the correct choice:

   a. Use a line graph because it's easy to understand and shows discrete data. ______

   b. Use a pictograph with a happy–face symbol to represent 500 students, and one stack of symbols for each language, because it's easy to understand. ______

   c. Use a circle graph because it shows discrete data and represents each language as part of the whole student population. ______

   d. Use a bar graph because it shows continuous data and is easy to understand. ______

4. A group of students is studying the bounciness of different types of balls. Each student chooses a ball, and measures the height it bounces after being dropped. Each ball is dropped from various starting heights. The results are going to be graphed.

   a. What type of graph should a student use?

   ____

   ____

   b. Give a reason for your choice of graph type.

   ____

5. Which type of graph would you consider to be best for a data set that shows the numbers of different types of fish found in a river system?

___

Give two reasons for your answer.

___

___

___

___

6. a. What type of graph is easiest to understand?

___

b. Give two reasons for your answer.

___

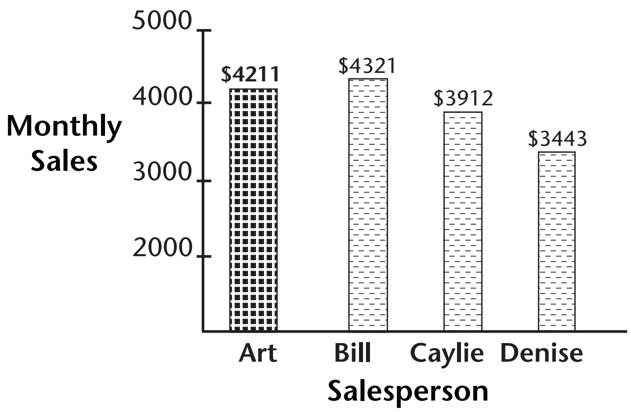
___

___

___

Lesson C: Misleading Graphs

7. Observe the bar graph below.



a. Point out three ways in which it is misleading.

1. ______________________________

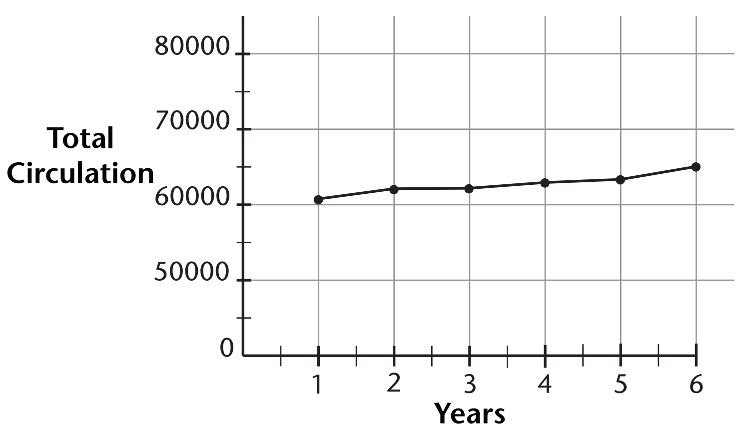
2. ______________________________

3. ______________________________

b. Which salesperson was the most likely creator of the graph?

____

8. Examine the line graph below.



a. Describe the misleading feature of this graph.

____

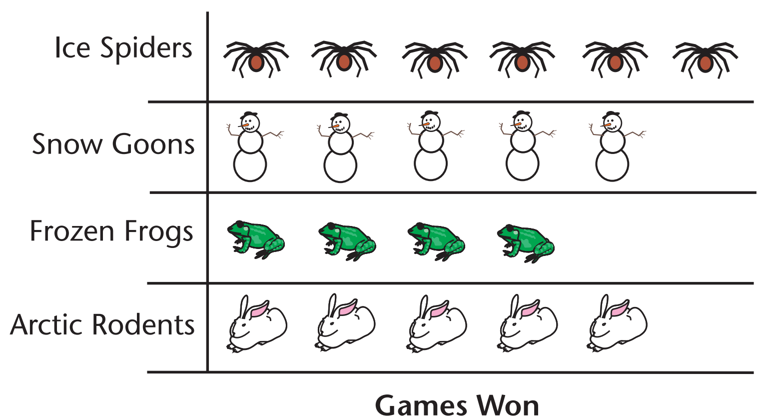
____

b. What feature needs to be added to the graph's vertical axis to make it less misleading?

____

c. Sketch this feature at the correct location on the graph.

9. Look at the pictograph below.



What are two ways it could be formatted so that it looks like the Snow Goons are winning the league?

1. ____

2. ____

10. Explain two ways to format a circle graph so that it is misleading.

    1. ____
    2. ____

Turn to the Solutions at the end of the module and mark your work.

Lesson A

Types of Graphs and Data

For this lesson you will need:

- ruler

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

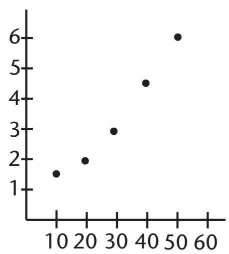
When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

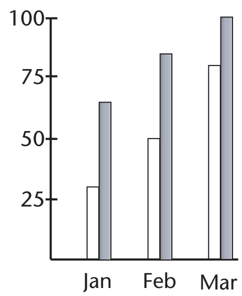
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What kinds of data are there?		
What kinds of graphs should be used for which kinds of data?		

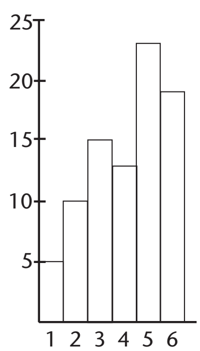
Thinking Space

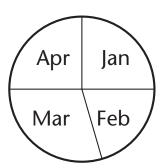
Warm–up

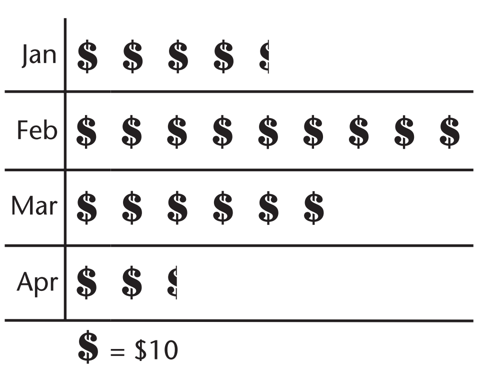
1. Match each data set with its graph.

______ 

______ 

______ 

______ 

______ 

a.

(1, 5)	
(2, 10)	
(3, 15)	
(4, 13)	
(5, 23)	
(6, 19)	

b.

Jan	3
Feb	2
Mar	4
Apr	3

c.

Jan	45
Feb	90
Mar	60
Apr	25

d.

Jan	30
Feb	50
Mar	80

Jan	65
Feb	85
Mar	100

e.

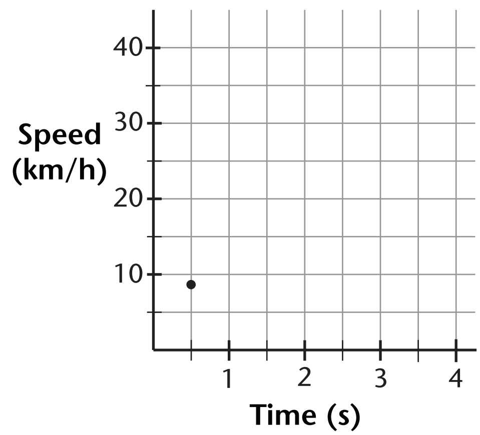
(10, 1.5)
(20, 2)
(30, 3)
(40, 4.5)
(50, 6)

Thinking Space

2. Finish drawing the following graphs.

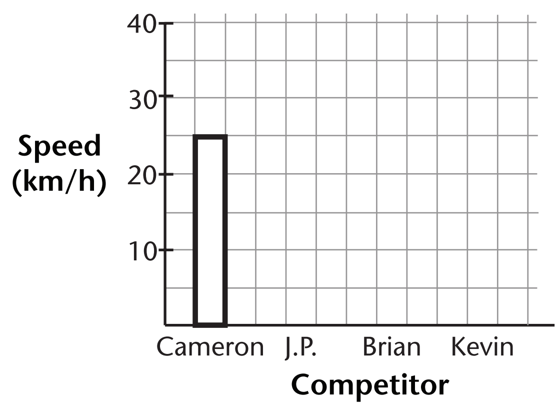
   a. Speed of a race car after the starting gun.

Time (s)	Speed (km/h)
0.5	8
1.0	21
2.5	32



Remember to join the points with straight lines once you have them all plotted.

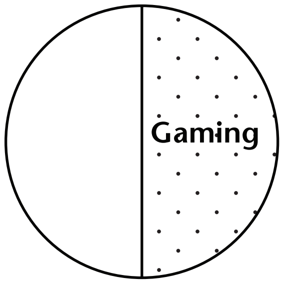
   b.



Cameron	25
J.P.	38
Brian	14
Kevin	32

Thinking Space

c.



Leisure Time

Gaming	50%
Reading Books	25%
Listening to Music	25%

d.

Favourite Snacks

Chocolate Bars	
Chips	
Gum	

Key  **= 5 people**

Chocolate bars	**20**
Chips	**30**
Gum	**15**

Turn to Solutions at the end of the module and mark your work.

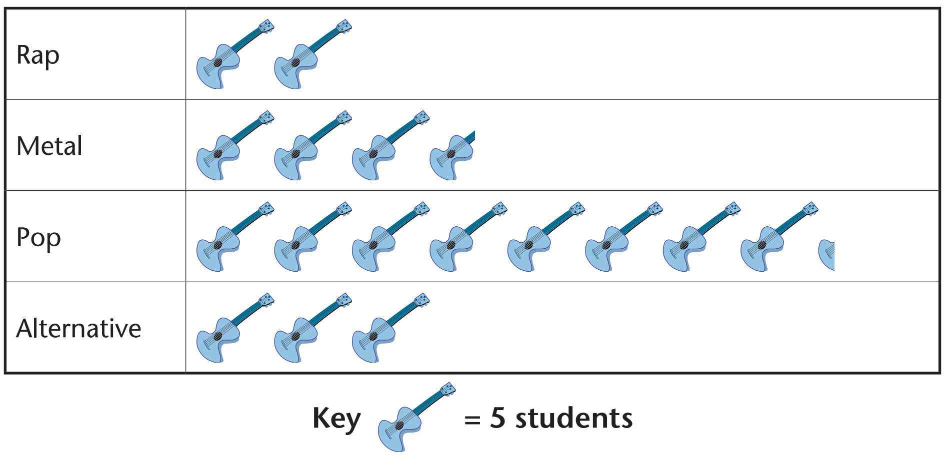
Thinking Space

Explore
A Gaggle of Graphs

Let's review some different types of graphs that you have studied in the past.

Pictographs use an icon to show a measurement or amount for each thing being measured.

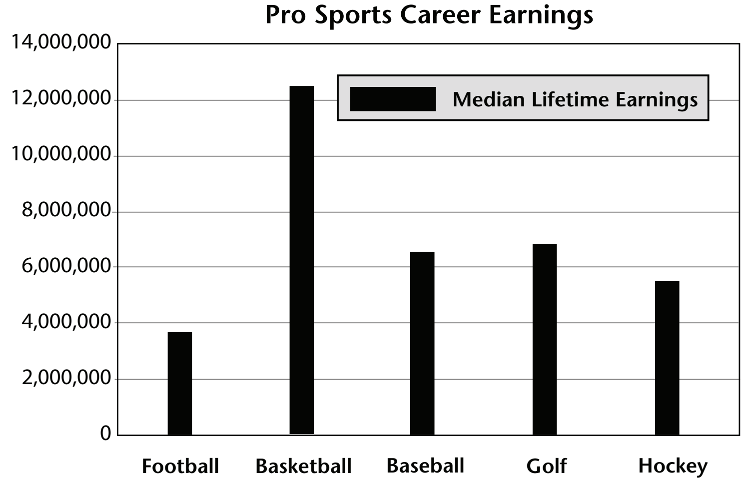
Students in Grade 8 Who Like Each Type of Music



Pictographs are for data that's in categories or separate groups. Notice there are no numbers on the graph except for the key. Numbers aren't needed on this type, because the symbols tell us each amount.

Bar graphs show a single number amount for each of the different things being measured. They compare *different* situations.

Pro Sports Career Earnings



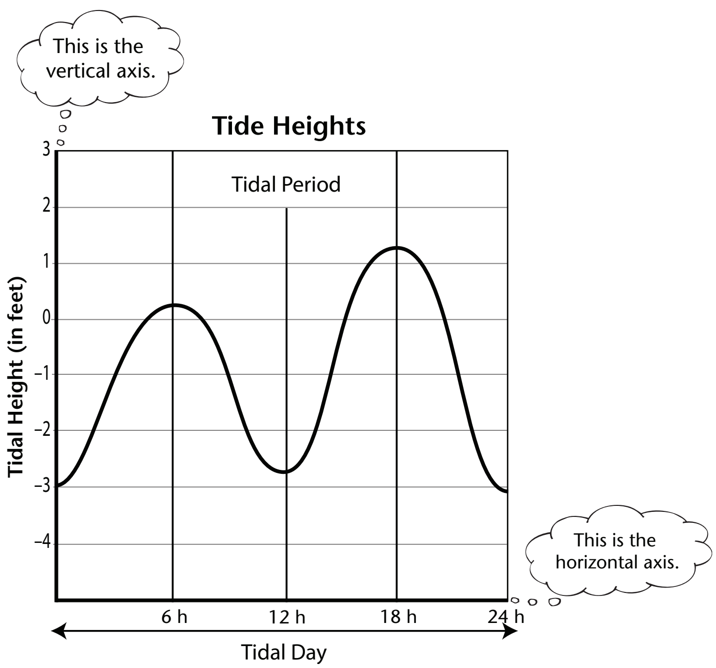
Thinking Space

Bar graphs should have numbers up the left margin.

Bar graphs are like pictographs, but instead of symbols, we use the height of each bar and the numbers up the side of the graph to tell us the amount for each data entry. Note that the numbers up the side start at zero. This is important, because a graph that starts at some other number could be misleading.

Line graphs show the relationship between *two different* sets of numbers.

What are two ways that line graphs differ from the other types of graphs?



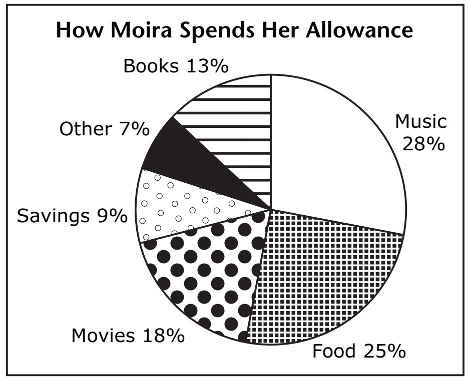
The left margin (**vertical axis**) shows one set of numbers, and the lower margin (**horizontal axis**) shows the other set. The line is the place where the number sets match up. Line graphs are often used to show things that change as time passes.

We put points on the graph that represent the data we measured, then draw a line between the points to represent all the data we *couldn't* or *didn't* measure.

Which of the situations use "time" in some form as part of the data set? Do all time–related situations belong to the same category of graph?

Thinking Space

Circle graphs show the fraction or percent amounts that create one whole thing.



Can you think of data situations that would work well with each graph type? Try looking in the newspaper, or on websites. Can you find an example of a good situation for each graph type?

Each wedge (or pie slice) represents one data entry, and the wider it is, the larger the fraction it represents. We usually show the percent of the total that the wedge represents.

Circle Graph or Bar Graph?

Many times, you might notice data sets that could be graphed with a circle graph or a bar graph. It just depends on your preference—do you want the reader to think of separate items (bar graph) or parts of a total (circle graph)?

If you would like some practice drawing graphs and you have Internet access, go to the Math 8 website at http://www.openschool.bc.ca/courses/math/math8/mod3.html and click on the link under *Lesson 3.1A: Types of Graphs and Data*.

Thinking Space

Try It!
Activity 1

1. For each item below, read the statement, then decide which kind of graph is most appropriate. Put a **B** (bar), **P** (pictograph), **L** (line), or **C** (circle) in the blank in front of each description. The first one is done for you.

   L a. sets of numbers that represent the hourly temperature during each day in July

   ______ b. temperature of water in a kettle as it heats up.

   ______ c. amount of wood in a tree as it grows.

   ______ d. average cost of living compared to location.

   ______ e. average income versus years of education.

   ______ f. types of garbage that goes into the landfill

   ______ g. number of households that have each type of pet

2. Look over the data we measured at an orchard.

Tree Type	Kilograms of Fruit
pear	1929
apple	2011
plum	1224

   a. What type of graph should **NOT** be used to represent this data?

   ____

Thinking Space

b. Why do you think this type of graph would not represent this data?

c. What type of graph would you choose to represent this data?

d. Why would you choose to use this type of graph?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Discrete or Continuous?

continuous data
Continuous data consists of data that can be infinitely divided into smaller and smaller fractions.

discrete data
Discrete data is grouped into separate categories, with no information existing between the categories.

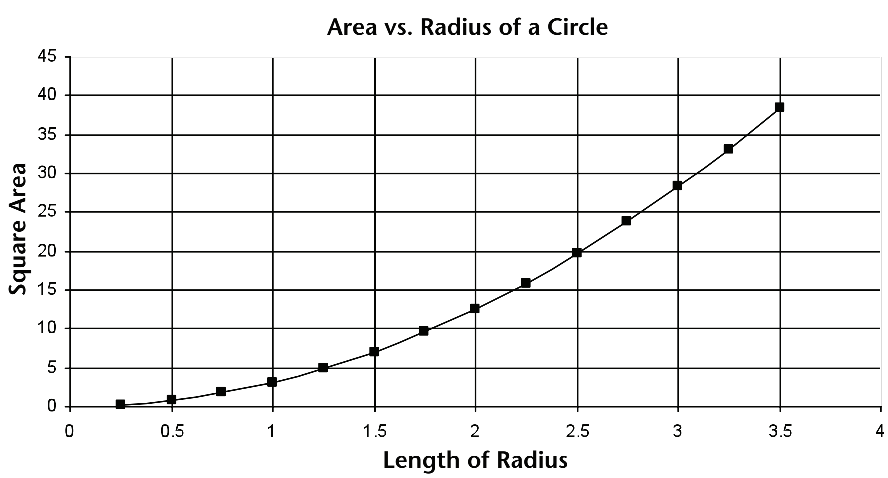
Read each description that follows. Think about the type of data—discrete or continuous—and why it's best shown by a particular type of graph.

Line graph: Use a line graph in any case where you measure how something changes over time, and you're measuring time in minutes or seconds. Because time always continues without stopping, anything that changes over time can be considered continuous data.

Another example of continuous data (that doesn't use time) is people's heights. Someone's height can be measured very exactly, in metres and centimetres and millimetres—or even fractions of a millimetre. We could show the average number of people for any given height on a line graph.

Can you think of an example of continuous data that does not depend on time?

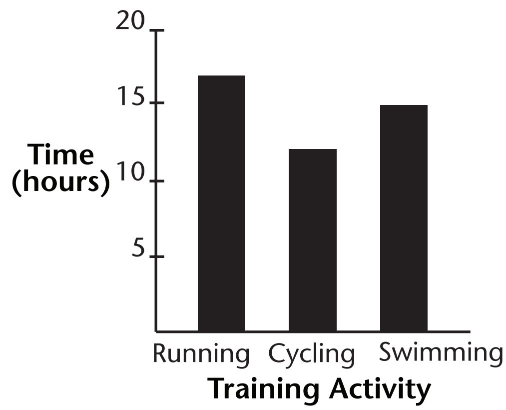
Here's an example of a line graph.



Thinking Space

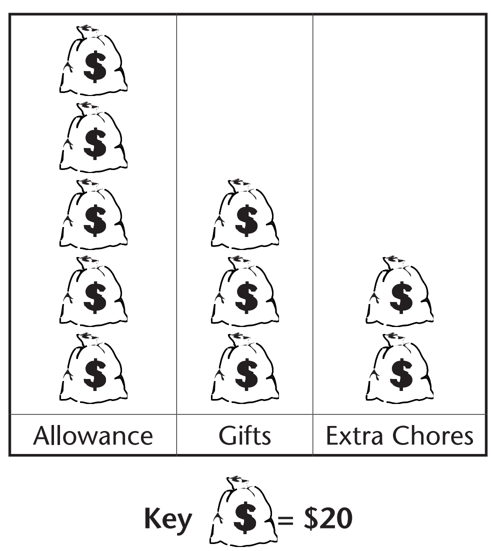
Bar graphs: These are used for discrete data groups. A bar represents a complete measurement for one group or type of thing.

For example, the amount of time an athlete spends on different training activities could go on a bar graph. Each activity gets its own bar.



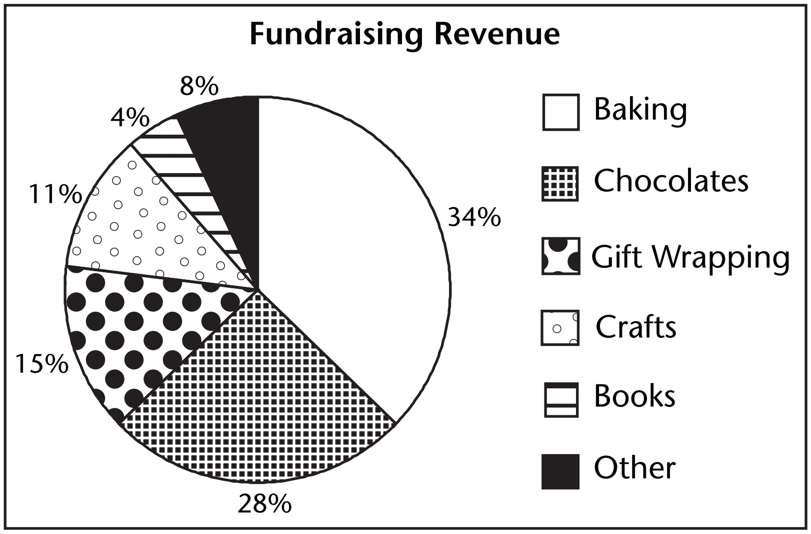
Pictograph: Like bar graphs, pictographs are used for data in discrete groups. Instead of using numbers up the left–side scale, a pictograph uses symbols to represent a certain number of things for each "bar." The reader counts the number of symbols in a bar to get an approximate value.

For example, you could use a pictograph to show how much money Cameron collected from different sources.



Thinking Space

Circle graphs: These show a comparison of the parts of a total. Each part is a discrete data group. For example, a fundraising group might show the various sources where their total revenue has come from, one wedge for each source.



How can you describe the difference between "continuous" data and "discrete" data in plain words?

Thinking Space

Try It!
Activity 2

1. Complete the chart.

   Fill in the middle column by using the words below to describe the data. There can be more than one word for each graph type, and words can be used more than once.

   Describe the data: **discrete**, **continuous**, **number**, **percent**, **part**, **total**, **symbol**

   For each graph type, fill in the last column with all of the following examples of data that could be graphed with that graph type.

Examples: **votes**: number of votes for each candidate in the school election

track: number of students participating in each of eight track and field events (students can participate in more than one event)

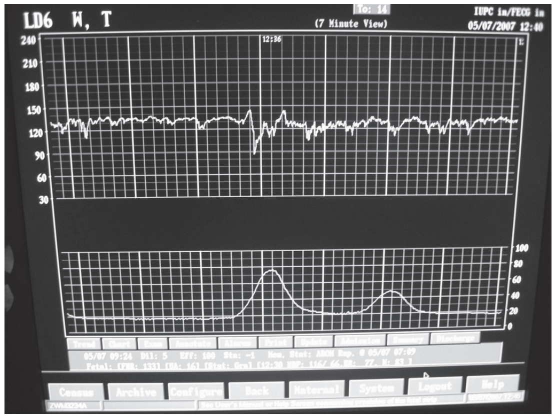
heights: people's heights vs. shoe size

summer: different activities Jeremy did on his summer vacation

Graph Type	Describe the Data	Example
line graph		
bar graph		
pictograph		
circle graph		

Thinking Space

2.



a. What type of graph is made by a heart monitor?

b. Why does it make sense to this this type of graph to measure heart rate?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

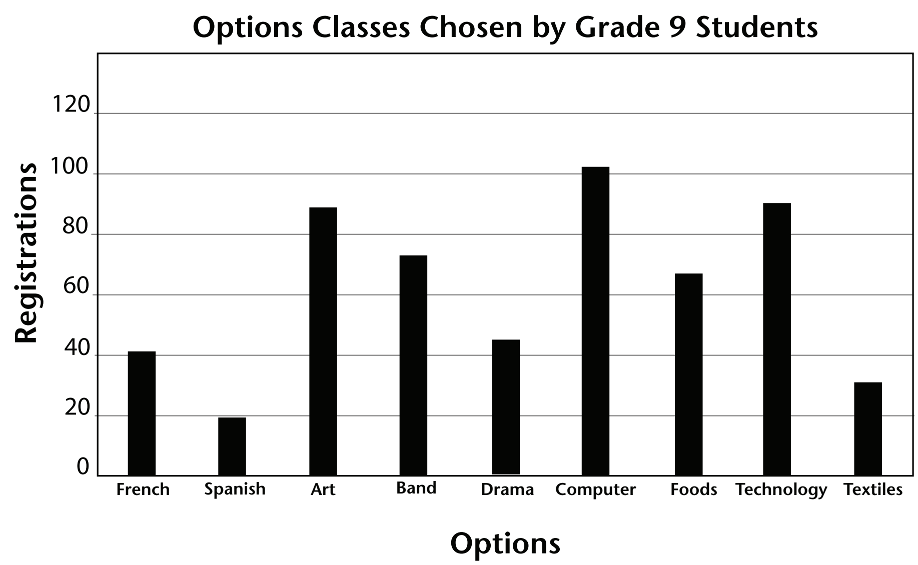
Explore
Strengths and Limitations of Graph Types

Let's look at one data set and see how it can be graphed in different ways.

Options Classes Chosen by Grade 9 Students
(each student gets three options)

French	41
Spanish	18
Art	88
Band	73
Drama	44
Computer	103
Foods	66
Technology	90
Textiles	32
Total	555

The Data As a Bar Graph



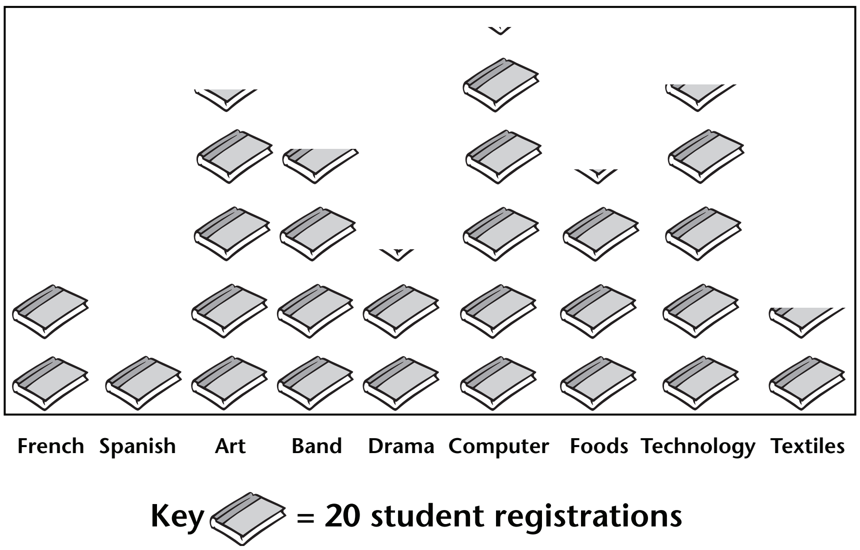
On this bar graph, it's easy to see approximately how many student registrations there are for each course without doing any calculations. You can tell at a glance that Computers is the most popular with over 100 registrations, and that Spanish is the least popular with fewer than 20.

Thinking Space

You can also see how each category compares to the others. For example, it looks as though Art is about twice as popular as Drama.

The Data As a Pictograph

Options Classes Chosen by Grade 9 Students



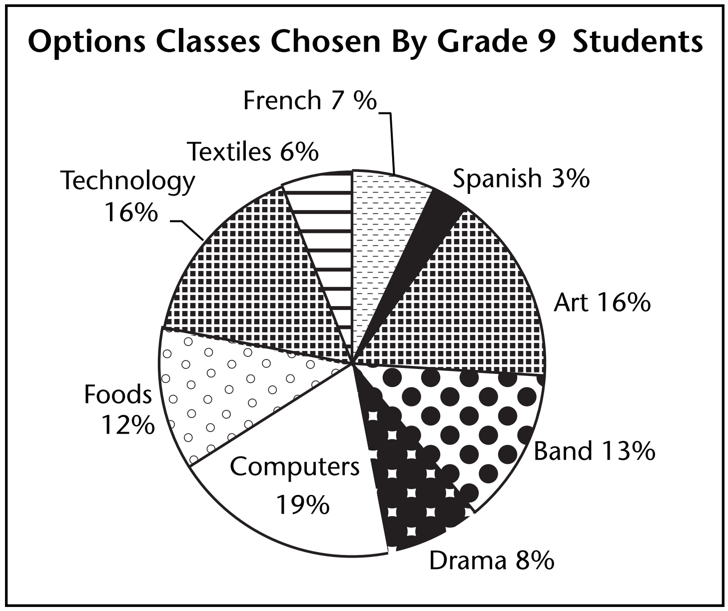
A pictograph can be more interesting to look at than a bar graph, and definitely more interesting than the raw data (the numbers in the chart). Like a bar graph, you can easily compare the different categories. For example, you can see that French is about twice as popular as Spanish.

Pictographs don't usually provide accurate numbers, but they are easy to read, and show the approximate data.

A pictograph can be hard to read if you're looking for exact numbers; you'll need to do some calculations.

Thinking Space

The Data as a Circle Graph



Circle graphs are also easy to read, and show the data in an approximate way.

On a circle graph, it's easy to see which slices are the biggest or smallest. This tells you which categories make up the biggest or smallest part of the total.

It can be hard to tell how the wedges of the graph compare to each other. For example, can you tell how Band compares to Technology without looking at the percentages?

What other courses could be combined, and what would be the category names?

A circle graph can become cluttered and hard to read if there are too many categories, like the one above. This graph might look better if we combined some of the different courses into fewer categories. For example, Art, Band, and Drama could be combined into "Fine Arts."

Thinking Space

The Data as a Line Graph?

Now let's see what it looks like as a line graph. But wait—should we even use a line graph for this data? Let's quickly review what we know about line graphs. They are used for graphing:

- two sets of numbers against each other, or one set of numbers against time
- continuous data, not discrete data

We don't have two sets of numbers, nor are we plotting anything against time. And we also don't have continuous data because our numbers are in categories (the categories are the subjects); so, it's discrete data. All of this means that we shouldn't use a line graph to display this information.

Thinking Space

Try It!
Activity 3

Look back through the lesson, and use this table to summarize the strengths and limitations of each graph type. Choose from the descriptions that follow the table. Descriptions can be used more than once. Add descriptions of your own if you have any that aren't included in the list.

Graph Type	Strengths	Limitations
bar graph		
circle graph		
line graph		
pictograph		

Thinking Space

Possible Descriptions

- used to compare parts to the total
- partial icons make calculating difficult
- can compare two sets of discrete data
- more difficult to compare one category to another
- shows data that changes over time
- shouldn't be used for discrete data
- can be used for discrete data
- interesting to look at
- can be difficult to draw: requires a protractor
- must use a key to calculate
- provides approximate numbers using scale on left side
- can only use with discrete data
- used to compare two sets of numbers

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B
Advantages and Disadvantages of Graph Types

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
In what ways do graphs help us understand data?		
How can you decide which type of graph to use?		
What are the advantages and disadvantages of each graph type (pictograph, line graph, circle graph, and bar graph)?		

Thinking Space

Warm-up

1. Classify each description of data as continuous (C) or discrete (D). Look in the glossary to review these terms first if you need to.

   ______ The diameter of a ball determines its circumference.

   ______ Students at our school eat different types of foods for breakfast.

   ______ The distance from home depends on how long we've been travelling.

   ______ A store tracks the sales of different brands of marmalade.

   ______ The height of water in a reservoir goes up and down depending on the season.

2. Which graph type…

   a. shows fractions of a total?

      ____

   b. shows data changing over time?

      ____

   c. has a vertical scale to measure separate categories of data?

      ____

   d. uses icons instead of numbers?

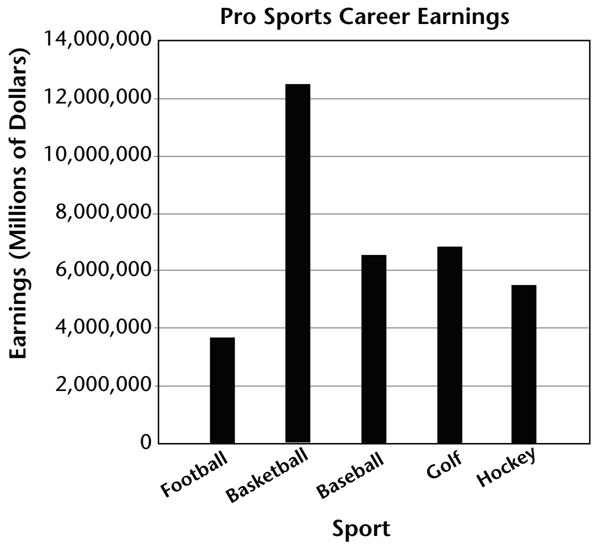
      ____

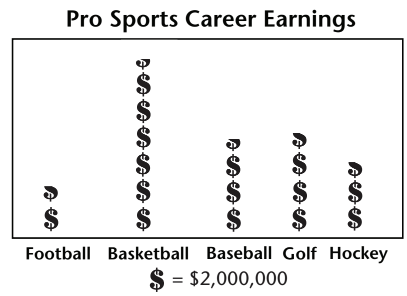
Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
The Challengers: Beneficial Bar Graphs vs. Popular Pictographs

Sometimes data is better shown using a particular format. Compare how the same data set looks for the bar graph and pictograph below.





Because bar graphs have numbers on the vertical axis, the height of each bar can be read from the graph. If you were asked what the lifetime earnings were for each of the pro sports, you could probably do it fairly quickly from the bar graph.

Is a bar graph more or less interesting to look at than a pictograph? Why?

However, if you had to use the pictograph, you might have a bit of trouble. You need to count the icons for each bar, and then multiply by the number given in the key. You also have to decide what to do with partial icons.

Thinking Space

Try It!
Activity 1

Read each scenario and choose between a bar graph or a pictograph according to the reasons given. Circle the letter of the best response.

1. You want to convince your parents to give you more allowance, so you poll your classmates and draw a graph showing the top five allowances compared to yours. You draw a:

   a. pictograph with funny icons because you want your parents to be in a good mood

   b. bar graph that shows what each of your classmates gets, because you want your parents to easily see the approximate amounts

2. In a biking club newsletter, you read that the number of accidents is increasing each year due to increased traffic around the stunt area. The biking club wants the city to block off some of the roads to vehicles around the stunt area. They should present this information in the form of a:

   a. bar graph because it will clearly show the trend of increasing accidents as the bars get bigger

   b. pictograph because it's interesting to look at

3. You're babysitting your six–year–old neighbour and helping him to put together a picture book. He wants to draw a graph that shows that he has a lot more action figures than teddy bears. You help him draw a:

   a. bar graph because it's important to show the numbers accurately

   b. pictograph because icons are a natural fit with the subject matter and audience

Turn to Solutions at the end of the module and mark your work.

Thinking Space

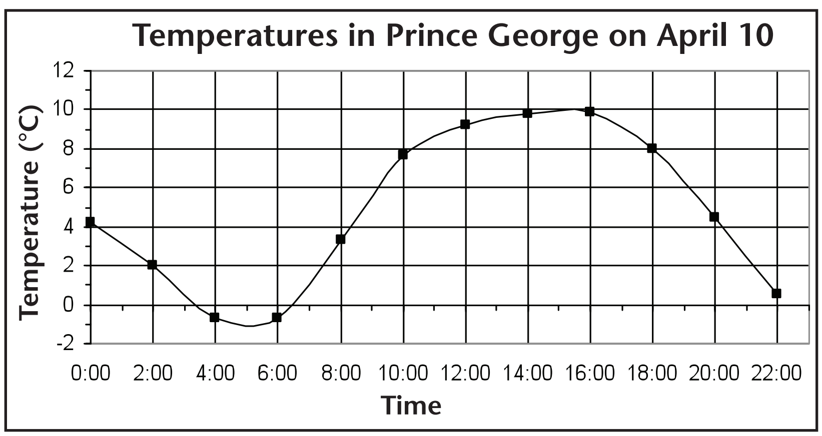
Explore

The Challengers: Legendary Line Graphs vs. Beneficial Bar Graphs

The most common type of line graph is one that measures a changing quantity against time. Time is usually shown along the bottom margin (the horizontal axis) and the other quantity is measured on the left margin (the vertical axis).

Line graphs are also the only type that can handle continuous data (the type that's not in separate categories). For these two reasons, there are really no decisions to be made about when to use a line graph. That's its main advantage.

Let's look at something else we can do with line graphs. The following graph shows the temperatures in Prince George one day in April. The graph plots the temperatures every two hours from midnight (0:00) to 10 p.m. (22:00).



Source: Environment Canada. Hourly Data Report from Climate Data Online

On the graph, times are shown with the 24–hour clock, so 12:00 is noon, and 16:00 is 4 p.m..

Although we don't have a measured temperature for 5 a.m., we can cleverly estimate its value by looking at the graphed line. Find the graphed line at the time between 4:00 and 6:00, and then draw a line with your finger to the temperature scale on the left. It looks as though the temperature at that time is about –1° C.

Thinking Space

We can also answer the question, "When did the highest temperature of the day happen?" The graphed line is highest some time between 14:00 and 16:00. Put your finger on the highest point, and then move it down to the time scale on the bottom. It looks a bit closer to 16:00 than 14:00, so it's probably about 15:30 (or 3:30 p.m.).

Here's another question that we can cleverly estimate the answer to by reading the graph: What do you think the temperature was at 23:00? We don't have a data point at 23:00 because the graph stops at 22:00. But we can see from the beginning of the graph that the temperature seems to keep falling until about 05:00, so it's safe to assume that the temperature in our graph keeps falling, too.

Mark 23:00 on the graph and put a ruler or straightedge on the graphed line. Follow it along the downward slope until you reach 23:00. At that point, it looks like the temperature should be somewhere between –1° C and –2° C, probably about –1.5° C.

Did you know?

When you look "in between" the data points on the graph in order to estimate, you are interpolating data. You cleverly estimate data that you do not have because it was not actually measured.

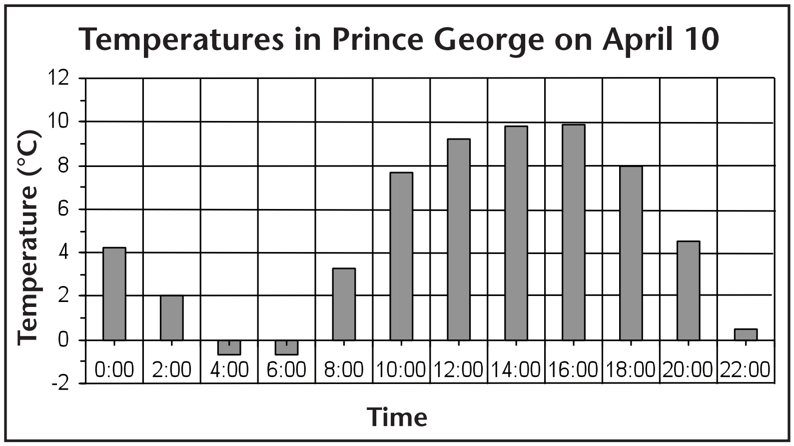
When you look beyond your measured data to estimate, you are extrapolating data. Again, you didn't measure it, but you use your original data to make an intelligent guess.

Is it possible to interpolate or extrapolate on a bar graph or pictograph?

Thinking Space

The Data As a Bar Graph?

To compare what this data looks like in another format, we could plot it as a bar graph.



Although data can be plotted this way, it doesn't show the trend of the temperature as well as a line graph does. It's also harder to interpolate or extrapolate.

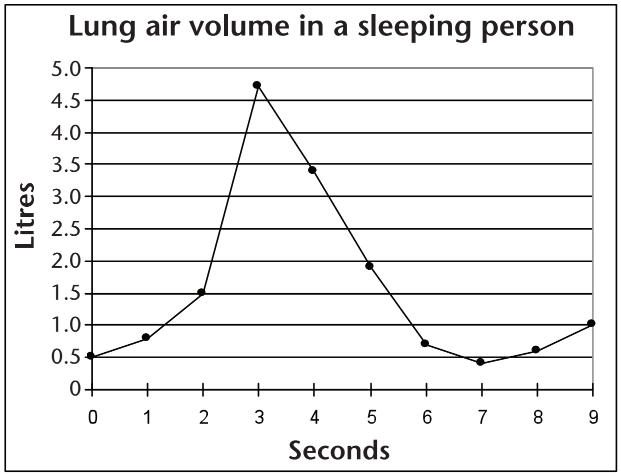
This bar graph looks okay because there are not a lot of data points. If there are a lot, it's better to use a line graph.

Thinking Space

Try It!
Activity 2

Here's a set of data that shows a comparison between time and the litres of air in a sleeping person's lungs.

Seconds	Air Volume
0	0.5
1	0.8
2	1.5
3	4.7
4	3.4
5	1.9
6	0.7
7	0.4
8	0.6
9	1



1. Answer the following questions about the graph.

   a. Describe what happens to air volume.

   ____

   ____

   b. How long does it take to fill the lungs?

   ____

   c. How long does it take to empty the lungs?

   ____

2. We assume the person keeps breathing. Approximate the time the lungs will be full again.

   ____

3. How long is the cycle of breathing for this person?

   ____

Why might the breathing cycle vary from person to person?

Thinking Space

4. Estimate from the graph the air volume at:

   a. 2.5 seconds ______________________

   b. 7.5 seconds ______________________

   c. 15 seconds ______________________

Turn to Solutions at the end of the module and mark your work.

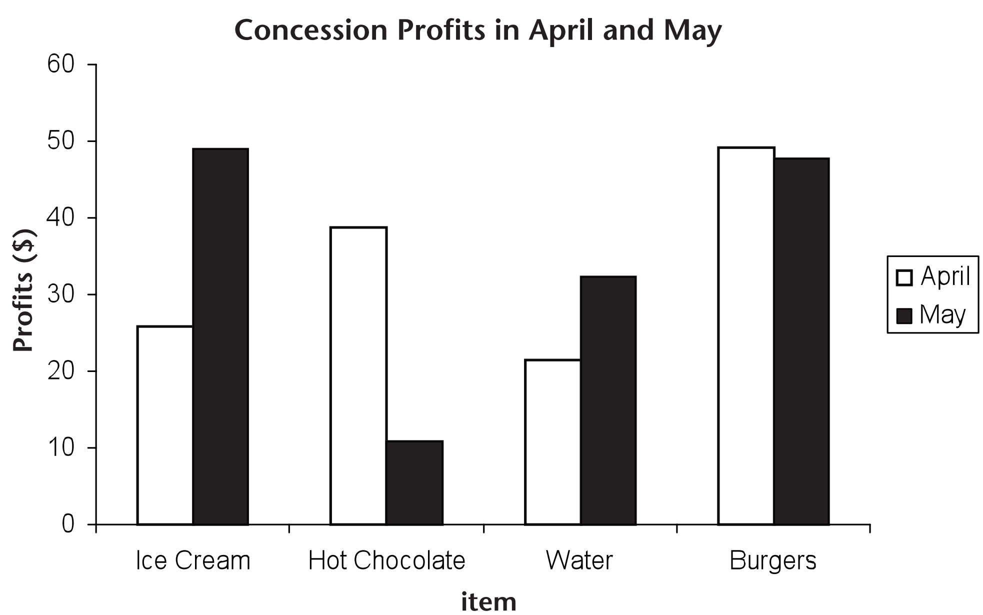
Thinking Space

Explore
The Challengers: Sizzling Circle Graphs vs. Dazzling Double Bar Graphs

Look at the data table, and compare how the data is shown in the following graphs.

Concession Profits in April and May

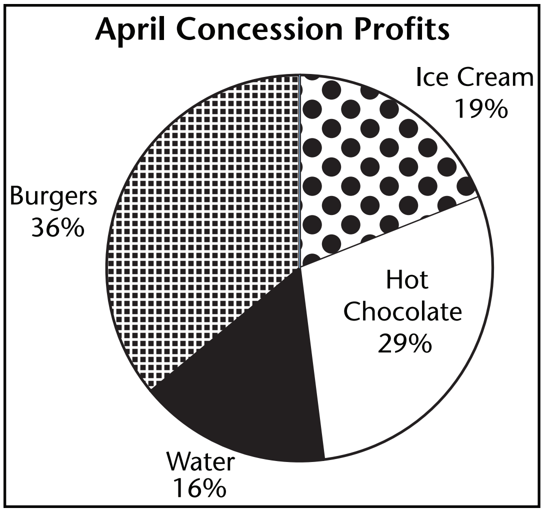
	April	May
Ice Cream	25.75	48.90
Hot Chocolate	38.65	10.80
Water	21.40	32.30
Burgers	49.15	47.75
Total	85.80	92.00

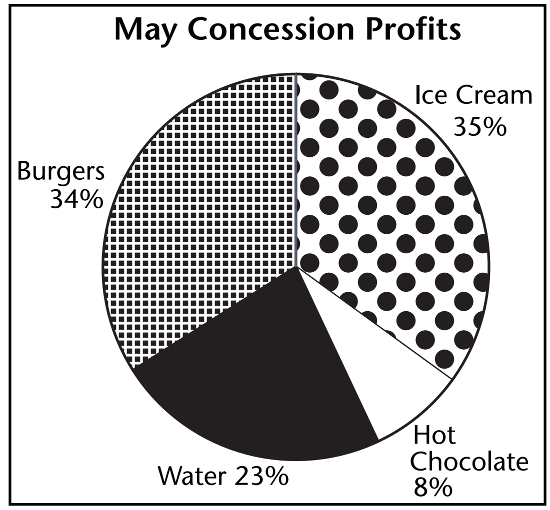


In this double bar graph, it's easy to see the difference between April and May in the profits of the different types of food and drink sold at the concession. The profits for ice cream and water went up in May, and profits for hot chocolate went down. Burger profits stayed almost the same.

What does this same information look like as a circle graph? Well, to start, we have to do two circle graphs to show the same data as the one double bar graph above.

Thinking Space





It's a little bit harder to see the difference between the April and May profits when you have to look at two graphs.

Double bar graphs are good for comparing two related sets of category (or discrete) data.

On the other hand, if we want to know how much the burger profits contributed to the total for the month of May, it would be hard to find that information on the double bar graph. We'd have to add up all the profit amounts for each bar to get the total, divide the burger profits by the total, and then convert it into a percentage. That's a lot of work! Circle graphs show that information right away without any calculations.

Thinking Space

Try It!
Activity 3

You are writing a report for your club that will include some graphs. This will help the club members see the situation quickly without having to read through data tables or a lot of numbers.

For each situation, write which type of graph you would create, and give two reasons why you chose that type. Your reasons can include reasons why you would not choose a different graph type. Use each graph type only once.

Graph types: line, bar, double bar, pictograph, circle

1. Compare the concession sales of pizza to burgers and sandwiches and show approximate values for each.

   Graph type: ______________________________

   Reasons: 1. ______________________________

   2. ______________________________

2. See if there is any connection between the resting heart rate and the height of all players on the basketball team. Try to figure out what the heart rate might be for a 7–foot tall player.

   Graph type: ______________________________

   Reasons: 1. ______________________________

   2. ______________________________

3. Show, at a quick glance and in a humourous way, who got the most votes for "Craziest Outfit on Backwards Day."

   Graph type: ______________________________

   Reasons: 1. ______________________________

   2. ______________________________

Thinking Space

4. Show that well over half the student population participated in the Spring Fair, and that only about 3% of the students didn't participate because they were away on the date of the fair.

   Graph type: ______________________________

   Reasons: 1. ______________________________

   2. ______________________________

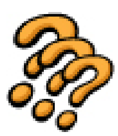
5. Compare the sales of school–themed memorabilia (thermal mugs, T–shirts, sweat pants, and pens) between this year and last year.

   Graph type: ______________________________

   Reasons: 1. ______________________________

   2. ______________________________

Turn to Solutions at the end of the module and mark your work.



You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson C
Misleading Graphs

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

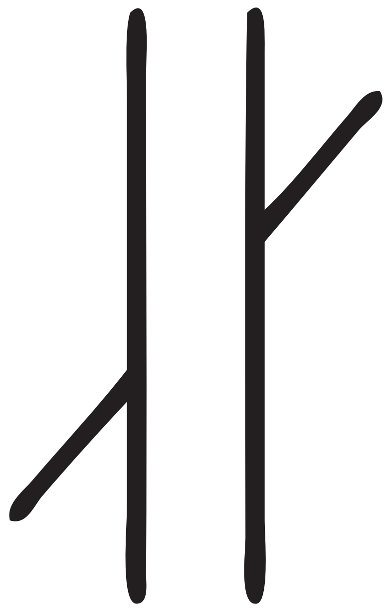
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How do I read graphs critically, in order to avoid being fooled by misleading graphs?		
What does it mean to mislead, misrepresent, or misinterpret?		
How do formatting decisions affect the way a graph is read?		

Thinking Space

Warm–up

What different ways are there to fool the brain, without actually lying? Look at the optical illusions below and answer the questions that follow.

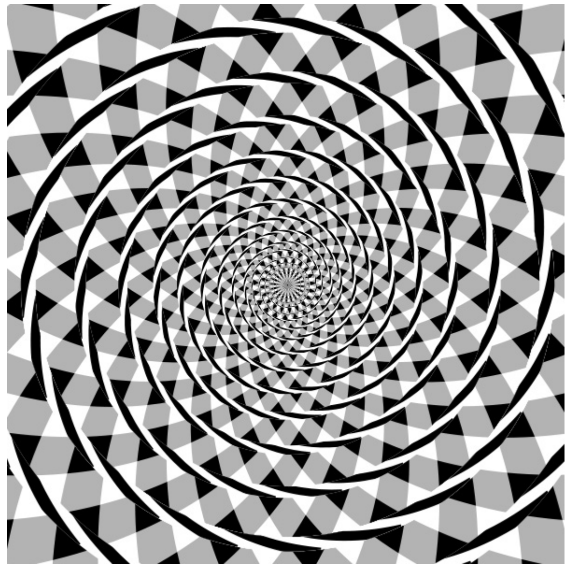
Poggendorff Illusion (1860)



1. The diagonal line is:
   a. one straight line connected at the back
   b. two separate lines not connected at the back

Thinking Space

Fraser's Spiral (1808)



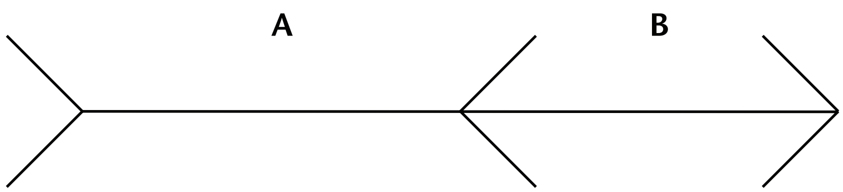
2. Does this figure form a spiral?

   a. yes

   b. no

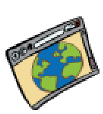
Müller–Lyer Illusion (1889)

3. Which part of the line is longer, A or B?



   a. A is longer

   b. B is longer



If you want to see more optical or visual illusions and you have Internet access, go to the Math 8 website at http://www.openschool.bc.ca/courses/math/math8/mod3.html and click on the link under *Lesson 3.1C: Misleading Graphs*.

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Misleading Statistics and Statements

In Canada, it's illegal to lie in advertising. Sometimes, though, it seems like it happens anyway. What's going on?

It is legal to advertise in truthful, yet misleading ways, and sometimes this occurs. If we are to be wise consumers and voters, we must be aware of the ways in which people can be misled. Otherwise, how can we make good choices?

For example, in a political debate, a candidate who wanted to criticize the school system said "It's a shame that in this country, 15% of our children can't even read!" He probably had some data to support this statement. It might even be true—but is it still misleading?

"Of course that's true," said his opponent, "but it doesn't mean our school system is a failure." She went on to point out that the 15% of children who can't read are the 15% of all children in our society who are under 5 years old. They haven't started school yet. The figure for school–aged illiteracy is much lower, indicating the success of the schools in teaching children to read.

This is just one example of many statements made in debates and advertisements. People naturally will say the things that support their views, and without lying, or without even knowing, they might warp the numbers (or graphs!) to make them more convincing. And as in the case above, a phrase that is omitted from the argument can be crucial information.

Thinking Space

Try It!
Activity 1

Analyze the following statements by finding ways they might mislead. On the lines following, write at least one statement or question in response to each statement.

1. I'm a professional painter, and I use Bling! brand paint on my house!

2. Almost half of our staff was absent from the meeting!

3. 4 out of 5 doctors use Aspelin for their headaches!

4. 3 out of 4 hairdressers use Grot shampoo.

Turn to Solutions at the end of the module and mark your work.

Explore
Foiled and Fooled by Formatting

Just like the optical or visual illusions in the warm–up, graphs can be formatted in ways that encourage people to make false assumptions. The graphs don't lie, but they manipulate the reader into inferring something that isn't true.

Formatting includes the eye–catching features of a document or graph:

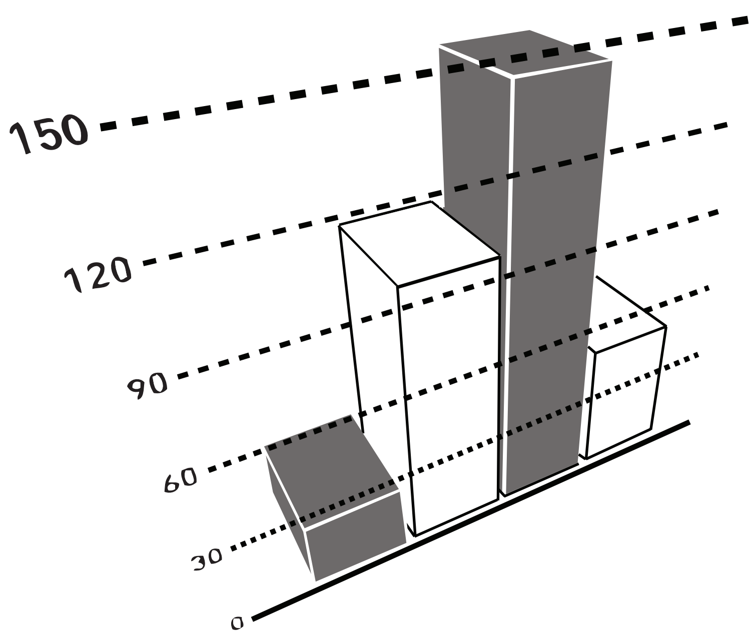
- colours, shadings, patterns
- sizes of objects and text
- positions of objects and text
- use of white space or blank areas

Graphic designers are trained to make use of formatting to improve the readability of documents and images.

Formatting to Fool

On bar graphs, making a bar wider, darker, or separated from the other bars will make it appear more important. Using colour, or a brighter colour, on a bar can do the same thing.

3–D graphs are sometimes used to make the view more interesting.



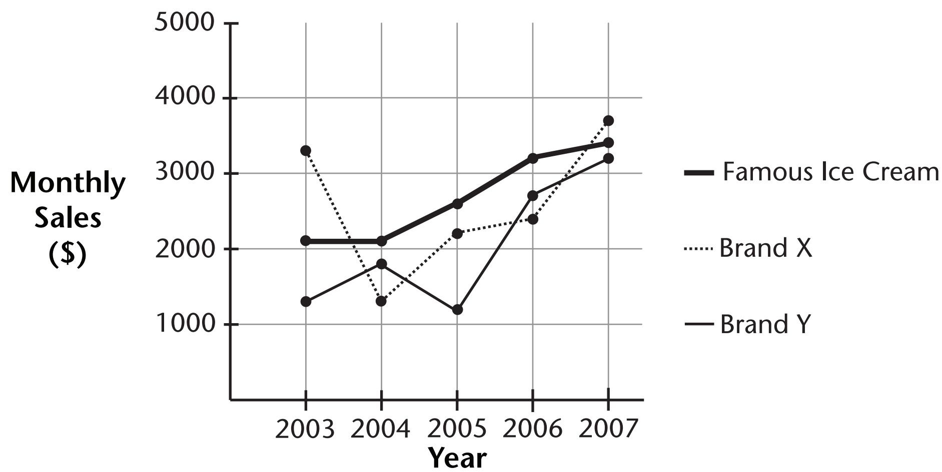
Thinking Space

Can you think of a book or document that was so poorly formatted you didn't want to read it? Can you find an example of interesting formatting?

Thinking Space

However, these can be misleading, because the nearest bars look larger than the ones at the back of the graph.

With line graphs, a line that changes thickness or colour can draw the reader's eye towards a particular area of the graph. Again, creating a 3–D effect can cause misperceptions of the data.



Pictographs are prime targets for formatting foibles. Using different colours, details, or shadings on icons can be hazardous to honest graphing! Icons of different sizes produce different–sized bars, which of course should be avoided.

Thinking Space

Try It!
Activity 2

1. Sketch two bar graphs that show the highest bar as being less important than the second–highest bar.

   a. use colours, shadings, or patterns

   b. use sizes of objects and text

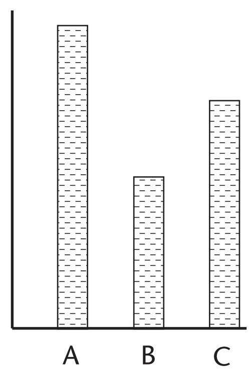
2. Sketch a pictograph of different flowers that results in different–sized columns, but the same number of icons.

Turn to Solutions at the end of the module and mark your work.

Thinking Space

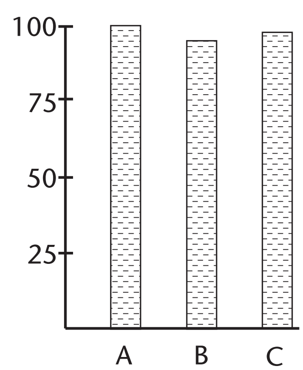
Explore
Struggling With Scale

A very common attempt to mislead a reader is the "scale scam." It works like this: draw a graph—bar or line—and leave the numbers off the vertical axis.



Now, many readers look at this and **assume** that bar A shows about twice as big a number as bar B. After all, the bar is twice as tall. But with no numbers along the left, the graph does not actually say anything at all. It's not, technically, lying to the reader.

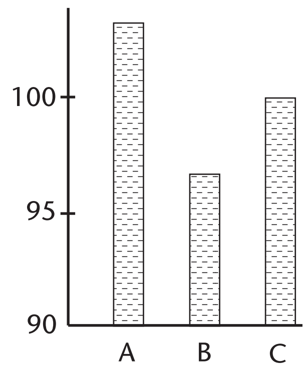
Here's a version of the previous graph which has been changed to allow accurate reading.



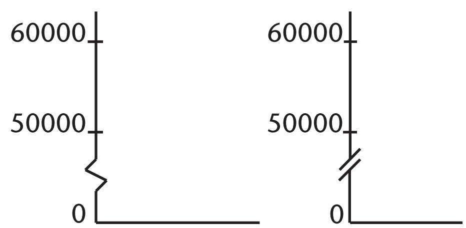
So the first graph is a "zoomed–in" version of the real graph. It only showed the very top of the accurate graph, and that's how it **emphasized the differences** between the bars. When we look at the accurate graph and compare the numbers, we see the differences aren't all that significant.

Thinking Space

This effect also occurs when the scale does not start at zero, or when a “break” is inserted into the vertical axis.



Here are two different ways that the break in a graph can be shown.



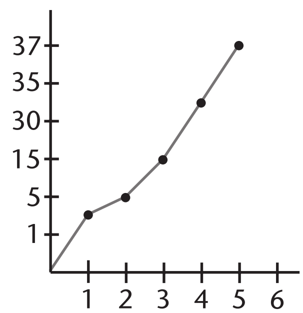
A similar misrepresentation is to have numbers along the scale that do not go up in even intervals. Each grid line, and each interval on the axes, must represent the same amount of data in order for our graph to be honest. Sometimes people forget this when making a graph, and only show the numbers they want to use from their data.

Thinking Space

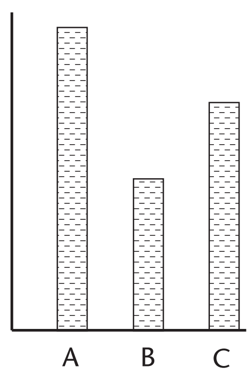
Try It!
Activity 3

Identify the misleading feature of each graph:

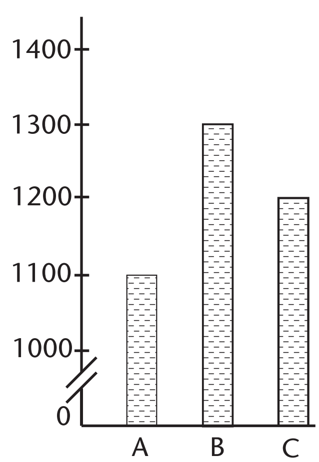
1.



2.



3.



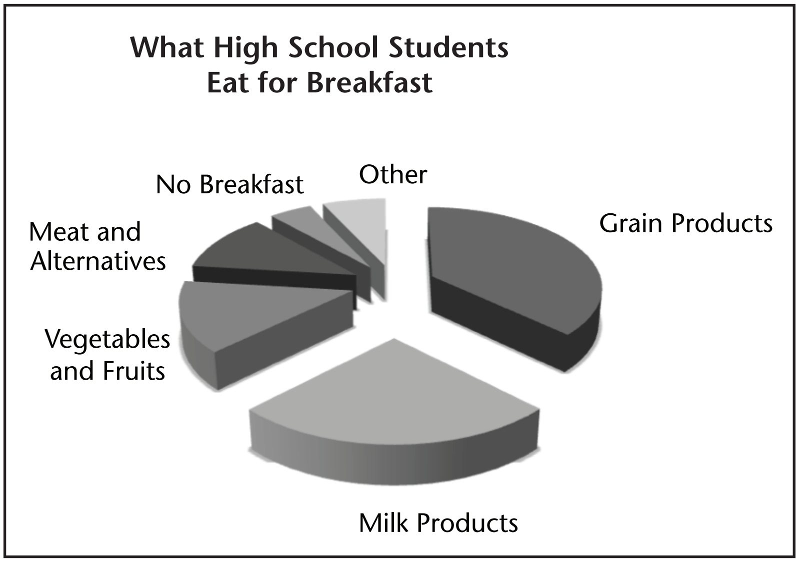
Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Problems With Your Pies?

Circle graphs have their own set of number and formatting problems. Circle or pie graphs are so easy to understand, many people don't bother to read the numbers or percents. Without the numbers, they can be easily mislead by the pie slices.

Sometimes a 3–D pie graph is used, perhaps with an "exploded" slice or slices:

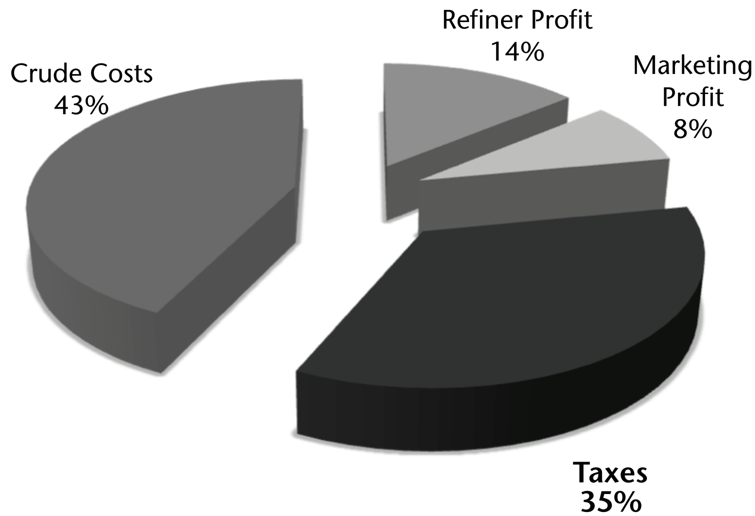


These kinds of pie charts are particularly easy to misunderstand, since the viewer's perspective makes the nearest pie slice the most important or biggest, regardless of the angle or percentage of the circle. Notice how the slice at the bottom seems closer than the others.

An example of this can be seen at gas stations. Some pumps have a circle graph that shows how the money we pay for gas is divided up. Why do you suppose gas companies use a circle graph to show us this information? They want to tell drivers why gas prices are so high, so they put the circle graph right there at the pump. The graph is easy to understand, and keeps the customers from pestering the staff with questions.

Thinking Space

Here's an example of the type of graph found at the pumps:



Reflect about the concept of honesty here. Mathematics (as well as many other skills) can be used honestly or dishonestly. With each new math concept you learn, you'll be more convincing and capable in the real world. Make a brief personal statement about the degree of honesty you feel you can commit to.

As well as informing, this graph can also be used to try to change attitudes. Do you see how the taxes part of the circle graph is nearest to the viewer, and formatted with a darker colour? It almost looks like taxes are the same amount as crude costs if you don't read the numbers. Also, the font for the category is bold instead of normal like the other wedges. All this leads us to think that the gas prices aren't the fault of the service station, but can be blamed on high taxes.

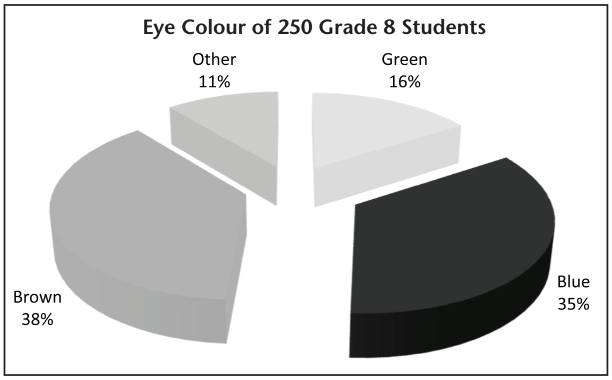
How else can the slices be presented to mislead? We could make a circle graph wedge or slice look bigger than it should be (on purpose or by "accident"). If we label the graph with the correct percentages, then technically we haven't lied—we just let the viewer get a false impression!

Thinking Space

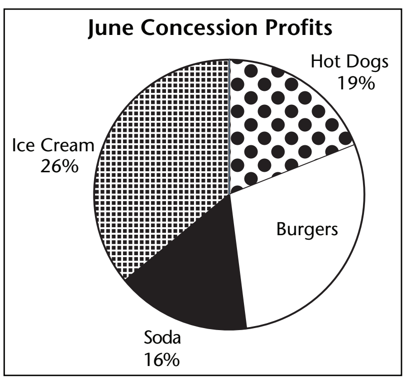
Try It!
Activity 4

Identify the misleading features (if any) in the graphs.

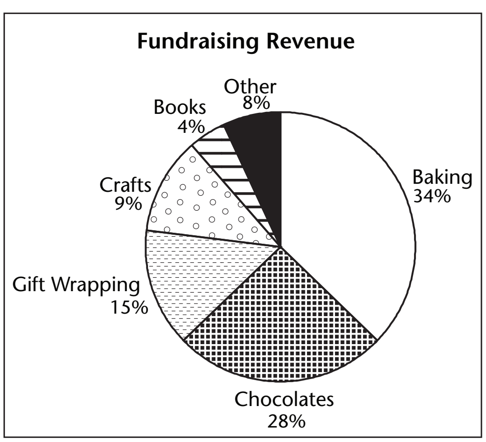
1.



2.



3.



Thinking Space

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson C. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Section Summary

Completing this section has helped you to:

- understand the advantages and disadvantages of each graph type
- identify data sets as continuous or discrete, and know which type(s) of graph to use for a given data set
- identify graphs that show information accurately, and identify those that are misleading
- identify common ways graphs are used to misrepresent information

Section 2
Graphing on the Coordinate Plane

In this section you will:

- plot points and create graphs
- understand the relationships between an ordered pair, points on a graph, and an equation
- find missing values for linear equations from tables of values or graphs
- identify linear equations on a graph

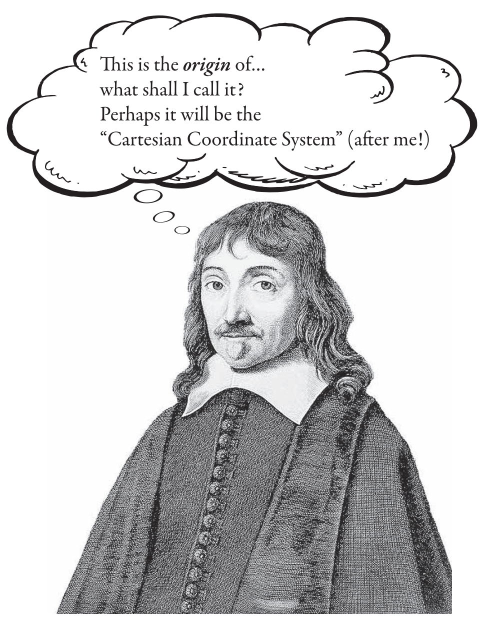
For this section you will need:

- ruler or straight edge

Where in the World...?

He lay in bed, almost delirious with a high fever. As he looked up, he saw a spider crawl across the ceiling… he couldn't tell if it was directly above him, or off to one side. He worried that it might drop onto his face. As he lay there, helpless, he thought about ways to pinpoint the spider's location.

This is how Rene Descartes, a French mathematician, scientist, and philosopher of the 1600s, came up with the idea for the coordinate plane. We use the coordinate plane in graphing to this day.



Section 2
Pretest

Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using the Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson A: Ordered Pairs and Linear Equations

1. A linear equation produces a set of ordered pairs that:

   ____

2. Calculate the matching y-values for the given x-values. Then list the ordered pairs.

   $y = -4x + 2$

   $x = -3$ $x = 0$

   $x = 2$ $x = 5$

   Ordered pairs: ______________________________

3. Finish the table of values for each equation:

$y = 9x$

x	y
–2	
0	
3	
4	

$d = -3c + 12$

c	d
–5	
–2	
0	
3	

$q = \frac{p}{3} + 1$

p	q
–6	
–3	
0	
3	

Lesson B: Tables of Values

4. For the equation $y = 4x + 2$, identify the following:

   a. variable or variables ______________________

   b. any coefficients ______________________

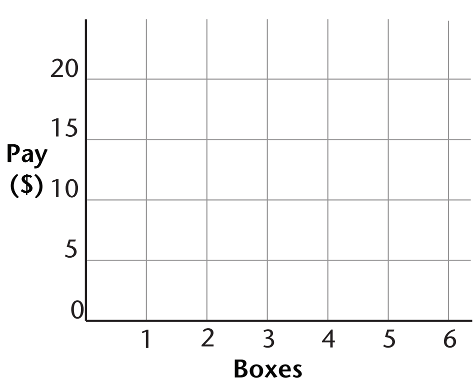
   c. the constant term ______________________

5. Morley constructs boxes at a fruit–packing plant. She gets paid $3.50 for each box she finishes.

   a. Fill in the table of values to show her pay compared to the number of boxes completed. Use the values of 0, 1, 2 and 5 boxes.

boxes				
pay				

   b. Plot the points from the table of values on to the graph.

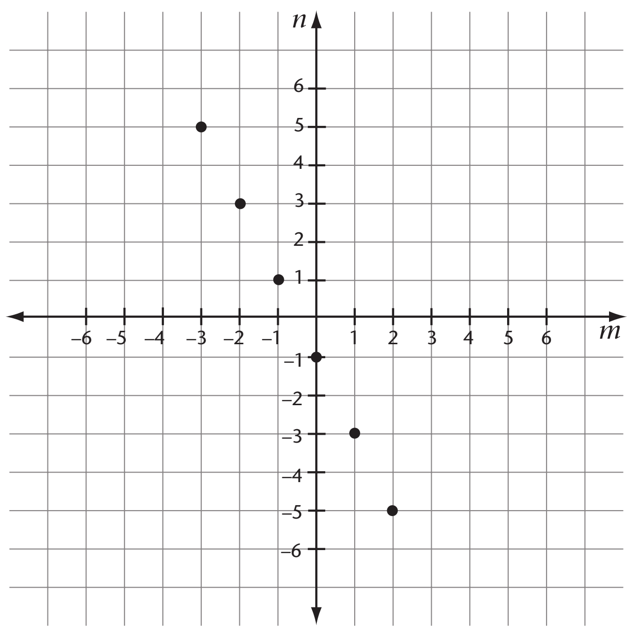


Lesson C: Relationships and Missing Values

6. a. For the following table of values, state the relationship between the two variables.

a	*b*
–3	6
0	0
1	–2
2	–4

b. For the following graph, describe the relationship between the two variables.



Turn to the Solutions at the end of the module and mark your work.

Lesson A
Ordered Pairs and Linear Equations

For this lesson you will need:

- ruler or straightedge

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

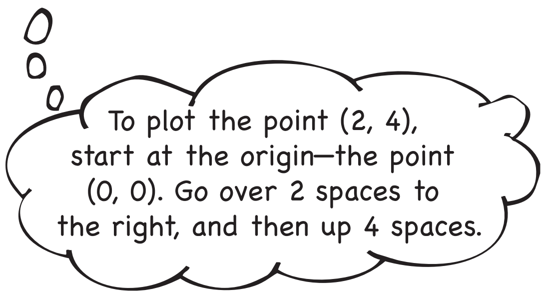
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What are linear equations?		
How do you find ordered pairs from a linear equation?		
How do ordered pairs get graphed?		

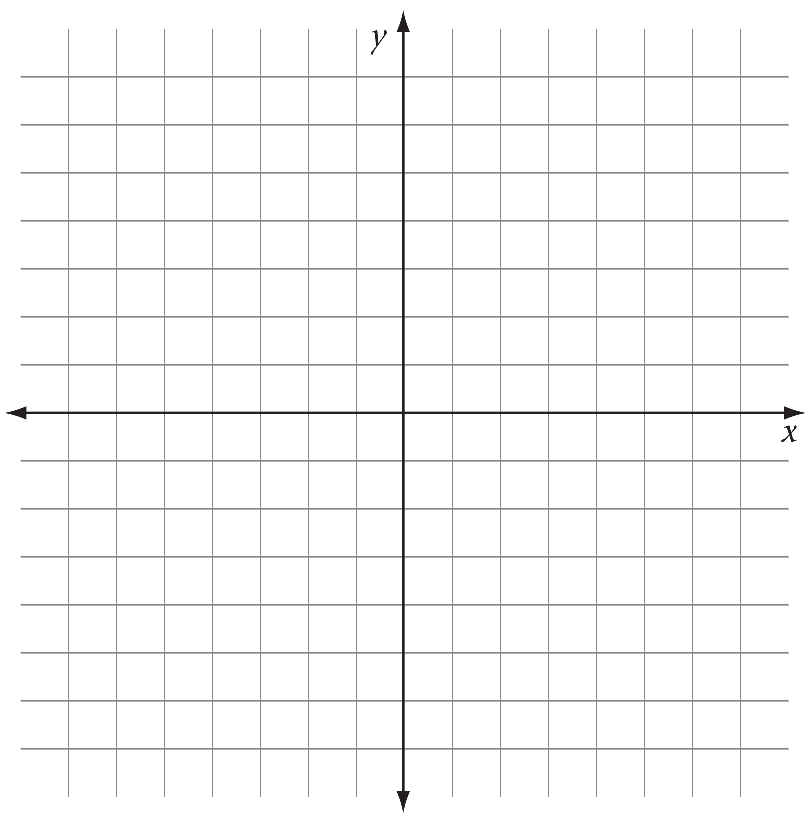
Thinking Space

Warm–up

1. Plot the following points on the coordinate plane. Use a ruler or straightedge to draw a line to connect the points in order as you plot them.

   (2,4) (0,4) (–2,2) (–2,0) (0,–2) (2,–2) (4,0) (4,2)

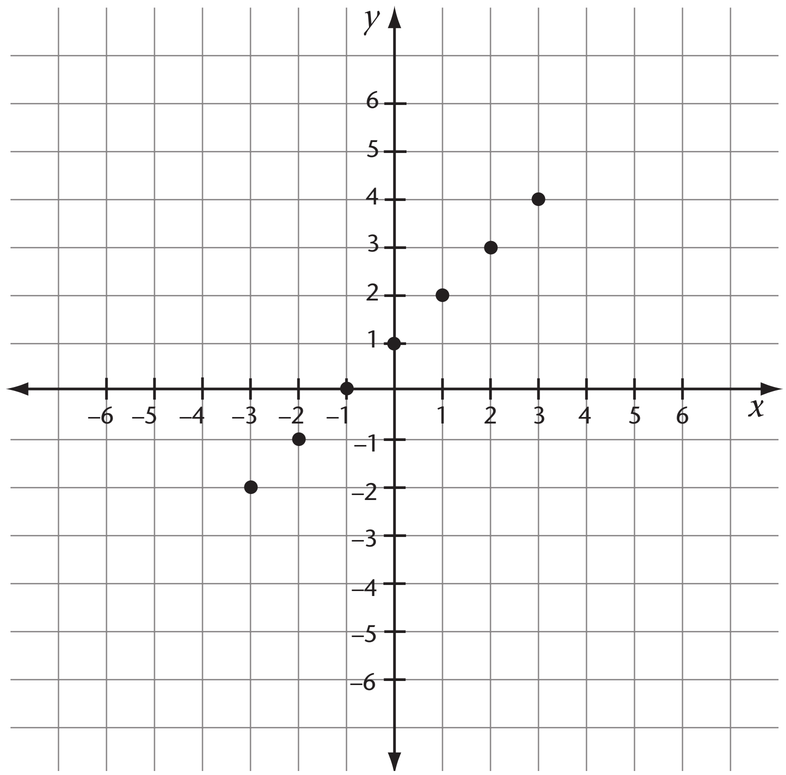




What geometric figure have you drawn? ____________________

Thinking Space

2. What are the coordinates of the following points? List them starting from the bottom left, and moving to the top right.



Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Equations, Graphs, and Points

An equation can be used to describe a group of points on a graph. A *linear* equation describes a group of points that line up. (Note the "line" in the word *linear*). These are examples of linear equations:

$y = 3x$ $\quad\quad$ $y = -4x + 12$ $\quad\quad$ $y = 5x + 2$

Linear equations have the form

$y =$ _____ $x +$ _____

where a number multiplies the *x* and some other number is added (or subtracted) at the end. You can see in the first example ($y = 3x$) that sometimes the last number is zero, and then we don't write it.

Linear equations can also look like the following, but we won't be dealing with them in this math course:

$x + y = 7$ $\quad\quad$ $x = -2y + 5$ $\quad\quad$ $3x - 5y = 21$

The following equations are NOT linear equations—they form curved patterns when graphed:

$y = \dfrac{1}{x}$ $\quad\quad$ $y = 3x^2 + 8$ $\quad\quad$ $x^2 + y^2 = 16$

> Note the exponents and the x in the denominator: these are NOT linear equations.

How can a simple equation represent a bunch of points? The *x* and *y* variables in the equation stand for pairs of numbers called ordered pairs. When you plot the pairs of numbers that make the linear equation into a true statement, they will line up together as points on the graph.

Thinking Space

Ordered Pairs

Let's find some ordered pairs for a linear equation.

$y = 2x - 1$

We can find ordered pairs by choosing a number for x, substituting that number into the equation, and then calculating the matching y-value.

For example, let's start with $x = 1$ and substitute it into the equation:

$y = 2x - 1$
$y = 2(1) - 1$

Now solve this equation to find the value of y.

$y = 2(1) - 1$
$y = 2 - 1$
$y = 1$

So an ordered pair for this equation is $x = 1$ and $y = 1$ or (1, 1).

Let's do it again with an x value of 2. Substitute 2 for x in the equation and do the math:

$y = 2x - 1$
$y = 2(2) - 1$
$y = 4 - 1$
$y = 3$

So when $x = 2$, $y = 3$ and our next ordered pair is (2, 3).

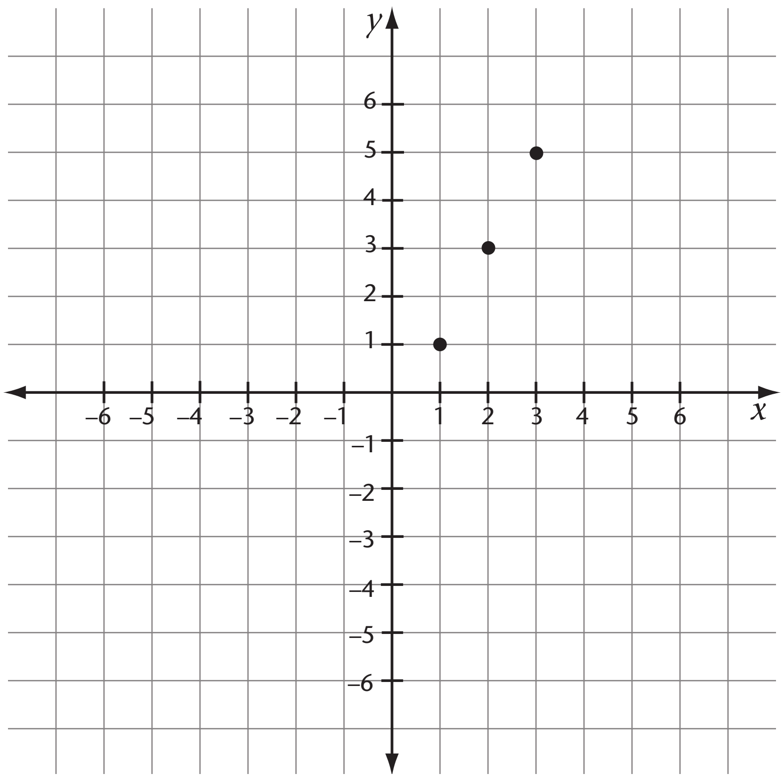
We'll do a third one: use $x = 3$.

$y = 2x - 1$
$y = 2(3) - 1$
$y = 6 - 1$
$y = 5$

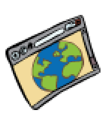
Thinking Space

So when $x = 3$, $y = 5$ and our third ordered pair is (3, 5).

If we plot all 3 points on a grid, we should see them line up nicely:



If a straight line can be defined by two points, why should we bother finding three or even four points for our graph?



To further explore finding ordered pairs for a linear equation, go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0832c1f_linear.html and open *Linear Equations*.

We'll find some ordered pairs for three linear equations, including the example from above, $y = 2x - 1$.

Click the Edit the Equation button on the right side.

The default equation (the one that shows when you first open the media) is $y = 2x + 1$. In order to see the ordered pairs from our first example, we need the +1 to read –1 instead. To do this we'll have to change the b value of the equation.

Click on the Edit b button at the bottom.

Thinking Space

When you see the + and – appear, click the – twice until the number reads –1.

Now the equation should read $y = 2x - 1$.

The table of values at the top shows you four points for this linear equation.

Table of Values

Point	x	y
A	–3	–7
B	–1	3
C	1	1
D	3	5

From this table we can find some ordered pairs for the linear equation. The x- and y-values for each point make up the ordered pairs.

For $y = 2x - 1$, they are

$$(-3, -7) \quad (-1, 3) \quad (1, 1) \quad (3, 5)$$

Practice finding some ordered pairs for other equations in Edit the Equation by changing m, changing b, or changing m and b. As you change the values, see how the line that is formed by the points changes also.

Thinking Space

Try It!
Activity 1

1. For each equation calculate the matching y-values for the given *x-value*s. Then list the ordered pairs. The first question is partly done.

   a. $y = x - 5$

   $x = 0$

   $x = -3$

   $y = x - 5$

   $y = (-3) - 5$

   $y = -8$

   $x = 2$

   $x = 7$

   Ordered pairs: (–3, –8), (0,____), (2,____), (7,____)

   b. $y = -3x + 2$

   $x = -5$

   $x = 0$

   $x = 1$

   $x = 3$

   Ordered pairs: ______________________________

Thinking Space

Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0832c1f_linear.html and open *Linear Equations*.

2. Click on the Edit the Equation button on the right side.

Use the Edit m, Edit b or Edit m and b buttons to change the linear equations to match the ones given. Then list the four ordered pairs that show in the Table of Values.

a. $y = 2x + 3$

Ordered pairs: ______________________________

b. $y = 5x$ (Make $b = 0$)

Ordered pairs: ______________________________

c. $y = 3x - 7$

Ordered pairs: ______________________________

d. $y = -x$ (The media will show $y = -1x$)

Ordered pairs: ______________________________

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

You can't satisfy everyone. But you can satisfy an equation.

Ordered pairs (x, y) can be substituted for the variables of a linear equation. For the equation $y = 8x - 9$, let's try substituting (3, 7) for the x- and y-values, and check to see if it makes the equation true.

We'll use the left side (LS) and right side (RS) method. The line down the middle represents the equal sign (=). We substitute the numbers into the equation, and put the parts to the left and right of the equal sign in their correct places.

Since the y-value is the only thing on the left side, we won't have to do any calculations there.

$y = 8x - 9$ Does the point (3, 7) satisfy the equation?

$7 = 8(3) - 9$

Left Side (LS)	Right Side (RS)
7	8(3) – 9
7	24 – 9
7	15

LS ≠ RS so the point (3, 7) doesn't satisfy the equation $y = 8x - 9$.

Does the point (3, 15) satisfy the equation?

$y = 8x - 9$

$15 = 8(3) - 9$

Left Side (LS)	Right Side (RS)
15	8(3) – 9
15	24 – 9
15	15

LS = RS so the point (3, 15) satisfies the equation $y = 8x - 9$.

Thinking Space

Try It!
Activity 2

1. Use the Left Side (LS) and Right Side (RS) method to decide if the point satisfies the equation. The first one is set up for you.

   a. (2, 11)

   $$y = 2x + 8$$
   $$11 = 2(2) + 8$$

Left Side (LS)	Right Side (RS)

   Does the LS = RS? ___________

   Does the point (2, 11) satisfy the equation $y = 2x + 8$? ___________

   b. (15, 3)

   $$y = \frac{1}{5}x$$

   $\frac{1}{5}x$ is the same as $\frac{x}{5}$ or $x \div 5$.

Left Side (LS)	Right Side (RS)

   Does the LS = RS? ___________

   Does the point (15, 3) satisfy the equation $y = \frac{1}{5}x$? ___________

Thinking Space

c. (–2, 8)

$y = -3x + 2$

Left Side (LS)	Right Side (RS)

Does the LS = RS? ___________

Does the point (2, 11) satisfy the equation $y = -3x + 2$? ___________

2. Which ordered pairs satisfy which equation? Underline or draw a circle or box around the ordered pairs that match each equation. One has been done for you.

$y = \frac{1}{2}x + 4$

$\underline{y = 3x - 2}$

$\boxed{y = -x - 1}$

(–2, 3) (4, 10) (2, 4) (4, 6) (0, –1) (–3, 2) (2, –3) (–4, 2) (–3, –11)

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B
Tables of Values

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What are the parts of a linear equation?		
What is a variable?		
What is a table of values?		
How do I use ordered pairs to make a graph?		

Thinking Space

Warm–up

1. a. Amrit sells her monster cookies for $1.50 each. How much money will she make selling 35?

   b. How many cookies will she need to sell to earn $150?

2. Rowan helps at a vegetable stand at a farmers' market on Saturday mornings. He earns $8 for helping set up the stand, and $0.50 for each kilogram of vegetables that he sells. Calculate how much he earns for the morning if he sells:

   a. 35 kg

   b. 50 kg

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Term Time

Important words to know when dealing with linear relations are **coefficient**, **constant**, **term**, and **variable.**

coefficient
A coefficient is a number that multiplies a variable in a mathematical expression.

constant/constant term
A constant is a number in a mathematical expression that has no variable attached to it. The number can't be changed.

term
A term is an item in an expression that is a constant, variable, or coefficient-and-variable combination.

variable
A variable is a value that is unknown or that could change. It is often represented in an expression by a letter such as x, but could be represented by a word or other symbol.

Why is the constant term called by that name?

What does "variable" mean in a non-mathematical definition?

variables

$y = 3x + 7$ ← constant

coefficient

The terms in this equation are y, $3x$, and 7.

$t = \frac{r}{5} + 2$

This can be rewritten as $t = \frac{1}{5}r + 2$.

Its terms are t, $\frac{1}{5}r$, and 2. The variables are t and r, and the r has a coefficient of $\frac{1}{5}$. The constant term is 2.

$y = -3x$

This equation has no constant term (or we could say that the constant is 0). Its variables are x and y, and the terms are y and $-3x$. The co-efficient of x is -3.

Thinking Space

Try It!
Activity 1

For the following equations, identify the variables, constant, coefficient and terms. Separate the variables and terms with commas. The first one has been done for you.

Equation	Variables	Constant	Coefficient	Terms
$y = -2x + 3$	y, x	3	–2	y, $-2x$, 3
$a = \frac{1}{2}p + 8$				
$c = 4q - 6$				
$m = \frac{n}{3}$				
$s = -4t - \frac{1}{2}$				

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Time to set the table... and plot the graph

A list of several ordered pairs that satisfy an equation is called a table of values. You've seen tables of values already, so we'll just review how to fill them out.

We'll work with the equation $y = \frac{1}{2}x + 6$.

Choose a number to substitute for x, such as 2.

Because we have to take half of the number, we'll start out with even numbers so it's easier to calculate.

$$y = \frac{1}{2}(2) + 6$$

One half of 2 is 1.

Calculate using the order of operations.

$$y = 1 + 6$$

$$y = 7$$

Now you have an entry for the table of values.

(2, 7) →

x	y
2	7

This process should be repeated to make at least three entries for the table. Four or five entries are even better.

Thinking Space

Here are two more points:

Choose $x = -4$

$y = \frac{1}{2}x + 6$

$y = \frac{1}{2}(-4) + 6$

$\frac{1}{2}(4) = 2;\ \frac{1}{2}(-4) = -2$

$y = -2 + 6$

$y = 4$

Choose $x = 5$

$y = \frac{1}{2}x + 6$

$y = \frac{1}{2}(5) + 6$

$\frac{1}{2}(5) = 2\frac{1}{2}$; try 0.5 x 5 on your calculator if it helps

$y = 2\frac{1}{2} + 6$

$y = 8\frac{1}{2}$

We can add these numbers to the table of values. Tables of values can be either vertical or horizontal.

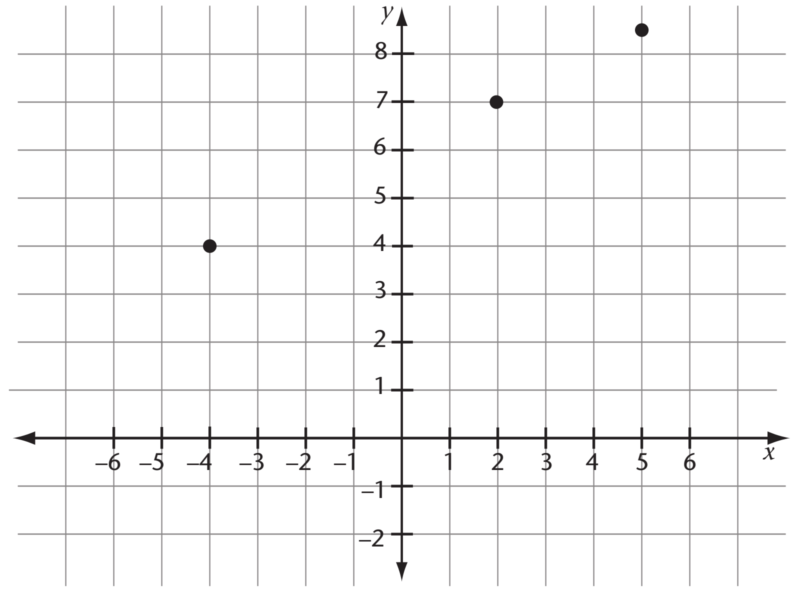
Is zero a good number to choose as a substitute for x? Why or why not?

The value that goes on the horizontal axis of the graph is listed first in the table of values

x	–4	2	5
y	4	7	$8\frac{1}{2}$

Thinking Space

Once you have the table of values, you can use the ordered pairs that you made to plot the points on a graph.



Costs For a Lemonade Stand

Sebastian is going to sell lemonade from a stand in his yard. To help him calculate how much the lemonade will cost to make, we'll create a table of values and draw a graph.

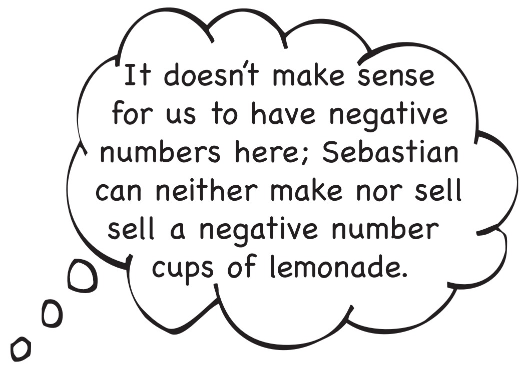
For a day of selling lemonade, Sebastian will have to buy one package of paper cups for \$4. His mom will charge him \$0.20 for the ingredients (lemon juice and sugar) for each glass of lemonade that he makes.

This means that his fixed costs are \$4 (the constant) and his variable costs are \$0.20 per cup. From this information we can make a table of values that shows the number of cups and the cost.

Here is the calculation for two cups of lemonade:

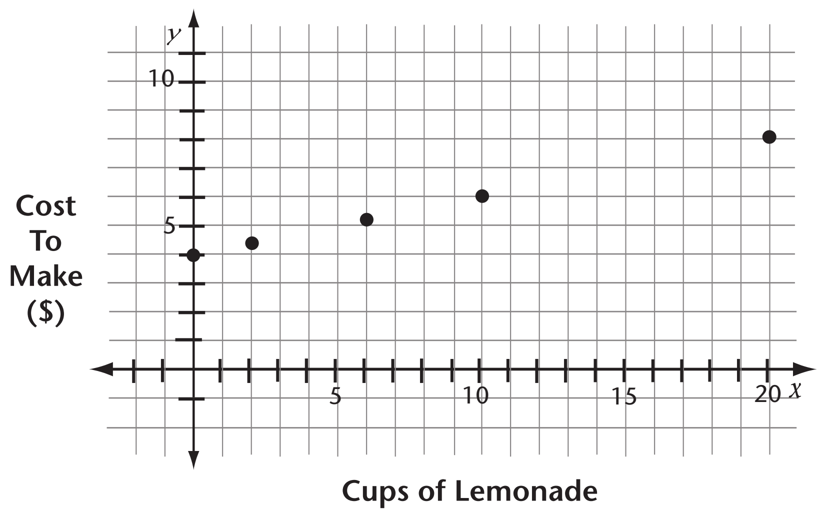
cost = price for cups + \$0.20 for each cup of lemonade
cost = \$4.00 + \$0.20 (2)
cost = \$4.00 + \$0.40
cost = \$4.40

Thinking Space



Cups of lemonade	0	2	6	10	20
Cost	\$4.00	\$4.40	\$5.20	\$6.00	\$8.00

Then we'll plot the graph to see Sebastian's costs.



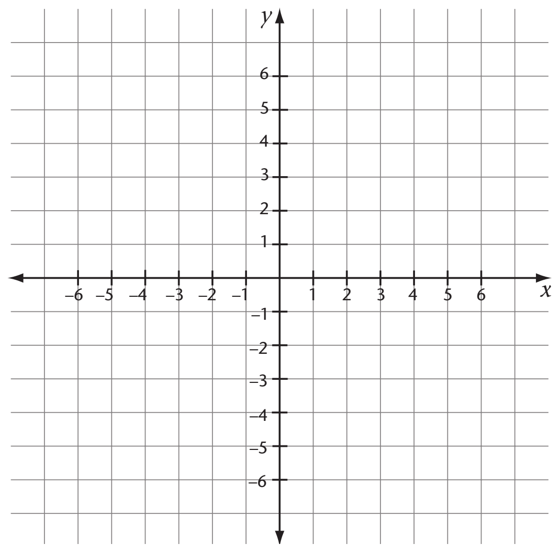
Thinking Space

Try It!
Activity 2

1. For the following linear equations, fill in the table of values using the given *x-value*s. Then plot the graph.

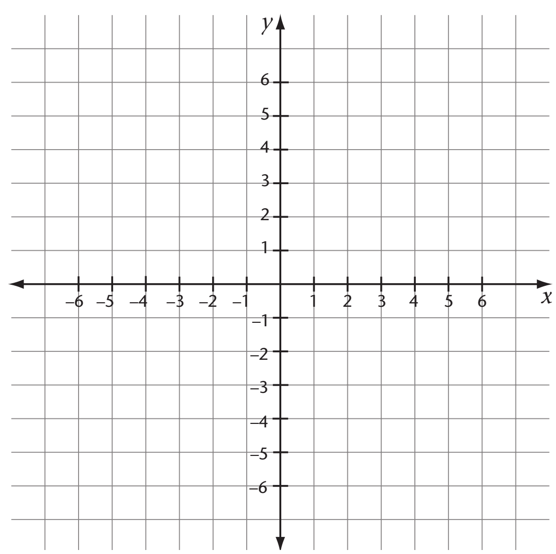
a. $y = x - 2$

x	y
–2	
0	
3	
6	



b. $y = -x$

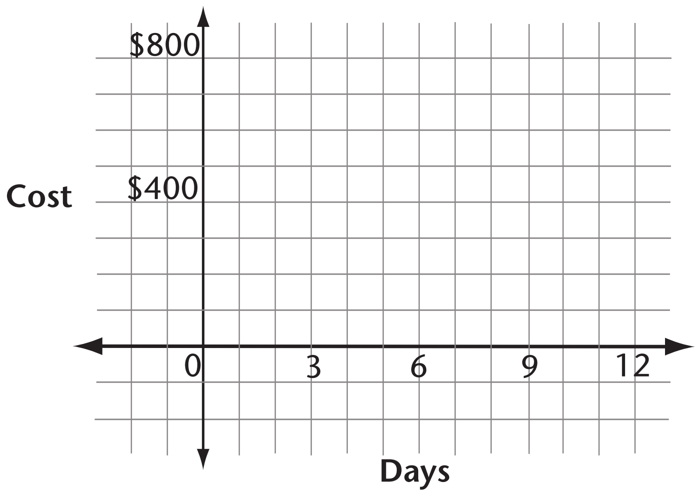
x	y
–4	
–1	
2	
5	



Thinking Space

2. Katie is getting a "value pass" for a local ski resort. She'll pay $250 plus $30 per day of skiing. Fill in the table to show her skiing costs based on how many days she skis. Show the cost for 6 days, 10 days, and two more numbers of your choice. Then plot the graph of the costs.

Days					6	10		
Cost								



Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

An Extra Idea: Graphing on a Spreadsheet (Optional)

If you have a computer with spreadsheet software, you can make excellent graphs very easily. For this example, we'll graph $y = \frac{1}{2}x + 3$

Here are the steps:

Open your spreadsheet program.

Enter the *x*-values into column A by doing the following:

Type –5 in A1 (the top left cell) and press the Enter or Return key.

Continue typing values, pressing the Enter or Return key after each one:

in A2, type –4

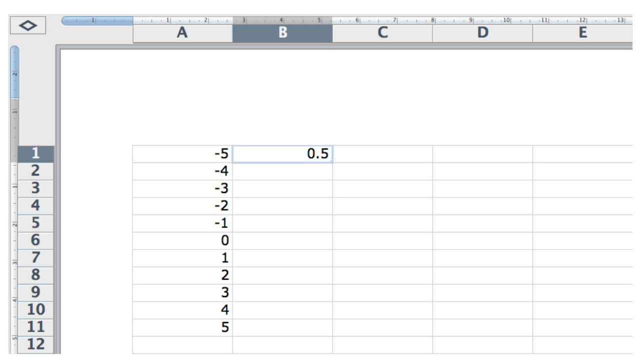
in A3, type –3… and so on, until you finish typing 5 into A11 (don't forget 0).

Enter the formula (linear equation) into column B. Click in cell B1 and type the following:

=(1/2)*A1+3

Press the Enter key.

You should see 0.5 appear in cell B1.



Copy this formula to cells B2 to B11 by doing the following:

Click on cell B1.

Use your mouse to go to the Edit menu and choose Copy.

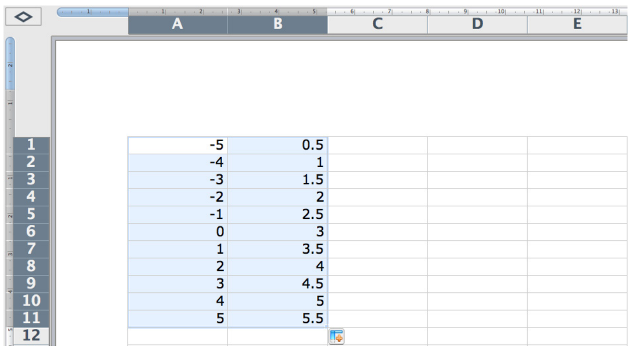
Thinking Space

Use your mouse to drag down through cells B2 to B11 to highlight them.

Use your mouse to go to the Edit menu and choose Paste.

(You may have to press the Esc key to deselect cell B1.)

Compare the resulting numbers with the following:



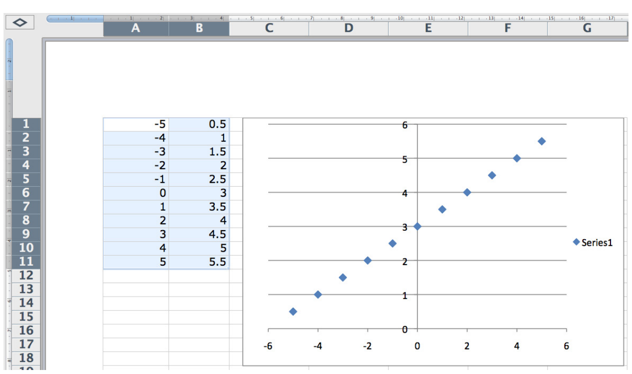
	A	B
1	-5	0.5
2	-4	1
3	-3	1.5
4	-2	2
5	-1	2.5
6	0	3
7	1	3.5
8	2	4
9	3	4.5
10	4	5
11	5	5.5
12		

Use your mouse to drag through cells A1 to B11 (both columns) to highlight them.

Find the command for "Insert Chart" in your spreadsheet program. It may show as a Chart Wizard on the tool bar, or you may find it in the Insert menu.

Choose an XY Scatter plot with no lines and just click the Next button through the rest of the steps.

You should see something like this:



	A	B
1	-5	0.5
2	-4	1
3	-3	1.5
4	-2	2
5	-1	2.5
6	0	3
7	1	3.5
8	2	4
9	3	4.5
10	4	5
11	5	5.5

If you like, you can customize your graph by adding titles and labels for the axes.

Thinking Space

You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson C

Relationships and Missing Values

For this lesson you will need:

- ruler or straightedge

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How are the two variables on a graph related to each other?		
How can you find missing values for a linear equation?		

Thinking Space

Warm–up

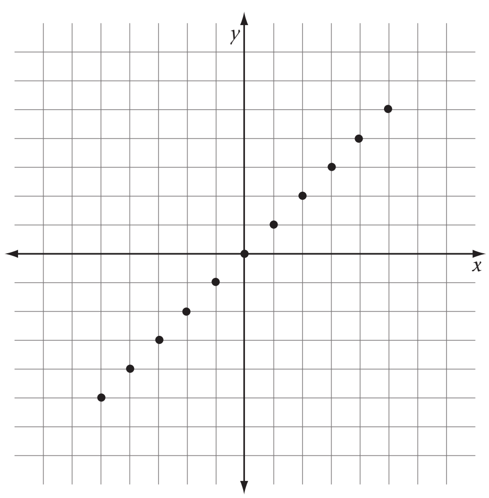
1. Draw a line from the expression to the phrase that describes it.

$2p$	one–quarter of p
$\frac{p}{2}$	one added to p
$p + 1$	twice p
$\frac{1}{4}p$	five added to eight times p
$8p + 5$	p divided by 2

2. Write the correct equation in the blank under each graph. Choose from the following equations:

$y = x + 2$ $\qquad$ $y = x$ $\qquad$ $y = 3x$

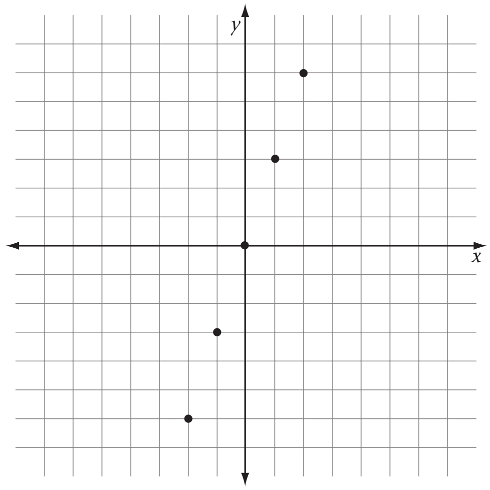
a.



Equation: ______________________

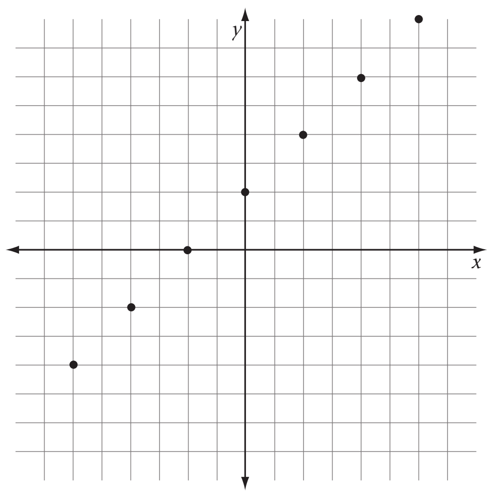
Thinking Space

b.



Equation: ____________________

c.



Equation: ____________________

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Describing Relationships: Start With the Table of Values

Linear relationships might sometimes be given by a table of values.

x	y
2	1
4	2
6	3
8	4

If we look at the values, we can discover the relationship between the two variables.

Look at the first row in the table of values (2, 1). Two possibilities for the relationships are:

1. Take away 1 from x to get y ($2 - 1 = 1$) OR
2. Divide x by 2 to get y ($2 \div 2 = 1$).

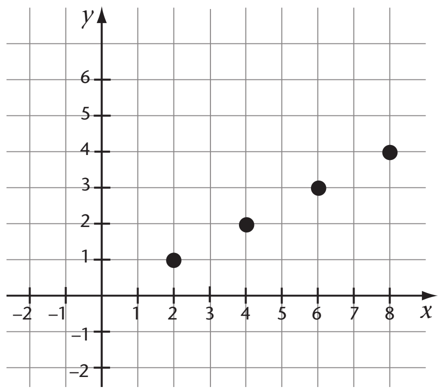
We won't know which one it is until we test another point.

Look at the point (8, 4). We can confirm that you divide x by 2 to get y, not subtract ($8 - 1$ is not 4). So we can say that x is twice as big as y, or the other way around, that y is one–half the value of x.

Be sure to check it against the other points that you have listed in order to confirm your work.

Thinking Space

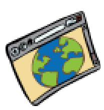
Here's the graph:



x	y
2	1
4	2
6	3
8	4

We can describe the graph by saying that for every one that y increases, x increases by 2.

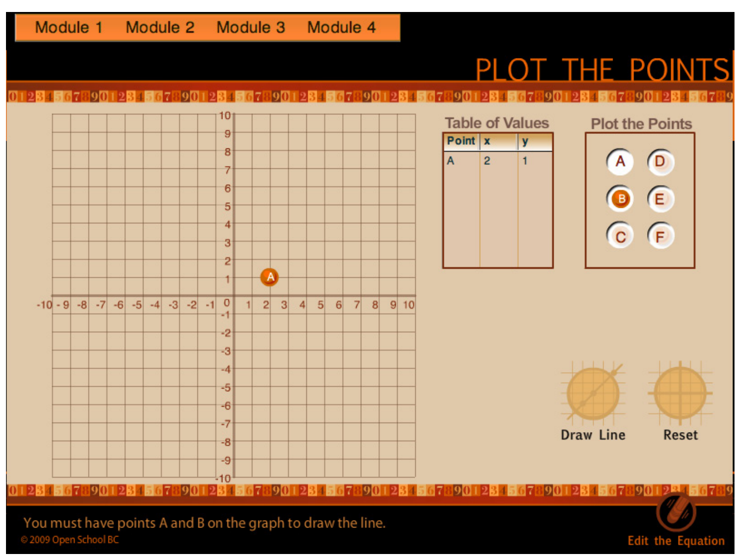
How does this description of the graph compare with the explanation from the table of values?



To further explore the relationships shown in tables of values, go to your http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0832c1f_linear.html and open *Linear Equations*.

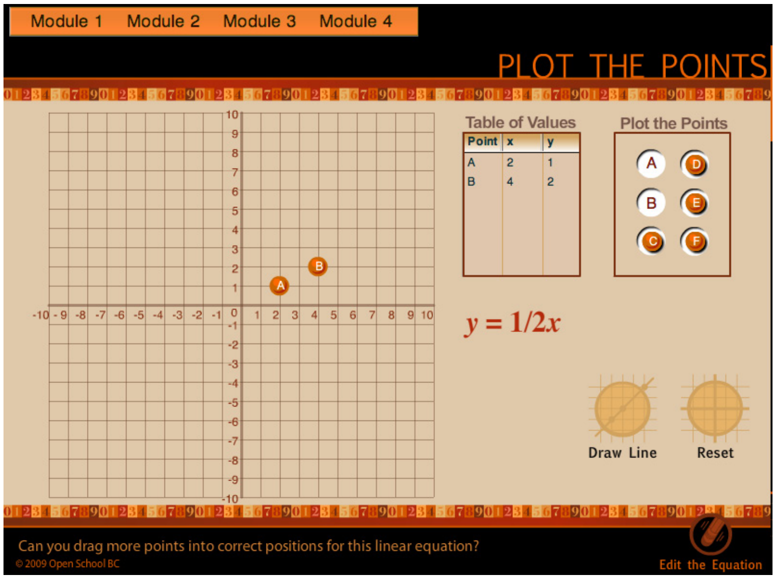
Click on the Plot the Points button on the right side.

Start by dragging Point A to (2, 1). Your screen will look like this:



Then drag Point B to (4,2). The screen will look like this:

Thinking Space



Finish by plotting the last two points shown in this table of values. When you have dragged the third and fourth points onto the grid into the correct positions, you will hear a "ping" from the media.

Point	x	y
A	2	1
B	4	2
C	6	3
D	8	4

After you have plotted two points, you'll see the equation for the line that is made by the points.

For this set of points, the equation will read $y = \frac{1}{2}x$.

How does the equation shown in the media relate to our description of the relationship between the two variables?

Thinking Space

Here's another example. The following table shows Corey's costs when he went to the summer fun fair.

Number of Rides	0	1	2	5	8	10
Cost	\$8	\$10	\$12	\$18	\$24	\$28

Looking at the table of values, you can see the following:

When he went on one ride, it cost him \$2 more than going on no rides (\$10 – \$8 = \$2).

When he went on two rides, it cost him \$2 more than going on one ride (\$12 – \$10 = \$2).

It looks as though each ride cost him \$2.

You can also see from the table that it cost Corey \$8 even when he didn't go on any rides. It probably cost him \$8 just to get into the fairgrounds.

You can test this out by calculating what it would cost Corey to go on 10 rides, and then compare your answer to the one given in the table of values.

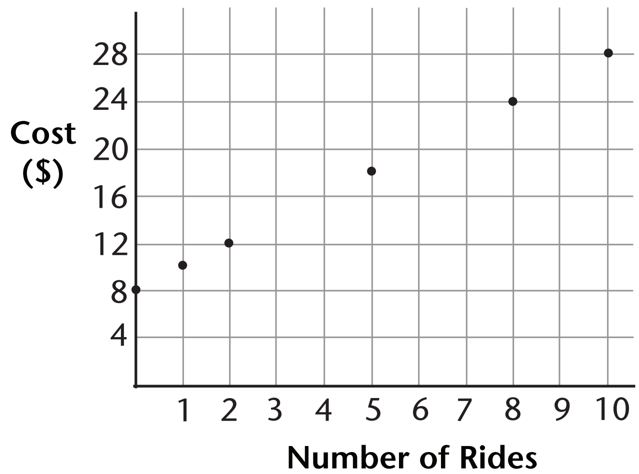
Our guess: admission to the fairgrounds is \$8, and each ride is \$2

For 10 rides: cost = admission + amount for 10 rides
cost = \$8 + 10 (\$2)
cost = \$8 + \$20
cost = \$28

This matches the value given in the table of values, so our description of the cost is probably accurate. To be sure, you could check it against more points.

You can also check it against the graph.

Thinking Space



To describe this graph, we would say that it starts at $8 on the *y*–axis. Then for every one it goes to the right, it goes up two.

How does the description of the graph compare to the explanation from the table of values?

Thinking Space

Try It!
Activity 1

Look at the following tables of values. Describe the relationships between the variables.

1.

x	y
2	–2
8	4
10	6
12	8

Relationship: ____________________

2.

Number of Snacks Consumed at Dance	0	1	3	5
Cost to Attend Dance	$5	$6	$8	$10

Relationship: ____________________

3.

r	t
–3	9
–1	3
0	0
4	–12

Relationship: ____________________

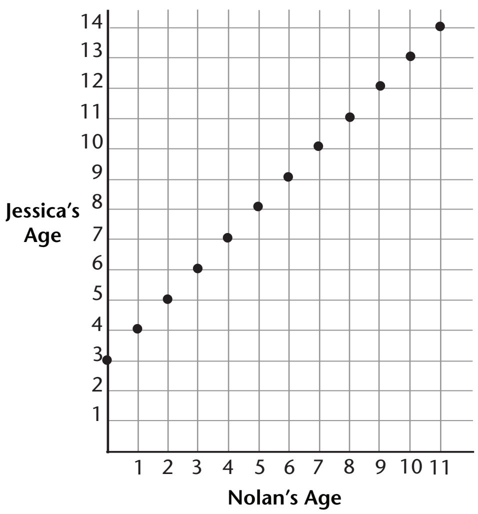
Turn to Solutions at the end of the module and mark your work.

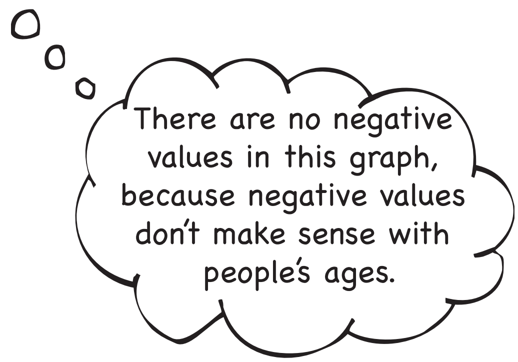
Thinking Space

Explore

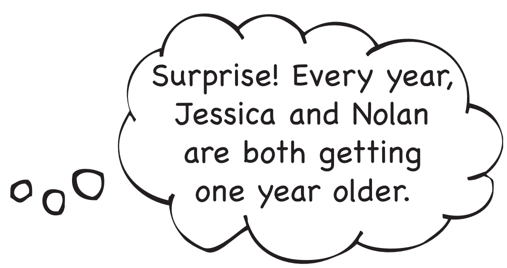
Describing Relationships: Start With the Graph

Here's a graph of the relationship between Jessica's age and the age of her little brother, Nolan.





This graph starts at three on the y–axis. Following the points from left to right, you can see that every time you move over one space, you also move up one space.



When you're given only the graph, it's a good idea to make a table of values containing at least three ordered pairs. These points will give you a better idea how these two are related.

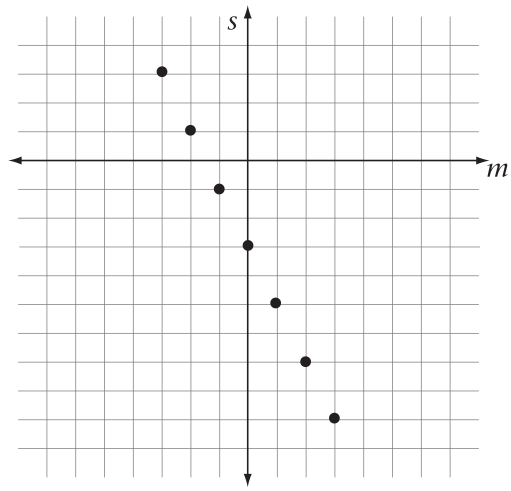
Nolan's Age	0	2	11
Jessica's Age	3	5	14

This means that when Nolan was 0 (just born), Jessica was three years old. Then when Nolan was two, Jessica was five. And now Nolan is 11, and Jessica is 14.

Thinking Space

You may have already noticed that Jessica is three years older than her brother Nolan. To put it into more algebraic terms, Jessica's age is Nolan's age plus three years.

Here's a more general graph:



This graph touches the vertical axis at –3. Looking at the points from left to right, you can see that for every one space the graph goes to the right, it goes down two spaces.

In order to describe the relationship between *m* and *s*, find some ordered pairs and list them in a table of values:

m	0	1	2	3
s	–3	–5	–7	–9

How does this compare with our description of the graph given above?

You can see from the table of values that for every one that *m* increases, *s* decreases by two.

Thinking Space

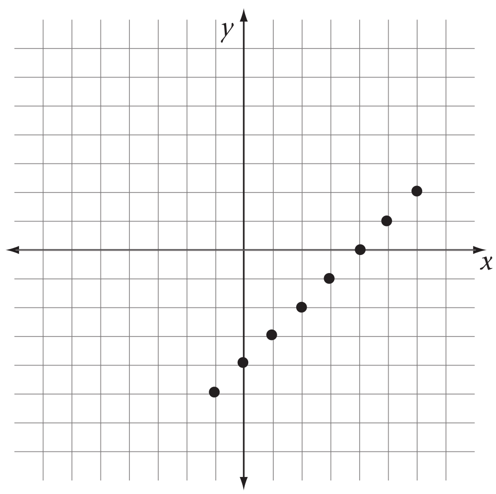
Try It!
Activity 2

Examine the graphs and describe each one by explaining:

- at which point it touches the *y*–axis
- how the position of the points change as you move from left to right

Then list four ordered pairs in the table of values, and describe the relationship between the two variables.

1.



This graph touches the *y*–axis at ______________.

Moving from left to right: ____

____

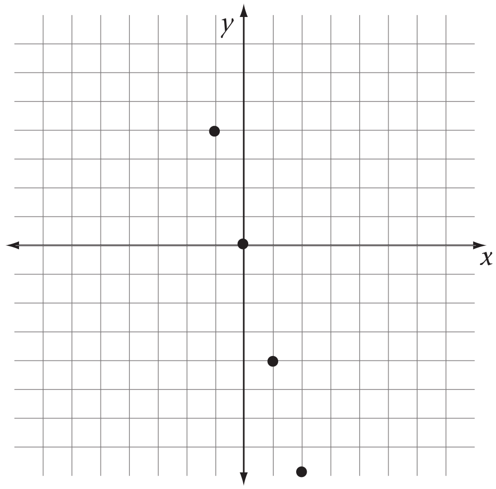
x	–2	0	2	4
y				

Describe the relationship from the table of values: ________________

____

Thinking Space

2.



This graph touches the y–axis at ______________.

Moving from left to right: ____

____

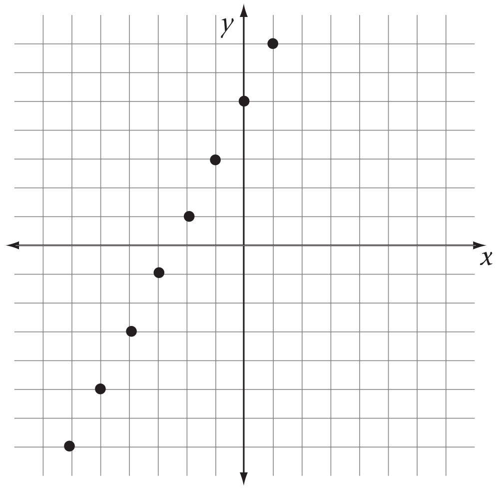
x	–1	0	1	2
y				

Describe the relationship from the table of values: ________________

____

Thinking Space

3.



This graph touches the y–axis at ______________.

Moving from left to right: ____

____

x	–2	–1	0	1
y				

Describe the relationship from the table of values: __________________

____

Turn to Solutions at the end of the module and mark your work.

Thinking Space

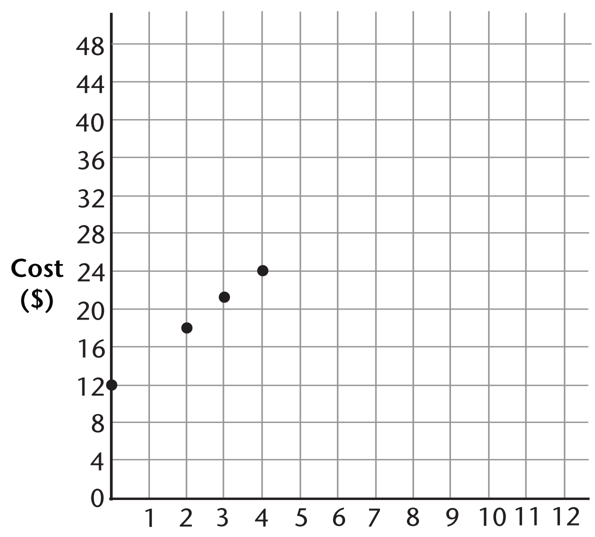
Explore
Find Missing Values

How do we find missing values on a graph or in a table of values for a linear equation? We can use two different methods:

Method 1: We can describe the relationships between variables in a linear equation, and we can use that information to discover missing values.

Method 2: If we're given some points on the graph, we can find others. Since all of the points that satisfy a linear equation will fall on a line, we can use a ruler or straightedge to help us discover other points.

Here's a graph that shows the costs of playing the pro version of an online game.



To start calculating the missing values, read the points that are plotted on the graph. Copy them into a table of values, and include some other values for the number of months.

Thinking Space

Months	0	1	2	3	4	5	6	7	8	9	10	11	12
Cost ($)	12		18	21	24								

Method 1: Look at the values given in the table and describe the relationship.

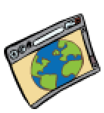
It looks as though every time the month goes up by 1, the cost goes up by 3. Carry on this pattern by adding 3 to the previous cost entry to the get the next one. Be sure to go back and fill in any missing values.

If you like, you can fill in the table above, and then compare your answers to this completed table:

Months	0	1	2	3	4	5	6	7	8	9	10	11	12
Cost ($)	12	15	18	21	24	27	30	33	36	39	42	45	48

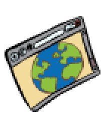
Looking at the table we can see that it will cost us $48 for one year (12 months) of playing the online game.

Method 2: Use the graph and a ruler or straightedge to help find missing values.



Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html, click on Module 3, and watch *Using a Ruler to Find Missing Points*.

Your Turn



To practice finding missing points on a graph, go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0832c1f_linear.html and open *Linear Equations*.

Click on the Plot the Points button on the right side.

Plot the following ordered pairs:

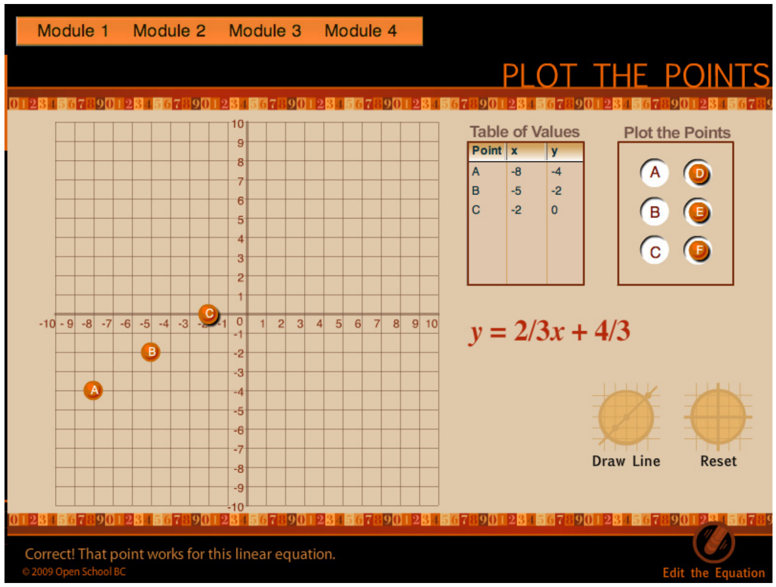
Point A: (–8, –4)

Point B: (–5, –2)

Point C: (–2, 0)

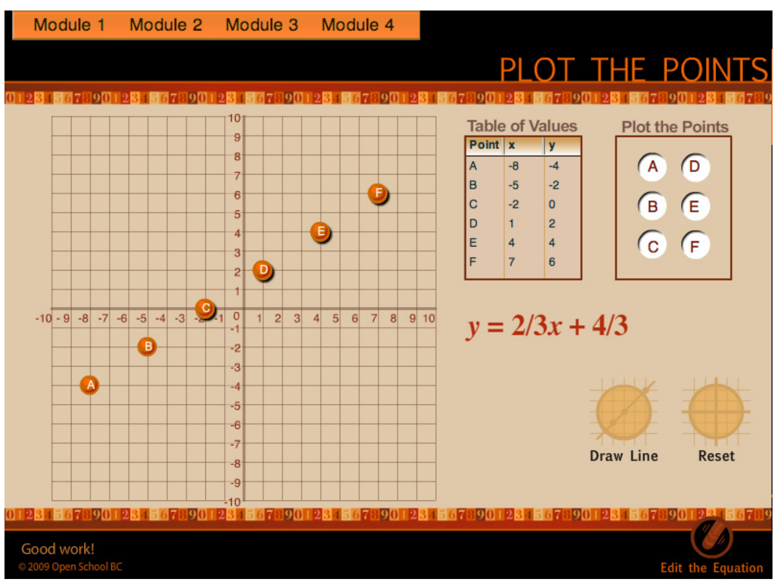
Thinking Space

Your screen should look like this:



You'll notice that when you plot Point C, the media "pings" to tell you that the point is in the correct position.

Now, using the same relationship between pairs of points (A to B, and B to C), try to plot Points D, E and F. Here's one possibility:



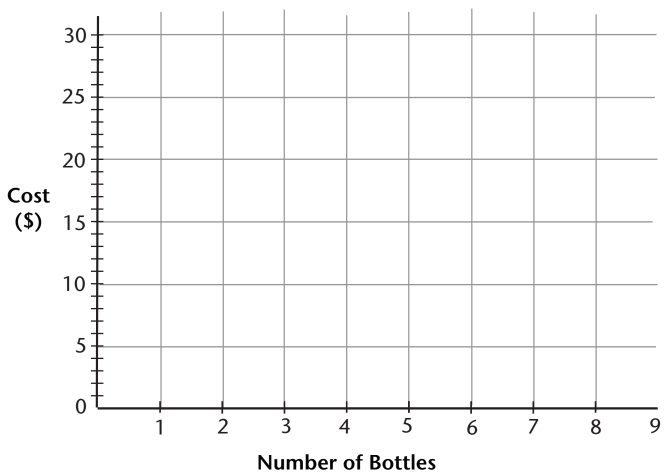
Click the Reset button and practice plotting points for other linear equations.

Thinking Space

Try It!
Activity 3

1. Matteo builds and sells Star Wars models. When he earns a bit more money, he buys paints for his models. Last month he paid the following amounts: $11.00 for 4 bottles, $8.25 for 3 bottles, and $19.25 for 7 bottles.

   a. Plot the points on the grid.



   Lay a straightedge or ruler along the points that you have graphed to make sure that they fall on a line. If they don't, check your calculations, and plot the points again.

   b. Using the information above, fill in the table of values for 3, 4, and 7 bottles.

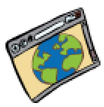
# of Bottles	Cost
3	
4	
7	

Thinking Space

c. Fill in the rest of the table. Use either method explained in the previous Explore.

d. How much does one bottle of paint cost? ________________

e. How much will nine bottles of paint cost? ________________



Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0832c1f_linear.html and open *Linear Equations*.

2. Click on the Plot the Points button on the right.

Drag Point A to (2, –4).

Drag Point B to (–2, 0)

a. What does the equation read? ________________

b. Fill in the missing coordinates in the following table of values:

Point	x	y
A	2	–4
B	–2	
C	0	
D	–5	
E	6	
F	–8	

3. Use Plot the Points to complete the following table of values.

$y = -2x + 3$

x	y
–1	5
	3
1	
2	–1
3	
4	

Thinking Space

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson C. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Thinking Space

Section Summary

Completing this section has helped you to:

- plot points and create graphs
- understand the relationships between an ordered pair, points on a graph, and an equation
- find missing values for linear equations from tables of values or graphs
- identify linear equations on a graph

Section 3
Linear Equations

In this section you will:

- recognize a linear equation
- use algebra tiles to model solutions to linear equations
- solve a linear equation with a fractional coefficient
- apply the distributive property to solve linear equations
- check your solutions to linear equations

For this section you will need:

- scissors
- small plastic bag or container

Where in the World...?

Balance is a hard thing to achieve. We need to balance the demands of school, family, friends, volunteer activities... and when our lives are out of balance we don't feel very happy.

The good news is that balance in mathematics is easier to achieve. In ancient Persia, the process of solving mathematical equations was called *al-jabur*, which meant "balance." From this term we derive our word *algebra*.

Muhammad ibn Mūsā al-Khwārizmī was a Persian mathematician, astronomer and geographer. His book Algebra was the first on the systematic solving of linear and quadratic equations.

Section 3
Pretest

Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using the Solutions found at the end of the module.

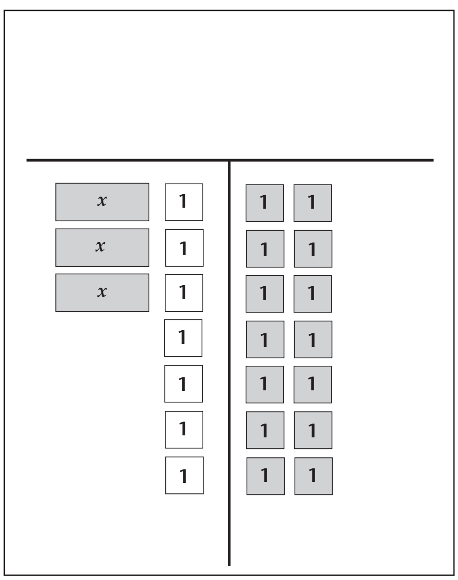
If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson A: Algebra Tiles

Use the algebra tile pictures to complete the following:

1.



a. What is the equation that is represented by the algebra tiles? Write the equation at the top of the equation sheet.

b. Explain in words how you would use algebra tiles to solve the equation.

____

____

____

____

c. What is the solution to the equation?

Lesson B: Solving Linear Equations Symbolically

2. Solve the following equations.

a. $3x - 11 = 22$

b. $\frac{a}{3} + 9 = 15$

c. $4x - 7 = 25$

d. $-5m - 7 = -22$

Lesson C: More Linear Equations

3. Solve each equation:

   a. $-2(x + 7) = -6$

   b. $-2y + 5 - 4y = 1$

   c. $3(x + 8) = 18$

   d. $\frac{r}{6} - 9 = -3$

4. Describe in words the steps to solve $4(t - 9) = 3t$

Turn to the Solutions at the end of the module and mark your work.

Lesson A
Algebra Tiles

For this lesson you will need:

- scissors
- small plastic container or bag

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What is a linear equation?		
What are some ways to “model” a linear equation?		
How can we solve a linear equation using a model?		

Thinking Space

Warm–up

Ward has a problem at his new job as a waiter. He has to put several objects from the kitchen on a tray, walk them over to a table, then take them off without unbalancing the tray. He must support the tray with one hand, and take the objects on and off with the other.

1. Suggest some strategies to help Ward cope with his new job.

For one order, Ward has the following on his tray:

- Four drinks (all weighing the same)
- Two plates (weighing the same)
- One bowl of soup
- Two salads

The two salads together weigh as much as one plate and the soup together.

2. What mathematical operations do you find described in this situation?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Working With Algebra Tiles

For this Explore you will need:

- Algebra Tiles Template (in the back of the module)
- scissors
- small plastic bag or other container

Before we work with numbers and variables, we'll explore how to keep equations balanced and solve them by using algebra tiles. Go to the Algebra Tiles Template in the Appendix and remove the page. The algebra tiles look like this: (but you'll have two on the page, one clear set and one shaded set).

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	x	x			
1	1	x	x			
1	1	x	x			
1	1	x	x			
1	1	x	x			
1	1	x	x			
1	1	x	x			

Cut out the tiles carefully, and put them in a small plastic bag or other container. Note what they are:

- Small clear squares are "unit" tiles that each represent 1.
- Small shaded squares represent –1 (or subtracting 1).
- Clear rectangles represent the variable x.
- Shaded rectangles represent $-x$ or $(-1)x$.

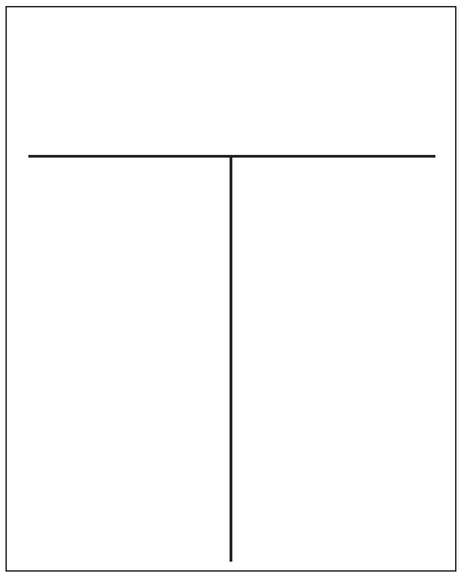
In order to model linear equations, we need to know how to set up the algebra tiles.

Thinking Space

Create an Equation Sheet

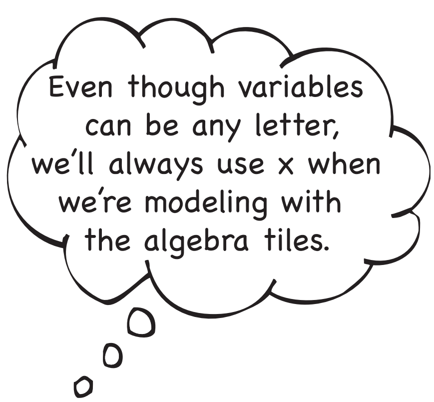
On a blank piece of paper, draw a horizontal line about one-third the way down. You'll put the equation in the top part and use the bottom part to set out the algebra tiles. In the bottom part, draw a vertical line half way across the page. This is our equation sheet.

It should look like this:



The vertical line down the middle of the bottom part represents the equals sign (=) in an equation.

Write the following equation on a sticky note or small piece of paper and place it in the top section of the equation sheet, making sure you place the equals sign (=) over the vertical line.



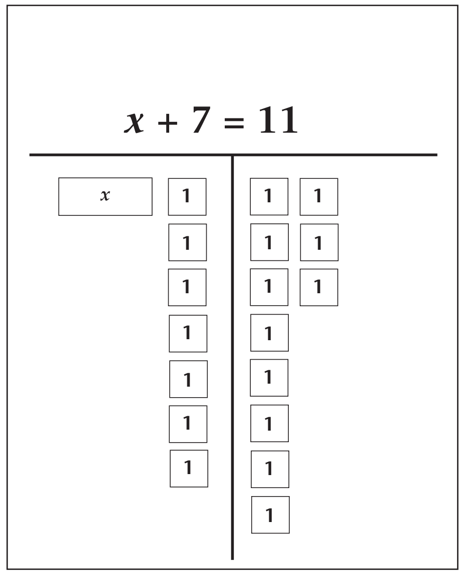
$$x + 7 = 11$$

Thinking Space

Now let's arrange our algebra tiles on the equation sheet.

- On the left side, put a $\boxed{\quad x \quad}$ and seven $\boxed{1}$.
- On the right side, put eleven $\boxed{1}$.

Your model should look like this:



In the model above, the numbers and the variable are positive. Let's try another model with some negatives. On another sticky note or small piece of paper, write this equation:

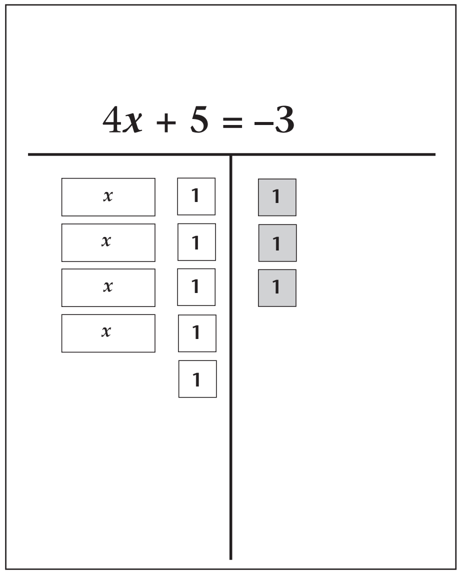
$$4x + 5 = -3$$

Put the paper in the top section of the equation paper.

Then set out the tiles to represent the equation.

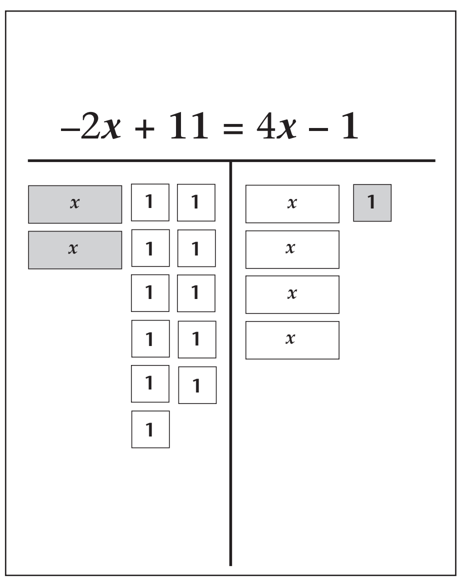
Thinking Space

Compare your model with the following:



Try one last example that is a little more complex.

$$-2x + 11 = 4x - 1$$

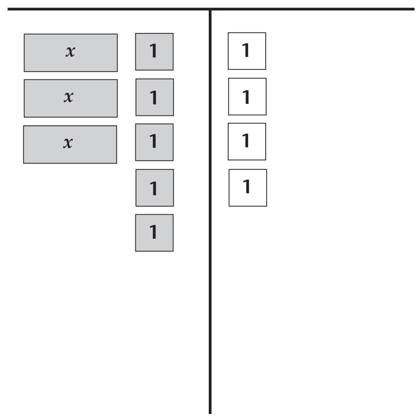


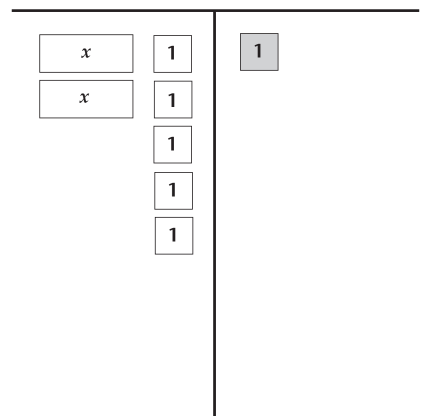
Thinking Space

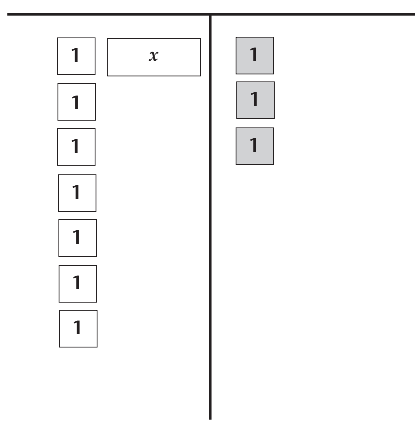
Try It!
Activity 1

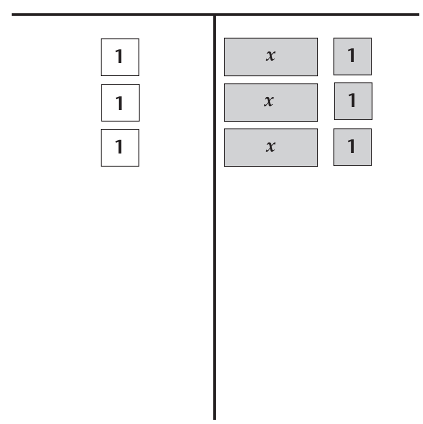
Write the correct equation for each picture. The first one is done for you.

$-3x - 5 = 4$









Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

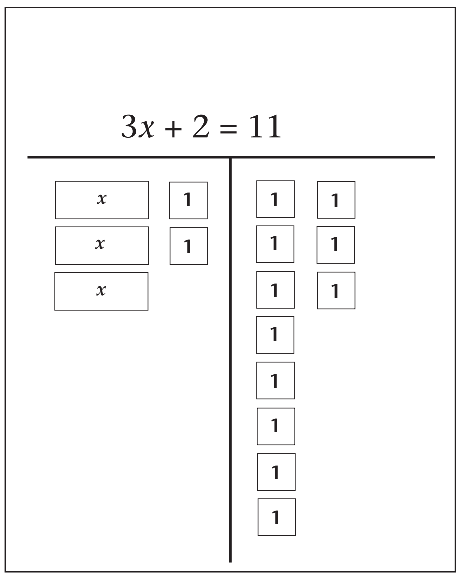
Solving a Linear Equation Using Algebra Tiles

AKA The *X*-Tiles: The Truth is Out There

For this Explore you will need:

- algebra tiles

Now we're going to work the algebra tiles a little harder and use them to solve some linear equations. Let's start by setting out tiles to represent $3x + 2 = 11$ on your equation sheet. Write the equation on a sticky note or small piece of paper, and set it at the top.

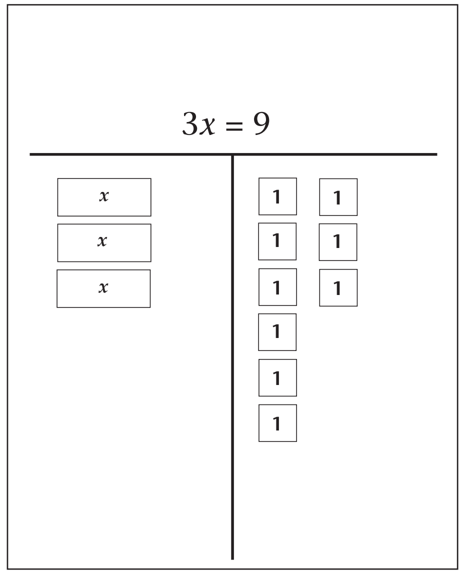


We'll make changes to the tiles, making sure that we make the same change to each side to keep it balanced. We'll keep making changes until each *x*-tile on one side can be balanced with an equal number of 1-tiles on the other side.

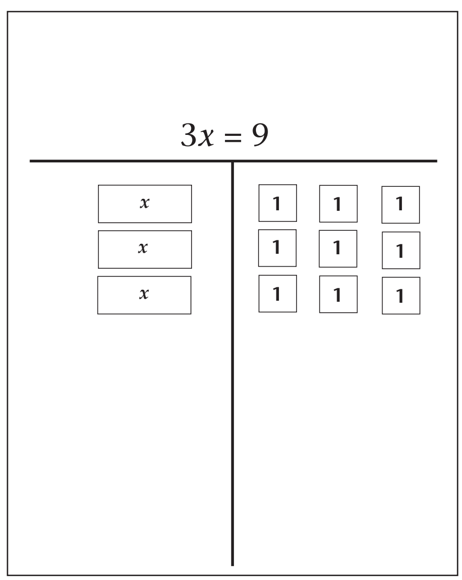
- Start by taking away two $\boxed{1}$ on the left.
- Take away two $\boxed{1}$ on the right as well, to keep the equation balanced.

Thinking Space

You should now have three $\boxed{x}$ on the left and nine $\boxed{1}$ on the right.



- Put the three $\boxed{x}$ on the left side in a column.
- Arrange the $\boxed{1}$ on the right side into three equal groups. Put one group next to each $\boxed{x}$.



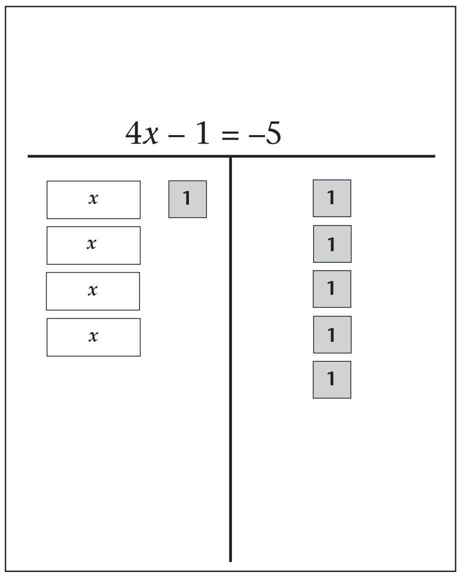
Each $\boxed{x}$ matches exactly with three $\boxed{1}$, which means that $x = 3$. We've solved the equation!

Thinking Space

Go to your http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html, click on Module 3, and watch *Working with Algebra Tiles*. **Note**: The video shows two different equations, one simple and one more complex. If you like, you can just watch the first part of the video at this time.

Let's try an example with some negative numbers:

$$4x - 1 = -5$$



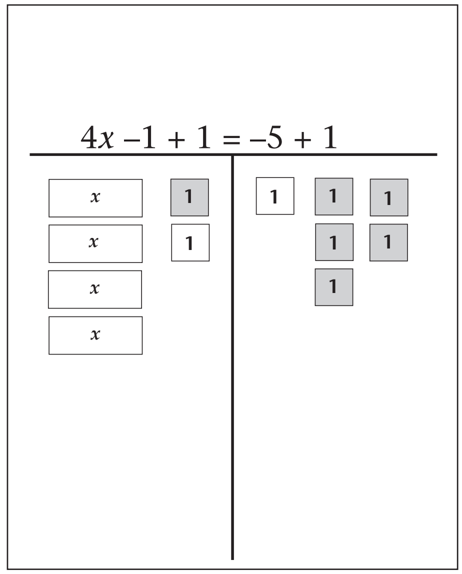
To get rid of the five $\boxed{1}$ on the right, we could try taking them away. However, we can't go ahead with operations in algebra unless we can do the same thing to the other side. Looking at the left side, we see that there is only one $\boxed{1}$ to remove.

What do we do now? You might remember from the video that we created zero pairs, and this is how we're going to solve our problem.

We'll add a $\boxed{1}$ on the left to cancel out the negative one. We can do this because $+1 + (-1) = 0$. When we match a positive tile with a negative one, we've created a zero pair, so you will be able to remove the pair from that side of the equation sheet. But before you remove anything, you must remember to add the same number of tiles to the right side to keep everything balanced (in this case it's only one).

Thinking Space

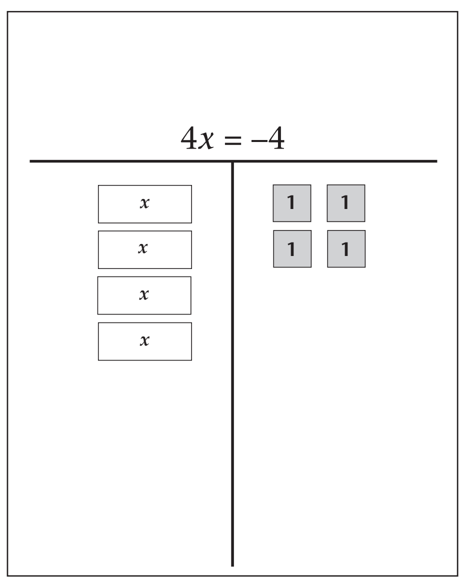
- Add one $\boxed{1}$ to the left and one $\boxed{1}$ to the right.



Each $\boxed{1}$ (+1) cancels out one $\boxed{1}$ (–1).

- Take the zero pair off your equation sheet on both sides.

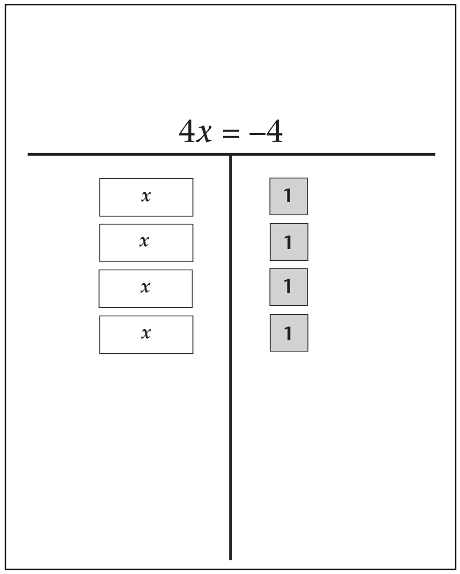
Your equation sheet should now look like this:



Since we only have *x*-tiles on the left side, we can now arrange the tiles to find our answer.

Thinking Space

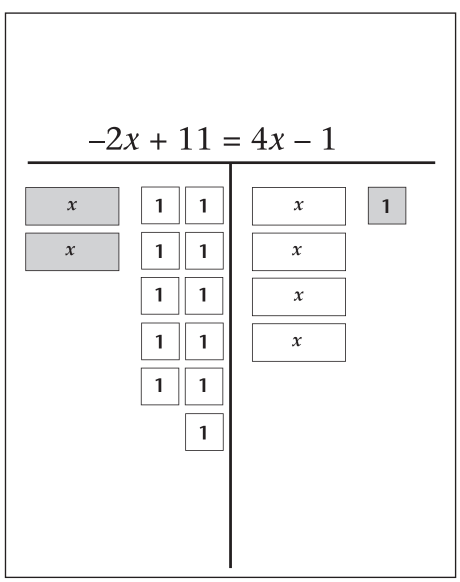
- Arrange the $\boxed{x}$ in a column on the left side.
- Arrange the $\boxed{1}$ in a column on the right side.



Each $\boxed{x}$ matches with one $\boxed{1}$, so we've solved our equation!

$$x = -1$$

We'll do one last example. Write $-2x + 11 = 4x - 1$ on a sticky note or small piece of paper and put it at the top of your equation sheet. Remember to put the equals sign over the vertical line on the page. Then set up the equation by placing the correct algebra tiles in the left and right columns.

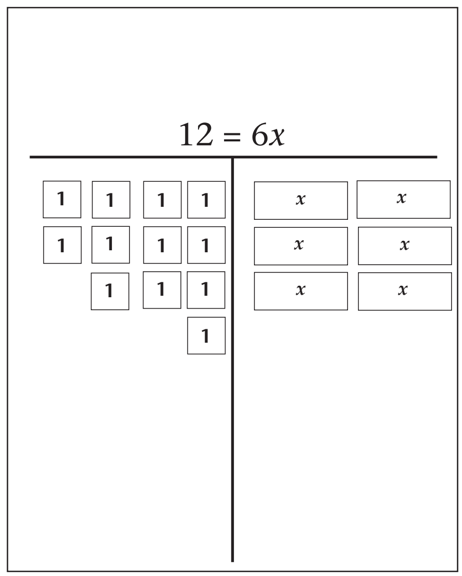


Thinking Space

Follow these steps to solve the equation:

- Start by adding a $\boxed{1}$ on the right. You can put it right next to the $\boxed{1}$ that's already there.
- Add a $\boxed{1}$ to the left as well, to keep the equation balanced.
- Remove the zero pair – the $\boxed{1}$ and the $\boxed{1}$ pair on the right,
- Get rid of the –2x on the left side by adding two $\boxed{x}$.
- Keep the equation balanced by adding two $\boxed{x}$ to the right side.
- Remove the zero pairs on the left created by the $\boxed{x}$ and $\boxed{x}$.

Now you should have twelve $\boxed{1}$ on the left and six $\boxed{x}$ on the right.

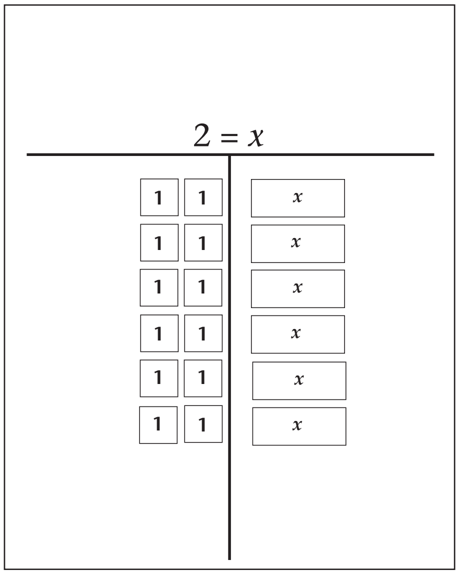


Since we only have x-tiles on the right side, we can now arrange the tiles to find our answer.

- Put the six $\boxed{x}$ on the right side in a column.
- Arrange the $\boxed{1}$ on the left side into six equal groups. One group matches each $\boxed{x}$ on the right.

Each group will have two $\boxed{1}$.

Thinking Space



Each $\boxed{x}$ matches exactly with two $\boxed{1}$, which means that $x = 2$.
We've solved the equation!

Thinking Space

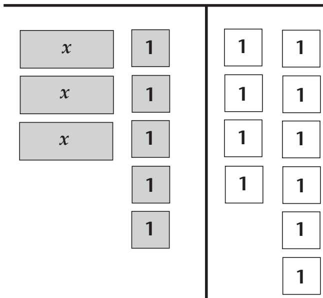
Try It!
Activity 2

For this Activity you will need:

- equation sheet
- algebra tiles

Use your equation sheet to model and solve the following equations. The first one has been set up for you.

1. $-3x - 5 = 10$



2. $-2x - 3 = -7$

3. $-5x - 1 = 9$

4. $x + 7 = -5$

Thinking Space

5.

$$7 + 3x = -2$$

6.

$$-5 = -2x + 3$$

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B
Solving Linear Equations Symbolically

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How do I solve a linear equation?		

Thinking Space

Warm–up

1. 4 times q is 12.

    a. Write this in mathematical symbols.

    b. What is q?

    c. What did you do to 4 and 12 to calculate q?

2. 15 divided by p is 3.

    a. Write this in mathematical symbols.

    b. What is p?

    c. What did you do to 15 and 3 to calculate p?

3. x is divided by 3 and the answer is 7.

    a. Write this in mathematical symbols.

    b. What is x?

    c. How can you rearrange 7 and 3 to equal x?

Turn to Solutions at the end of the Module and mark your work.

Thinking Space

Explore

Solving Equations by "Undoing" Them

It can be awkward to solve some equations using models. This is why we usually use algebra for complex equations. We'll examine each equation to learn what is being done to the variable, and then we'll "undo" what we see happening. Our goal is to get the variable by itself on one side of the equals sign. This is known as isolating the variable.

Example 1:

$$4x = 24$$

We see that something's happening to x: it's being multiplied by 4. To solve an equation we must *undo* everything that's happening to x, so we'll undo the multiplication by 4 with another operation—division by 4.

Undo multiplication with division, the opposite or inverse operation.

Think of a number. Add 2, then subtract 2. What is the result?

We do the opposite action, and of course we must do that operation on both sides to keep the equation balanced.

$$\frac{4x}{4} = \frac{24}{4}$$

$\frac{4}{4} = 1$

$1x = x$

Now let's write the results of our operation. On the left side, the result is that x is no longer multiplied by 4; it's just plain x, all alone. On the right, 24 is divided by 4, resulting in 6.

$$x = 6$$

***Isolated** means "by itself."*

When x is isolated on one side of the equation, we've solved the equation.

Thinking Space

Example 2:

$$\frac{x}{5} = 3$$

We see that x is being divided by 5. We'll undo the division by 5 with another operation—multiplication by 5 (the inverse operation). Of course, we must do that same operation on both sides to keep the equation balanced.

$$\frac{(5)x}{5} = 3(5)$$

$$\frac{5}{5} = 1$$

$$1x = x$$

Now let's write down the results of our operation. On the left, x is no longer divided by 5; it's just a plain x all alone. On the right 3 is multiplied by 5, resulting in 15.

$$x = 15$$

When x is isolated on one side of the equation, we've solved the equation.

Thinking Space

Try It!
Activity 1

1. For the expression or equations in the following table, answer the following questions:
   - What is happening to x?
   - What's the opposite action?

   The first one is done for you.

Expression or Equation	What's happening to x?	What's the opposite?
$3x$	multiplied by 3	divided by 3
$x - 5$		
$\frac{x}{7}$		
$-2x$		
$14 + x$		
$-5x = 10$		
$\frac{x}{-4} = 8$		

2. Perform the "undoing" actions to each side of the equations to solve them:

   a. $3x = 21$ b. $x + 16 = 8$ c. $x - 9 = 8$

Thinking Space

3. Solve. Be careful with the negative numbers.

   a. $\frac{x}{8} = 2$ b. $5x = -35$ c. $x - 16 = -3$

Turn to Solutions at the end of the Module and mark your work.

Thinking Space

Explore

Oh, My Dear *x*! What's Happening to You?

Often more than one thing is happening to x. In these situations, we must do more than one "undoing" action, and we have to do them in *reverse-BEDMAS* order. This is because we're not performing calculations, we're *reversing* the calculations that are in the equation.

Brackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction

Look at $4x - 6 = 2$. x is being multiplied by 4, and then 6 is subtracted. BEDMAS tells us to do multiplication before subtraction. To go in reverse BEDMAS order, we'll first undo the subtraction by adding 6, and then undo the multiplication by dividing by 4.

Note: It's very important to show your steps on the line below the equation, then write the new equation that results from your steps, like we've shown here.

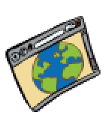
$$\begin{array}{cc} 4x - 6 = & 2 \\ {\scriptstyle (+6)} & {\scriptstyle (+6)} \end{array}$$

Add 6 to the left side, then add 6 to the right side to keep it balanced.

$$\frac{4x}{4} = \frac{8}{4}$$

Divide the left side by 4 to get x by itself, and then divide the right side by 4 to keep it balanced.

$$x = 2$$



Go to your http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html, **click on Module 3,** and watch *Solving a Two-Step Linear Equation* to see a complete solution for this next example.

$$\begin{array}{c} 19 + 7x - 6 = 40 \\ {\scriptstyle (-19) \qquad\qquad (-19)} \\ 7x = 21 \\ \frac{7x}{7} = \frac{21}{7} \\ x = 3 \end{array}$$

Thinking Space

Try It!
Activity 2

1. For the two solved equations below, fill in the blanks to explain how they were solved.

   a.

   $\frac{x}{5} - 8 = 2$ — x is being __________ by 5 and then _____ is subtracted.

   $(+8) \quad (+8)$ — To undo the "subtract 8," add 8 to the left side, and then add _____ to the __________ side to keep it __________.

   $\frac{x}{5} = 10$

   $(5)\frac{x}{5} = 10(5)$ — To undo the "divided by 5," multiply the left side by _____, and then __________ the right side by _____ to keep it balanced.

   $x = 50$

   b.

   $14 = -3x + 20$ — x is multiplied by _____ and then _____ is __________.

   $(-20) \quad (-20)$ — Undo the "plus 20" by __________ 20 from the _____ side, and then _______________ 20 from the _______________ side to keep it balanced.

   $-6 = -3x$

   $\frac{-6}{(-3)} = \frac{-3x}{(-3)}$ — Undo the "multiplied by –3" by _______________ by –3 on the right side, and then _______________ by _____ on the left side to keep it _______________.

   $2 = x$

   $x = 2$

Thinking Space

Does it make a difference in the final step if x is isolated on the left or the right? Why or why not?

2. Using the same procedure as above, solve these equations.

a. $-14 = 3x + 4$

b. $-14x + 1 = 43$

c. $\frac{y}{9} - 5 = -3$

d. $13 = 3 - 5x$

Turn to Solutions at the end of the Module and mark your work.

Thinking Space

Explore
Just Checking

Once we've solved a linear equation, it's a good idea to check the answer. To do this, we can simply substitute the solution into the original equation, then do the equation's arithmetic according to BEDMAS.

When we solve

$$\begin{aligned} -14 &= 7x + 7 \\ \scriptstyle(-7) & \qquad\ \scriptstyle(-7) \\ \frac{-21}{(7)} &= \frac{7x}{(7)} \\ -3 &= x \end{aligned}$$

We get $x = -3$

To check the answer, substitute –3 in place of x in the original equation, and do the math. Here we'll use a Left Side (LS) and Right Side (RS) format.

LS	RS	
–14	7(–3) + 7	
–14	–21 + 7	
–14	–14	LS = RS

The left side equals the right side, so we know our solution was correct. If the two sides had different numbers, we'd have to go over our work to find the error.

In solving the following equation, we've made an error. Can you spot it? We'll use the check that follows to show how we can catch the mistake.

$$\begin{aligned} -2x + 7 &= -3 \\ \scriptstyle -7 & \quad \scriptstyle -7 \\ -2x &= 4 \\ \frac{-2x}{-2} &= \frac{4}{-2} \\ x &= -2 \end{aligned}$$

Check:

LS	RS	
$-2x + 7$	–3	
–2(–2) + 7	–3	
4 + 7	–3	
11	–3	LS ≠ RS, so our answer is not correct.

Thinking Space

Let's go back and solve it properly.

$$\begin{aligned} -2x + 7 &= -3 \\ {\scriptstyle -7} \quad & \quad {\scriptstyle -7} \\ -2x &= -10 \\ \frac{-2x}{-2} &= \frac{-10}{-2} \\ x &= 5 \end{aligned}$$

−3 + (−7) ≠ 4
Last time we
must have added
+7, not −7.

Check:

LS	RS
$-2x + 7$	-3
$-2(5) + 7$	-3
$-10 + 7$	-3
-3	-3

LS = RS, so our answer is correct.

Thinking Space

Try It!
Activity 3

Answers are provided for each of the equations below. Some are correct, and some are not. Start by providing a check for each equation. If the provided answer is correct, place a check mark beside the equation. If the answer is not correct, solve the equation for the correct answer.

1. $6m + 4 = 28$
   $m = 3$

   Check:

LS	RS
$6m + 4$	28

2. $-8 = -24 - 4x$
   $x = -4$

   Check:

LS	RS
-8	$-24 - 4x$

3. $-5 - 9s = 13$
   $s = -2$

   Check:

LS	RS
$-5 - 9s$	13

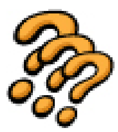
Thinking Space

4. $7 = 4x - 13$
   $x = -5$

Check:

LS	RS
7	$4x - 13$

Turn to Solutions at the end of the module and mark your work.



You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson C
More Linear Equations

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How can we undo equations that have two or more variable terms?		
What do we do about brackets in an equation?		
What is the distributive property?		

Thinking Space

Warm–up

1. Mentally add or subtract the following items:
   a. 10 cm + 8 cm ____________

   b. six oranges plus 11 oranges ____________

   c. 12 mangoes minus 7 mangoes ____________

   d. 4 walnuts plus 9 walnuts ____________

   e. 7 grapes minus 3 bananas ____________

2. What rule can you make about subtracting and adding things?

3. What are the “units” that go with your first 4 answers?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

We Like Terms Alike

Hopefully the warm-up prompted you to realize that things must be alike in order for us to add or subtract them, and that the “units” stay the same in the answer as they were in the question. So 8 goats plus 5 goats is 13 goats, not 13 of something new. If this all seems too obvious for you, apply the same thinking to expressions with variables in them:

$8x + 5x$ has to equal $13x$, not 13 of something else.

$7y - 10y$ is $-3y$

$10q + (-2q) = 8q$ (just add the integer coefficients)

$9x + 12y$ is not possible. We can’t group unlike terms together.

When we see more than one variable item in an equation, we have to group them before we can solve it, so they’d better be alike! Look at the following example:

$$5x + 6 - 2x = 15$$

The variable x is in two places, so we’ll rearrange the terms so that the ones with x in them appear together. Then it will be easier to combine them. Just remember to keep them on the correct side of the equals sign.

Terms are separated by + or –.

We rewrite –2x as + (–2x) so that we can use the commutative property of addition: 5 + 3 = 3 + 5. It doesn’t work the same way for subtraction: 5 – 3 ≠ 3 – 5

Rewrite $-2x$ to $+ (-2x)$ (Instead of subtracting, we’re adding the opposite.)

$$5x + 6 - 2x = 15$$
$$5x + 6 + (-2x) = 15$$

Rearrange to put like terms together.

$$5x + 6 + (-2x) = 15$$
$$5x + (-2x) + 6 = 15$$

$5x + (-2x) = 3x$, so we’re left with $3x$ and the constant 6 on the left.

$$3x + 6 = 15$$

Thinking Space

Now that it looks familiar, we can go through the usual steps to solve it.

$$\begin{aligned} 3x + 6 &= 15 \\ -6 \quad &\quad -6 \\ 3x &= 9 \\ \frac{3x}{3} &= \frac{9}{3} \\ x &= 3 \end{aligned}$$

Check:

$$\begin{aligned} 5x + 6 - 2x &\stackrel{?}{=} 15 \\ 5(3) + 6 - 2(3) &\stackrel{?}{=} 15 \\ 15 + 6 - 6 &\stackrel{?}{=} 15 \\ 15 &= 15 \quad ✔ \end{aligned}$$

Let's try another one.

$$-4x - 7 + 7x - 9 = 32$$

Change subtractions to additions: $-4x + (-7) + 7x + (-9) = 32$

Rearrange to put like terms together: $-4x + 7x + (-7) + (-9) = 32$

Simplify: $3x + (-16) = 32$

Rewrite: $3x - 16 = 32$

Solve:

$$\begin{aligned} 3x - 16 &= 32 \\ +16 \quad &\quad +16 \\ 3x &= 48 \\ \frac{3x}{3} &= \frac{48}{3} \\ x &= 16 \end{aligned}$$

Check:

$$\begin{aligned} -4x - 7 + 7x - 9 &= 32 \\ -4(16) - 7 + 7(16) - 9 &\stackrel{?}{=} 32 \\ -64 - 7 + 112 - 9 &\stackrel{?}{=} 32 \\ -71 + 112 - 9 &\stackrel{?}{=} 32 \\ 41 - 9 &\stackrel{?}{=} 32 \\ 32 &= 32 \quad ✔ \end{aligned}$$

Thinking Space

Try It!
Activity 1

Rearrange these equations so that all of the similar terms are grouped together, and then simplify them. Solve the equation and check your answers. The first one has been done for you.

1.
$$\begin{aligned} 2x + 3 + x &= 15 \\ 2x + x + 3 &= 15 \\ 3x + 3 &= 15 \\ -3 \quad & \;\; -3 \\ \frac{3x}{3} &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

Check :
$$\begin{aligned} 2(4) + 3 + (4) &= 15 \\ 8 + 3 + 4 &= 15 \\ 15 &= 15 \checkmark \end{aligned}$$

2. $3x - 2 - x = 6$

3. $3x + 7 + 6x = 61$

4. $-8a + 5 - 2a + 1 = -44$

Thinking Space

5. $4x - 1 - x - 3 = 5$

6. $-4g + 3 - 7g = 25$

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
There are Two Sides to Every Story

What if an x-term appears on both sides of the equation? Are we allowed to just remove some of them?

It turns out we are allowed, if we remember the principle of balance. We must remove an equal number of the variable from the other side as well.

We can remove $4x$ from each side of this equation.

Then it becomes a simple 2-step equation to solve.

$$\begin{aligned}
6x - 8 &= 4x + 12 \\
-4x \quad & \quad -4x \\
2x - 8 &= 12 \\
+8 \quad & \quad +8 \\
2x &= 20 \\
\frac{2x}{2} &= \frac{20}{2} \\
x &= 10
\end{aligned}$$

Check:

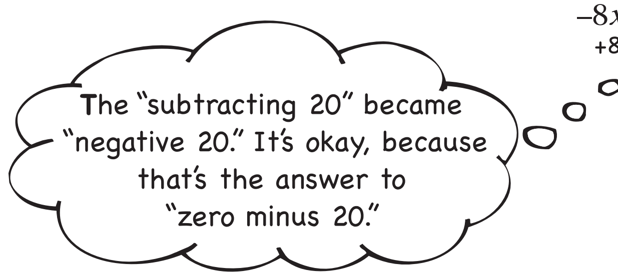
$$\begin{aligned}
6(10) - 8 &\stackrel{?}{=} 4(10) + 12 \\
60 - 8 &\stackrel{?}{=} 40 + 12 \\
52 &= 52 \quad ✔
\end{aligned}$$

Another example:

It's our choice: subtract $7x$ from both sides, or add $8x$ to both sides. Let's try the second option.

$$-8x - 20 = 7x - 5$$

$$\begin{aligned}
-8x - 20 &= 7x - 5 \\
+8x \quad & \quad +8x \\
-20 &= 15x - 5 \\
+5 \quad & \quad +5 \\
\frac{-15}{15} &= \frac{15x}{15} \\
-1 &= x \\
\text{or } x &= -1
\end{aligned}$$



Check:

$$\begin{aligned}
-8(-1) - 20 &\stackrel{?}{=} 7(-1) - 5 \\
8 - 20 &\stackrel{?}{=} -7 - 5 \\
-12 &= -12 \quad ✔
\end{aligned}$$

Thinking Space

We'll do the same example again, but this time we'll start by subtracting $7x$ from both sides:

$$\begin{aligned} -8x - 20 &= 7x - 5 \\ {\scriptstyle -7x} \quad &\quad {\scriptstyle -7x} \\ -15x - 20 &= -5 \\ {\scriptstyle +20} &\quad {\scriptstyle +20} \\ -15x &= 15 \\ \frac{-15x}{-15} &= \frac{15}{-15} \\ x &= -1 \end{aligned}$$

So it works out the same, no matter which option we choose!

Thinking Space

Try It!
Activity 2

Solve these equations by working with the variable term first. Check your answers. To check, you can use either the Left Side—Right Side method, or put a question mark over the equals sign.

1. $6h + 19 = 4h + 29$

2. $-5x + 20 = 8x - 6$

3. $-9x - 13 = 11x + 7$

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Bursting the Brackets

Some expressions and some equations have brackets in them. The order of operations tells us to do what's in the brackets first, but if the brackets contain unlike terms, we can't!

$7(x+2) = 3x - 6$ Help! We can't add x and 2.

So we'll use another technique. The **distributive property** allows us to rewrite expressions that have brackets in them so they don't have brackets anymore. It works like this:

$7(x+2)$ means 7 times $(x+2)$. We'll *distribute* the 7 to the items inside the brackets: 7 times x and 7 times 2. Then we won't need the brackets anymore.

$$7(x+2) = 7(x) + 7(2) = 7x + 14$$

When you finish multiplying, there are no brackets.

Here are a few more examples of the distributive property:

$-4(m-5) = -4m + 20$ Remember: $(-4)(-5) = +20$

$3(s-12) = 3s - 36$

$-5(2a+3) = -10a - 15$

-5 x 2a = -10a
Just multiply the number in front by the coefficient of the variable.

Thinking Space

Here's an example where we use the distributive property to help us solve a linear equation:

$$7(x + 2) = 3x - 6$$ Distribute the 7 over the x and the 2.

$$7x + 14 = 3x - 6$$

$$\begin{aligned} 7x + 14 &= 3x - 6 \\ \underset{-3x}{} \qquad &\quad \underset{-3x}{} \end{aligned}$$ Subtract $3x$ from each side.

$$4x + 14 = -6$$

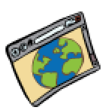
$$\begin{aligned} 4x + 14 &= -6 \\ -14 \quad &\quad -14 \end{aligned}$$ Subtract 14 from each side.

$$4x = -20$$

$$\frac{4x}{4} = \frac{-20}{4}$$ Divide each side by 4.

$$x = -5$$

Remember, the number in front of the brackets has to multiply all the items within the brackets.



Go to your http://media.openschool.bc.ca/osbcmedia/ma08/course/html/math08_ui.html, **click on Module 3,** and watch *Solving a Linear Equation Using the Distributive Property.*

Here's another example:

$$2(x - 6) = 4$$

$$\begin{aligned} 2x - 12 &= 4 \\ +12 \quad &\quad +12 \end{aligned}$$

$$2x = 16$$

$$\frac{2x}{2} = \frac{16}{2}$$

$$x = 8$$

Thinking Space

Try It!
Activity 3

Solve these equations. Check your work for each using either the Left Side—Right Side method or by putting a ? over the =.

1. $-3(x + 9) = x + 1$
2. $-1(6x + 6) = 2x + 18$
3. $3(x - 8) = 9$
4. $-4(x + 3) = x - 62$
5. $-2(3x - 1) = x + 9$
6. $7(m - 5) = m + 1$

Turn to Solutions at the end of the module and mark your work.

Thinking Space

You've finished Lesson C. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Section Summary

Completing this section has helped you to:

- recognize a linear equation
- use algebra tiles to model solutions to linear equations
- solve a linear equation with a fractional coefficient
- apply the distributive property to solve linear equations
- check your solutions to linear equations

Appendix

Solutions

Section 1

Pretest

1.

c	Bar graph	a. not very accurate, but easy to understand, shows data as portions of a whole
b	Line graph	b. can be very accurate, but can only be used with continuous data
a	Circle graph	c. shows data for categories, easy to read, can show two or more data sets on one graph
d	Pictograph	d. shows data in categories, easy to read, uses icons instead of numbers on a scale

2. a. The data is discrete

   b. It falls into different categories, and there is no possible data "between" the types, e.g. between police officers and nurses.

   c. A bar graph would best describe the data.

3. c. ✓

4. a. The student should use a line graph, since there are many possible starting heights and therefore many possible bounce heights.

   b. A line graph is the best type for graphing continuous data.

5. To show fish in a river system, use a bar graph, pictograph or circle graph.

   Reasons: (any two)

   Use these types of graphs because the data is discrete.

   For circle graph: Use this graph if you want to include the total number of fish in the system.

   For pictograph: It's the easiest to understand.
   It's the most visually appealing (looks the nicest).

   For bar graph: It's the most accurate.

6. a. Answers will vary. Two most likely answers: pictographs or circle graphs. (But bar graph is acceptable as long as you provide two reasons.)

   b. Answers will vary. Some examples of acceptable answers are:

   If you chose pictograph:
   - It's attractive and interesting.
   - The pictures make it easy to compare two quantities.

   If you chose circle graph:
   - The sections show how each category relates to the whole.
   - The sections are easy to compare.

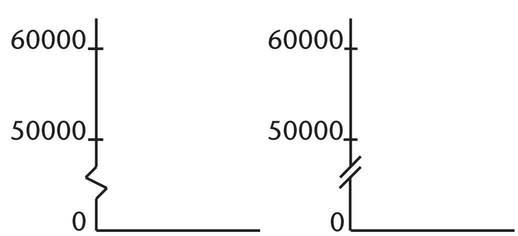
7. a. Answers will vary. Any three of the following
   - The scale does not start at zero.
   - Art's bar is wider than others.
   - Art's bar is separated from the others.
   - Art's bar is more darkly shaded.
   - Art's bar has bolder text.

   b. Art

8. a. The scale is not even: the first interval is from 0 to 50 000, but the next interval is 50 000 to 60 000 and they go up by 10 000 from there.

   b. A break in the scale needs to be added.

   c. It can be a small zigzag or a pair of parallel lines, e.g.



9. Answers will vary. Possible answers: (Any two)
   - Make pictograph in black and white except for the snowman.
   - Make snowman icon larger than the others so the Snow Goons column takes up more room.

- Make the snowmen smiling, or with darker lines, and other icons frowning, faded.
- Move Snow Goons row to the top so it appears first.
- Write “5 Wins!!!” next to the Snow Goons row.

10. Answers will vary. Possible answers: (Any two)

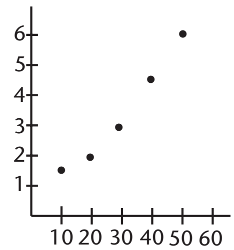
- Make one slice actually bigger than its label indicates it is.
- Make it a 3-D graph and orient largest label towards the viewer, pulled out of the pie towards the viewer.
- Leave off the labels.
- Colour the slices to make one more appealing

Lesson A: Types of Graphs and Data

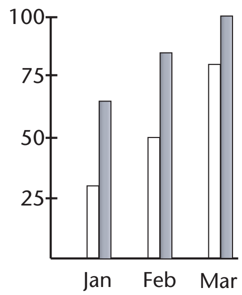
Warm-up

1.

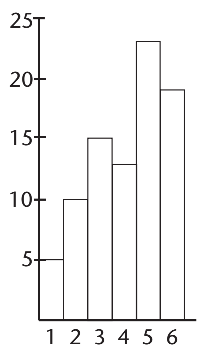
e



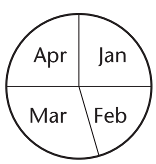
d



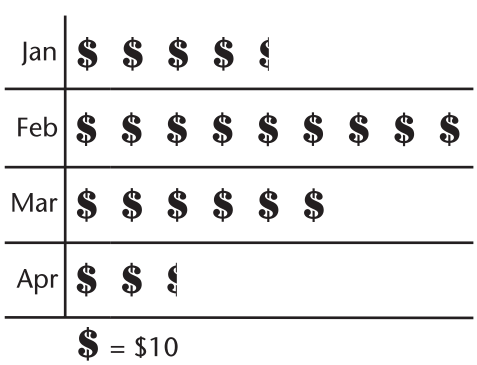
a



b



c



a.

(1, 5)
(2, 10)
(3, 15)
(4, 13)
(5, 23)
(6, 19)

b.

Jan	3
Feb	2
Mar	4
Apr	3

c.

Jan	45
Feb	90
Mar	60
Apr	25

d.

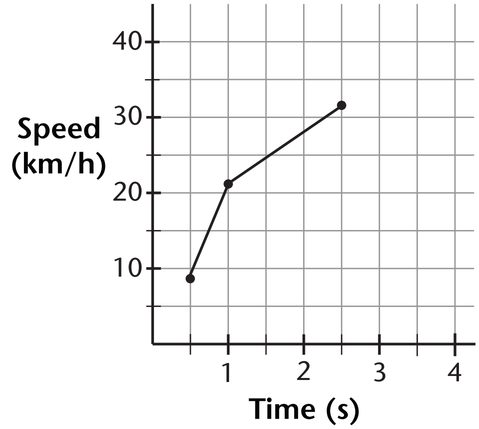
Jan	30
Feb	50
Mar	80

Jan	65
Feb	85
Mar	100

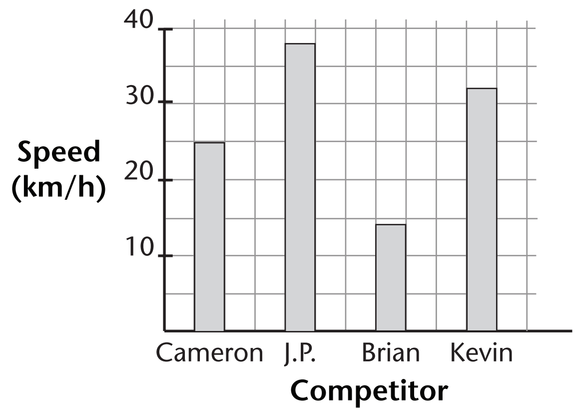
e.

(10, 1.5)
(20, 2)
(30, 3)
(40, 4.5)
(50, 6)

2. a.


40
30
20
10
Speed (km/h)
1
2
3
4
Time (s)

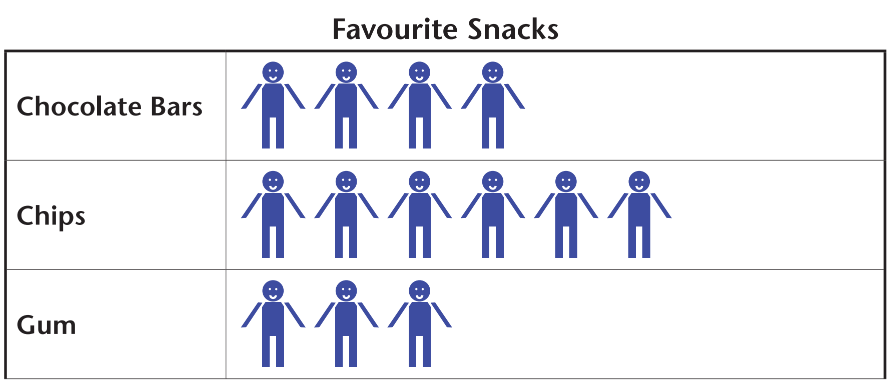
b.


40
30
20
10
Speed (km/h)
Cameron
J.P.
Brian
Kevin
Competitor

c.


Reading Books
Gaming
Listening to Music
Leisure Time

d.


Favourite Snacks
Chocolate Bars
Chips
Gum

Try It! Activity 1

1. For each item below, read the statement, then decide which kind of graph is most appropriate. Put a B (bar), P (pictograph), L (line), or C (circle) in the blank after each description. The first one is done for you.

    L a. sets of numbers that represent the hourly temperature during each day in July

    L b. temperature of water in a kettle as it heats up.

    L c. amount of wood in a tree as it grows.

    B d. average cost of living compared to location.

    B e. average income versus years of education.

    C f. types of garbage that goes into the landfill

    B or P g. number of households that have each type of pet

2. a. a line graph

    b. This data describes different categories of fruit. It doesn't show anything changing with time, nor does it graph two sets of numbers.

    c. a bar graph, pictograph, or circle graph

    d. Answers will vary. Some possibilities

    For circle graph: It shows each type of fruit as a fraction of the total amount of fruit, and it's easy to understand.

    For bar graph: It's easy to make and shows the actual data.

    For pictograph: It's attractive and interesting.

Try It! Activity 2

1.

Graph Type	Describe the Data	Example
line graph	**continuous, number**	**heights**
bar graph	**discrete, number**	**track, summer, or votes**
pictograph	**discrete, symbol**	**track, summer, or votes**
circle graph	**discrete, percent, part, total**	**votes or summer**

2. a. a line graph

   b. A person's heart keeps beating all the time so it is continuous data.

Try It! Activity 3

Graph Type	Strengths	Limitations
bar graph	**can be used for discrete data** **provides approximate numbers using scale on left side**	**can only use with discrete data**
circle graph	**used to compare parts to the total** **can be used for discrete data** **interesting to look at**	**can be difficult to draw: requires a protractor** **more difficult to compare one category to another**
line graph	**shows data that changes over time** **used to compare two sets of numbers**	**shouldn't be used for discrete data**
pictograph	**can be used for discrete data** **interesting to look at**	**partial icons make calculating difficult** **must use a key to calculate**

Lesson B: Advantages and Disadvantages of Graph Types

Warm-up

1.

C The diameter of a ball determines its circumference.

D Students at our school eat different types of foods for breakfast.

C The distance from home depends on how long we've been travelling.

D A store tracks the sales of different brands of marmalade.

C The height of water in a reservoir goes up and down depending on the season.

2. a. circle graph
   b. line graph
   c. bar graph
   d. pictograph

Try It! Activity 1

1. b. bar graph that shows what each of your classmates gets, because you want your parents to easily see the approximate amounts

2. a. bar graph because it will clearly show the trend of increasing accidents as the bars get bigger

3. b. pictograph because icons are a natural fit with the subject matter and audience

Try It! Activity 2

1. a. Air volume changes from about 0.4 L to 4.7 L, goes down to 0.4 L, and then starts to go back up again. This happens over the course of 9 seconds.
   b. It takes 3 seconds to fill the lungs.
   c. It takes 4 seconds to empty the lungs.

2. The lungs will be full again at 10 or 11 seconds.
3. The breathing cycle is about 7 seconds.
4. a. 2 ½ seconds, about 3 litres

   b. 7 ½ seconds, about 0.5 litres

   c. 15 seconds, about 0.5 litres

Try It! Activity 3

1. Graph type: bar
   Reasons: (Any two of the following)
   - The data is discrete.
   - It easily shows how one item compares to another by the height of the bars.
   - It has a scale up the vertical axis where you can read the approximate values.
   - A circle graph won't work, because it shows percentages, not actual values.
   - It's harder to compare "wedge" sizes on a circle graph.
2. Graph type: line
   Reasons: (Any two of the following)
   - The data is continuous. A line graph is the only choice.
   - A line graph easily shows connections or trends.
   - It's easiest to extrapolate data on a line graph.
3. Graph type: pictograph
   Reasons: (Any two of the following)
   - The data is discrete.
   - The number of icons easily shows who got the most.
   - You don't need a lot of detail.
   - You don't need a bar graph because you don't need to know approximate numbers.
   - It would be okay to have partial icons on the graph.
   - The graph could be made very attractive with different icons for each person.
4. Graph type: circle
   Reasons: (Any two of the following)
   - The data is discrete.
   - Circle graphs easily show how portions relate to the total.
   - Circle graphs usually show percentages.

5. Graph type: double bar graph

   Reasons: (Any two of the following)

   - The data is discrete.
   - Double bar graphs are the best at comparing two related sets of category data.
   - A circle graph wouldn't work as well because you would have to draw two of them.

Lesson C: Misleading Graphs

Warm-up

The diagonal line is:

a. **one straight line connected at the back**

Does this figure form a spiral?

b. no

Which part of the line is longer, A or B?

Neither! They're both the same length. (This wasn't really a fair question!)

Try It! Activity 1

Answers will vary. Some examples of good answers are:

1. What does this person paint? (Houses, art?)

   Does the painter use the paint for all of the house, or just parts?

2. Most of the staff was present at the meeting.
3. What kind of doctors?

   Were only 5 doctors surveyed?

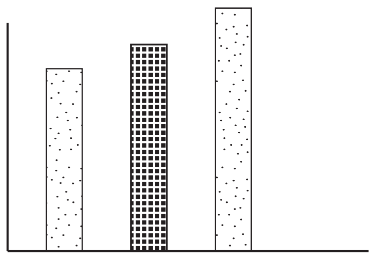
4. Does this mean three-quarters of all hairdressers?

   Were only four hairdressers surveyed?

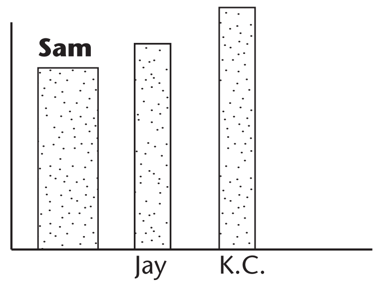
   Do they use it on themselves, someone else (or even a family pet)?

Try It! Activity 2

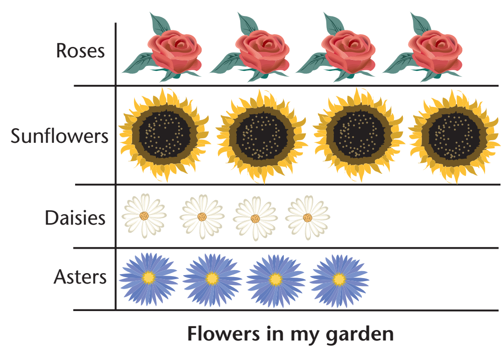
1.
   a. Here is one possibility:



   b. Here is one possibility:



2.



Try It! Activity 3

1. uneven scale numbers
2. no numbers on scale
3. break inserted on scale

Try It! Activity 4

1. Exploded slice and darker colour makes the slice seem bigger.
2. The ice cream wedge is larger than its percentage seems to indicate; 26% is approximately one quarter. That wedge is much bigger than one quarter of the circle. The burger wedge has no numbers on it.
3. No misleading features.

Section 2

Pretest

Lesson A: Ordered Pairs and Linear Equations

1. form a straight line

2. $y = -4(-3) + 2$
$y = 12 + 2$
$y = 14$

$y = -4(0) + 2$
$y = 0 + 2$
$y = 2$

$y = -4(2) + 2$
$y = -8 + 2$
$y = -6$

$y = -4(5) + 2$
$y = -20 + 2$
$y = -18$

Ordered pairs: (–3, 14) (0, 2) (2, –6) (5, –18)

3.

$y = 9x$

x	y
–2	**–18**
0	**0**
3	**27**
4	**36**

$d = -3c + 12$

c	d
–5	**27**
–2	**18**
0	**12**
3	**3**

$q = \frac{p}{3} + 1$

p	q
–6	**–1**
–3	**0**
0	**1**
3	**2**

Lesson B: Tables of Values

4. a. ***y* and *x***

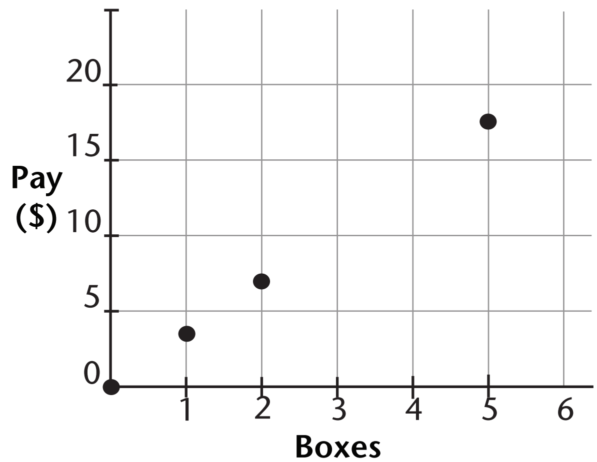
   b. **4**

   c. **2**

5. a.

Boxes	**0**	**1**	**2**	**5**
Pay ($)	**0**	**3.50**	**7.00**	**17.50**

b. Plot the points from the table of values on to the graph.



Lesson C: Relationships and Missing Values

6. a.

a	*b*
–3	6
0	0
1	–2
2	–4

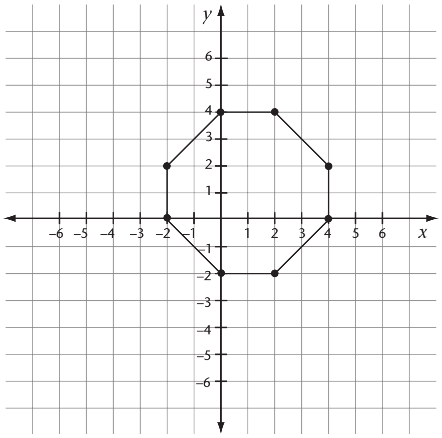
b is –2 times *a* OR
b is the negative of twice *a* OR
a times –2 is *b* OR
for every 1 that *a* goes up, *b* goes down by 2

b. For every one that *m* increases (or moves to the right), *n* decreases (or goes down) by two.

Lesson A: Ordered Pairs and Linear Equations

Warm-up

1.



octagon

2. (–3, –2) (–2, –1) (–1, 0) (0, 1) (1, 2) (2, 3) (3, 4)

Try It! Activity 1

1. a. $y = x - 5$

$x = 0$
$y = x - 5$
$y = (0) - 5$
$y = -5$

$x = -3$
$y = x - 5$
$y = (-3) - 5$
$y = -8$

$x = 2$
$y = x - 5$
$y = (2) - 5$
$y = -3$

$x = 7$
$y = x - 5$
$y = (7) - 5$
$y = 2$

Ordered pairs: (–3, –8) (0, –5) (2, –3) (7, 2)

b. $y = -3x + 2$

$x = -5$
$y = -3x + 2$
$y = -3(-5) + 2$
$y = 15 + 2$
$y = 17$

$x = 0$
$y = -3x + 2$
$y = -3(0) + 2$
$y = 0 + 2$
$y = 2$

$x = 1$
$y = -3x + 2$
$y = -3(1) + 2$
$y = -3 + 2$
$y = -1$

$x = 3$
$y = -3x + 2$
$y = -3(3) + 2$
$y = -9 + 2$
$y = -7$

Ordered pairs: (–5, 17) (0, 2) (1, –1) (3, –7)

2. a. Ordered pairs: (–3, 9) (–1, 5) (1, 1) (3, –3)

   b. Ordered pairs: (–3, –15) (–1, –5) (1, 5) (3, 15)

   c. Ordered pairs: (–3, –16) (–1, –10) (1, –4) (3, 2)

   d. Ordered pairs: (–3, 3) (–1, 1) (1, –1) (3, –3)

Try It! Activity 2

1. a.

$$y = 2x + 8$$
$$11 = 2(2) + 8$$

Left Side (LS)	Right Side (RS)
11	**2(2) + 8**
11	**4 + 8**
11	**12**

Does the LS = RS? **No**

Does the point (2, 11) satisfy the equation $y = 2x + 8$? **No**

   b. (15, 3)

$$y = \frac{1}{5}x$$

Left Side (LS)	Right Side (RS)
3	$\mathbf{\frac{15}{5}}$
3	**3**

Does the LS = RS? **Yes**

Does the point (15, 3) satisfy the equation $y = \frac{1}{5}x$? **Yes**

   c. (–2, 8)

$$y = -3x + 2$$

Left Side (LS)	Right Side (RS)
8	**–3(–2) + 2**
8	**6 + 2**
8	**8**

Does the LS = RS? **Yes**

Does the point (2, 11) satisfy the equation $y = -3x + 2$? **Yes**

2.

$y = \frac{1}{2}x + 4$

(–2, 3) $\quad$ <u>(4, 10)</u>

<u>(2, 4)</u>

<u>$y = 3x - 2$</u> $\quad$ (4, 6) $\quad$ $\boxed{(0, -1)}$ $\quad$ $\boxed{(-3, 2)}$

$\boxed{(2, -3)}$

(–4, 2)

$\boxed{y = -x - 1}$ $\quad$ <u>(–3, –11)</u>

Lesson B: Tables of Values

Warm-up

1. a. 35 × \$1.50 = \$52.50

   b. \$150.00 ÷ \$1.50 = 100

2. a. = \$8.00 + 35 × \$0.50
   = \$8.00 + \$17.50
   = \$25.50

   b. = \$8.00 + 50 × \$0.50
   = \$8.00 + \$25.00
   = \$33.00

Try It! Activity 1

Equation	Variables	Constant	Coefficient	Terms
$\mathbf{y = -2x + 3}$	$\mathbf{y, x}$	**3**	**–2**	$\mathbf{y, -2x, 3}$
$a = \frac{1}{2}p + 8$	a, p	8	$\frac{1}{2}$	$a, \frac{1}{2}p, 8$
$c = 4q - 6$	c, q	–6	4	$c, 4q, -6$
$m = \frac{n}{3}$	m, n	0	$\frac{1}{3}$	$m, \frac{1}{3}n$ or $m, \frac{n}{3}$
$s = -4t - \frac{1}{2}$	s, t	$-\frac{1}{2}$	–4	$s, -4t, -\frac{1}{2}$

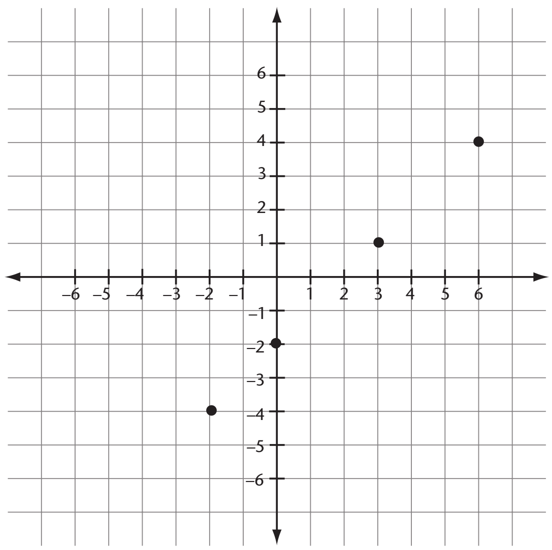
Try It! Activity 2

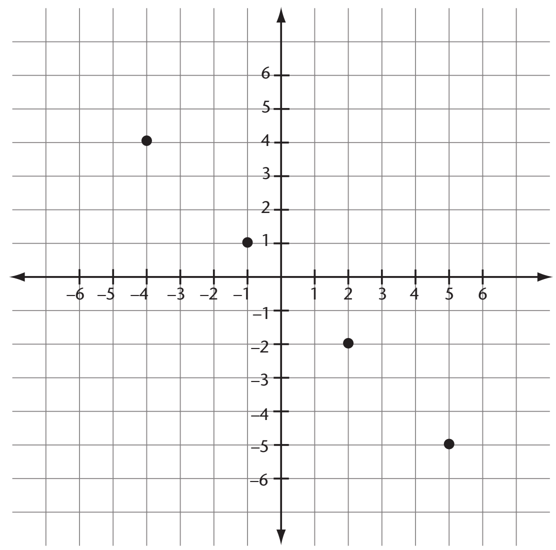
1. a. $y = x - 2$

x	y
–2	**–4**
0	**–2**
3	**1**
6	**4**

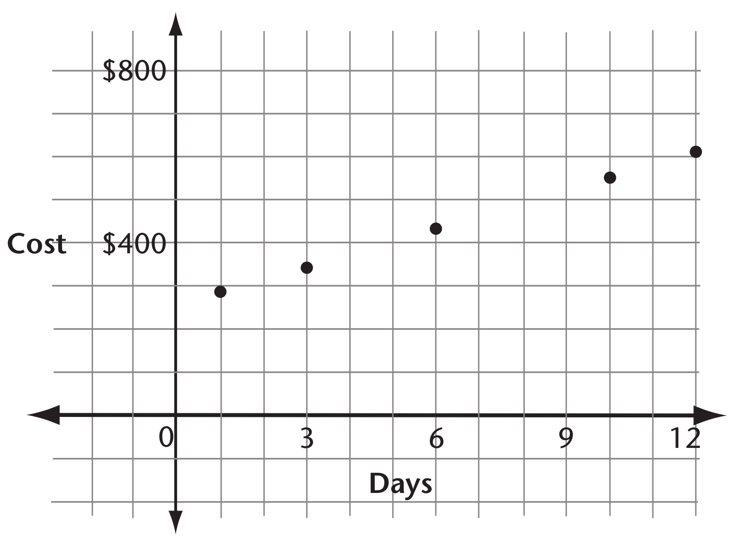
b. $y = -x$

x	y
–4	**4**
–1	**1**
2	**–2**
5	**–5**





2. **Example solutions:**

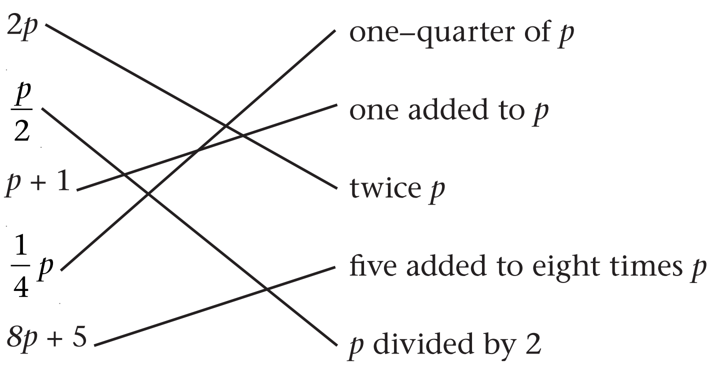


Days	1	2	3	4	5	6	10	11	12	13	14	15
Cost	280	310	340	370	400	430	550	580	610	640	670	700

Lesson C: Relationships and Missing Values

Warm-up

1.



2. a. Equation: $y = x$
   b. Equation: $y = 3x$
   c. Equation: $y = x + 2$

Try It! Activity 1

Answers will vary. Some examples of correct answers are:

1. Subtract 4 from each x-value to get the y-value. Each x is 4 more than y. $y = x - 4$.

2. It costs $5 to attend the dance. Each snack cost $1.

3. Multiply r by –3 to get t. $t = -3r$

Try It! Activity 2

1. This graph touches the y–axis at –4

   Moving from left to right: For every one that x goes to the right, y goes up one

x	–2	0	2	4
y	–6	–4	–2	0

   Describe the relationship from the table of values: For every one that x increases, y increases by one. Or, y is 4 less than x.

2. This graph touches the y–axis at 0

   Moving from left to right: For every one that x goes to the right, y goes down by 4

x	–1	0	1	2
y	4	0	–4	–8

Describe the relationship from the table of values: <u>Whenever *x* goes up by 1, *y* goes down by 4. Or, *y* is –4 times *x*.</u>

3. This graph touches the *y*–axis at <u>5</u>

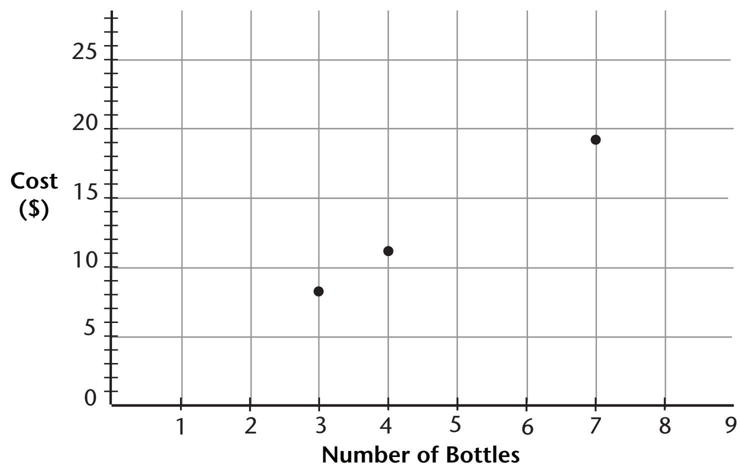
Moving from left to right: <u>For every one that *x* goes to the right, *y* goes up by 2</u>

x	–2	–1	0	1
y	1	3	5	7

Describe the relationship from the table of values: <u>Whenever *x* goes up by 1, *y* goes down by 2. Or, *y* is 5 more than twice *x*.</u>

Try It! Activity 3

1. a.



b.

# of Bottles	Cost
3	\$8.25
4	\$11.00
7	\$19.25

c.

# of Bottles	Cost
1	\$2.75
2	\$5.50
3	\$8.25
4	\$11.00
5	\$13.75
6	\$16.50
7	\$19.25

d. 2.75

e. \$24.75

2. a. $y = -x - 2$

   b.

Point	x	y
A	2	–4
B	–2	0
C	0	-2
D	–5	3
E	6	–8
F	–8	6

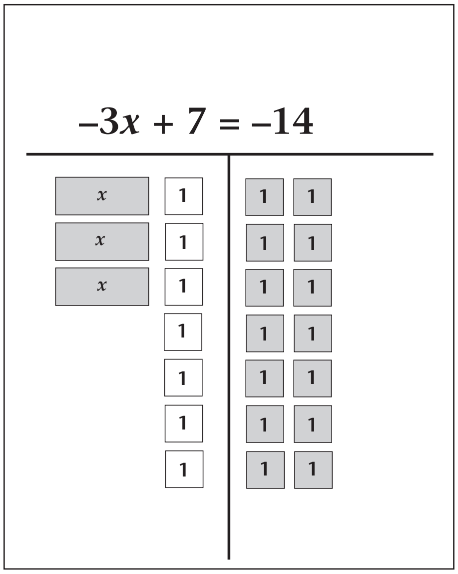
3.

x	y
–1	5
0	3
1	1
2	–1
3	–3
4	–5

Section 3

Pretest

1. a.



b. Add 3 positive x-tiles to both sides. Take the zero pairs made by the x-tiles away from the left side.

Add 14 positive x-tiles to both sides. Take the 14 zero pairs from the right side.

Arrange the 1-tiles on the left (21) into 3 groups of 7 tiles each.

Match each group with one x-tile.

c. $x = 7$

Lesson B: Solving Linear Equations Symbolically

2.

a.
$$\begin{aligned} 3x - 11 &= 22 \\ {\scriptstyle +11} \quad & \quad {\scriptstyle +11} \\ 3x &= 33 \\ \frac{3x}{3} &= \frac{33}{3} \\ x &= 11 \end{aligned}$$

b.
$$\begin{aligned} \frac{a}{3} + 9 &= 15 \\ {\scriptstyle -9} \quad & \quad {\scriptstyle -9} \\ \frac{a}{3} &= 6 \\ \frac{(3)a}{3} &= 6(3) \\ a &= 18 \end{aligned}$$

c.
$$\begin{aligned} 4x - 7 &= 25 \\ {\scriptstyle +7} \quad & \quad {\scriptstyle +7} \\ 4x &= 32 \\ \frac{4x}{4} &= \frac{32}{4} \\ x &= 8 \end{aligned}$$

d.
$$\begin{aligned} -5m - 7 &= -22 \\ {\scriptstyle +7} \quad & \quad {\scriptstyle +7} \\ -5m &= -15 \\ \frac{-5m}{-5} &= \frac{-15}{-5} \\ m &= 3 \end{aligned}$$

Lesson C: More Linear Equations

3. Solve each equation:

a.
$$-2(x+7) = -6$$
$$\underset{+14}{-2x - 14} = \underset{+14}{-6}$$
$$-2x = 8$$
$$\frac{-2x}{-2} = \frac{8}{-2}$$
$$x = -4$$

b.
$$-2y + 5 - 4y = 13$$
$$-6y + 5 = 13$$
$$-6y = 8$$
$$y = \frac{8}{-6}$$
$$y = -\frac{4}{3} \quad \text{or} \quad -1\frac{1}{3}$$

c.
$$3(x + 8) = 18$$
$$3x + 24 = 18$$
$$3x = -6$$
$$x = -\frac{6}{3}$$
$$x = -2$$

d.
$$\frac{r}{6} - 9 = -3$$
$$\frac{r}{6} = 6$$
$$r = 36$$

4. $4(t - 9) = 3t$

Multiply 4 by $(t - 9)$ to get $4t - 36$ on the left. $4t - 36 = 3t$

Add 36 to both sides to get $4t = 3t + 36$

Subtract 3t from both sides to get $1t = 36$ or $t = 36$.

Lesson A: Algebra Tiles

Warm-up

1. Answers will vary, but might include suggestions to

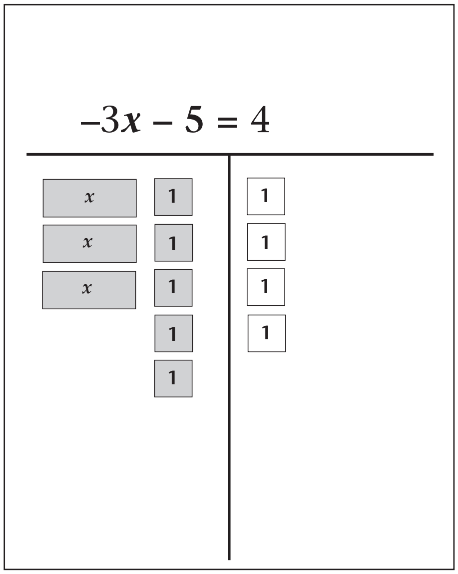
   - take items off in the reverse order that they went on
   - place and remove items of similar mass across from each other on the tray
   - arrange items from the center of the tray outwards

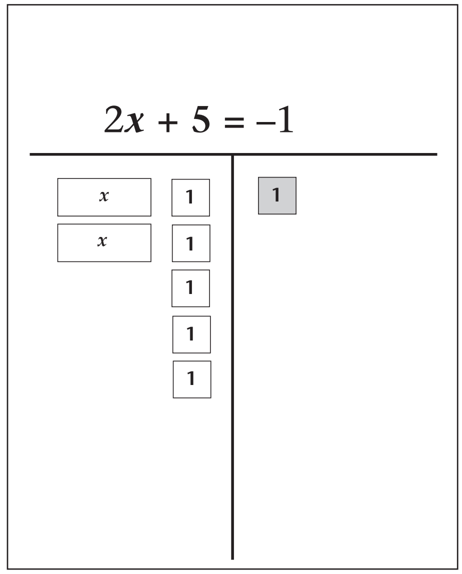
2. Adding and subtracting items from the tray.

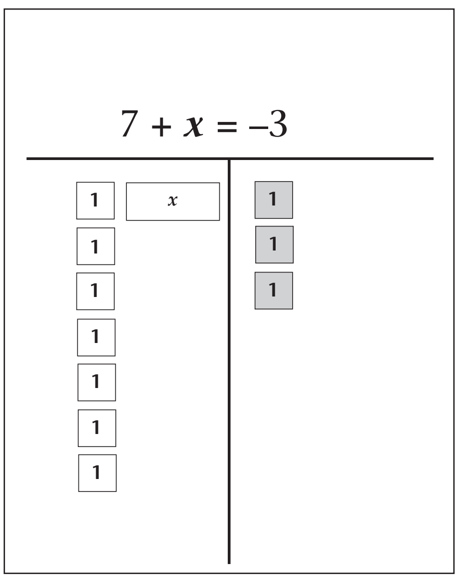
   The "two plates" mentioned above, and "both the salads" imply multiplication by 2.

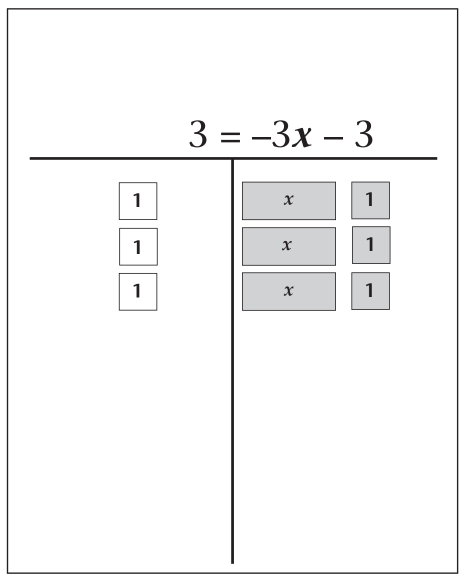
   "Weigh the same" implies equality.

Try It! Activity 1





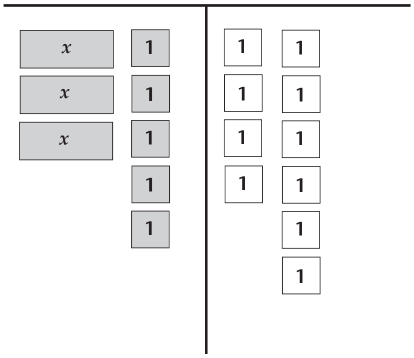




Try It! Activity 2

1.

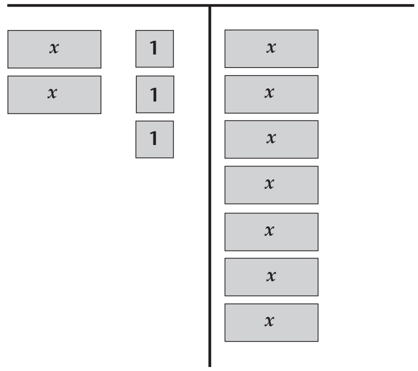
$-3x - 5 = 10$



2.

$-2x - 3 = -7$

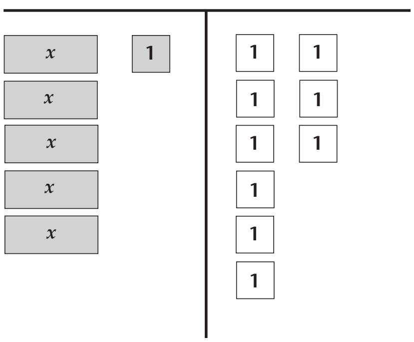
$x = 2$ or $2 = x$



3.

$-5x - 1 = 9$

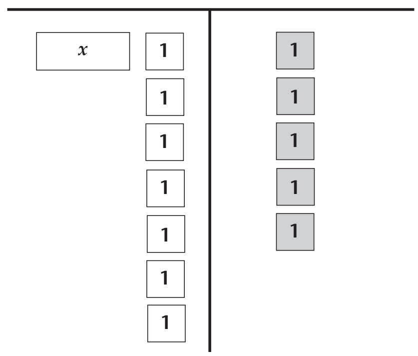
$x = -2$ or $-2 = x$



4.

$x + 7 = -5$

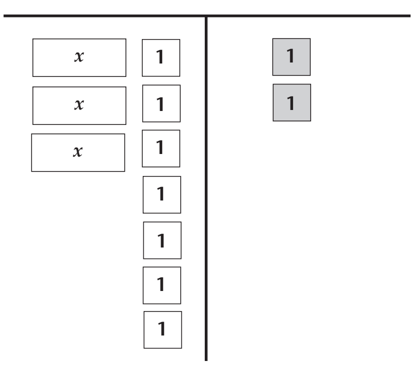
$x = -12$ or $-12 = x$



5.

$7 + 3x = -2$

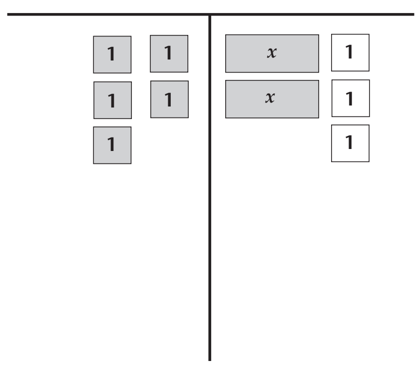
$x = -3$ or $-3 = x$



6.

$-5 = -2x + 3$

$x = 4$ or $4 = x$



Lesson B: Solving Linear Equations Symbolically

Warm-up

1. a. $4q = 12$

   b. q is 3

   c. q is 12 divided by 4

2. a. $15 \div p = 3$

   b. $p = 5$

   c. $15 \div 3 = 5$ or $\frac{15}{3} = 5$

3. a. $\frac{x}{3} = 7$ OR $x \div 3 = 7$

   b. $x = 21$

   c. $(7)(3) = 21$

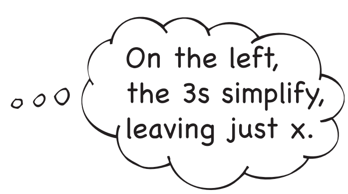
Try It! Activity 1

1.

Expression or Equation	What's happening to x?	What's the opposite?
$3x$	**multiplied by 3**	**divided by 3**
$x - 5$	**5 is subtracted**	**5 is added**
$\frac{x}{7}$	**divided by 7**	**multiplied by 7**
$-2x$	**multiplied by –2**	**divided by –2**
$14 + x$	**14 is added**	**14 is subtracted**
$-5x = 10$	**multiplied by –5**	**divided by –5**
$\frac{x}{-4} = 8$	**divided by –4**	**multiplied by –4**

2.

a.
$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$



b.
$$\begin{array}{rcl} x + 16 & = & 8 \\ -16 & & -16 \\ x & = & -8 \end{array}$$

c.
$$\begin{array}{rcl} x - 9 & = & 8 \\ +9 & & +9 \\ x & = & 17 \end{array}$$

3.

a.
$$\frac{x}{8} = 2$$
$$\frac{8x}{8} = 2$$
$$x = 16$$

b.
$$5x = -35$$
$$\frac{5x}{5} = \frac{-35}{5}$$
$$x = -7$$

c.
$$x - 16 = -3$$
$${+16} \qquad {+16}$$
$$x = 13$$

Try It! Activity 2

1.

a. $\frac{x}{5} - 8 = 2$ — x is being divided by 5 and then 8 is subtracted.

$(+8) \quad (+8)$ — To undo the "subtract 8," add 8 to the left side, and then add 8 to the right side to keep it balanced.

$$\frac{x}{5} = 10$$
$$(5)\frac{x}{5} = 10(5)$$
$$x = 50$$

To undo the "divided by 5," multiply the left side by 5 and then multiply the right side by 5 to keep it balanced.

b. $14 = -3x + 20$ — x is multiplied by –3 and then 20 is added.

$-20 \qquad -20$ — Undo the "plus 20" by subtracting 20 from the right side, and then subtracting 20 from the left side to keep it balanced.

$$-6 = -3x$$
$$\frac{-6}{(-3)} = \frac{-3x}{(-3)}$$

Undo the "multiplied by –3" by dividing by –3 on the right side, and then dividing by –3 on the left side to keep it balanced.

$$2 = x$$
$$x = 2$$

2.

a.

$$-14 = 3x + 4$$
$$(-4) \qquad (-4)$$
$$\frac{-18}{(3)} = \frac{3x}{(3)}$$
$$-6 = x$$

b.

$$-14x + 1 = 43$$
$$(-1) \qquad (-1)$$
$$\frac{-14x}{(-14)} = \frac{42}{(-14)}$$
$$x = -3$$

c.

$$\frac{y}{9} - 5 = -3$$
$$(+5) \qquad (+5)$$
$$\frac{(9)y}{9} = 2(9)$$
$$y = 18$$

d.

$$13 = 3 - 5x$$
$$(-3) \qquad (-3)$$
$$\frac{10}{5} = \frac{5x}{5}$$
$$2 = x$$

Try It! Activity 3

1. $6m + 4 = 28$ This question is wrong. ×
   $m = 3$

Check:

LS	RS
$6m + 4$	28
$6(3) + 4$	28
$18 + 4$	28
22	28

Correct solution:

$$6m + 4 = 28$$
$$-4 \qquad -4$$
$$\frac{6m}{6} = \frac{24}{6}$$
$$m = 4$$

2. $-8 = -24 - 4x$ This question is correct. ✓
   $x = -4$

Check:

LS	RS
-8	$-24 - 4x$
-8	$-24 - 4(-4)$
-8	$-24 + 16$
-8	-8

3. $-5 - 9s = 13$ This question is correct. ✓
   $s = -2$

Check:

LS	RS
$-5 - 9s$	13
$-5 - 9(-2)$	13
$-5 + 18$	13
13	13

4. $7 = 4x - 13$ This question is wrong. ✗
   $x = -5$

Check:

LS	RS
7	4(–5) – 13
7	–20 – 13
7	–33

Correct solution:

$$\begin{aligned} 7 &= 4x - 13 \\ {\scriptstyle +13} &\quad\;\; {\scriptstyle +13} \\ \frac{20}{4} &= \frac{4x}{4} \\ 5 &= x \end{aligned}$$

Lesson C: More Linear Equations

Warm-up

1. a. 18 cm
   b. 17 oranges
   c. 5 mangoes
   d. 13 walnuts
   e. no answer possible
2. You can only add or subtract things that are of the same type.
3. cm, oranges, mangoes

Try It! Activity 1

1.
$$\begin{aligned} 2x + 3 + x &= 15 \\ 2x + x + 3 &= 15 \\ 3x + 3 &= 15 \\ {\scriptstyle -3} &\quad {\scriptstyle -3} \\ \frac{3x}{3} &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

Check:

$$\begin{aligned} 2(4) + 3 + (4) &\stackrel{?}{=} 15 \\ 8 + 3 + 4 &\stackrel{?}{=} 15 \\ 15 &= 15 \checkmark \end{aligned}$$

2.
$$\begin{aligned} 3x - 2 - x &= 6 \\ 3x + (-x) + (-2) &= 6 \\ 2x - 2 &= 6 \\ {\scriptstyle +2} &\quad {\scriptstyle +2} \\ \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

Check:

$$\begin{aligned} 3(4) - 2 - (4) &\stackrel{?}{=} 6 \\ 12 - 2 - 4 &\stackrel{?}{=} 6 \\ 10 - 4 &\stackrel{?}{=} 6 \\ 6 &= 6 \checkmark \end{aligned}$$

3.
$$
\begin{aligned}
3x + 7 + 6x &= 61 \\
3x + 6x + 7 &= 61 \\
9x + 7 &= 61 \\
\scriptstyle -7 \quad & \scriptstyle -7 \\
\frac{9x}{9} &= \frac{54}{9} \\
x &= 6
\end{aligned}
$$

Check:
$$
\begin{aligned}
3(-6) + 7 + 6(-6) &\overset{?}{=} 61 \\
18 + 7 + 36 &\overset{?}{=} 61 \\
61 &= 61 \checkmark
\end{aligned}
$$

4.
$$
\begin{aligned}
-8a + 5 - 2a + 1 &= -44 \\
-8a + (-2a) + 5 + 1 &= -44 \\
-10a + 6 &= -44 \\
\scriptstyle -6 \quad & \scriptstyle -6 \\
\frac{-10a}{-10} &= \frac{-50}{-10} \\
a &= 5
\end{aligned}
$$

Check:
$$
\begin{aligned}
-8(-5) + 5 - 2(-6) + 1 &\overset{?}{=} -44 \\
-40 + 5 - 10 + 1 &\overset{?}{=} -44 \\
-44 &= -44 \checkmark
\end{aligned}
$$

5.
$$
\begin{aligned}
4x - 1 - x - 3 &= 5 \\
4x + (-x) + (-1) + (-3) &= 5 \\
3x + (-4) &= 5 \\
\scriptstyle +4 \quad & \scriptstyle +4 \\
\frac{3x}{3} &= \frac{9}{3} \\
x &= 3
\end{aligned}
$$

Check:
$$
\begin{aligned}
4(3) - 1 - 3 - 3 &\overset{?}{=} 5 \\
12 - 1 - 3 - 3 &\overset{?}{=} 5 \\
5 &= 5 \checkmark
\end{aligned}
$$

6.
$$
\begin{aligned}
-4g + 3 - 7g &= 25 \\
-4g + (-7g) + 3 &= 25 \\
-11g + 3 &= 25 \\
\scriptstyle -3 \quad & \scriptstyle -3 \\
\frac{-11g}{-11} &= \frac{22}{-11} \\
g &= -2
\end{aligned}
$$

Check:
$$
\begin{aligned}
-4(-2) + 3 - 7(-2) &\overset{?}{=} 25 \\
8 + 3 - (-14) &\overset{?}{=} 25 \\
11 + 14 &\overset{?}{=} 25 \\
25 &= 25 \checkmark
\end{aligned}
$$

Try It! Activity 2

1. $6h + 19 = 4h + 29$

$$
\begin{aligned}
6h + 19 &= 4h + 29 \\
\scriptstyle -4h \quad & \scriptstyle -4h \\
2h + 19 &= 29 \\
2h &= 10 \\
h &= 5
\end{aligned}
$$

Check:
$$
\begin{aligned}
6(5) + 19 &\overset{?}{=} 4(5) + 29 \\
30 + 19 &\overset{?}{=} 20 + 29 \\
49 &= 49 \checkmark
\end{aligned}
$$

2. $-5x + 20 = 8x - 6$

$$
\begin{aligned}
-5x + 20 &= 8x - 6 \\
\scriptstyle -8x \quad & \scriptstyle -8x \\
-13x + 20 &= -6 \\
-13x &= -26 \\
x &= 2
\end{aligned}
$$

Check:
$$
\begin{aligned}
-5(2) + 20 &\overset{?}{=} 8(2) - 6 \\
-10 + 20 &\overset{?}{=} 16 - 6 \\
10 &= 10 \checkmark
\end{aligned}
$$

3. $-9x - 13 = 11x + 7$

$$
\begin{aligned}
-9x - 13 &= 11x + 7 \\
\scriptstyle +9x \quad & \scriptstyle +9x \\
-13 &= 20x + 7 \\
-20 &= 20x \\
-1 &= x
\end{aligned}
$$

Check:
$$
\begin{aligned}
-9(-1) - 13 &\overset{?}{=} 11(-1) \cdot \\
9 - 13 &\overset{?}{=} -11 + 7 \\
-4 &= -4 \checkmark
\end{aligned}
$$

Try It! Activity 3

1.
$$\begin{aligned} -3(x+9) &= x+1 \\ -3x+(-27) &= x+1 \\ -4x+(-27) &= 1 \\ -4x &= 28 \\ x &= -7 \end{aligned}$$

Check:
$$\begin{aligned} -3(-7+9) &= -7+1 \\ -3(2) &= -6 \\ -6 &= -6 \ \checkmark \end{aligned}$$

2.
$$\begin{aligned} -1(6x+6) &= 2x+18 \\ -6x+(-6) &= 2x+18 \\ -8x+(-6) &= 18 \\ -8x &= 24 \\ x &= -3 \end{aligned}$$

Check:
$$\begin{aligned} -1(6(-3)+6) &= 2(-3)+18 \\ -1(-18+6) &= -6+18 \\ -1(-12) &= 12 \\ 12 &= 12 \ \checkmark \end{aligned}$$

3.
$$\begin{aligned} 3(x-8) &= 9 \\ 3x-24 &= 9 \\ 3x &= 33 \\ x &= 11 \end{aligned}$$

Check:
$$\begin{aligned} 3(11-8) &= 9 \\ 3(3) &= 9 \\ 9 &= 9 \ \checkmark \end{aligned}$$

4.
$$\begin{aligned} -4(x+3) &= x-62 \\ -4x+(-12) &= x-62 \\ -5x+(-12) &= -62 \\ -5x &= -50 \\ x &= 10 \end{aligned}$$

Check:
$$\begin{aligned} -4(10+3) &= 10-62 \\ -4(13) &= -52 \\ -52 &= -52 \ \checkmark \end{aligned}$$

5.
$$\begin{aligned} -2(3x-1) &= x+9 \\ -6x+2 &= x+9 \\ -7x+2 &= 9 \\ -7x &= 7 \\ x &= -1 \end{aligned}$$

Check:
$$\begin{aligned} -2(3(-1)-1) &= (-1)+9 \\ -2(-4) &= 8 \\ 8 &= 8 \ \checkmark \end{aligned}$$

6.
$$\begin{aligned} 7(m-35) &= m+1 \\ 6m-35 &= 1 \\ 6m &= 36 \\ \frac{6m}{6} &= \frac{36}{6} \\ m &= 6 \end{aligned}$$

Check:
$$\begin{aligned} 7(6-5) &= 6+1 \\ 7(1) &= 7 \\ 7 &= 7 \ \checkmark \end{aligned}$$

Glossary

area
Area is the number of square units that fit inside a 2-D shape.

axis (axes)
The axes are the lines that show the number scale on a graph. The x-axis is horizontal, and the y-axis is vertical. Axis is singular and axes is plural.

bar graph
A bar graph is a graph that uses vertical or horizontal rectangular bars to show the quantity being measured. The longer (or higher) the bar, the higher value it represents.

basic operations
Basic operations include addition, subtraction, multiplication, and division.

bias
Bias occurs when a particular outcome is favoured over another.

circle graph/pie chart
A circle graph or pie chart are visual representations of data amounts that together form a total amount or a single quantity.

circumference
Circumference is the distance around a circle.

coefficient
A coefficient is a number that multiplies a variable in a mathematical expression.

For example, in the expression $3x - 7$, the number 3 is a coefficient. In the expression $\frac{x}{5} + 8$, the coefficient is $\frac{1}{5}$.

constant/constant term
A constant or constant term is a number in a mathematical expression that has no variable attached to it. The number can't be changed.

For example, in the expression $3x - 7$, the constant is 7. In the expression $\frac{x}{5} + 8$, the constant is 8.

continuous data
Continuous data is data that is part of a set of numbers that can be infinitely divided into smaller and smaller fractions.

For example, time or distance information can be thought of as continuous because they exist in units smaller than we can measure.

coordinates
Coordinates are a set of numbers that can be used to describe a location of a point on a coordinate plane.

coordinate plane or Cartesian plane
A coordinate plane or Cartesian plane is a rectangular area with one or more axes. The plane is designed to show data in a visual way. It is named after its inventor, Rene Descartes.

congruent
Congruent means “equal to.”

cross section
A cross section is a section cut from a prism or a cylinder. The cut is made parallel to the base.

cylinder
A cylinder is a three-dimensional or 3-D shape which has two circular bases that are parallel to each other and the same distance apart.

data
Data are numbers that represent measurements. Data may represent money, time, distances, or any other amounts.

degrees
Degrees are the measurement of the size of an angle or part of a circle. A full circle is 360 degrees, also written as 360°.

denominator
The denominator is the bottom number in a fraction. It represents the total number of equal parts.

For example, in the fraction $\frac{3}{4}$ where 4 is the denominator, an object or group has been divided into 4 equal parts. (See also **numerator**.)

diameter
In a circle, the diameter is a straight line from one edge of the circle to the other, which passes through the centre of the circle.

discrete data
Discrete data is data that is grouped into separate categories, with no information existing between the categories.

distributive property
The distributive property states that if you add two numbers and then multiply the sum by another number, you'll get the same result as if you multiply each of the two numbers by the other number and then add the products.

For example, $4(2 + 5) = (4)(2) + (4)(5)$.

equation
An equation is a pair of mathematical expressions that are joined by an equals sign (=), and so they represent the same amounts. An equation is a mathematical "complete sentence".

equilateral triangle
An equilateral triangle is a triangle with three equal sides. In an equilateral triangle, all of the interior angles are 60°.

equivalent
When two things are equivalent they have the same value.

For example $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent expressions

event
An event is a specific outcome from the sample set of all possible outcomes.

For example, drawing a five of hearts from a normal deck of 52 cards is an event.

expression
An expression is a mathematical phrase. An expression is made of terms. Terms are joined by the mathematical operators plus or minus (+ or –) into expressions.

For example, $5x - 7$ is a two-term expression.

extrapolate
To extrapolate means to estimate quantities or data beyond the last amounts measured; to extend a graph line beyond the last data point. (See also **interpolate**.)

favourable outome
A favourable outcome means achieving a desired result in a probability experiment.

fraction
A fraction is a number that represents part of a whole.

For example, $\frac{1}{2}$ represents one part out of a total of two parts.

graph
A graph is a visual representation of data using lines, bars, symbols, or areas.

heptagon
A heptagon is a seven-sided closed figure.

hexagon
A hexagon is a six-sided closed figure.

histogram
A histogram is a vertical bar graph.

hypotenuse
1. the side of a right triangle that is not a leg.
2. the longest side of a right triangle.
3. the side of a right triangle that is opposite the right angle.

icon
An icon is a small symbol that represents a quantity of items for a pictograph or in a graph legend. Usually a picture or line drawing of the item is used as an icon.

improper fraction
An improper fraction is a fraction where the numerator is larger than the denominator.

For example, $\frac{7}{5}$ is an improper fraction.

independent event
In a probability experiment, an independent event is when the outcome of one event does not influence or change the possible outcome of another event.

intercept
The intercept is the location where a line graph intersects an axis.

interior angles
Interior angles are angles that are inside a figure. For polygons, interior angles are at each vertex.

interpolate
To interpolate means to estimate the data amounts between data points that were measured. (See also **extrapolate**.)

interval
An interval is the amount between two values; their difference.

irregular polygon
An irregular polygon is a closed figure where all the sides are not equal and all the angles are not equal.

isosceles triangle
An isosceles triangle is one with two equal sides.

legs
Legs refer to:
1. the sides of a right triangle that form the right angle.
2. the parts of the body that the feet are attached to.

line graph
A line graph is a graph using a straight, bent, or curved line to show continuous data.

linear equation/linear relation
A linear equation or linear relation is an equation, table, description or graph that shows the relationship between two variables and forms a straight-line graph.

misinterpret
To misinterpret means to misunderstand or to gain a false impression from a conversation, picture, data or text.

misleading information
Misleading information is information (such as a graph) that is technically correct but would give most viewers an inaccurate impression.

misrepresent
To misrepresent is to present information falsely, visually or in words.

mixed number
A mixed number is a number composed of a whole number and a fraction.

For example, $2\frac{1}{3}$ is a mixed number.

model
1. To model is to create a representation of real-life data.
2. A model is the graph, map, computer program or another item that represents data.

net
A net is a two-dimensional or 2-D construction of a three-dimensional or 3-D object.

numerator
The numerator is the top number in a fraction. It represents the number of equal parts you are working with.

For example, in the fraction $\frac{3}{4}$ where 3 is the numerator, you are working with only 3 of the parts out of 4 total. (See also **denominator**.)

octagon
An octagon is an eight-sided closed figure.

operations
When we do something with a number or numbers, it is called an operation. Addition, subtraction, multiplication, and division are basic operations.

ordered pair
An ordered pair is a pair of numbers (x, y) that represent the values that satisfy a relation and also represent a location on the graph of the relation.

origin
The origin is the point (0,0) on a two-dimensional graph at which the axes intersect.

outcome
The outcome is the result of a single trial or experiment.

pentagon
A pentagon is a five-sided closed figure.

percent
A percent is a fraction of a whole, expressed as a fraction out of 100.

perfect square
A perfect square is a number that represents the area of a square whose sides are whole numbers.

For example, if a square has sides of length 3, its area is 9, and 9 is a perfect square.

It is also the result when a whole number is multiplied by itself.

For example, $5 \times 5 = 25$, and 25 is a perfect square.

perspective
Perspective is the viewer's perception, visually or psychologically.

pictograph
A pictograph is a graph that uses icons or symbols to represent the amount measured in each category, instead of using an axis to show the measurements.

pie chart
See **circle graph**.

plane
A plane is a two-dimensional or 2-D surface.

point
A point is a location on a coordinate plane which can be represented by an ordered pair (x, y).

polygon
A polygon is a closed geometric shape made of 3 or more line segments.

prism
A prism has three-dimensional or 3-D shapes that have the same cross section along a length.

proper fraction
A proper fraction is a fraction whose numerator is less than its denominator.

For example, $\frac{2}{3}$ is a proper fraction.

probability
Probability is the chance or likelihood that a particular event will occur. Probabilities are often listed as ratios (e.g. 1:2 or 2 to 5), fractions (e.g. $\frac{3}{5}$) or percents (e.g. 15%)

proportion
A proportion is a pair of equal ratios.

Pythagorean Theorem
The Pythagorean Theorem describes the relationship among the lengths of the three sides of a right triangle: $a^2 + b^2 = c^2$

Pythagorean Triple
A Pythagorean Triple is a set of three whole numbers that satisfy the Pythagorean Theorem.

> For example, the numbers 3, 4, and 5 form one Pythagorean Triple. The first two numbers in a Pythagorean Triple are the measurements of the legs, and the third (the largest number) is the measurement of the hypotenuse.

quadrilateral
A quadrilateral is a four-sided closed figure.

radius
In a circle, the radius is the distance from the center to the edge of the circle.

random experiment
A random experiment is a a process leading to at least two outcomes with some uncertainty about which will occur.

rate
A rate is a comparison of two quantities in which each quantity is measured in different units. For example \$8 per dozen roses (or \$8.00/12 roses) is a rate. (See also **unit rate**.)

ratio
A ratio is a comparison of two or more numbers. Ratios are written with a ":" (e.g. 2:3), using words (e.g. 2 to 3), or as a fraction (e.g. $\frac{2}{3}$).

reciprocal
A reciprocal is a number that you multiply a fraction by so that the result equals one. If you start with a whole number, put it over 1 first. The easiest way to find it is to just flip the fraction over.

> For example, the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.

rectangular prism
A rectangular prism is a six-sided three-dimensional or 3-D shape made up of rectangles.

regular polygon
A regular polygon is a closed figure with all sides equal and all angles equal.

right angle
A right angle is an angle that measures 90°.

right triangle
A right triangle has one right angle.

round/round off
To round or round off is to remove unwanted place values at the right end of a number, adjusting the first remaining place value if necessary. (See also **truncate**.)

sample space
A sample space includes all the possible outcomes resulting from a probability experiment.

satisfy
To satisfy means to replace variables with values that make an equation into a true statement.

For example, $y = 3x$ can be satisfied with the ordered pair (2, 6), but cannot be satisfied with (4, 9).

square root
The square root symbol tells us to take the square root of the number that's inside.

For example, $5^2 = 25$. The square root of 25 is 5.

square root symbol
$\sqrt{}$ This symbol tells us to take the square root of the number that's inside.

For example: $\sqrt{4} = 2$

surface area
Surface area refers to the total area of the net of a three-dimensional or 3-D object. The units are squared, for example, cm^2, m^2.

term
A term is an item in an expression that is a constant, or variable, or coefficient-and-variable combination. (See also **expression**.)

tessellation
A tessellation is a tiling pattern that covers an entire plane without overlapping or leaving gaps.

three-dimensional (3-D)
Three-dimensional refers to an object that has length, width and depth, or a representation of an object that has the appearance of depth.

triangular prism
A triangular prism is a five-sided three-dimensional or 3-D shape with two triangles that are parallel and equal to each other and joined by rectangles.

truncate
To truncate means to remove unwanted place values at the right end of a number without adjusting the remaining place value. (See also **round/round off.**)

two-dimensional (2-D)
Two-dimensional refers to an object that has length and width, but no depth.

unit rate
A unit rate is a rate where the second term is 1.

For example, wages are often given as a unit rate.

$10.00/hr represents $10.00 earned for every 1 hour worked.

unknown
An unknown is the value(s) that provide the solution to an equation. (See also **variable.**)

variable
A variable is a value that is unknown or that could change. It is often represented in an expression by a letter such as x, but could be represented by a word or other symbol. (See also **unknown**.)

vertex (vertices)
In a closed figure, the vertex refers to the point where two sides meet. Vertex is singular and vertices is plural.

view
The view refers to a two-dimensional or 2-D drawing of a three-dimensional or 3-D object from one particular position—front view, side view, top view, bottom view, etc.

volume
The volume is the amount of space an object takes up. The units are cubed, for example, cm^3, m^3.

x-axis
The *x*-axis is the horizontal axis of a coordinate plane. (See also **coordinate plane** and **axis**.)

y-axis
The *y*-axis is the vertical axis of a coordinate. See also **coordinate plane** and **axis**.)

Templates

Section 3 | Lesson A: Algebra Tiles