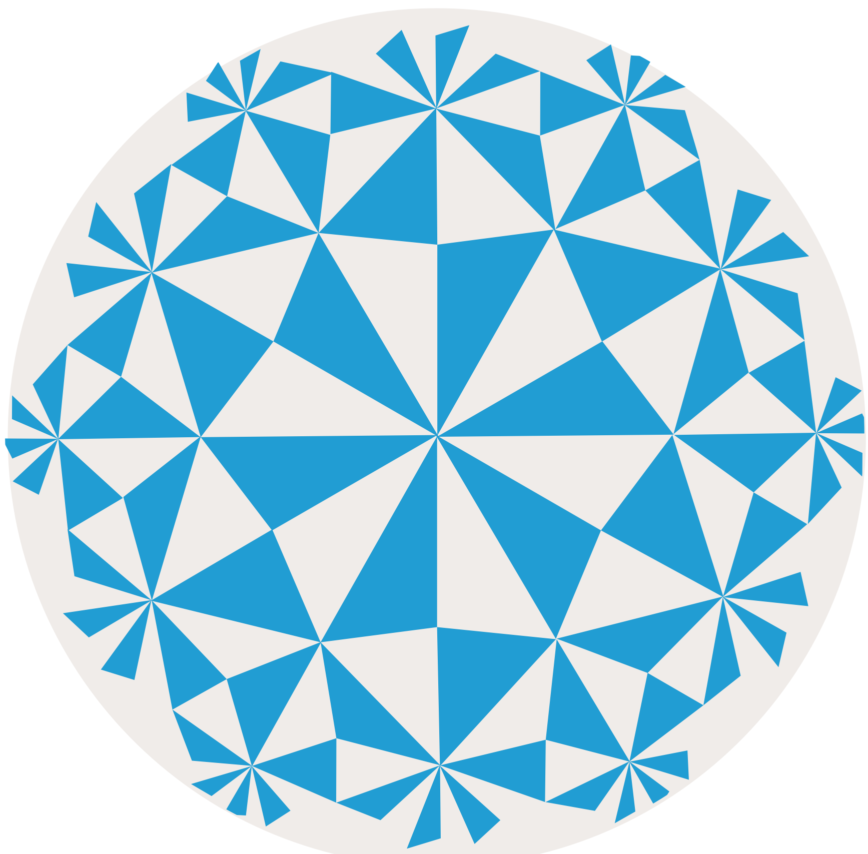
Module 1
Version 03
Section 1-3

Math 8

Module 1

Exploring 2-D and 3-D Connections



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Contents

Course Overview

Welcome to Mathematics 8!

In this course you will continue your exploration of mathematics. You'll have a chance to practice and review the math skills you already have as you learn new concepts and skills. This course will help you to increase your ability to think mathematically.

Organization of the Course

The Mathematics 8 course is made up of four modules. These modules are:

Module 1: Exploring 2-D and 3-D Connections

Module 2: Squares, Integers, and the Pythagorean Theorem

Module 3: Data, Graphing, and Linear Equations

Module 4: Fractions, Ratios, and Probability

Organization of the Modules

Each module has three sections. The sections have the following features:

Pretest

This is for students who feel they already know the concepts in the section. It is divided by lesson, so you can get an idea of where you need to focus your attention within the section.

Lessons

Each section is divided into lessons. Each lesson is made up of the following parts:

Essential Questions

Essential questions are based on the concepts in each lesson. This activity will help you organize information and reflect on your learning.

Warm-up

This is a brief drill or review to get ready for the lesson.

Explore

This is the main teaching part of the lesson. Here you will explore new concepts and learn new skills.

Try it! Activities

These are activities for you to complete to solidify your new skills. You will mark these using Solutions at the end of each module.

At the end of each module you will find:

Solutions

This contains all of the solutions to the Pretests, Warm-ups and Try it! Activities.

Templates

Templates to pull out, cut, colour, or fold in order to complete specific activities. You will be directed to these as needed.

Glossary

This is a list of key terms and their definitions.

More about the Pretest

There is a pretest at the beginning of each section. This pretest has questions for each lesson in the section. Complete this pretest if you think that you already have a strong grasp of the topics and concepts covered in the section. Mark your answers using Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Thinking Space

The column on the outside edge of most pages is called the Thinking Space. You can use this space to

- write questions about things you don't understand
- note things that you want to look at again
- respond to a question in the Thinking Space or the text
- draw pictures that help you understand the math
- identify words that you don't understand
- connect what you are learning to what you already know
- make your own notes or comments

Materials and Resources

There is no textbook required for this course. All of the necessary materials and exercises are found in the modules.

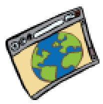
In some cases, you will be referred to templates to pull out, cut, colour, or fold. These templates will always be found near the end of the module, just in front of the answer key.

You will need a scientific calculator for some of the activities. A geometry set would also be helpful, although for many activities you can use a straightedge rather than a ruler. A protractor is available in the Appendix if you don't have one.

If you have Internet access, you might want to do some exploring online. The Math 8 Course Website will be a good starting point. Go to **http://www.openschool.bc.ca/courses/math/math8/** and find the lesson that you're working on. You'll find relevant links to websites with games, activities, and extra practice.

Icons

You will see a few icons used on the left-hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.



Explore Online

Essential Questions

Solutions

Use a Calculator

Module 1 Overview

Module 1 consists of three sections on nets, views, calculations in 2-D and 3-D, and tessellations. You'll learn how to draw nets of 3-D objects and create a 2-D representation of a 3-D object. Following this you'll study these 3-D objects in more detail by calculating their volume and surface area. Finally, you'll use rotations, reflections and translations to unravel the mysteries of tessellations and create some of your own.

Section Overviews

Section 1.1: Nets and Views

In this first section you'll compare 2-D drawings with 3-D objects. You'll learn how to draw a representation of a 3-D figure on isometric dot paper and how to visualize the different views of 3-D objects. You'll draw nets of prisms and cylinders by starting with a common household article and then move to drawing the net just by visualizing the shape. When shown a representation of a 3-D object, you'll be able to sketch its views. You'll also learn to create a net and construct or sketch a 3-D object when given the views.

Section 1.2: Surface Area and Volume

In the second section, you'll focus on 3-D objects by finding their surface area and volume. In addition to regular 3-D objects like prisms and cylinders, you'll also work with irregular shapes and composite figures.

Section 1.3: Tessellations

How can you tell if a shape will tessellate? This is one of the questions that you'll be able to answer after studying Section 1.3. You'll see how to use transformations – translations, reflections, rotations – to create a tessellation. You'll learn how to design a tessellation tile with irregular edges and use the tile to make a tessellation with translation and rotation.

Course Map

On the following page you'll find a course map. If you colour in the box for each section and lesson as you complete it, you'll easily be able to see how much of the course you've finished, and how much is still left to complete.

Mathematics 8

Module 1

Section 1: Nets and Views

- Lesson A: Visualizing in Three Dimensions
- Lesson B: Prisms and Cylinders and their Nets
- Lesson C: Top View, Side View, and Front View

Section 2: Calculations in 2-D and 3-D

- Lesson A: Total Area of the Net
- Lesson B: More about Area in 2-D and 3-D
- Lesson C: The Amount of Space

Section 3: Shapes and Design

- Lesson A: What is a Tessellation Anyways?
- Lesson B: Transformations in Tessellations
- Lesson C: Creating a Tessellation

Module 2

Section 1: Multiplying and Dividing Integers

- Lesson A: Should I Multiply or Divide?
- Lesson B: Mult. and Div. with Negative Numbers
- Lesson C: Expressions–More than one Operation

Section 2: Squares and Square Roots

- Lesson A: Perfect Squares
- Lesson B: Square Roots of Perfect Squares
- Lesson C: Estimating Square Roots
- Lesson D: Calculating Square Roots

Section 3: The Pythagorean Theorem

- Lesson A: The Pythagorean Theorem
- Lesson B: Pythagorean Triples
- Lesson C: Using the Pythagorean Theorem

Module 3

Section 1: Analyzing statistical graphs

- Lesson A: Types of Graphs and Data
- Lesson B: Adv. and Dis. of Graph Types
- Lesson C: Misleading Graphs

Section 2: Graphing on the Coordinate Plane

- Lesson A: Ordered Pairs and Linear Equations
- Lesson B: Tables of Values
- Lesson C: Relationships and Missing Values

Section 3: Linear Equations

- Lesson A: Algebra Tiles
- Lesson B: Solving Linear Equations Symbolically
- Lesson C: More Linear Equations

Module 4

Section 1: Multiplying and Dividing Fractions

- Lesson A: Proper, Improper Fractions, Mixed Numbers
- Lesson B: Multiplying Fractions
- Lesson C: Dividing Fractions
- Lesson D: Estimating and Solving Problems

Section 2: Ratios, Rates and Percents

- Lesson A: Ratios and Rates in Everyday Life
- Lesson B: Making Sense of Percents
- Lesson C: Solving Problems with Percents

Section 3: Probability

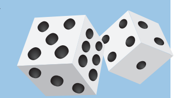
- Lesson A: The Probability of Independent Events
- Lesson B: Problem Solving with Probability

Section 1
Nets and Views

In this section you will:

- differentiate between 2-D and 3-D
- draw 3-D and 2-D images
- create different 2-D views and nets of a 3-D object
- construct a 3-D object, given various views and nets

For this section you will need:

- 10 dice—check that your numbers are arranged as

- rectangular tissue box
- scissors
- tape
- Metric ruler
- compass
- isometric dot paper and graph paper (in Appendix)

Where in the World...?

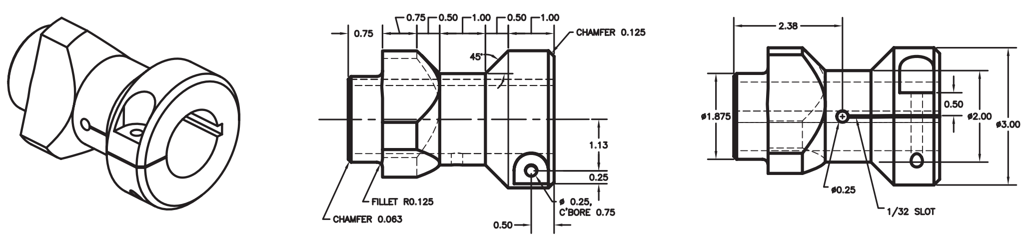
An object that takes up space can be seen in many ways.

Our eyes see a go-kart in three dimensions.

However, developers such as engineers, designers, and fabricators will create the go-kart from a two-dimensional drawing with various views. Each of these views will look different, depending on what side of the go-kart they are looking at. All these different views help build the go-kart to the right size with the right components so it all works.

Similarly, developers create each component of a go-kart from drawings that detail it. For example, below are three views of a wheel hub for a go-kart.

KART WHEEL HUB



ISOMETRIC VIEW TOP VIEW FRONT VIEW

Many professions, from architects to video game creators, use 2-D drawings to design and create various products.

Section 1
Pretest

Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

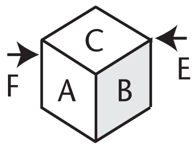
If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson A: Visualizing in Three Dimensions

1. Put each of the words in the list under either the 2-D or 3-D heading:
   - rectangle
   - cube
   - a box
   - oval
   - sphere
   - can of pop
   - circle

2-D Shape	3-D Shape

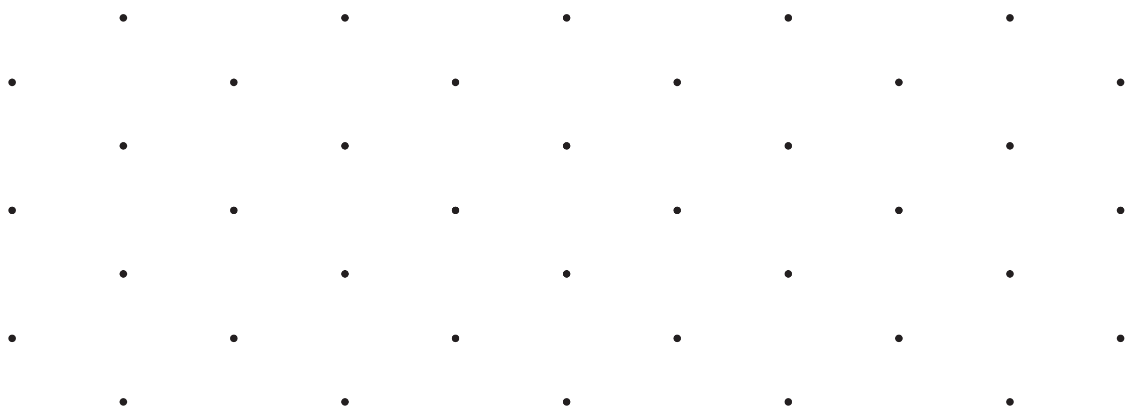
2. Visualize this cube in 3-dimensions.



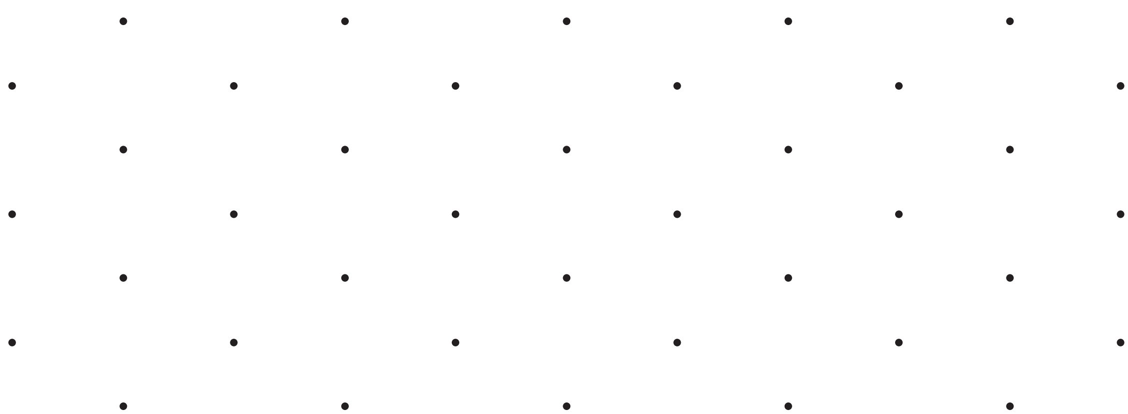
On the bottom is: D

Assume the cube begins in this position, then:

a. On the isometric dots, draw the cube as it would appear, keeping D on the bottom and turning it 90 degrees clockwise. Label all the sides.



b. Return the cube back to the way it started at the beginning of question 2. Then, on the isometric dots, draw the cube as it would appear, keeping D on the bottom and turning it 90 degrees counter clockwise. Label all the sides.



Lesson B: Prisms and Their Nets

3. Look at each picture and decide whether it contains one or more of the following:
   - triangular prism
   - rectangular prism
   - cylinder

   Circle your choice(s).

   a.

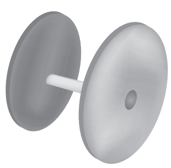
   triangular prism   rectangular prism   cylinder

   b.

   triangular prism   rectangular prism   cylinder

   c.

   triangular prism   rectangular prism   cylinder

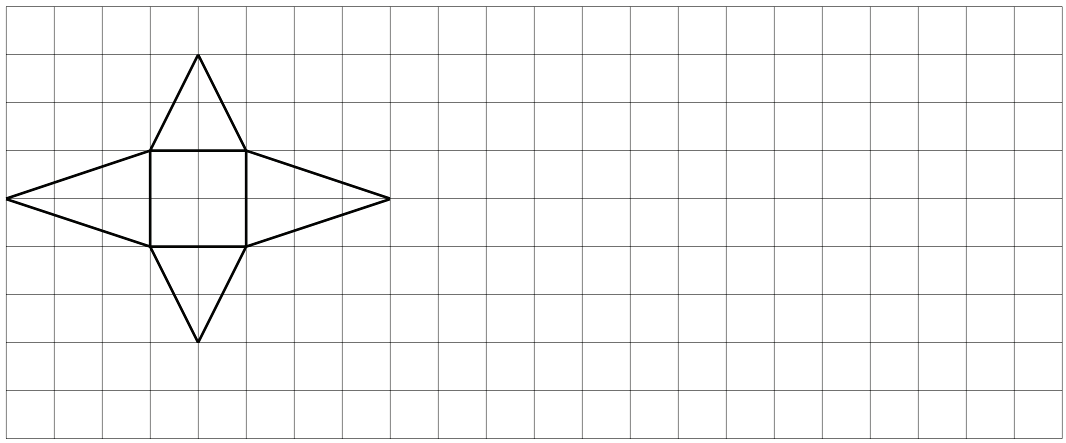
   d.

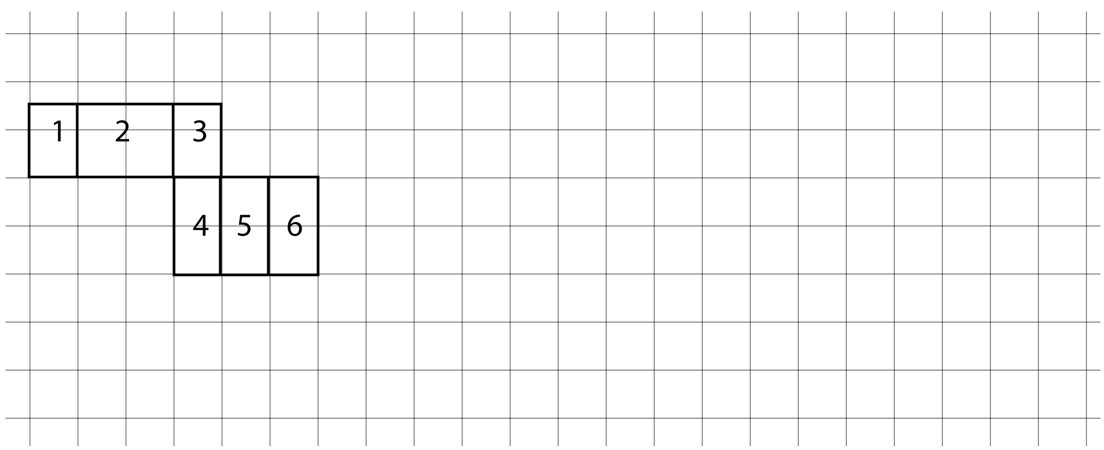
   triangular prism   rectangular prism   cylinder

4. Both nets are incorrectly drawn. Neither will construct a 3-D shape. For each net, explain how you would change the net so that it could build a 3-D shape. Then draw the net correctly.

a.

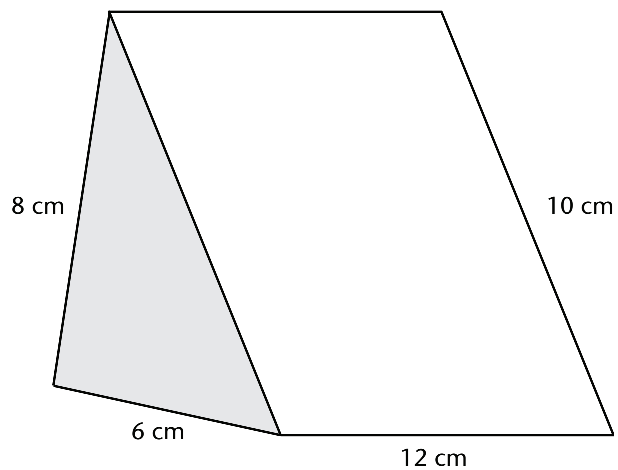


b.

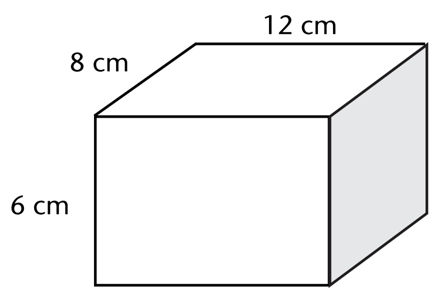


5. Draw a net for each of the following figures. Use a sheet of the graph paper provided at the end of the module. Label all the sides with their lengths.

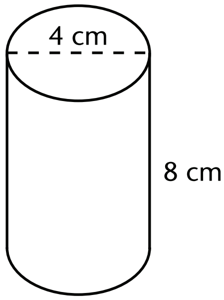
a.



b.



c.



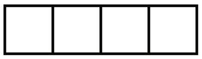
Lesson C: Top View, Side View, and Front View

6. Draw the top, side, and front views of this cell phone.

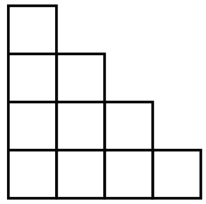
7. Using the following views, construct a 3-D object with 10 six-sided dice, building blocks, or sugar cubes.

a.

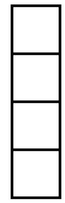
Top View



Front View

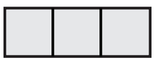


Side View

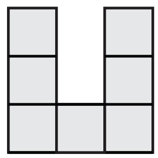


b.

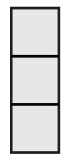
Top View



Front View

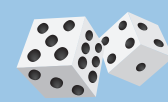


Side View



Turn to Solutions at the end of the module and mark your work.

Lesson A
Visualizing in Three Dimensions



For this lesson you will need:

- 10 six-sided cubes, blocks, or dice
- graph paper (from the Appendix at the back of the module)
- isometric dot paper (from the Appendix at the back of the module)
- tape
- scissors
- ruler or straight edge

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

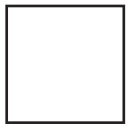
When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

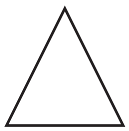
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What does 3-D mean?		
How is a 3-D object different from a 2-D object?		
What are views?		

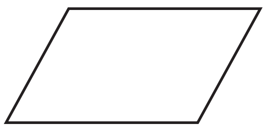
Thinking Space

Warm-up

1. Match the name to the shape.

____ 

____ 

____ 

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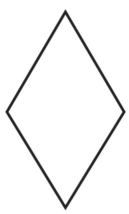
____ 

a. parallelogram

b. rectangle

c. square

d. rhombus

e. triangle

2. Which of these situations describes a rotation?

   a. A car makes a right hand turn.

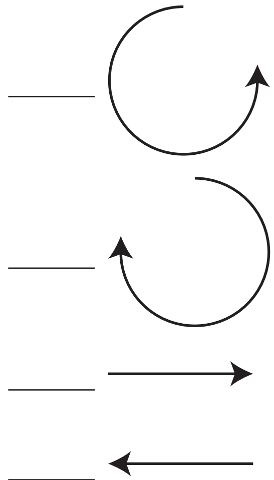
   b. A skater spins on one skate.

   c. A person skiis down a hill.

   d. A bus reverses down a driveway.

Thinking Space

3. Match the description to the arrow.



a. moving right

b. turning clockwise

c. moving left

d. turning counter-clockwise

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

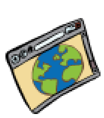
3-D Versus 2-D

For this Explore you will need:

- isometric dot paper (in Appendix)

Trace an outline of your hand onto a piece of paper. Looking at the drawing you can see the shape of your hand and how wide and long it is, but that's about all. The drawing is a two-dimensional (2-D) representation of your hand; it shows two dimensions, width and length.

Now place the hand you just traced beside the drawing. Run a finger from your other hand over the drawing and then your placed hand. Your finger bumps up when you come to your hand, right? That's because your hand has depth, in addition to length and width. Any object that has depth, width and length has three dimensions and therefore occupies physical space. If you drew your hand showing its width, length, and depth, you'd have a three-dimensional (3-D) representation of it.



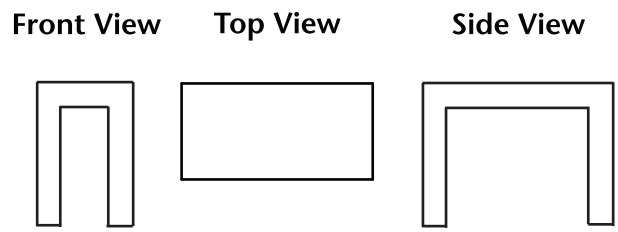
To learn more about 2-D and 3-D representations and how to draw them, go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0811b1f_dimensions.html and look at *Drawing Dimensions* now.

Thinking Space

It's Your Turn!

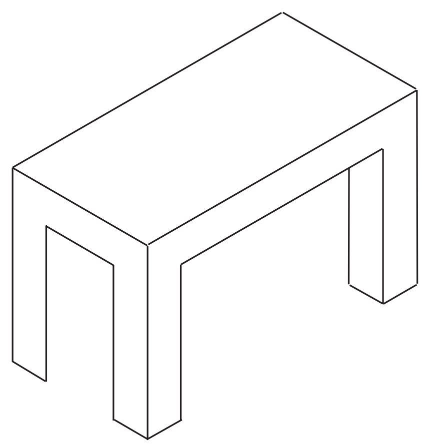
Try drawing the 2-D front, top and side views of a rectangular table in your home.

You may have views that look like these

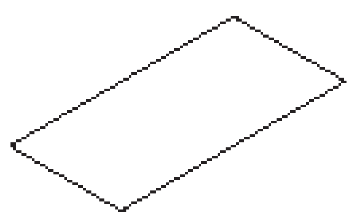


Now try drawing the table using isometric dot paper.

Your drawing may look similar to this.



Here the top of the table is drawn as a parallelogram, rather than a rectangle.

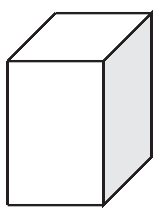


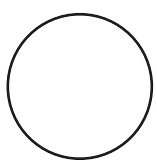
Thinking Space

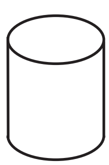
Try It!
Activity 1

1. What is the difference between a two-dimensional drawing and a three-dimensional drawing?

2. Beside each drawing, write 2-D or 3-D.

 a.


 b.


 c.


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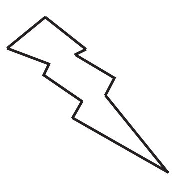

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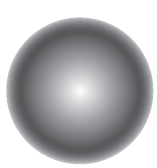


Thinking Space

3. Place a die in front of you. Have three dots facing up and five dots visible on your left and one dot visible on your right. Use the isometric dots to complete the following drawings.

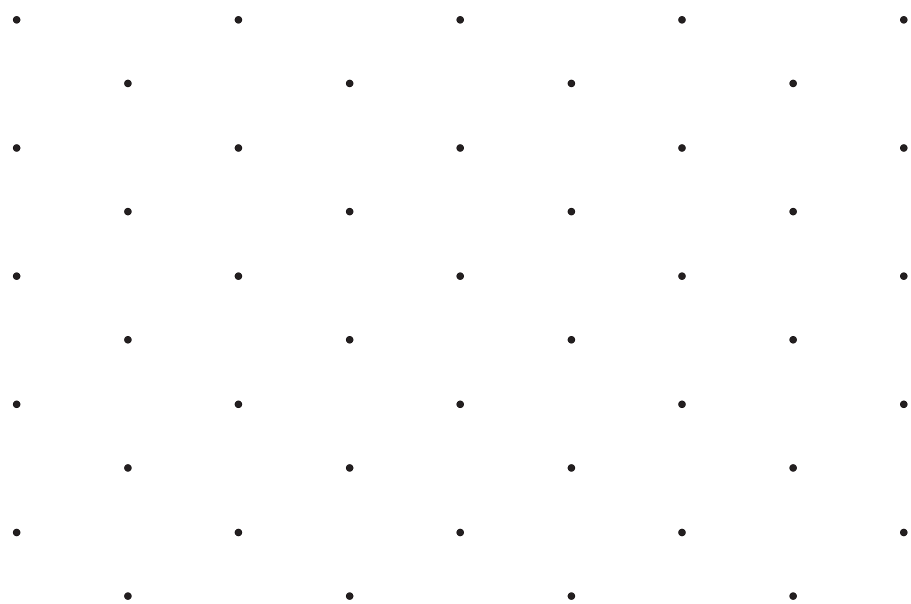
   a. Draw what you see.

   b. Turn the die 90 degrees clockwise. Draw what you see.

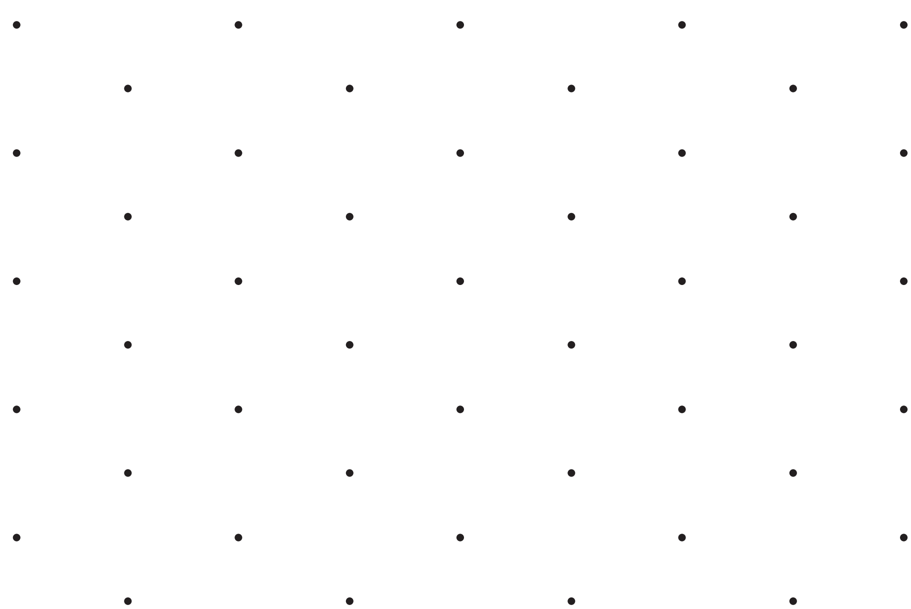
Thinking Space

4. Place a die in front of you. Place five dots face down, have three dots to the left, and one dot to the right. Use the isometric dots to complete the following drawings.

   a. Draw what you see.



   b. Turn the die 90 degrees counter-clockwise. Draw what you see.



Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Different Views

By sketching an outline of what you see, you can create a 2-D drawing. We'll try this with something familiar—your shoe. Take off your shoe and place it on a table. It's three-dimensional, but by sketching the outline of it, you're creating a 2-D drawing.

Draw your shoe now.

Did you draw a side view?

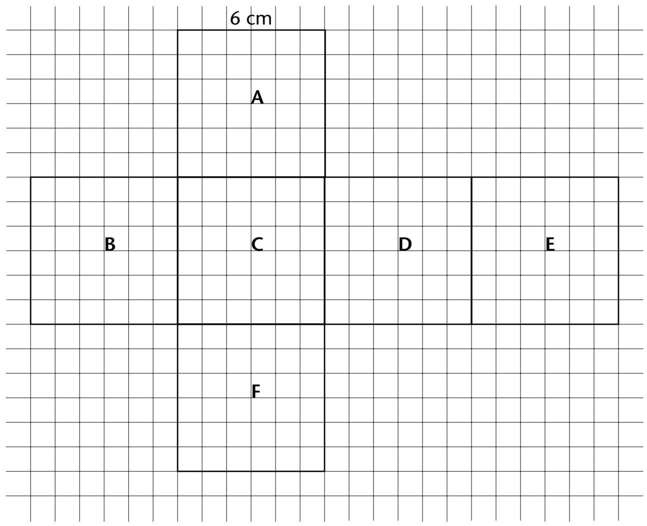
Or a view from the front?

Or a view from the top?

The view that you sketched will depend on where you were standing.

Thinking Space

On a piece of graph paper, draw the following six squares. Each side of the square should be 6 cm long and drawn connected as shown:



Cut this figure out and tape it together into a cube. Place it in front of you on your desk.

If D is facing up, what letter is facing down?

If A is facing up and E is to your left, what letter do you see to your right?

If B is on top and upside down, what letter is facing you?

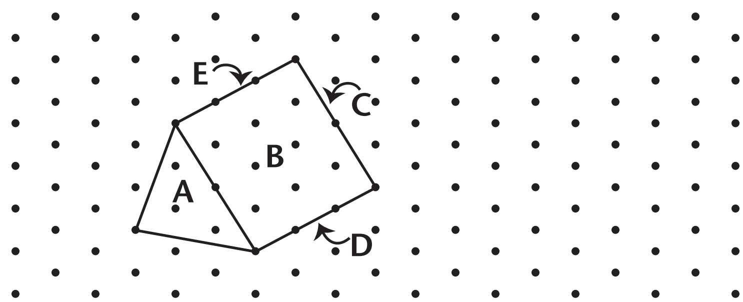
Notice that the orientation (position) the object is in affects what you are able to see. When architects or artists create images of 3-D objects they have to keep in mind all these various views.

Thinking Space

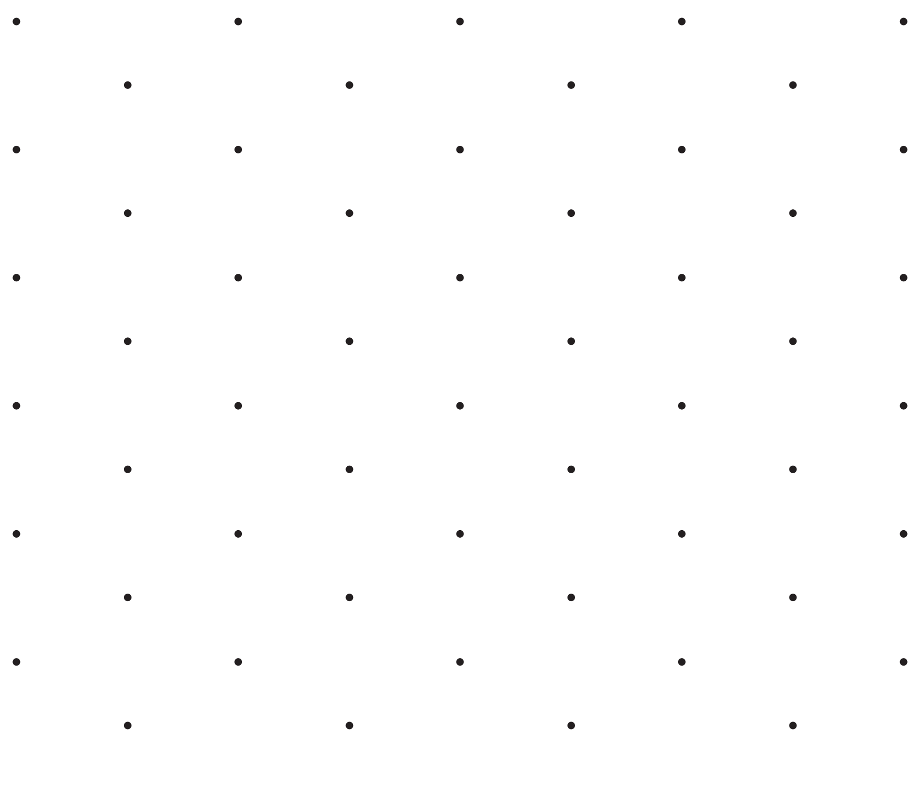
Try It!
Activity 2

1. Look at this shape.

   The faces have all been labelled.



   a. Imagine pushing this shape over so that B is on the bottom. Draw the figure again, and label the letters for all the sides.



Thinking Space

b. Imagine the original shape standing on the triangle side labelled A. Have E on the left and B on the right. Draw this on the isometric dots.

2. In the Appendix, a 2-D drawing has been made for you to cut out. Cut it out, and tape it together to form the shape in question 1. Use this to check your answers.

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B
Prisms and Cylinders and Their Nets

For this lesson you will need:

- rectangular tissue box
- scissors
- tape
- metric ruler
- compass

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What is a rectangular prism? How do I draw and label its net?		
What is a triangular prism? How do I draw and label its net?		
What is a cylinder? How do I draw and label its net?		

Thinking Space

Warm-up

1. Draw each of the following lengths using a ruler:
   a. 5 cm

   b. 3.4 cm

   c. 28 mm

2. Using a ruler and/or compass, draw the following:
   a. two 4 cm long lines that meet at 90º (a right angle)

   b. a triangle with one right angle and sides of the following lengths: 3 cm, 4 cm, and 5 cm

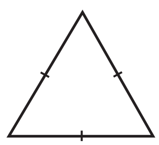
   c. a circle with a radius of 1.5 cm

Thinking Space

d. a circle with a diameter of 10 cm

3. Name each of these shapes.

a.



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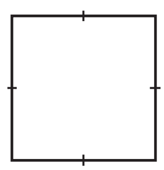
b.



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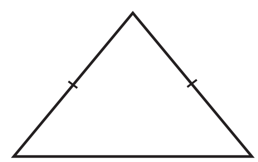
c.



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Turn to the Solutions at the end of the module and mark your work.

Thinking Space

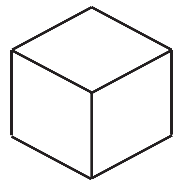
Explore

Nets of Rectangular Prisms

For this Explore you will need:

- rectangular tissue box (empty)
- scissors
- tape
- templates (in Appendix)
- graph paper (in Appendix)
- isometric dot paper (in Appendix)
- Metric ruler

A cube is a 3-D shape with six equal sized squares as faces.



A die is an example of a cube.

Each of these figures below can be made into a cube… or can they?

Shade in each figure that you think can be folded into a cube.

Figure 1

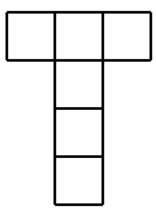


Figure 2

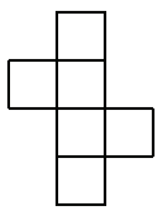


Figure 3

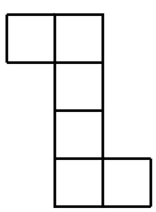
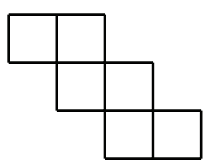


Figure 4



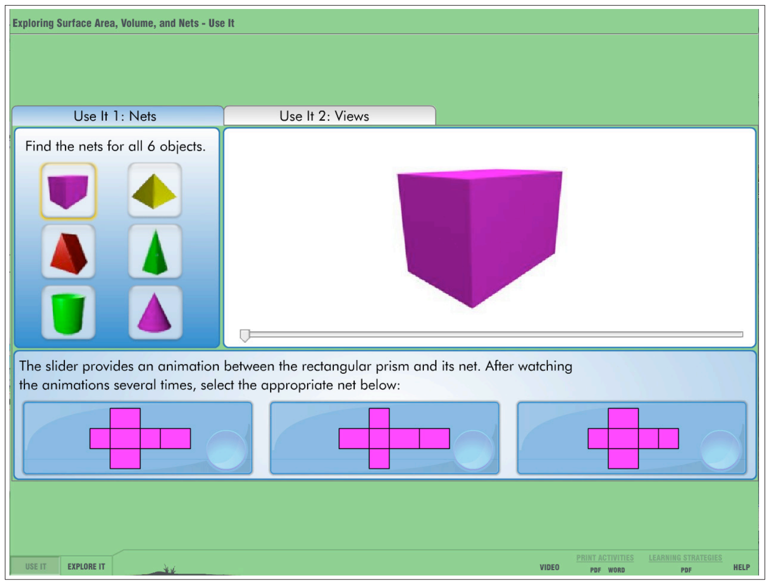
Now, go to the templates in the Appendix and cut out each figure. Fold along the lines to see if you can make a cube. Were you right?

Thinking Space

On graph paper, draw your own net made up of six connected squares that is different from figures 1–4. Use a ruler. Cut out your drawing and fold it to see if your figure creates a cube or not.

The figure you drew and the figures you cut out are all called nets. A **net** is a 2-D figure that makes a 3-D object when it's folded.

Go to **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0811b2f_netview.html** and open *Nets and Views*.



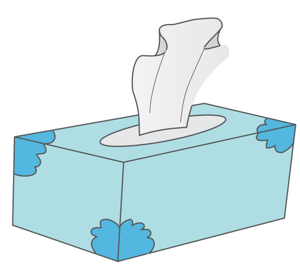
Click on a shape on the left and then use the slider under the shape to see its net. If you like, you can try the quiz at the bottom of the screen for each figure.

Drawing the Net of a Rectangular Prism

Can you draw the net for the tissue box?

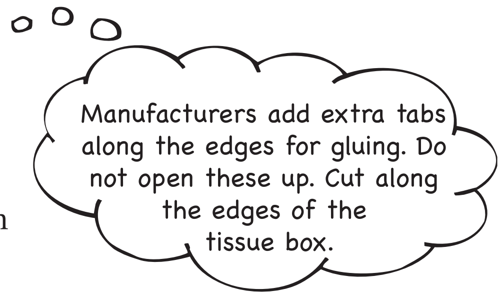
Here's another 3-D shape.

A tissue box is not a cube, but an example of a rectangular prism. One or more of its faces are rectangles that are all joined together at 90 degree angles.



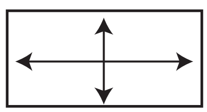
To draw a net of the tissue box, follow these instructions.

1. Take the tissue box you set aside for this lesson, and a pair of scissors.
2. Cut along each fold to flatten the tissue box.
3. Cut the flattened box into six rectangles.
4. Tape the pieces together to form the net of the tissue box.
5. Draw the net.

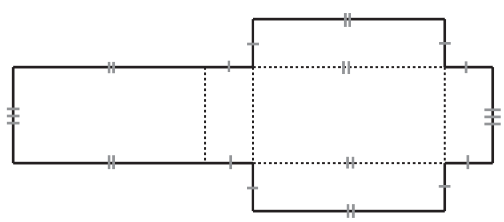


There are many edges of the rectangles above that end up taped together, and there are edges that don't. The edges that are taped together should be the same length as each other.

Also, you'll notice the opposite sides of each rectangle are the same length.



In the diagram below, the gray bars show you which edges have the same length. Taped edges are shown as dotted lines.



Look at your net again to see if each pair of taped edges is the same length.

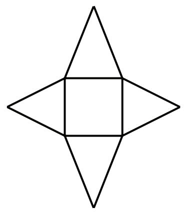
Thinking Space

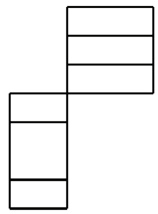
Of the edges that are taped together, how many are the same length?

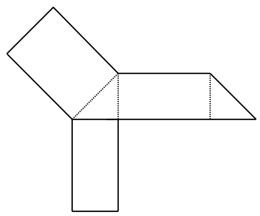
Thinking Space

Try It!
Activity 1

1. Can you construct a 3-D object from each of the nets? If you think you can, shade the net in.

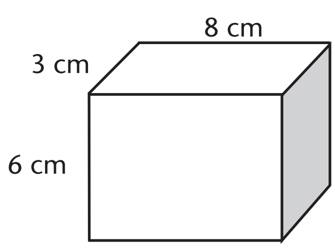
a. 

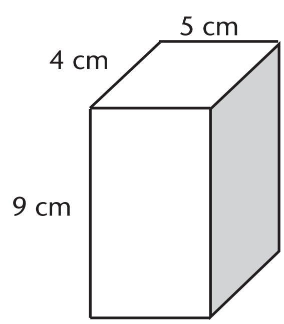
b. 

c. 

To check your answer, go to the templates in the Appendix. Cut out each net on this sheet and try to form a 3-D object to check your prediction(s).

2. Using a ruler and graph paper from the Appendix, draw a net for each of these 3-D objects.

a. 

b. 

3. Now cut out each net you drew in question 2. Construct your nets into shapes to confirm that they are correct.

Turn to Solutions at the end of the module and mark your work.

Thinking Space

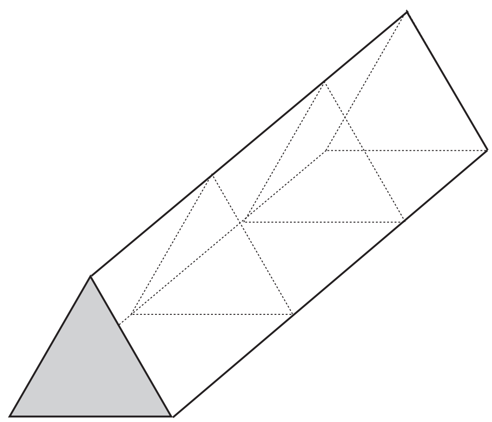
Explore

Nets of Triangular Prisms and Cylinders

The tissue box was an example of a rectangular **prism**. But what is a prism?

To understand the definition of a prism, we first need to understand the term **cross section**. When you slice bread, each slice has the same front and back view. When you have the same front and back view, you have a cross section.

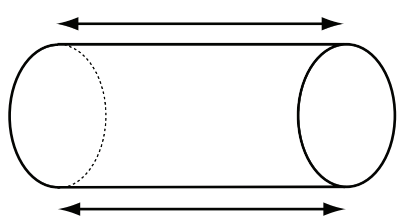
Prisms are 3-D shapes that have the same cross section along a length. In a triangular prism, all the cross sections are triangles.



In a rectangular prism, what shape are all the cross-sections?

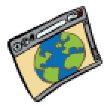
In **triangular** and **rectangular prisms**, the front and back views are the same distance apart and are joined by rectangles.

In the case of a **cylinder**, the cross sections are all circles, and the front and back views are identical and the same distance apart.



Thinking Space

Watch the following video that shows how to draw the net of a **triangular prism** and a **cylinder**.



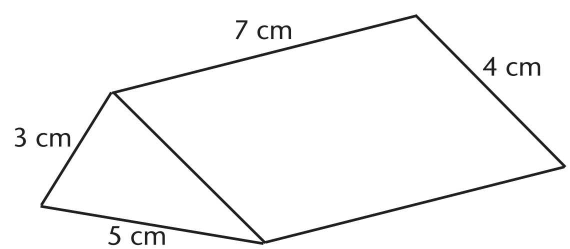
Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0811b2f_netview.html and find out *How to Make a Net for a Triangular Prism and a Cylinder.*

In the video, the triangular prism had an equilateral triangle for both the top and bottom face. This makes drawing a net quite simple.

But when the triangle does not have three equal sides like in an equilateral triangle, then there is more room for error.

Drawing a Net of a Triangular Prism

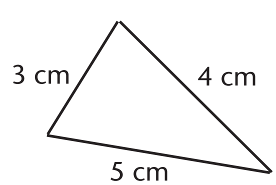
Use a ruler and graph paper to draw this net.



Note: It may be helpful to read through all of the directions first to see where to place the net on your paper.

Step 1: Draw the **top view**.

The **top view** of a triangular prism is one of the triangles.

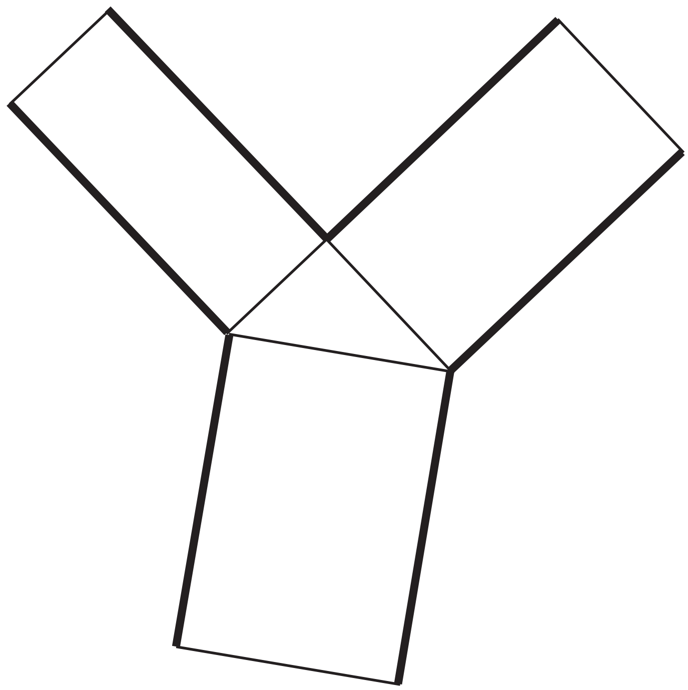


Thinking Space

Step 2: Draw the rectangles.

The easiest way to do this is to draw them on each edge of the triangle.

Important: These are drawn at a 90° angle to each edge of the triangle.



Notice how each of the rectangles have darkened edges. These lines are darkened to help you see that they are all the same length.

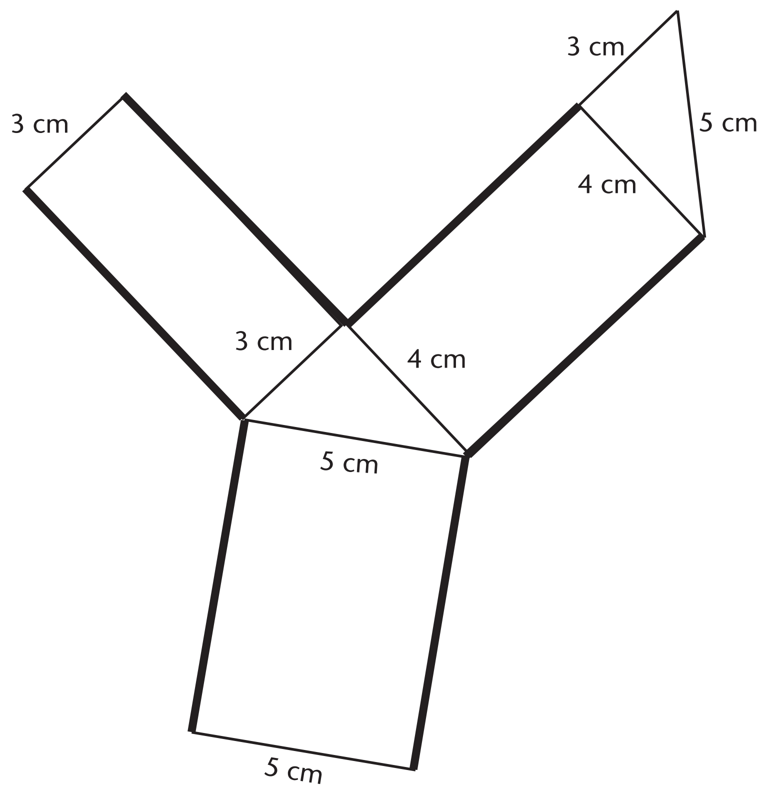
All of these lengths are 7 cm, which is the distance between the two triangular faces.

Thinking Space

Step 3: Draw the other triangle.

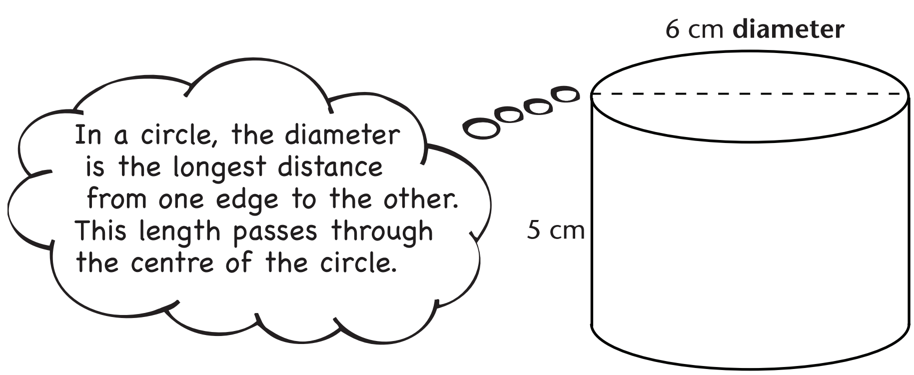
This face can be drawn onto the end of any of the rectangles.

Here it is drawn at the end of the largest rectangle.

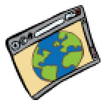


Cut it out and tape joining edges together. Does it rebuild into a triangular prism?

Drawing a Net for a Cylinder



Thinking Space



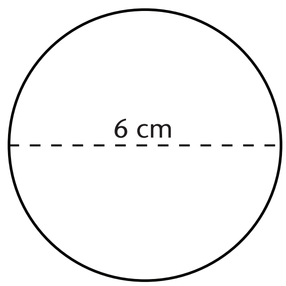
Go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0811b2f_cylinder.html and watch *Constructing Cylinders*.

The tricky part in creating the net of a cylinder is figuring out how long to make the rectangle.

Use a ruler and graph paper to draw this net.

Step 1: Draw the top view.

The **top view** of a cylinder is a circle with a **diameter** of 6 cm.



Step 2: Draw the rectangle.

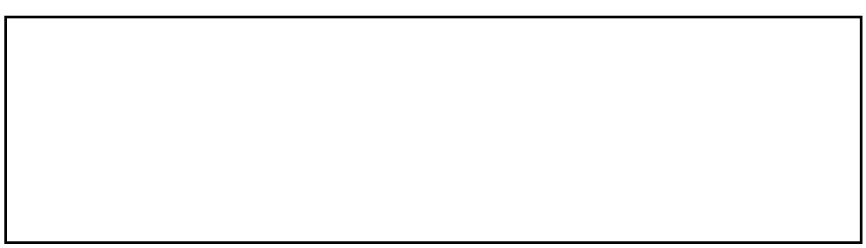
The height of the rectangle will be 5 cm. But what will the length be?

The length of the rectangle = length around the outside edge of the circle = πd

The diameter of the circle is 6 cm.

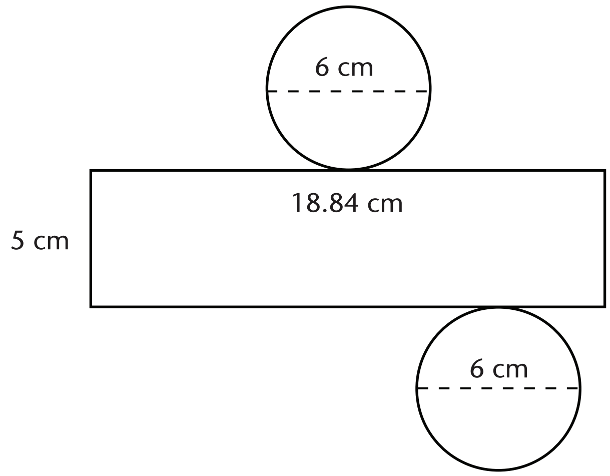
3.14×6 cm = 18.84 cm

Then the length around the outside edge of the circle is 18.84 cm.



Thinking Space

Step 3: Draw the **bottom view** of the cylinder, which will be identical to the top view.



Cut out your net, and tape the edges together. Does it build into a cylinder?

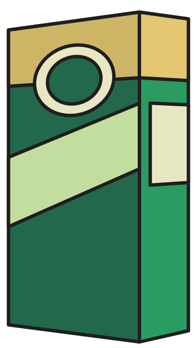
Thinking Space

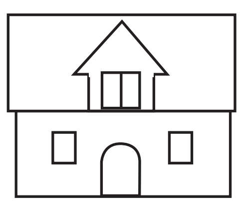
Try It!
Activity 2

1. Match the shape to the name. Draw lines. Each name can be used more than once.

______ 

______ 

______ 

______ 

a. rectangular prism

b. triangular prism

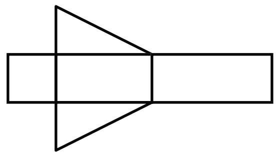
c. cylinder

d. rectangular prism and triangular prism

Thinking Space

2. Which type of prism does each net form?

a.



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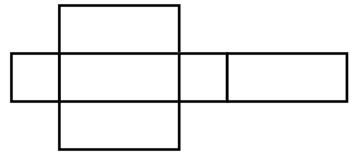
b.



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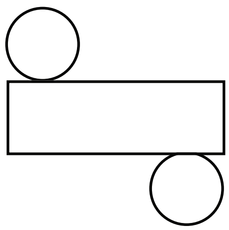
c.



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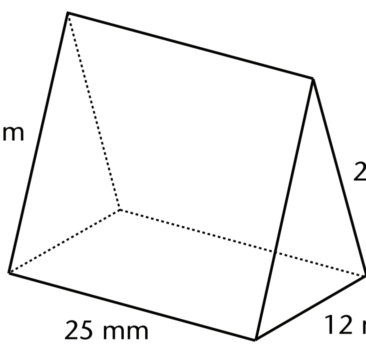
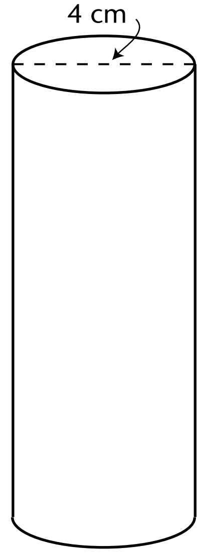
3. Use a ruler and graph paper found in the Appendix to sketch the net for each of the following. Label each edge with its length.

a.



b.



Thinking Space

4. Cut out the nets you drew in question 3 to check if they build into the 3-D figures above.

Turn to Solutions at the end of the module and mark your work.

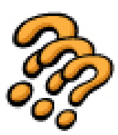
You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson C

Top View, Front View, and Side View

In this lesson you will need:

- scissors
- tape
- metric ruler
- graph paper (in Appendix)
- 10 dice, building blocks, or sugar cubes



When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

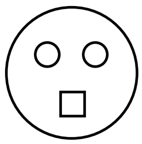
When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

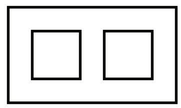
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
When is it useful to draw side views, top views, or front views of 3-D objects?		
What jobs involve using drawings with different views?		

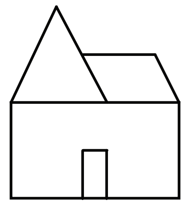
Thinking Space

Warm-up

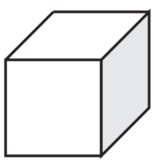
1. Name all the shapes you see in each picture.

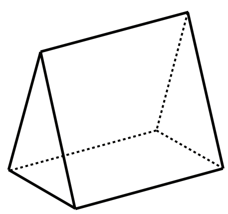
a. 

b. 

c. 

2. How many sides does each shape have?

a. 

b. 

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Top, Front, and Side Views

For this Explore you will need:

- graph paper in Appendix

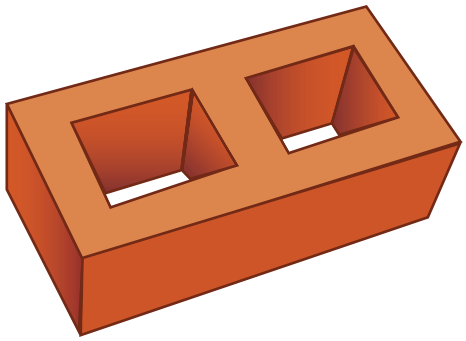
Imagine someone gave you some tools, a pile of wood and some electrical wire and asked you to build a garage. Where would you start? You'd need to know what the garage was for to know how big it should be. You'd also need to know where the doors go, where lights and light switches need to be located, and many, many other details. How much easier would it be if you also had a set of drawings showing all the details about what the finished garage should look like?

For whatever we want to build we first need a plan. This is true whether we are building large complex things like a building or a car, right down to the small components within them.

The plan for anything we build is known as a working drawing. A working drawing contains 2-D views of the object. At a minimum, there are views of the the top, front, and side. These views contain measurements and all the other details needed to build the object. A 3-D view, often an isometric drawing, can also be part of a working drawing to help us visualize what the object will look like.

Let's work through the process backwards, starting with the finished product, a clay brick.

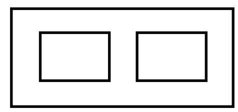
Thinking Space



Use graph paper for the following sketches:

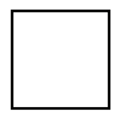
Sketch the **top view**.

Standing over top and looking down, what do you see?

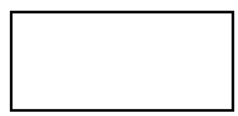


Sketch the **side view**.

Standing to the side, either left or right what do you see?



Sketch the **front view**.

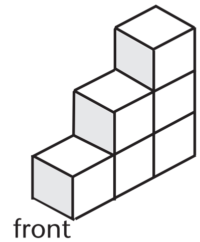


Standing in front, what do you see?

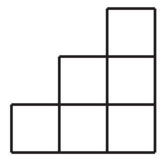
Thinking Space

If you stand on either side, does it look the same?

Here is a slightly more challenging 3-D shape.



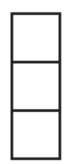
Sketch the **side view**.



Sketch the **front view**.

We know from the 3-D drawing that the cubes step back like stairs.

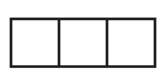
But since we are drawing in 2-D, there is no depth. So we still draw these faces of the cubes all in one column, one on top of the other.



Sketch the **top view**.

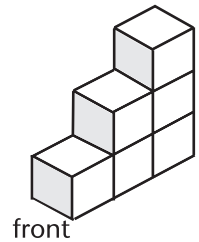
Looking from the top, one of the faces of the cubes starts closer to you and then as you move along, the next cube face is further away and the next is even further.

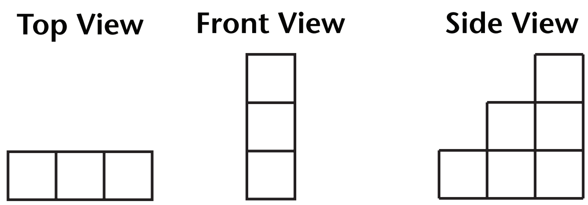
But, since we are drawing in 2-D, there is no depth. So we draw the faces of other cubes all in one row, one beside the other.



Thinking Space

Here is the tower of cubes again with all of the views:



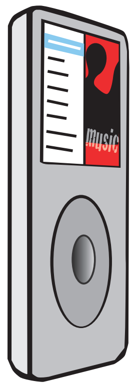


Thinking Space

Try It!
Activity 1

1. Draw the top, front, and side view of each shape on graph paper.

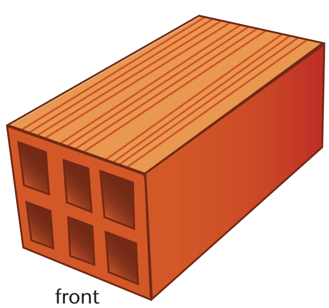
   a.



   b.

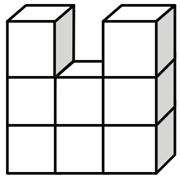
2. Draw the top, front, and side views of each of these 3-D objects on graph paper.

   a.

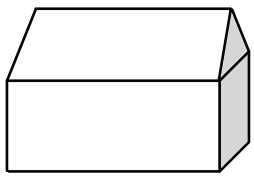


Thinking Space

b.



c.



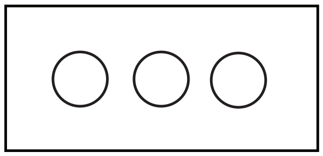
Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
From Views to 3-D

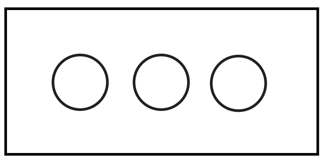
Use these views of a brick to create a net:

Top View



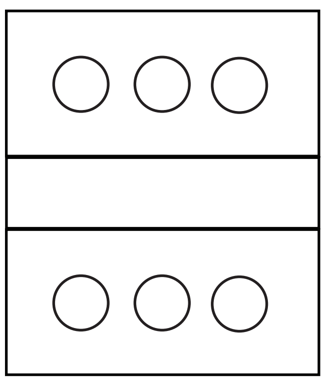
Front View

Side View



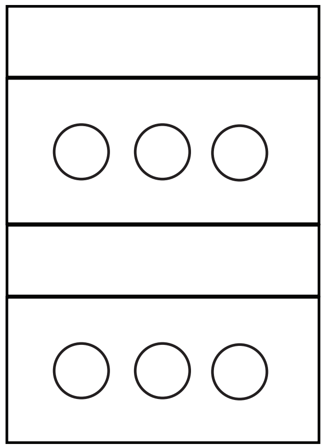
To put these together, start with the top view.

Now imagine moving your view from the top, down to the front, and then under to the bottom.

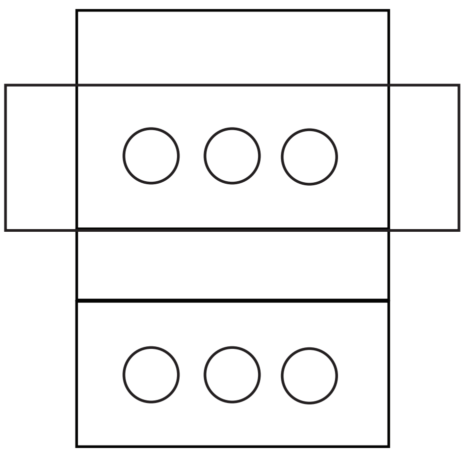


You see that the view from the bottom is the same as the Top View, but they're not connected to each other. Instead they are separated by the Front View piece.

Thinking Space

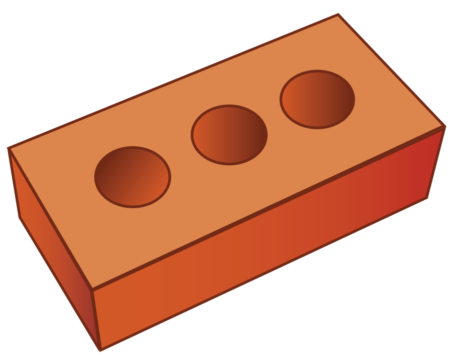


In the same way, move your view from the top and down the back (which looks the same as the Front View). Draw the back view piece attached to the Top View.



Lastly, look sideways at the Top View, then move your view from the top and down to the side. You see that the Side View piece is attached to the top. Draw both side view pieces attached to the Top View.

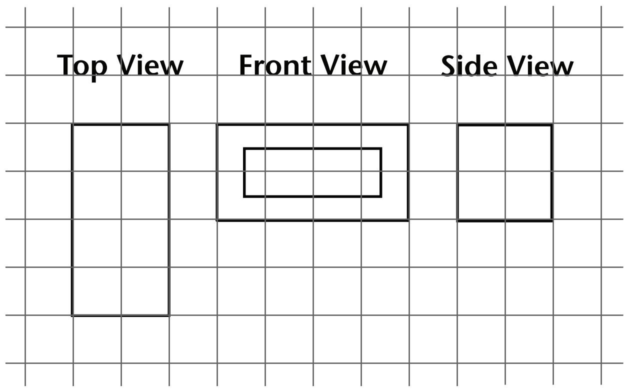
The result is a net. This net can be folded together to form a 3-D object.



Thinking Space

Try It!
Activity 2

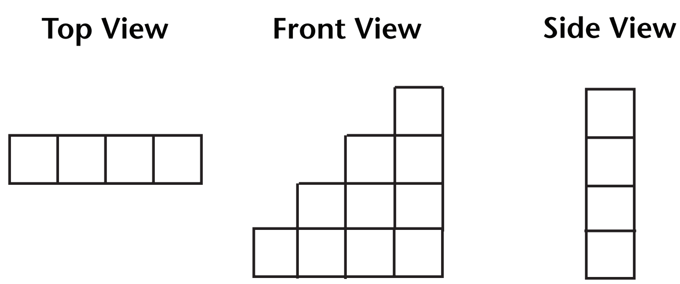
1. a. Use these drawings to create the net of its 3-D object. Use graph paper.



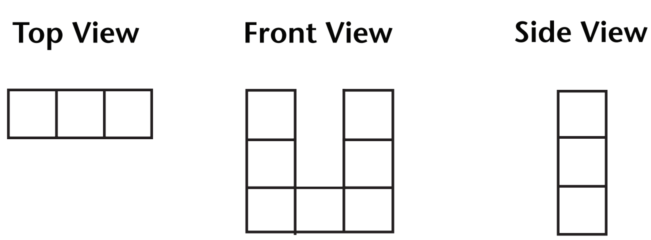
b. Cut out your net to check if it creates a 3-D object.

2. Get out ten building blocks, dice, or sugar cubes. Using the following views, construct the correct 3-D object.

a.



b.



Thinking Space

3. Draw all of the following views of this house on graph paper.

   - top view
   - front view
   - side view

   Include windows and doors in your different views.

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson C. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Thinking Space

Section Summary

Completing this section has helped you to:

- differentiate between 2-D and 3-D
- draw 3-D and 2-D images
- create different 2-D views and nets of a 3-D object
- construct a 3-D object, given various views and nets

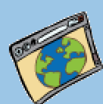
Section 2
Calculations in 2-D and 3-D

In this section you will:

- find the surface area (SA) of rectangular and triangular prisms and cylinders
- use SA to solve problems
- discover the formula for volume
- apply volume formulas for prisms and cylinders

For this section you will need:

- scientific calculator
- pencil
- metric ruler
- graph paper
- scissors
- tape



Note: you can also go to the Math 8 website at: http://www.openschool.bc.ca/courses/math/math8/mod1/.html to find a link to an online scientific calculator.

Where in the World...?

Packaging is the container or combination of materials used to wrap and protect a product. When packaging an item, there are two major things to consider. One is the amount of material you will use. The other is the amount of space your product will take up.

When constructing homes or other buildings, materials and space are major factors. Building costs will depend on the materials needed. The amount of materials depends on how large the building is.

Section 2
Pretest

Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using Solutions found at the end of the module.

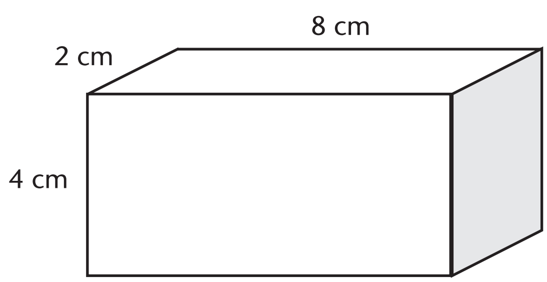
If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

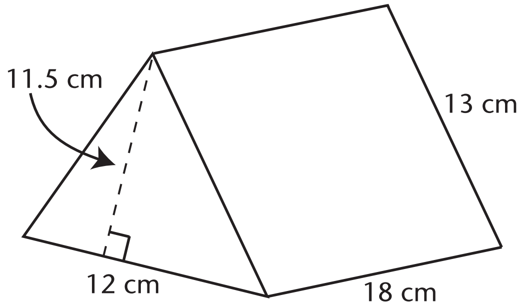
Lesson A: Total Area in the Net

1. Find the surface area. Round to the nearest tenth, if necessary.

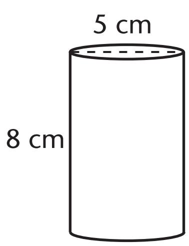
   a.



b.



c. Use the π button on your calculator for this question. See the Appendix for an example of where you might find this on your calculator.



2. A manufacturer is creating a label for his new brand of soup. The label will be 16 cm high and will wrap around the can, which has a diameter of 5.6 cm.

   Answer the following to the nearest tenth.

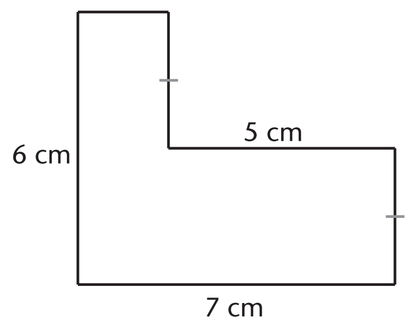
   a. How long will the label need to be to fit around the can?

   b. What is the area of the label?

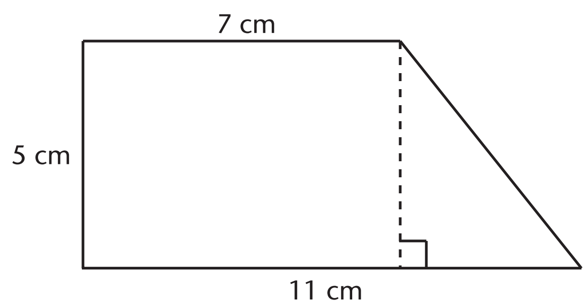
Lesson B: More About Area in 2-D and 3-D

3. Find the area of each irregular shape, to one decimal place.

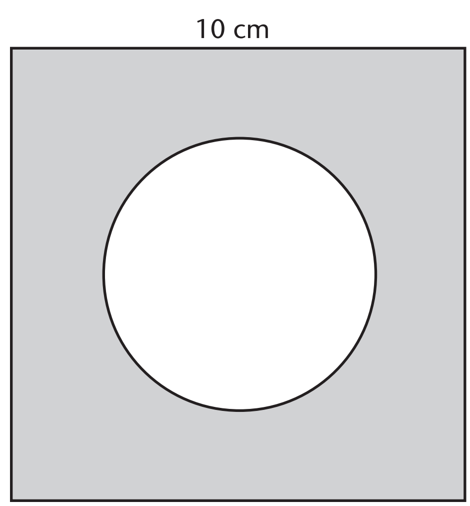
a.



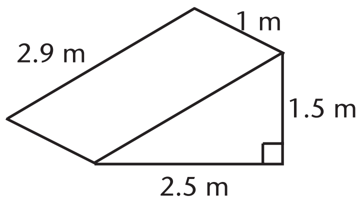
b.



c. Diameter of the circle = 6 cm



4. You make a skateboard ramp and need to paint all the surfaces except the bottom.



a. Draw the net of the surfaces you have to paint. Label all the necessary lengths on your net.

b. Draw each view. Label all the lengths on the views.

Top View **Side View** **Back View**

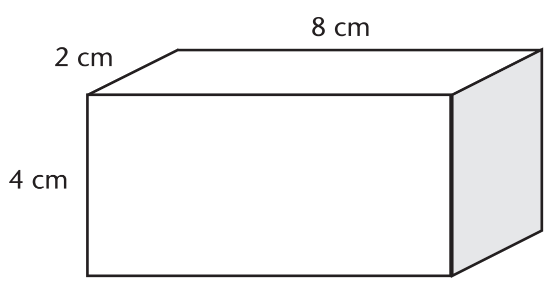
c. Find the total area that needs to be painted.

d. Will one can of paint that covers 10 m^2 be enough? Explain your answer.

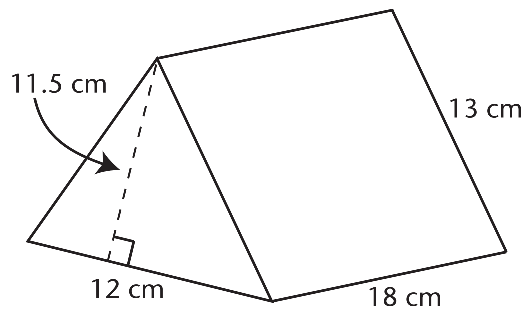
Lesson C: The Amount of Space

5. Find the volume for each object. Round to two decimal places if necessary.

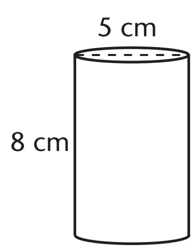
a.



b.



c. Use the π button on your calculator for this question.



6. A company has a new product and is deciding on packaging. There are two different box options that will fit the product.

Option 1: 3 cm by 4 cm by 5 cm **Option 2: 2 cm by 5 cm by 6 cm**

a. Do these containers have the same volume? What is the volume of each?

b. What is the surface area of each packaging option?

Option 1: ____________________ **Option 2:** ____________________

c. Which packaging option would cost less? Explain your answer.

____

____

7. What happens to the surface area of a prism if the volume increases? Explain your answer.

____

____

Turn to Solutions at the end of the module and mark your work.

Lesson A

Total Area of the Net

For this lesson you will need:

- scientific calculator (see Appendix | Templates: Section 3)
- graph paper (in Appendix)

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What is surface area?		
How do I find the surface area of a triangular prism?		
What formulas do I know for finding surface area?		

Thinking Space

Warm-up

1. Of the following 3-D shapes—cube, rectangular prism, triangular prism, and cylinder—which have:
   a. rectangles in the net? ______________________________

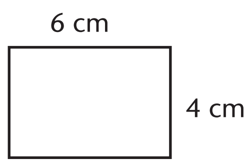
   ____

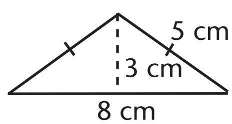
   b. triangles as faces? ______________________________

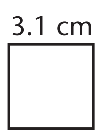
   c. circles as a surface? ______________________________

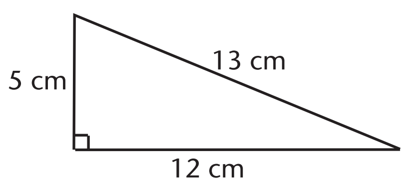
   d. only squares in the net? ______________________________

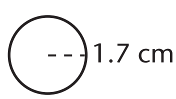
2. Match each shape to each area. You can use your formula sheet.

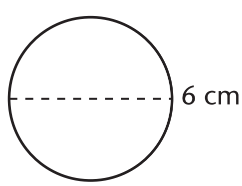
______ 

______ 

______ 

______ 

______ 

______ 

a. 9.1 cm^2
b. 113 cm^2
c. 9.6 cm^2
d. 30 cm^2
e. 60 cm^2
f. 28.3 cm^2
g. 12 cm^2
h. 24 cm^2

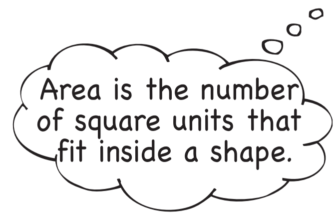
Thinking Space

Turn to Solutions at the end of the module and mark your work.

Explore

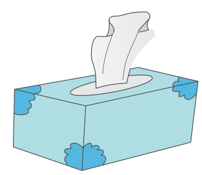
Surface Area of Prisms

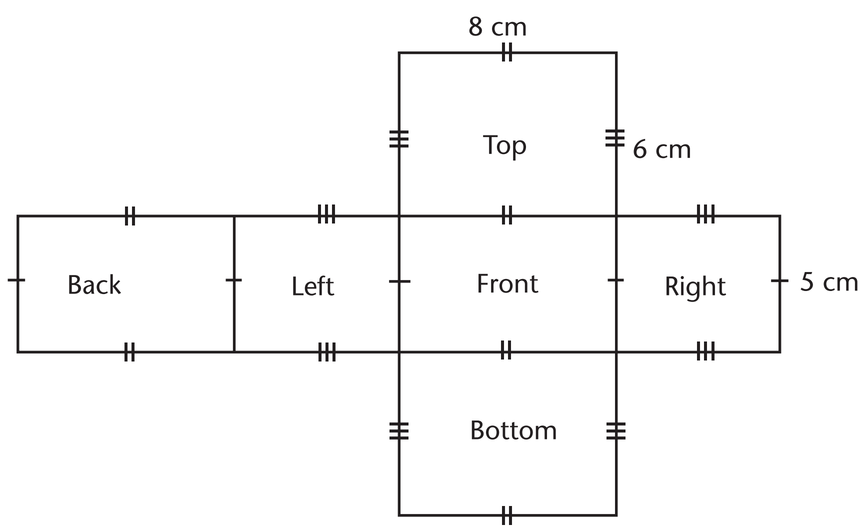
Every 3-D object is made up of 2-D faces. Each face has an **area**. All the faces together form a net.



Surface area is the area of all the faces in the net added together. The units of surface area are units squared, for example cm^2 or m^2.

A tissue box is made up of cardboard. Manufacturers figure out the surface area in order to figure out how much cardboard to buy.





The net of the tissue box helps you calculate the surface area.

Thinking Space

Fill in the missing numbers:

Front Area: ________ (8×5)

Back Area: ________ (8×5)

Left Side Area: ________ (6×5)

Right Side Area: ________ (6×5)

Top Area: ________ (8×6)

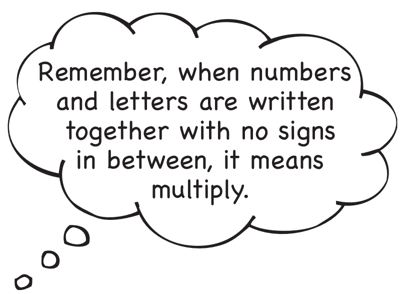
Bottom Area: ________ (8×6)

If you add up all the above areas, then the surface area is ________ cm^2.

Notice that:

- The **front and back views** are the same size. The areas were found by multiplying 8×5 or $l \times h$. The area of both views together is $2lh$.
- The **left and right views** are the same size. The areas were found by multiplying 6×5 or $w \times h$. The area of both views together is $2wh$.
- The **top and bottom views** are the same size. The areas were found by multiplying 8×6 or $l \times w$. The area of both views together is $2lw$.

So a formula for finding the surface area of a rectangular prism is:



$$SA = 2lw + 2wh + 2lh$$

To use this formula, plug in 8 for *l*, 6 for w, and 5 for h.

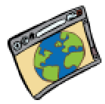
$$SA = 2\ l\ w\ + 2\ w\ h\ + 2\ l\ h$$
$$SA = 2(8)(6) + 2(6)(5) + 2(8)(5)$$

Enter these numbers in a scientific calculator exactly as shown here:

$$2 \times 8 \times 6 + 2 \times 6 \times 5 + 2 \times 8 \times 5 =$$
$$SA = 236 \text{ cm}^2$$

***Note**: If your calculation doesn't give the same surface area as shown here, refer to your calculator's instructions.*

Thinking Space

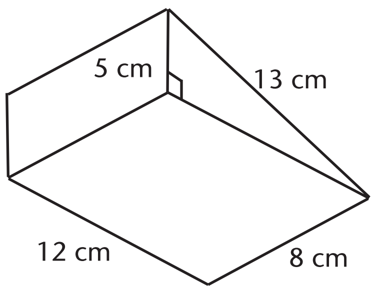


If you have access to the Internet, you can also use an online scientific calculator. Go to the Math 8 website at http://www.openschool.bc.ca/courses/math/math8/mod1.html and look for *Lesson 2A: Total Area of the Net*.

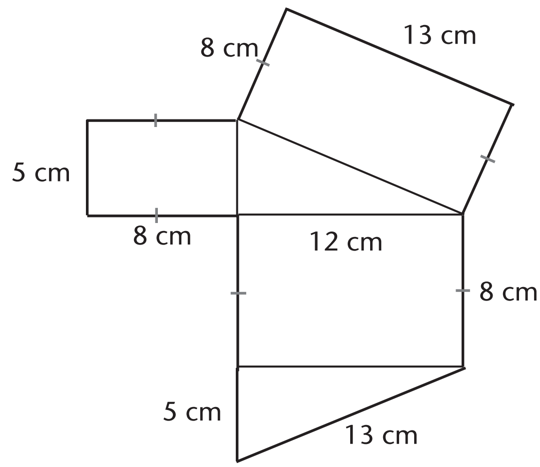
Surface Area of a Triangular Prism

The surface area of a triangular prism is found in the same way. Draw the net, find the area of each face, and then add them all together.

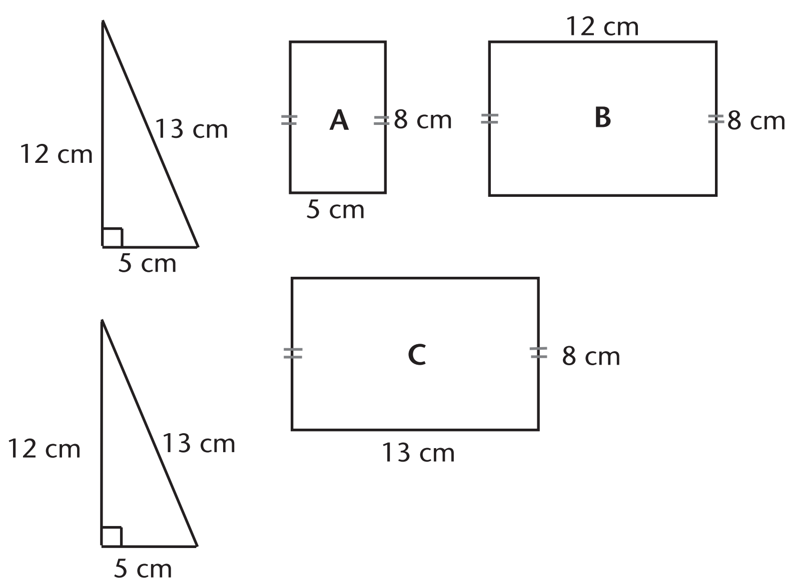
3-D object:



Its net:



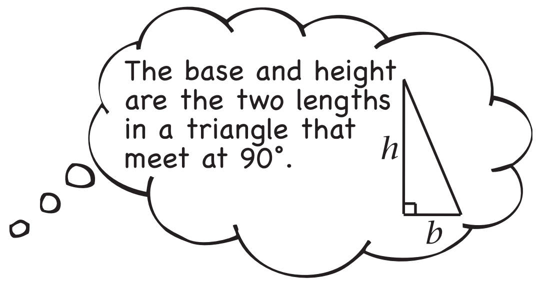
All the different faces:



Thinking Space

This net has two identical faces that are triangles and three different sized rectangles.

The area of one triangle is:



$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(5)(12)$$

$$A = 30 \text{ cm}^2$$

So the area of both triangles is:

$$30 + 30 = 60 \text{ cm}^2$$

The area of the 3 different rectangles in the net are:

Rectangle A	**Rectangle B**	**Rectangle C**
$A = lw$	$A = lw$	$A = lw$
$A = (5)(8)$	$A = (12)(8)$	$A = (13)(8)$
$A = 40 \text{ cm}^2$	$A = 96 \text{ cm}^2$	$A = 104 \text{ cm}^2$

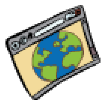
The area of all three rectangles is:

$$40 + 96 + 104 = 240 \text{ cm}^2$$

The surface area of the entire triangular prism is:

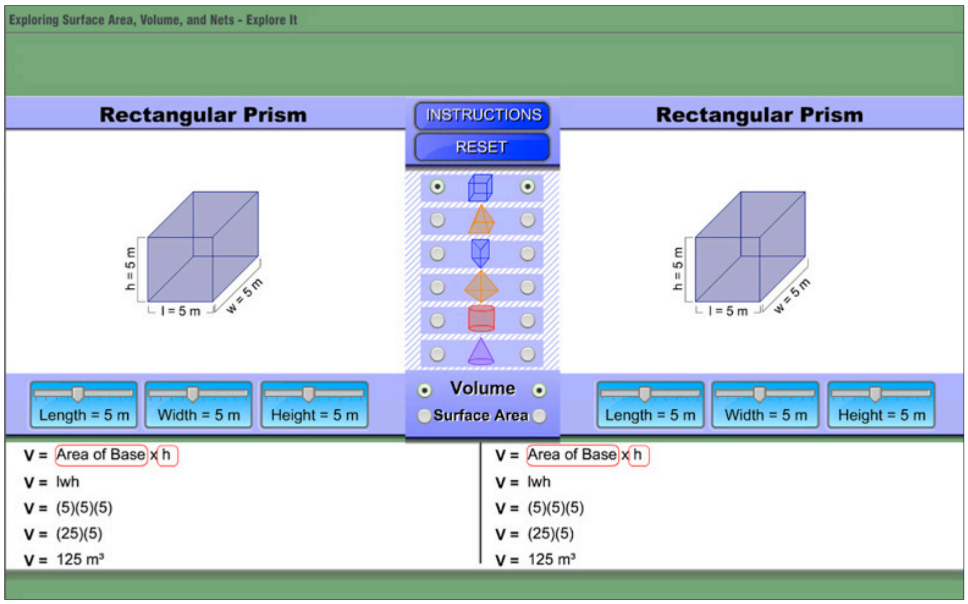
$$60 \text{ cm}^2 + 240 \text{ cm}^2 = 300 \text{ cm}^2$$

Thinking Space

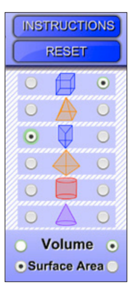


To explore further how to find the surface area of a triangular prism, go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0812a1f_areavolume.html and open *Exploring Surface Area and Volume*.

Close the instructions window to see a screen like the following:



There are two objects, one on the left and one on the right, but we'll just work with the object on the left.

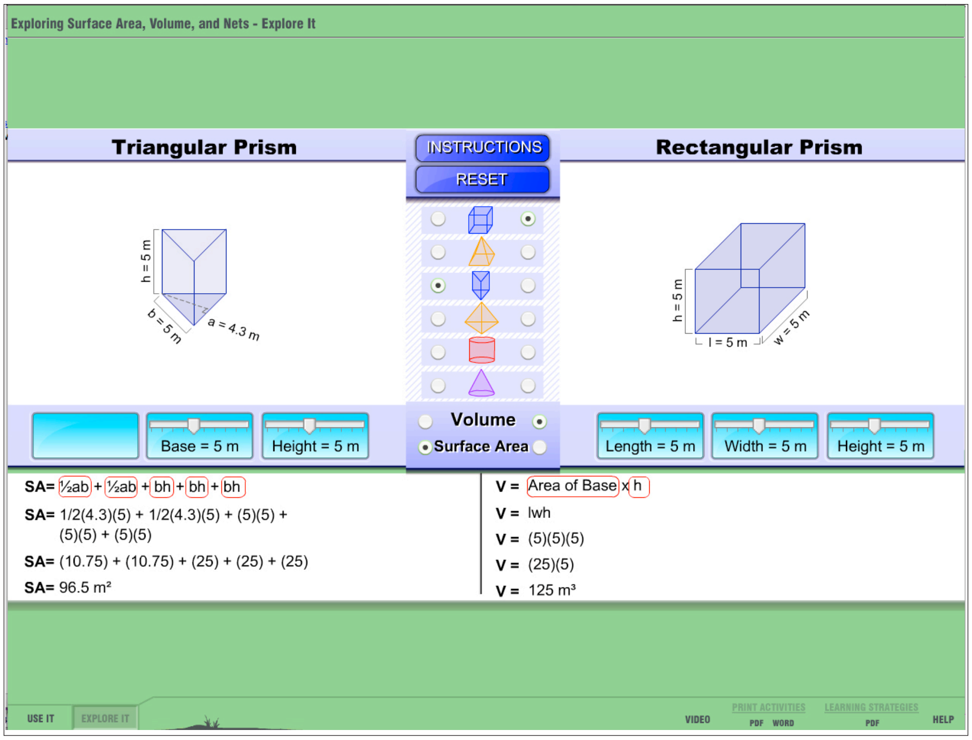


In the centre column under the words INSTRUCTIONS and RESET, click on the left side circle button for the third object down.

At the bottom of the column, click on the left side circle button for Surface Area.

The object on the left will now be a triangular prism and the bottom area will show the calculations for surface area (SA).

Thinking Space



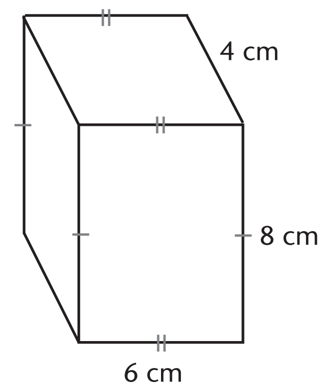
Investigate the changes in the surface area of a triangular prism by doing the following:

- Use the Base and Height sliders to change the measurements of the prism. Notice how these changes affect the drawing and the calculations.
- Move your mouse pointer over the red-circled portions of the SA formula to see which part of the figure is matched by the different calculations in the formula.

Thinking Space

Try It!
Activity 1

1. For this rectangular prism:



   a. Draw the net or the various views for the prism. Label the lengths of all the sides. Use the graph paper in the Appendix.

   b. Find the area of the entire net.

2. For the prism in question 1:

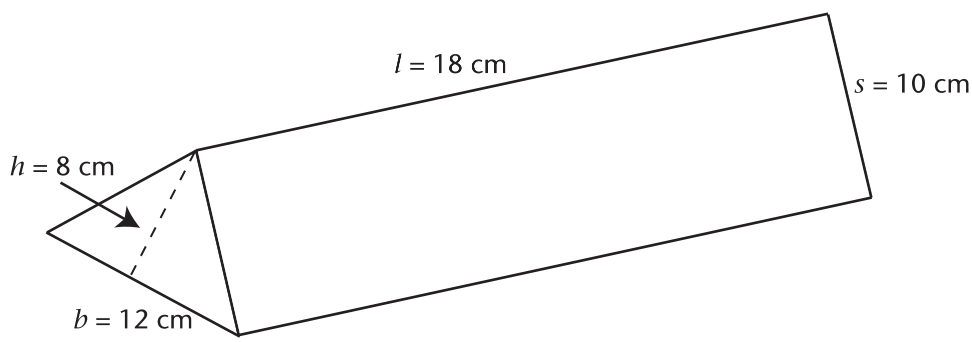
   a. Identify the length, the width, and the height of the prism.

   l = _______ w =_______ h = _______

Thinking Space

b. Using the SA formula for a rectangular prism, find the surface area. Show your work.

3. For this triangular prism:



a. Draw the net using the graph paper in the Appendix.

b. Find the surface area.

c. Create a formula for the surface area of this triangular prism, using the letters *b*, *h*, *s*, and *l*.

Turn to Solutions at the end of the module and mark your work.

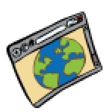
Thinking Space

Explore
Surface Area of a Cylinder

For this Explore you will need:

- graph paper

Pop cans come in the form of a cylinder. They typically have a height of 12.2 cm and a diameter of 6.4 cm.

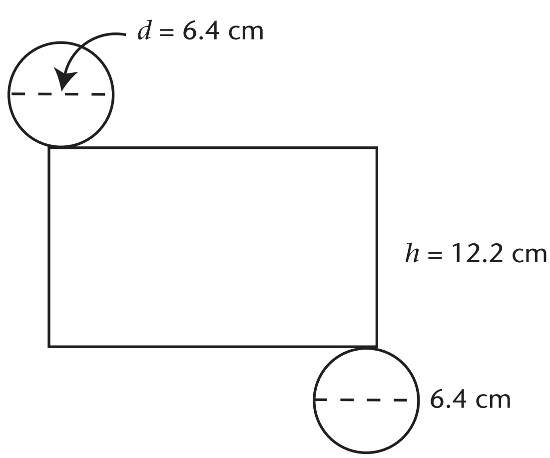


Go to your http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0811b2f_cylinder.html and watch *Constructing Cylinders*.

How much aluminum would cover this pop can?

*The **diameter** goes across the entire circle through the centre.*

To answer this question, we need to look at the net.



The net of a cylinder includes two circles and a rectangle.

Thinking Space

The formula for the area of a circle is:

$A_{circle} = \pi r^2$

$$A = \pi \times r \times r \text{ or } A = \pi r^2$$

The length of the diameter is 6.4 cm.

The radius goes from the centre of the circle to the edge. The radius is half as long as the diameter of the circle.

To use this formula you will plug in 3.2 cm for "*r*". Why? Because the radius is half the diameter (6.4 ÷ 2 = 3.2).

The area of each circle is:

Use the π button on your calculator, not 3.14

$$A = \pi r^2$$
$$A = \pi \times r \times r$$
$$A = \pi \times 3.2 \times 3.2$$
$$A = 32.16990877.....$$

You can round this to one decimal place, to get 32.2 cm^2.

The area of both circles is 32.2 + 32.2 = 64.4 cm^2

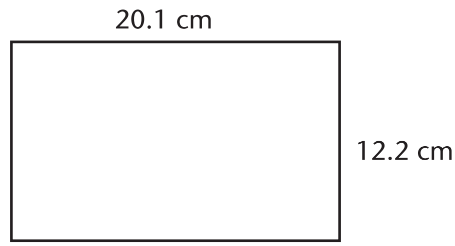
Let's work on the area of the rectangle next.

The **circumference of a circle** is the distance around the circle.

The length of the rectangle is equal to the circumference of the circle.

Thinking Space

Length of the rectangle
= circumference of the circle
= $\pi d = \pi \times (6.4 \text{ cm})$
= 20.1 cm



So the rectangle has dimensions 20.1 cm by 12.2 cm.

The area of the rectangle is:

$$A = lw$$
$$A = (20.1)(12.2)$$
$$A = 245.22 \text{ cm}^2$$

You can round this to one decimal place, to get 245.2 cm^2.

The surface area of the entire pop can is:

$$64.4 \text{ cm}^2 + 245.2 \text{ cm}^2 = 309.6 \text{ cm}^2.$$

Just over 300 cm^2 of aluminum, at the minimum, is needed to make a pop can.

In your calculations, notice that both of the circles are the same size, and the areas were found by multiplying $\pi \times r \times r$ which is the same as πr^2. The area of both circles together is $2\pi r^2$.

The length of the rectangle is equal to πd and the width of the rectangle is h. The area of the rectangle is length (πd) times width (h) so the area is πdh.

So a formula for finding the surface area of a cylinder is:

$$SA = 2\pi r^2 + \pi dh$$

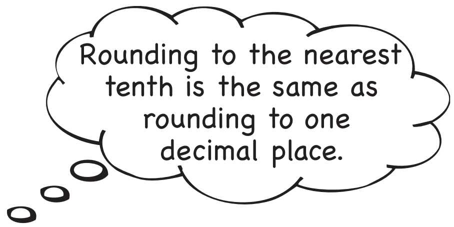
Thinking Space

To use this formula, plug in the 3.2 for *r*, 6.4 for *d*, and 12.2 for *h*.

$$SA = 2\pi \quad r \quad\quad r \; + \pi \quad d \quad\quad h$$
$$SA = 2\pi(3.2)(3.2) + \pi(6.4)(12.2)$$

Put these into a scientific calculator exactly as they appear.

SA = 309.6353719…



Round this to the nearest tenth.

SA = 309.6 cm^2

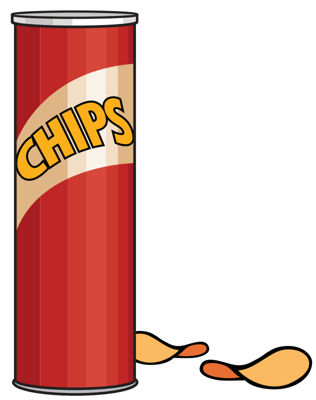
Thinking Space

Try It!
Activity 2

1. a. Using a ruler and graph paper, draw a cylinder with a diameter of 5 cm and a height of 7 cm.

   b. Using a ruler and graph paper, draw the net of this cylinder. Draw the length of the rectangle to the nearest millimetre.

2. A potato chip can is in the shape of a cylinder. The top and bottom are made of plastic. The rest is made of cardboard.



   a. Find the total area covered in plastic. Round to one decimal place.

   b. How many square centimetres of cardboard (to the nearest tenth) does the can use?

   c. Use the formula for the surface area of a cylinder to find the total surface area of the can of chips. Round to the nearest tenth.

Thinking Space

3. The formula for the SA of a cylinder is SA = $2\pi r^2 + \pi dh$, but another way of writing this formula is: SA = $2\pi r^2 + 2\pi rh$.

   a. What is the difference between these two formulas for the surface area of a cylinder?

   ____

   ____

   b. Explain why these two formulas will give you the same result.

   ____

   ____

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B

More About Area in 2-D and 3-D

For this lesson you will need:

- metric ruler
- scientific calculator

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

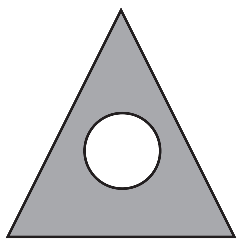
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How do I find the area of an irregular 2-D shape?		
How do I find the surface area of a composite 3-D shape?		

Thinking Space

Warm-up

1. Describe each of the following shapes.

a.

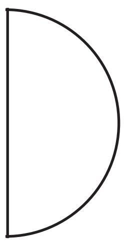


____

____

____

b.

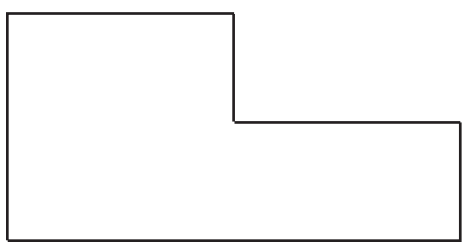


____

____

____

c.



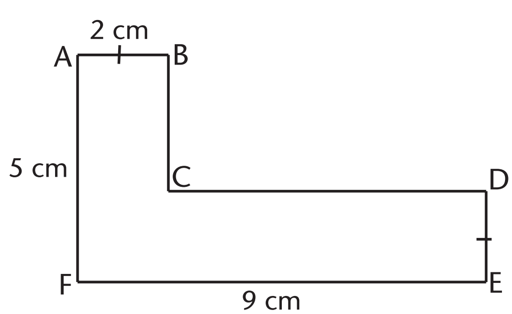
____

____

____

2. Find the missing lengths in each diagram.

a.



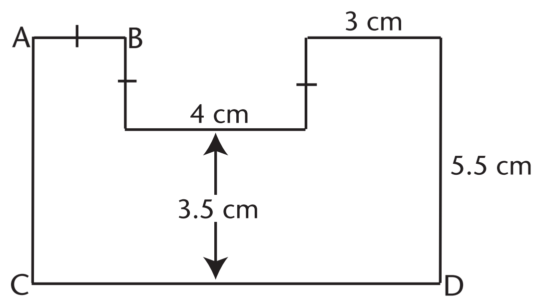
BC = ______

CD = ______

DE = ______

Thinking Space

b.



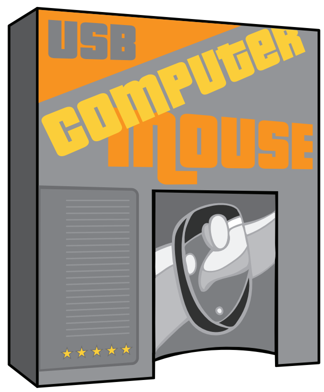
AB = ______
AC = ______
CD = ______

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Areas of Irregular Shapes

Packaging often looks like a prism or cylinder, and it may have cutouts.

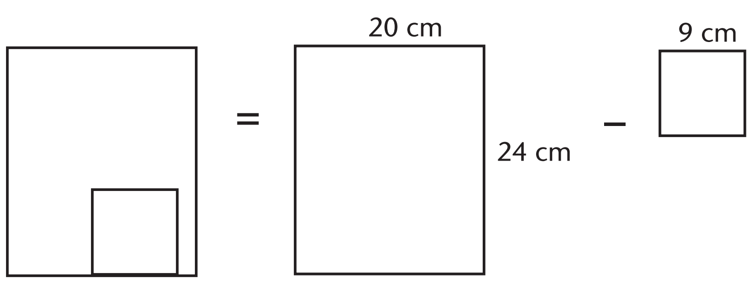


In the computer mouse package above, the front view has a square cut out of it. The square measures 9 cm by 9 cm. The front view has outer dimensions of 20 cm by 24 cm.

What is the area of the front face?

Here is a drawing of the front view with an explanation about how to find its area.

Front View

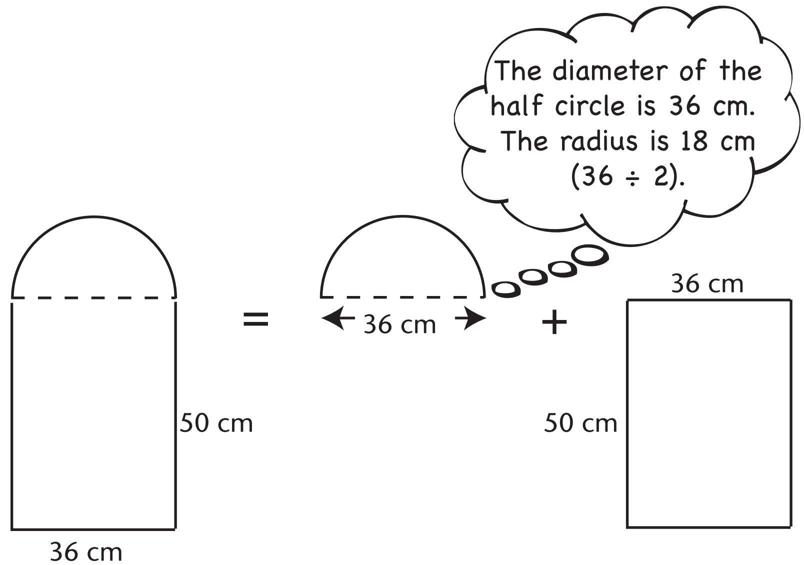


Area of the Irregular Shape = (Area of Rectangle) minus (Area of Square)

$$= lw \quad - \quad s^2$$
$$= 24 \times 20 \quad - \quad 9^2$$
$$= 480 \quad - \quad 81$$
$$= 299 \text{ cm}^2$$

Thinking Space

Mirrors often come in shapes that are not simple rectangles. A mirror could appear as a rectangle with a half circle on top.



Area of the Irregular Shape = (Area of Rectangle) plus (Area of Half Circle)

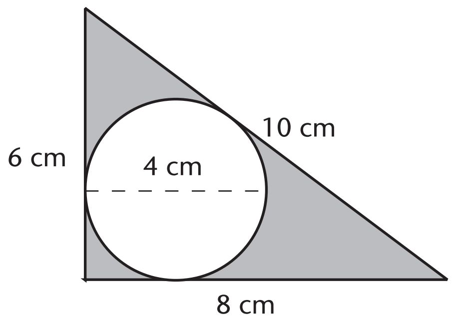
$$= lw \quad + \quad \pi r^2 \div 2$$
$$= 36 \times 50 \quad + \quad (\pi \times 18^2) \div 2$$
$$= 1800 \quad + \quad 508.9$$
$$= 2308.9\text{ cm}^2$$

Thinking Space

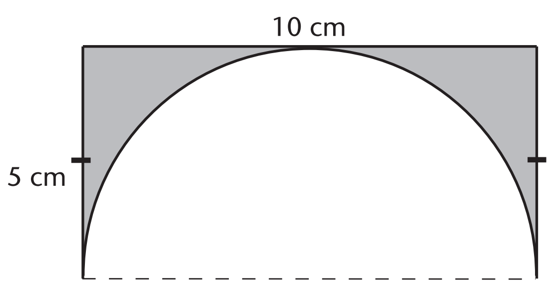
Try It!
Activity 1

1. Find the area of each shape without the removed section. The shape is shaded, and the removed section is white. Write answers to the nearest tenth.

   a.



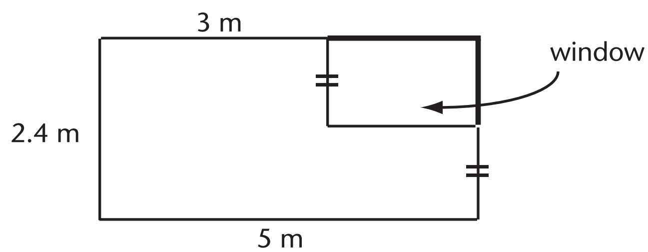
   b.



Thinking Space

2. You really want to paint your room a new colour, but you are only allowed to paint one wall. You choose the wall that has a window in the corner of it.

   a. Calculate the area that you need to paint.



   b. If a small can of paint will paint 8 m^2, will you have enough paint?

   c. If you need to do two coats of paint, how many small cans will you need?

   d. A large can of paint will paint 50 m^2 and costs $22. A small can of paint will paint 8 m^2 and costs $10. Would you buy a large can of paint, or several small cans? Explain your answer.

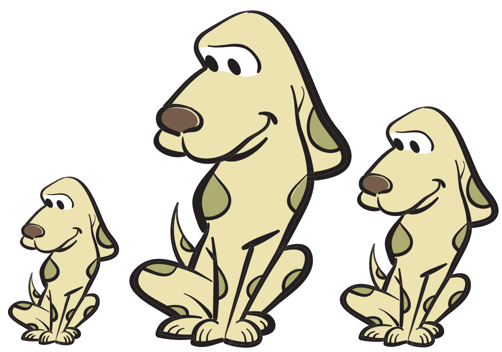
Turn to Solutions at the end of the module and mark your work.

Thinking Space

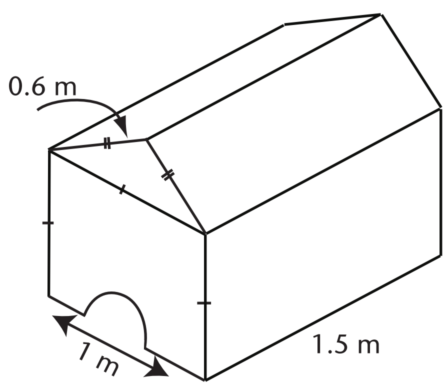
Explore
Area of Composite Figures

The size of a building or home will affect the cost to make it. In packaging, this is also the case, but the cost is not as great. Building materials cost much more than the paper and plastics needed to package a product.

To build a doghouse, the size of the dog will affect the size of the building.



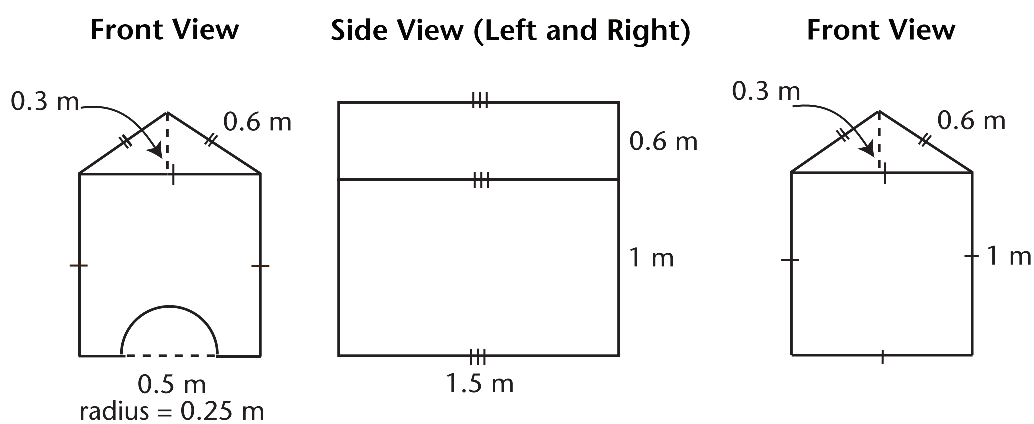
The doghouse you decide to build has a half circle opening at the front only. The half circle has a diameter of 0.5 m. There is no bottom on the doghouse.



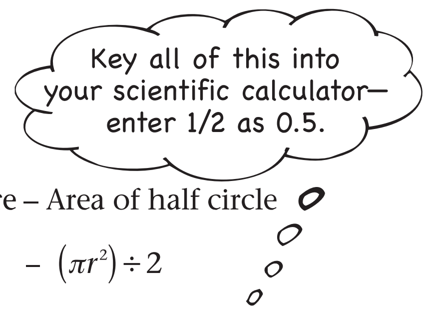
The height of the triangle on the roof is 0.3 m.

Thinking Space

The cost to build the doghouse depends on the amount of wood you need. The wood needed depends on the surface area of the doghouse. Let's find the surface area to two decimal places.



To find the area of the front:



Area of front = Area of triangle + Area of square – Area of half circle

$$
\begin{aligned}
&= \frac{1}{2}bh \qquad + s^2 \qquad - \left(\pi r^2\right) \div 2 \\
&= \frac{1}{2}(1)(0.3) \qquad + 1^2 \qquad - \left(\pi \times 0.25^2\right) \div 2 \\
&= 1.05182523\ldots
\end{aligned}
$$

Round this to two decimal places, so the area of the front = 1.05 m².

To find the area of the sides:

Area of the side = area of roof rectangle + area of bottom rectangle

$$
\begin{aligned}
&= lw \qquad + lw \\
&= (1.5)(0.6) \qquad + (1.5)(1.0) \\
&= 2.4
\end{aligned}
$$

Since there are two identical sides: Area of the sides = 2 × 2.4 = 4.8 m².

Thinking Space

To find the area of the back:

Area of back = Area of triangle + Area of square

$$= \frac{1}{2}bh \qquad + s^2$$

$$= \frac{1}{2}(1)(0.3) \qquad + 1^2$$

$$= 1.15$$

In total, the surface area of the doghouse is:

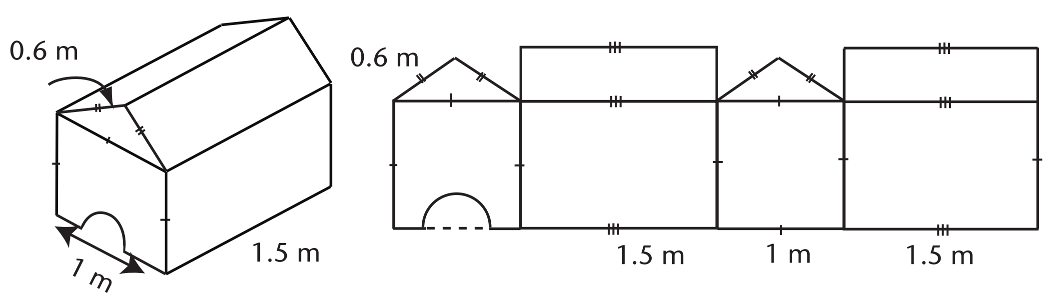
SA = area of front + area of sides + area of back
= 1.05 m² + 4.8 m² + 1.15 m² = 7 m²

So you need to purchase approximately seven square metres of wood to build this doghouse. If the wood costs $4.89 per square metre, then the cost of wood would be:

cost of wood = 7 m² × $4.89/m² = $34.23

What other supplies would you need to buy, if you already own a saw and a hammer?

You can also use a net to find the surface area of the doghouse.

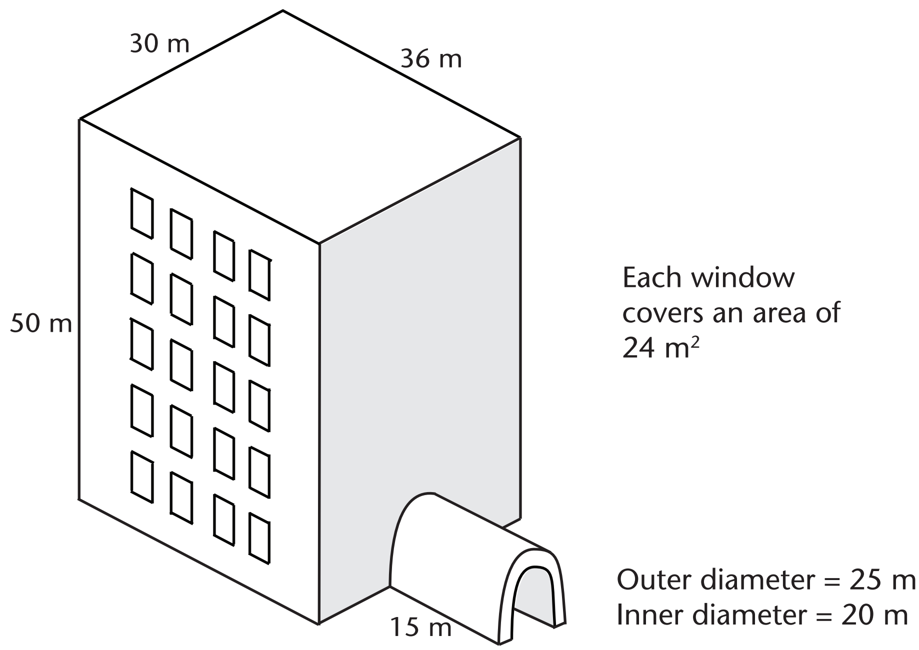


Draw the net, find the area of each face, and add all the areas together.

Thinking Space

Try It!
Activity 2

1. A cement building is to be painted. The sides and back of the building have no windows. The roof will not be painted, but the rest of the building and the parking garage entrance will be.



   a. Draw the front view, back view, left view, and right view of this building. Include any lengths that you know in your diagram.

   Front View **Back View**

Thinking Space

Left Side View **Right Side View**

b. For each view, find the area that will be painted. Do not include the parking garage in these calculations. Round to two decimal places.

Front view (without parking garage)

Back view

Left side view

Right side view

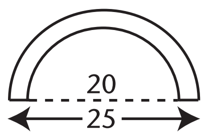
c. What is the total area of the building that must be painted, without including the parking garage?

Thinking Space

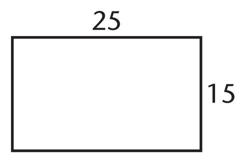
2. The two painters decide to figure out the area of the parking garage before painting.

Painter One uses views to find the surface area:

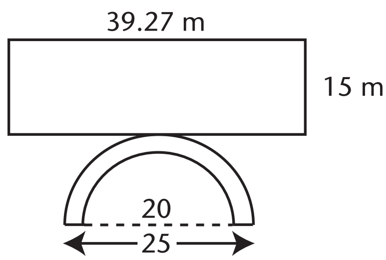
Front View



Top View



Painter Two draws the net:



a. Using the numbers in their drawings, Painter One ends up with the incorrect surface area. Painter Two ends up with the correct answer. Explain why Painter One arrives at the wrong answer.

____

____

b. Using Painter Two's diagram, find the area that needs to be painted (to two decimal places).

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson C

The Amount of Space

For this lesson, you will need:

- scientific calculator

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What is volume?		
How do you find the volume of different shapes?		
How are surface area and volume related?		
How does orientation affect volume?		

Thinking Space

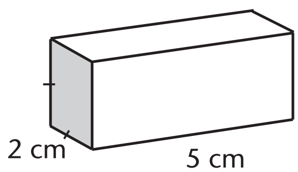
Warm-up

1. Which words and units match? Write the correct letter in each blank.

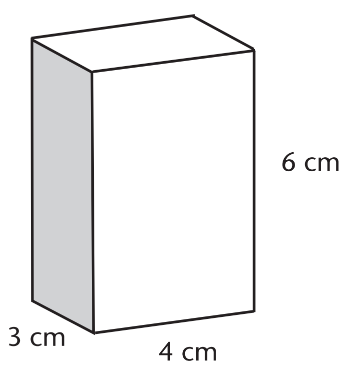
______ surface area		a. cm
______ volume		b. m^2
______ length		c. mm^3

2. Find the volume of each rectangular prism. Include units in your answer.

a.



b.



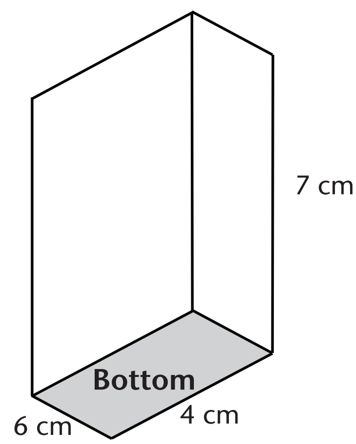
Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore
Volume of Prisms and Cylinders

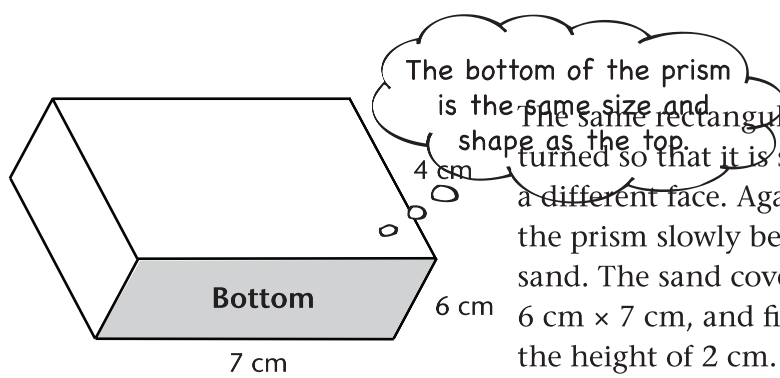
Volume is the amount of space an object takes up.

In a rectangular prism, you find volume by multiplying $l \times w \times h$.



Imagine pouring sand into the rectangular prism. The sand covers the area of 4 cm × 6 cm and fills up through the height of 7 cm.

Volume = (4 cm × 6 cm) × 7 cm
Volume = 168 cm^3



The same rectangular prism is turned so that it is standing on a different face. Again, imagine the prism slowly being filled with sand. The sand covers the area of 6 cm × 7 cm, and fills up through the height of 2 cm.

Volume = (6 cm × 7 cm) × 4 cm
Volume = 168 cm^3

Thinking Space

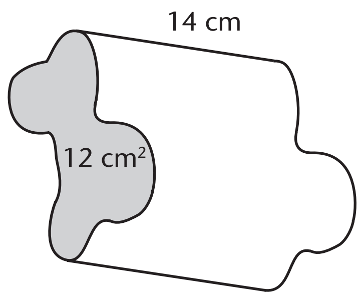
To find the volume of any prism or cylinder:

Step 1: Find the area of a face that has an identical face opposite it.

Step 2: Identify the distance between these two identical and parallel faces.

Step 3: Multiply the area of the face from the first step by the distance found in the second step to find the volume.

You can use these same steps to find the volume of an irregularly shaped prism.

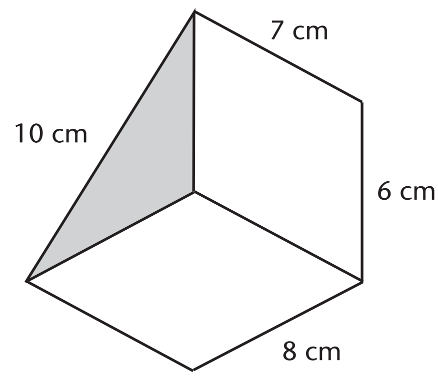


Volume is equal to $12\ \text{cm}^2 \times 14\ \text{cm} = 168\ \text{cm}^3$.

Thinking Space

Volume of a Triangular Prism

Use the following steps to calculate the volume of a triangular prism.



Step 1: Figure out the area of the triangle.

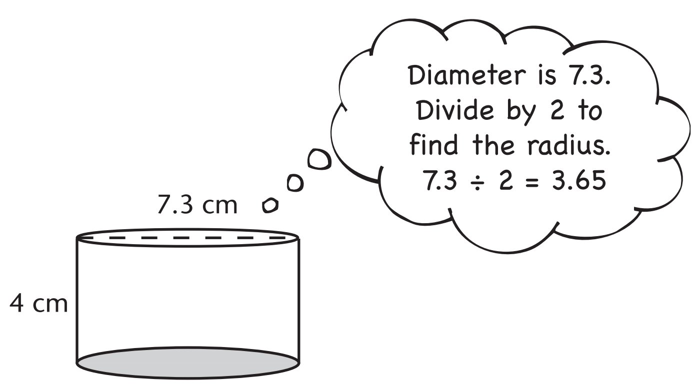
$$\text{Area of back} = \frac{1}{2}bh$$
$$= \frac{1}{2}(8)(6)$$
$$= 24 \text{ cm}^2$$

Step 2: Figure out the distance between the triangles.
= 7 cm

Step 3: Calculate the volume.
$= 24 \text{ cm}^2 \times 7 \text{ cm} = 168 \text{ cm}^3$

Volume of a Cylinder

You can find the volume of cylinders using these same steps.



Thinking Space

Step 1: Figure out the area of the circle.
Area = $\pi r^2 = \pi(r \times r)$
Area = $\pi(3.65)(3.65)$
Area = 41.85 cm^2

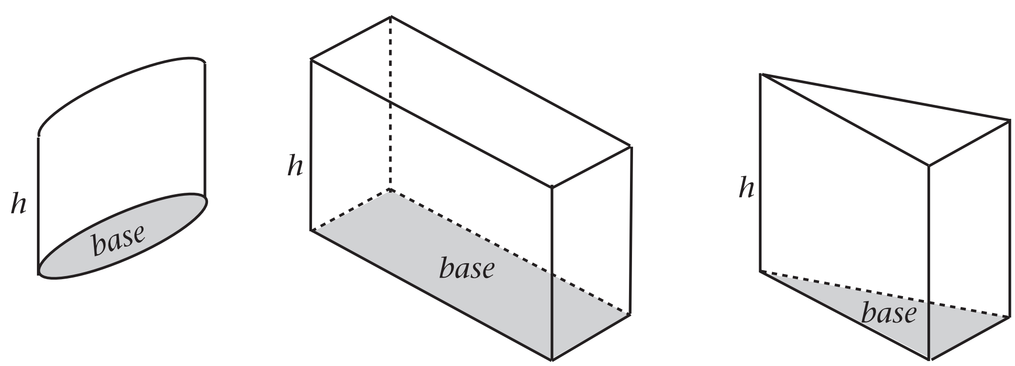
Step 2: Figure out the distance between the circles.
= 4 cm

Step 3: Calculate the volume.
= 41.85 cm^2 × 4 cm = 167. 4 cm^3

Notice that the volumes of all the prisms were close to 168 cm^3. So even though they all had different shapes, they could all hold about the same amount of sand.

Also, notice that it doesn't matter if the shape is standing up or lying down—the volume is found the same way:

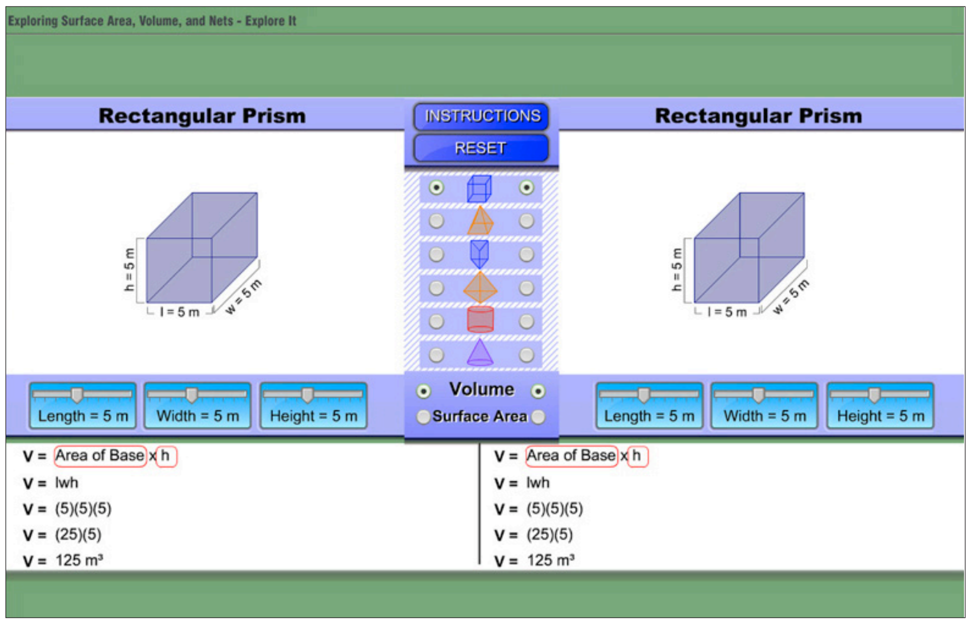
$$\text{Volume} = (\text{Area of Base}) \times h$$



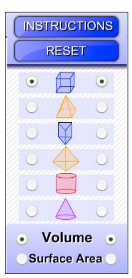
Thinking Space

To explore further how to find the volume of a rectangular prism, triangular prism, or cylinder, go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0812a1f_areavolume.html and open *Exploring Surface Area and Volume*.

Close the instructions window to see a screen like the following:



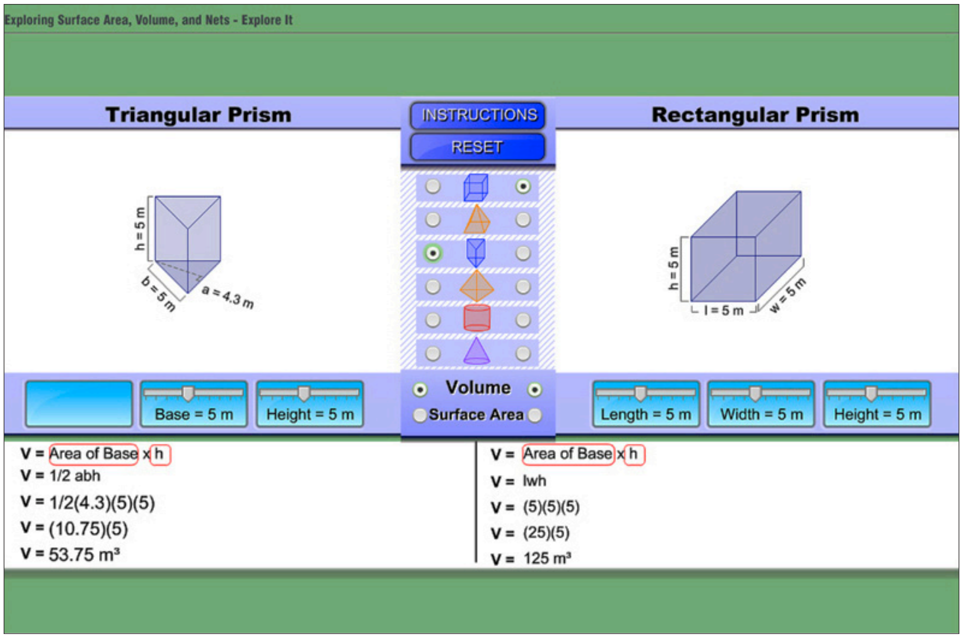
There are two objects, one on the left and one on the right, but we'll just work with the object on the left for now.



Change the object to a triangular prism: In the centre column under the words INSTRUCTIONS and RESET, click on the left side circle button for the third object down.

The object on the left will now be a triangular prism and the bottom area will show the calculations for Volume (V).

Thinking Space



Investigate the changes in the volume of a triangular prism by doing the following:

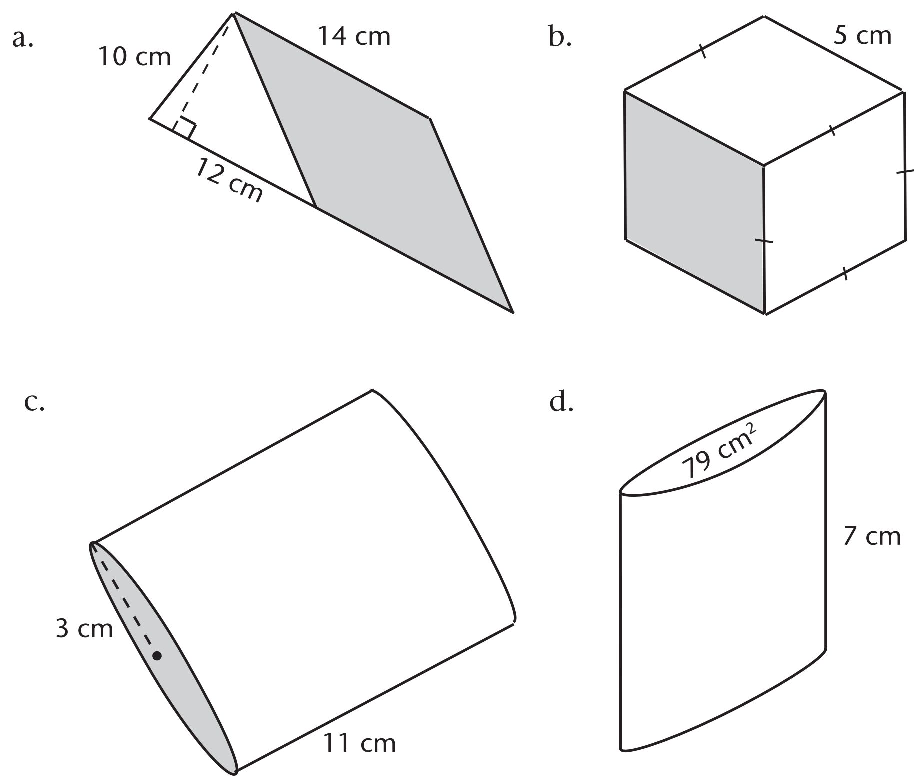
- Use the Base and Height sliders to change the measurements of the prism. Notice how these changes affect the drawing and the calculations.
- Move your mouse pointer over the red-circled portions of the V formula to see which part of the figure is matched by the different calculations in the formula.

Use the circle buttons in the centre column to change the object to a cylinder (fifth object from the top). You can also change the measurements of the rectangular prism shown on the right side of the screen. Investigate the changes in the volume of both of these objects by following the instructions for the triangular prism given above.

Thinking Space

Try It!
Activity 1

1. Find the volume of each prism. Round your answers to the nearest whole cm^3.



2. Explain why the formula for finding the volume of a cylinder is: $V = \pi r^2 h$

Thinking Space

3. A box display needs to be set up. The employee decides to stack the boxes sideways rather than upright. Will the display have more volume upright or sideways? Explain your answer.

____

____

____

Turn to Solutions at the end of the module and mark your work.

Thinking Space

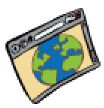
Explore

Are Surface Area and Volume Related?

The surface area of a 3-D object is the total area of all the faces in its net.

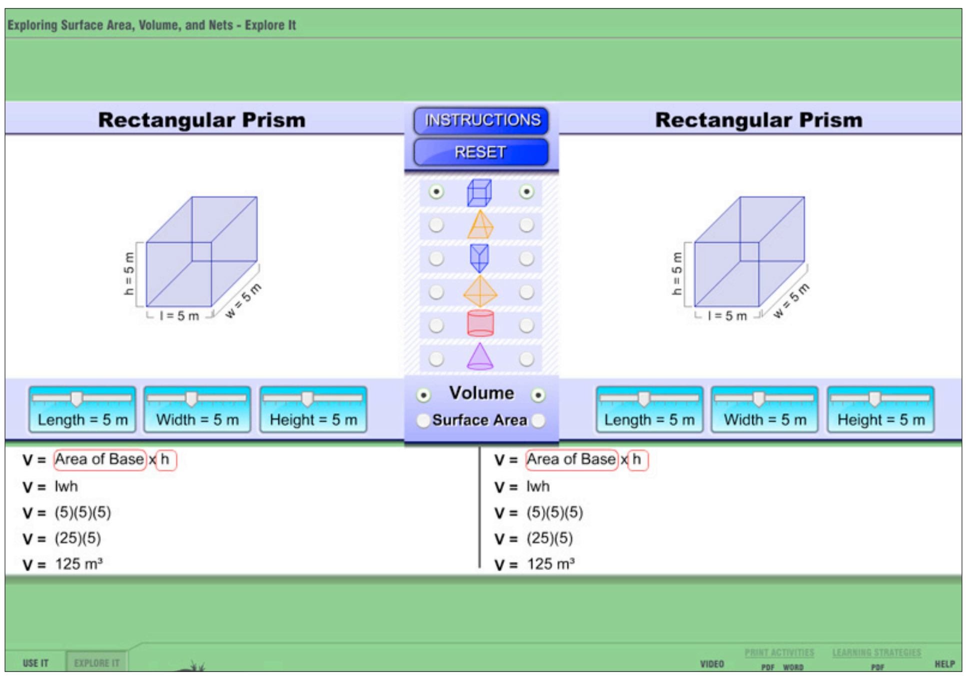
The volume of a 3-D object is the total space the object occupies.

How are surface area and volume related?

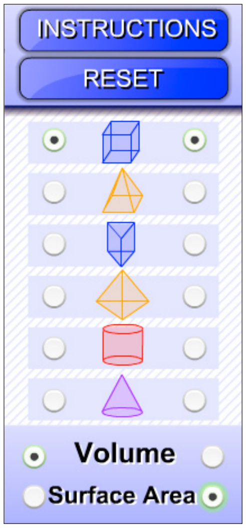


To help you answer this question, go to http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0812a1f_areavolume.html and open *Exploring Surface Area and Volume*.

Close the instructions window to see a screen like the following:

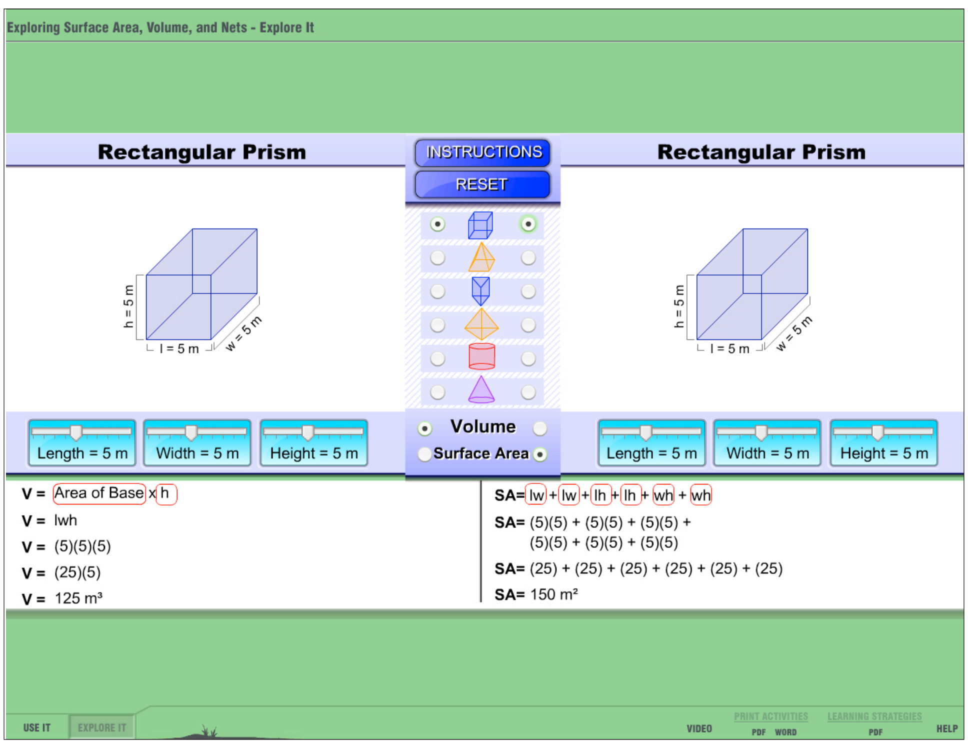


Thinking Space



There are two rectangular prisms, one on the left and one on the right. We'll leave the left one as it is, showing Volume, but we'll make the right one show Surface Area (SA). Click on the Surface Area circle button on the bottom right of the centre column.

Your screen will now look like this:



You can see that the volume of a 5 m × 5 m × 5 m rectangular prism is 125 m^3. The surface area of that same object is 150 m^2.

Investigate the changes in the volume and surface area of a rectangular prism by doing the following:

- Keep the Length and Width sliders both at 5 m.
- Change the Height of both prisms to 6 m, 7 m, 8 m, and 9 m and record the changes in the following table.

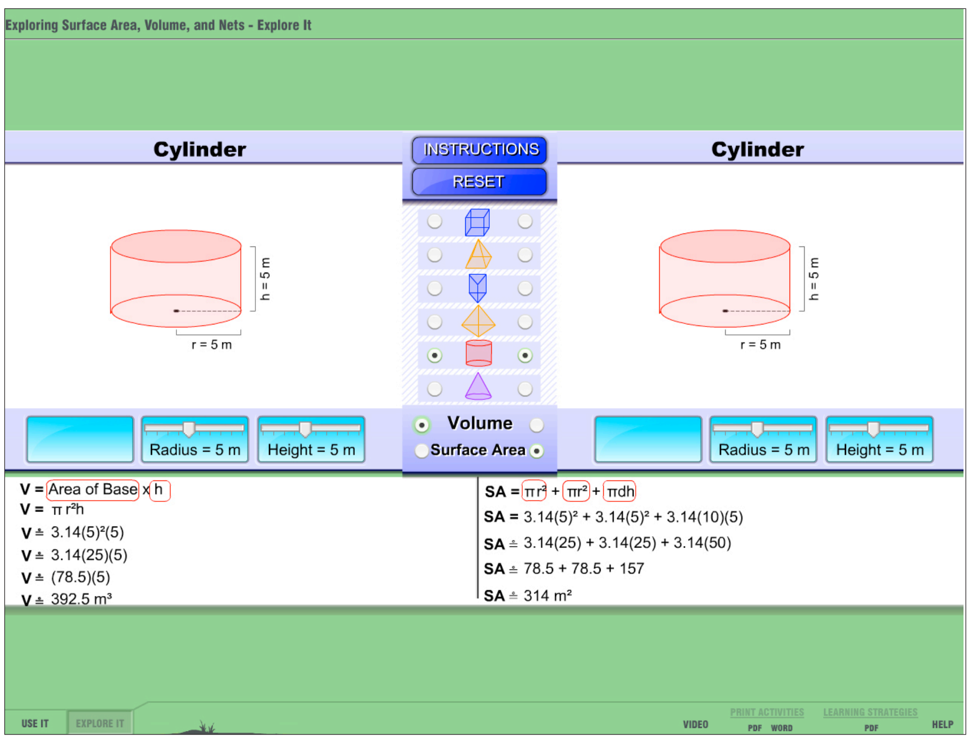
Thinking Space

What do you notice about the relationship between the volume and the surface area as the height gets bigger? Do they increase, decrease or do the opposite from each other?

	Left side Volume $V = lwh$ $V = (5)(5)h$	Right side Surface Area $SA = 2lw + 2lh + 2hw$ $SA = 2(5)(5) + 2(5)h + 2h(5)$
$h = 5$	125 m^3	150 m^2
$h = 6$		
$h = 7$		190 m^2
$h = 8$	200 m^3	
$h = 9$		

Volume and Surface Area of a Cylinder

Change both objects to a cylinder: In the centre column on both sides, click on the circle buttons for the fifth object from the top.



In the case of the cylinder, we'll change the radius (r). Remember that $2r$ is equal to diameter (d), which you'll see in the formula for surface area on the right.

Thinking Space

Investigate the changes in the volume and surface area of a cylinder by doing the following:

- Keep the Height slider at 5 m.
- Change the Radius of both cylinders to 6 m, 7 m, 8 m, and 9 m and record the changes in the following table.

	Left side **Volume** $V = \pi r^2 h$ $V = 3.14r^2(5)$	**Right side** **Surface Area** $SA = 2\pi r^2 + \pi dh$ $SA = 2(3.14)r^2 + (3.14)d(5)$
$h = 5$	392.5 m^3	314 m^2
$h = 6$		
$h = 7$		527.52 m^2
$h = 8$	1004.8 m^3	
$h = 9$		

What do you notice about the relationship between the volume and the surface area as the radius gets bigger? Do they increase, decrease or do the opposite from each other?

You may have discovered that both the volume and surface area of 3-D objects increase as any measurement of the object increases.

Thinking Space

Try It!
Activity 2

1. A metal box needs to have a volume of 60 cm^3. There are many different sized boxes with this volume.

   Option A: 3 cm by 2 cm by 10 cm

   Option B: 5 cm by 3 cm by 4 cm

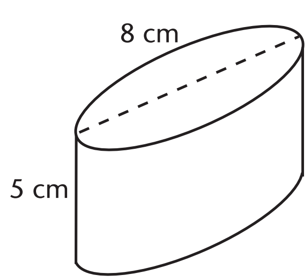
   a. Which box would have the smallest surface area?

   b. Write the dimensions of another rectangular prism that has a volume of 60 cm^3. Calculate the surface area.

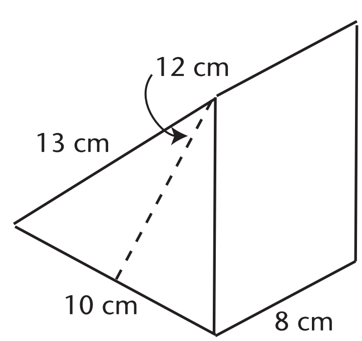
   l = __________ w = __________ h = __________ SA = __________

2. A newly designed product is ready for shipping. The choices of packaging are shaped as either a cylinder or a triangular prism.

Option 1:



Option 2:



Thinking Space

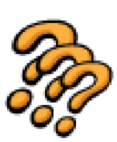
a. Without calculating, decide which you think has the greater volume and the smaller surface area. For what reason(s) did you choose this one?

b. Which option has the greater volume?

c. Which option has the smaller surface area?

d. Which option would you choose? Explain your answer.

Turn to Solutions at the end of the module and mark your work.



You've finished Lesson C. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Thinking Space

Section Summary

Completing this section has helped you to:

- find the surface area and volume of prisms and cylinders
- learn how surface area and volume affect decisions in building and in packaging

Section 3

Shapes and Design

In this section you will:

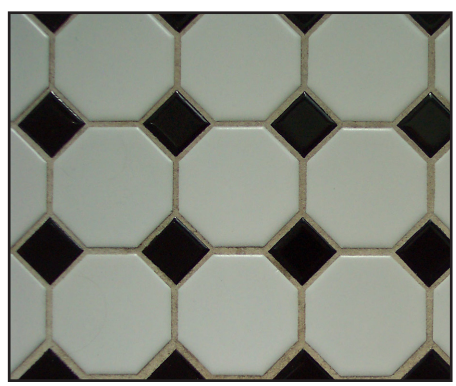
- learn which shapes can tessellate
- create tessellations
- recognize a tessellation in the environment

For this section you will need:

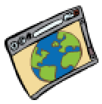
- scissors
- protractor
- ruler
- cardstock, construction paper, or index card (optional)
- coloured pencils
- tape
- graph paper (from Appendix)

Where in the World...?

Patterns and colour schemes improve the look of a place. In a bathroom, for example, tiles can really enhance the overall look. The symmetry of tiling patterns is used in backsplashes, bathroom floors, fabrics, and in laying bricks.



Tiling has inspired artists to create some extremely elaborate designs. M.C. Escher is a well known artist who used tiling patterns in many of his art pieces.



If you would like to learn more about M. C. Esher and you have internet access, go to the **http://www.openschool.bc.ca/courses/math/math8/mod1.html** and look for Section 1.3: *Shapes and Design*.

Section 3
Pretest

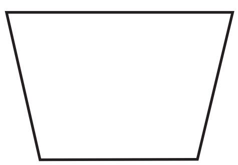
Complete this pretest if you think that you already understand the topics and concepts covered in this section. Mark your work using Solutions found at the end of the module.

If you get all the answers correct (100%), you may decide that you can omit the lesson activities.

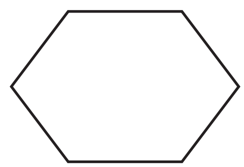
If you get all the answers correct for one or more lessons, but not for the whole pretest, you can decide whether you can omit the activities for those lessons.

Lesson A: What is a Tessellation Anyways?

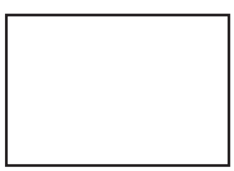
1. Decide if each polygon is regular or irregular. Give a reason for each.

a. 

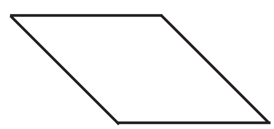
Reason:

b. 

Reason:

c. 

Reason:

d. 

Reason:

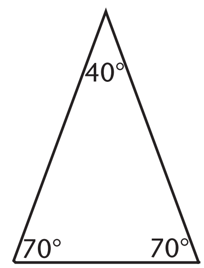
2. Will each regular polygon tessellate the plane? Answer yes or no.

   a. square ____________________

   b. pentagon ____________________

   c. hexagon ____________________

3. Can you create a tessellation with the following triangle?
   Explain your answer.

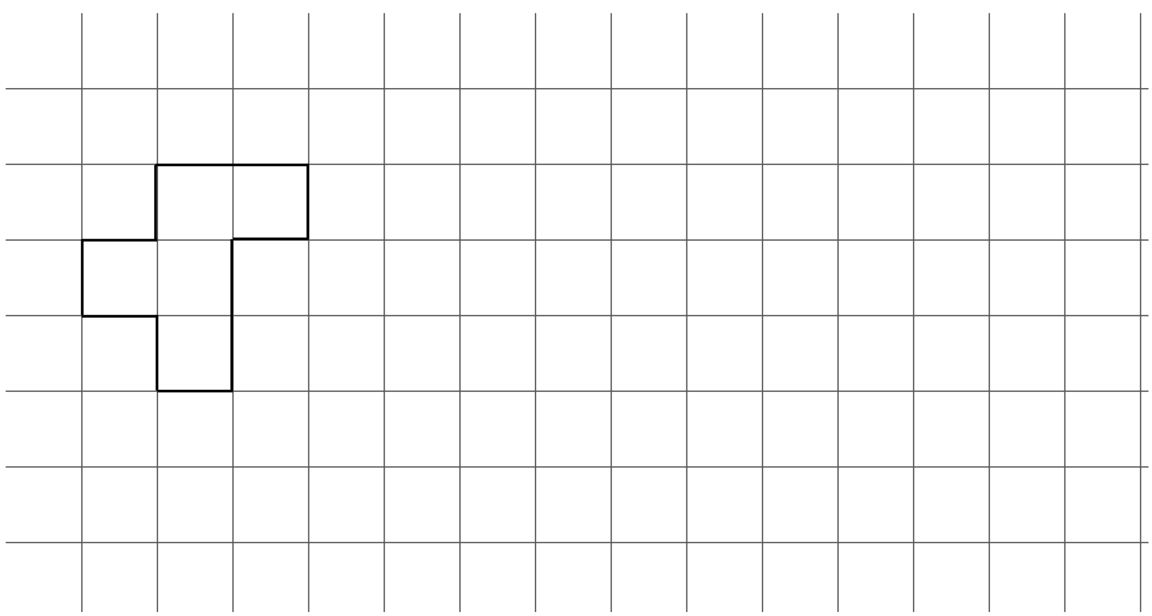


____

____

____

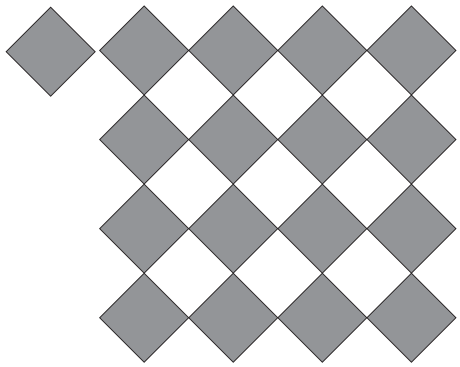
4. Tile the plane with this irregular shape.



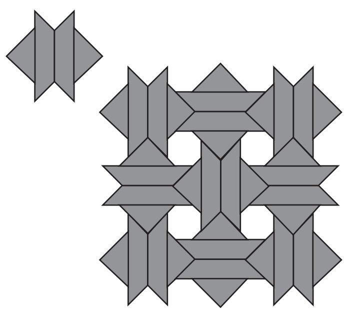
Lesson B: Transformations in Tessellations

5. Transformations are translations, reflections, or rotations. Identify the type(s) of transformation used in each tessellation. (You may have more than one answer.)

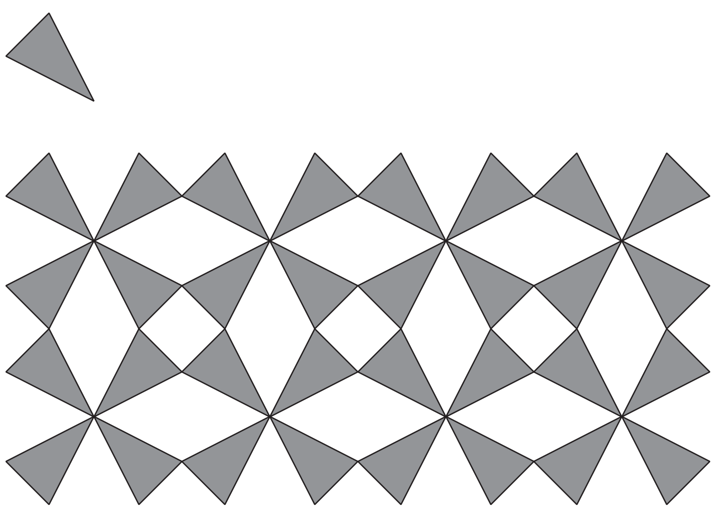
   a.



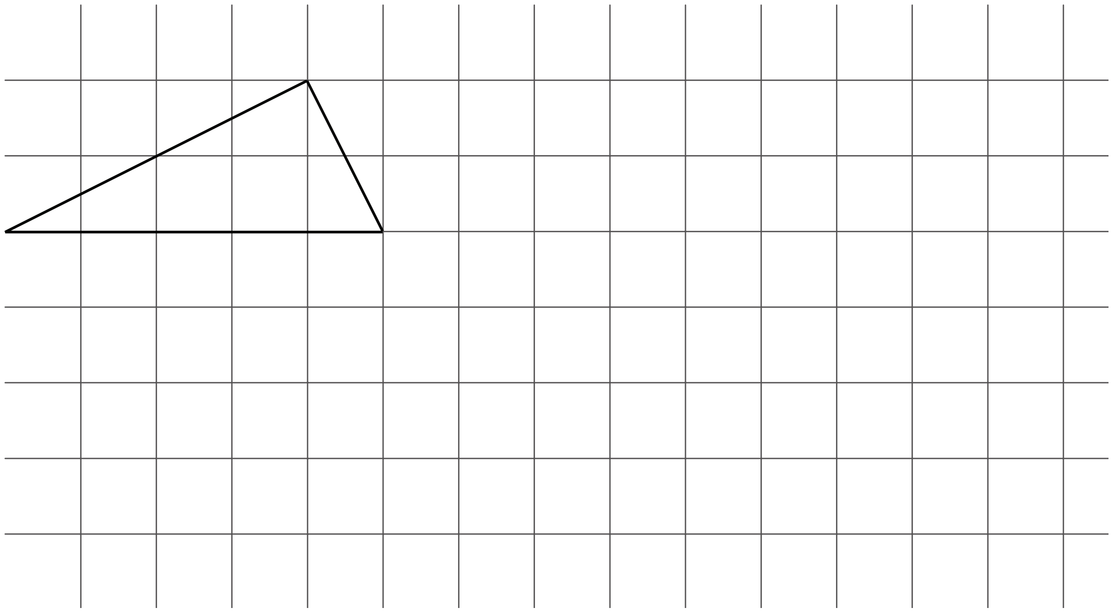
   b.



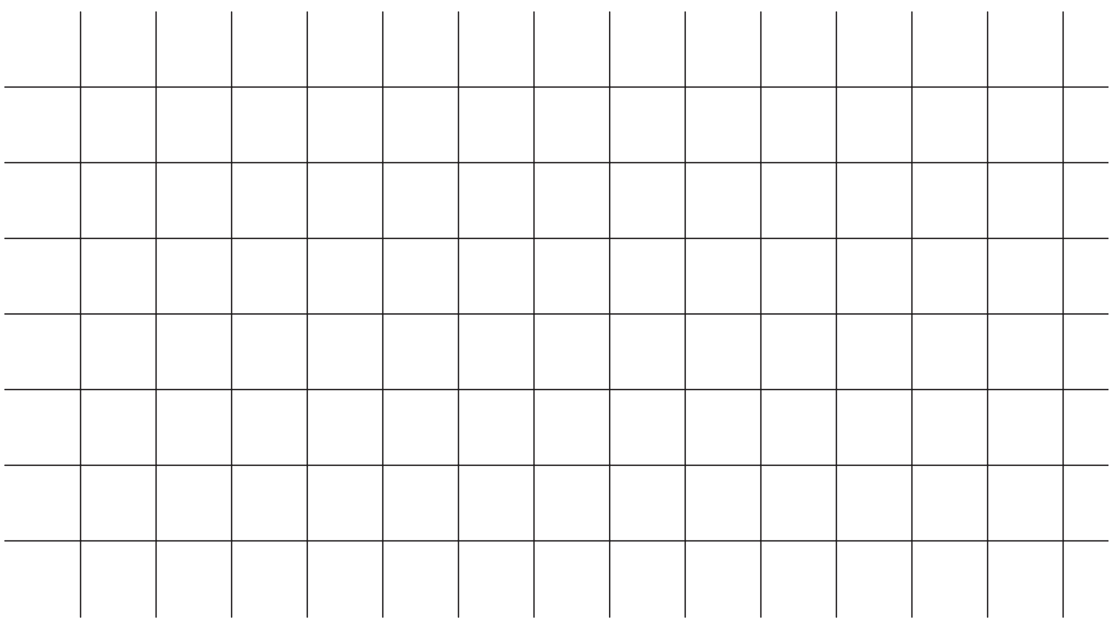
   c.



6. a. Reflect the triangle horizontally → or ←. Repeat until you reach the end of the grid. Then, reflect it vertically ↑ or ↓. Reflect it repeatedly. Continue to use reflections to tile the entire plane.

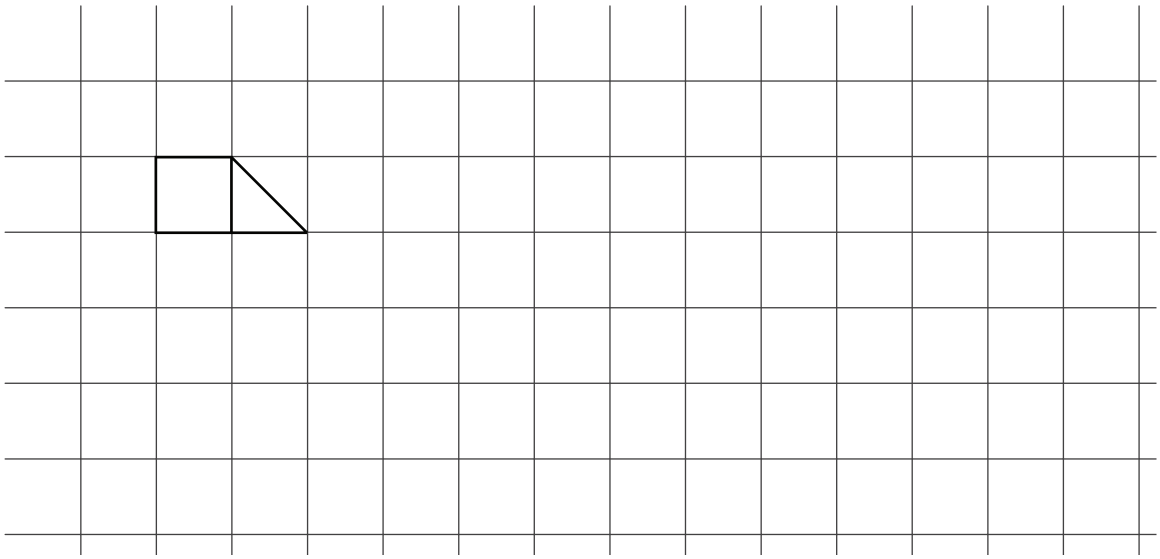


b. How many different polygons made up this tessellation? Draw them here.



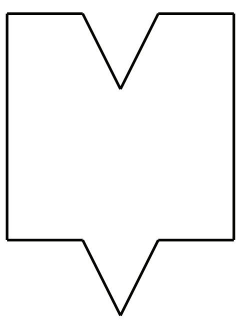
Lesson C: Creating a Tessellation

7. Use this shape to tile the entire plane. Complete this tessellation. Keep the square and triangle as one shape as shown, when creating the tessellation.



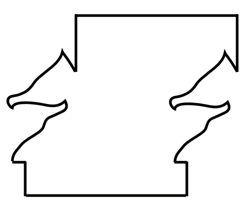
8. Which of the following shapes will tessellate? Answer yes or no.

a.



____

b.



____

Turn to Solutions at the end of the module and mark your work.

Lesson A

What is a Tessellation Anyway?

For this lesson you will need:

- protractor (you can find one in the Appendix if you don't have one)
- metric ruler

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

Essential Questions	Before the lesson: What I know	After the lesson: What I learned
What is a tessellation?		
Which polygons can tessellate?		
Where do I find tessellations in the world?		

Thinking Space

Warm-up

1. A regular pentagon has five equal sides and five equal angles. Write a definition for a:
   a. regular hexagon: ______________________________

   ____

   b. regular octagon: ______________________________

   ____

2. An irregular octagon has eight sides that are not all equal and eight angles that are not all equal. Write a definition for an:
   a. irregular pentagon: ______________________________

   ____

   b. irregular hexagon: ______________________________

   ____

3. Is a square a regular polygon or an irregular polygon? Explain.

   ____

   ____

Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Which Shapes Tessellate?

Some bathroom floors or shower stalls are examples of tiling patterns. The walls and floors are covered by tiles placed in a pattern over entire surface. The tiles do not overlap or leave gaps.

The surface of the wall or floor can also be called a **plane**. A plane is a 2-D flat surface.

A **tessellation** is a tiling pattern that covers an entire plane without overlapping or leaving gaps.

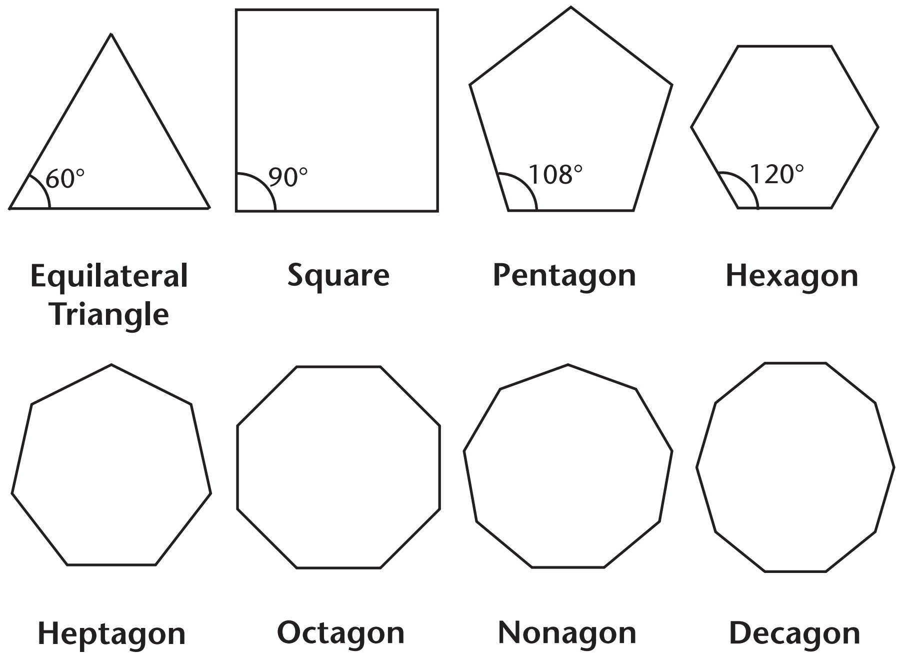
Not all regular polygons will tessellate. In the picture we can see that regular hexagons can tessellate an entire bathroom floor. We also see that rectangles can tessellate the entire wall.

What polygons will tessellate an entire plane, and which ones won't?

Before we can answer this question we need to review the parts of a polygon.

Thinking Space

Regular Polygons



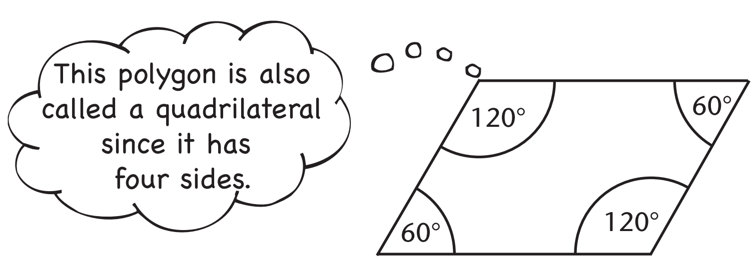
The **vertices** of these regular polygons are the corners of each shape. One corner is called a **vertex**.

One interior angle from each regular polygon in the first row has been labelled. **Interior angles** are the angles that are inside the figure and at each vertex of the polygon. A heptagon has seven interior angles, and an octagon has eight interior angles.

A polygon will tessellate if it covers an entire plane without overlapping or leaving gaps. That means that if vertices of the same polygon are all joined at a point, all the interior angles will add up to 360°.

Will These Polygons Tessellate?

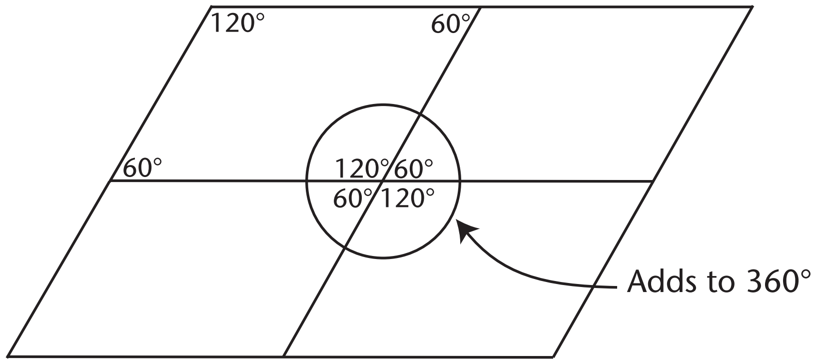
Will this polygon tessellate?



Thinking Space

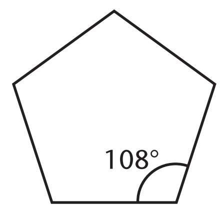
Do you think all quadrilaterals will tessellate?

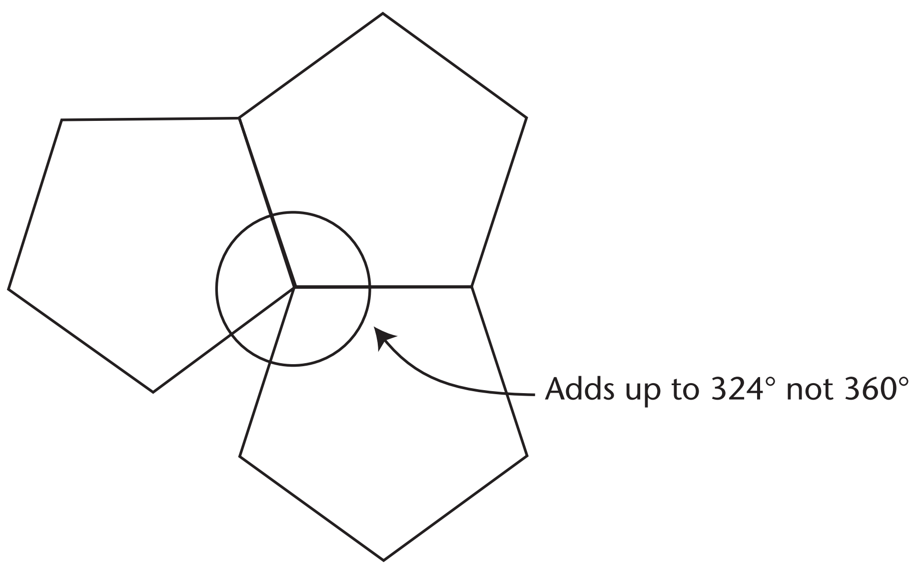
First, create more copies of the polygon. Then, turn and slide them together so that you can join vertices together and have the interior angles add to 360º.



Since the interior angles all add to 360º where they meet, this **quadrilateral** will tessellate.

Will this regular pentagon tessellate?





The pentagon will not tessellate since the interior angles do not add up to 360º. When this happens the edges overlap or leave a gap.

Thinking Space

So the question is:

Which polygons will tessellate an entire plane, and which ones won't?

For a regular or irregular polygon to tessellate an entire plane, you have to be able to arrange the vertices at a point without overlap or gaps. This means that the interior angles can be arranged so that they all add up to exactly 360º.

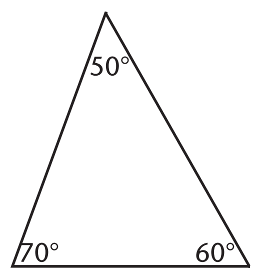
Thinking Space

Try It!
Activity 1

1. Squares tessellate an entire plane since all four angles add to 360°. Which other two regular polygons tessellate?

2. Explain why a regular pentagon cannot tessellate an entire plane.

3. Will this irregular polygon tessellate? Explain your answer.

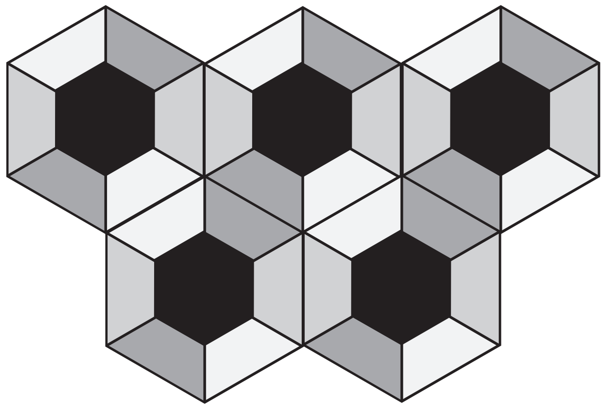


Turn to Solutions at the end of the course to mark your work.

Thinking Space

Explore
Tessellations Around Us

Not all tessellations need to be made with the same polygon in the tiling design.



Notice this tessellation is made up of quadrilaterals and hexagons.

Many different shapes combined can tile an entire surface. In many quilting projects, various shapes are used to make intricate designs.

Thinking Space

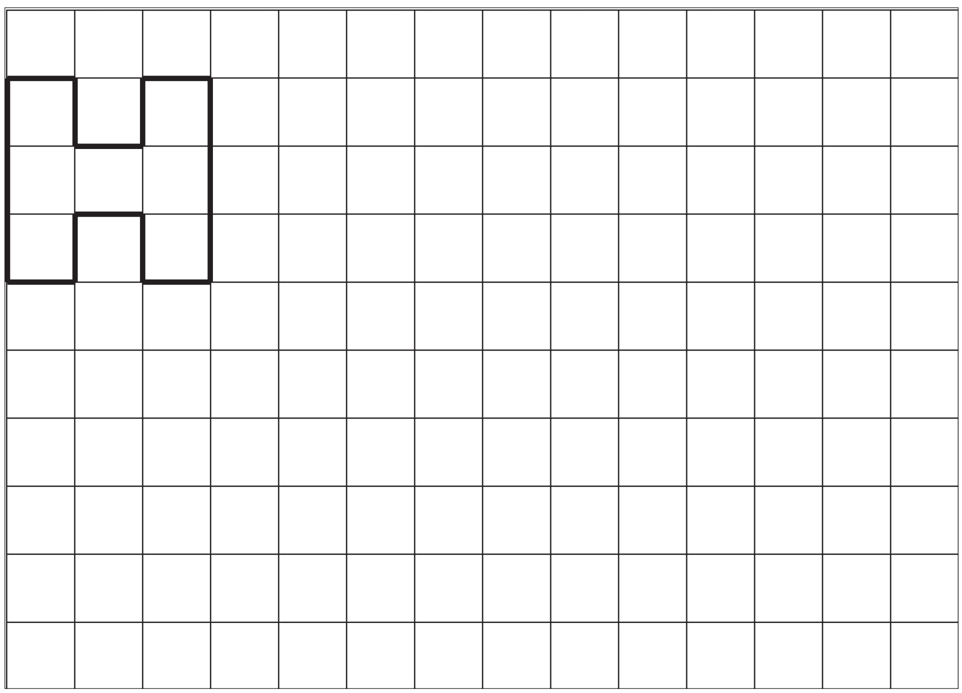
To create a tessellation, you start with a tessellation tile and use it to cover an entire plane.

If you would like to explore a tessellations creation program and you have Internet access, go to the Math 8 website at **http://www.openschool.bc.ca/courses/math/math8/mod1.html**. Look for *Lesson 1.3A: What Is a Tessellation Anyway?*

Thinking Space

Try It!
Activity 2

1. Use this irregular shape to tessellate the entire grid.



2. Tessellations appear in kitchen backsplashes, fabrics, clothing, upholstery, video games, and so much more.

   a. Find a tessellation in your home, and copy a part of the tiled pattern in the space below.

Thinking Space

b. Name the regular polygons, if any, that appear in the pattern.

____

____

c. Name the irregular polygons, if any, that appear in the pattern.

____

____

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson A. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson B

Transformations in Tessellations

For this lesson you will need:

- protractor (you can find one in the Appendix if you don't have one)
- metric ruler

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

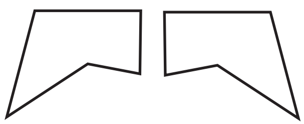
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How can you identify a translation in a tessellation?		
How can you identify a reflection in a tessellation?		
How can you identify a rotation in a tessellation?		

Thinking Space

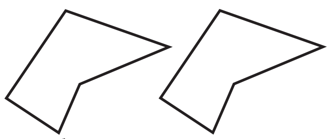
Warm-up

The starting polygon has been transformed five times. From one image to the next, describe the **transformation** as a **translation** (slide), **rotation** (turn), or **reflection** (flip).

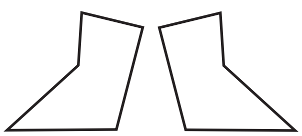
1.



a.

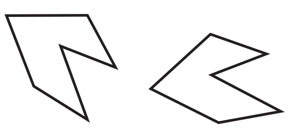


b.

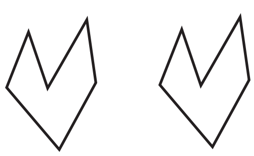


c.

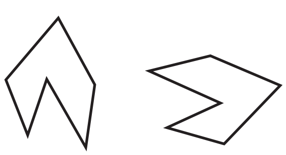
2.



a.



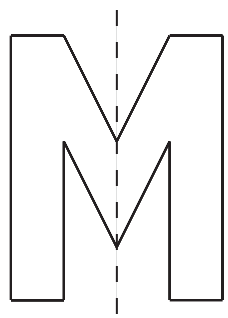
b.

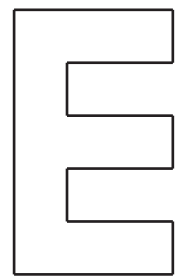


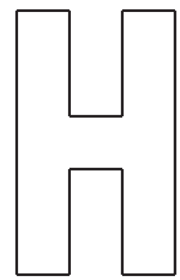
c.

Thinking Space

3. Draw in all the lines of symmetry for each letter. The first one has been done for you. Notice the image to the left of the line of symmetry. It is the mirror image of what you see to the right of this line.

a. 

b. 

c. 

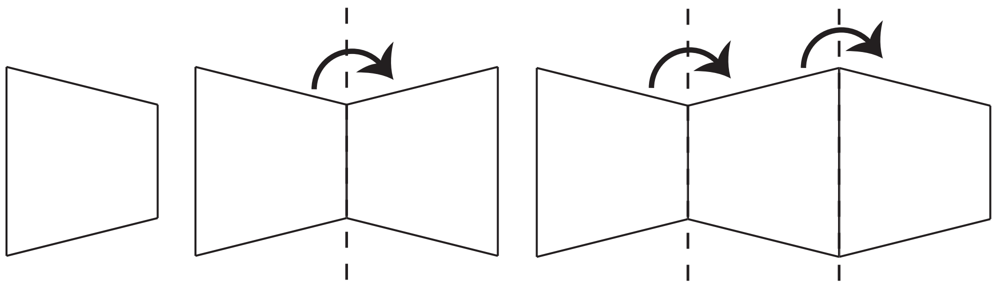
Turn to Solutions at the end of the module and mark your work.

Thinking Space

Explore

Using Reflections and Translations to Tessellate

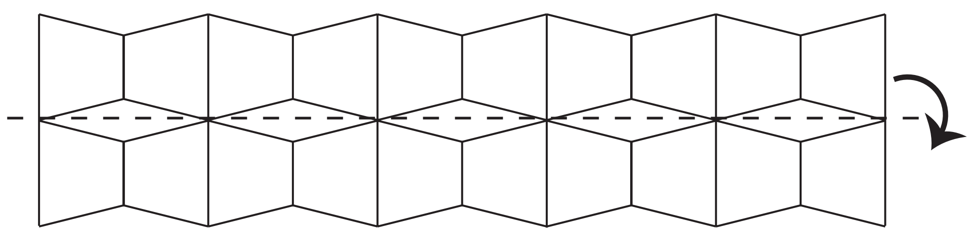
Tessellations must cover an entire plane. A simple way to tessellate is by repeatedly reflecting a polygon.



Polygon **Polygon and Its Vertical Reflection** **Polygon and Two Vertical Reflections**

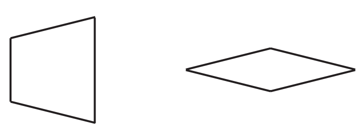
A repeated vertical reflection can create a row of patterned tiling.

A horizontal reflection creates a 2nd row.



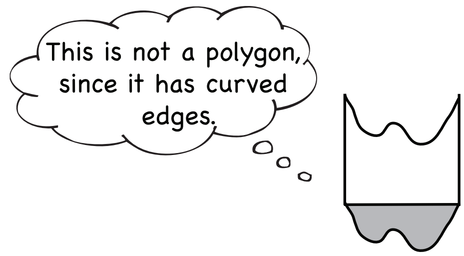
Could the second row of tiles have been created using a translation rather than a reflection?

Notice how the entire plane is covered by two different quadrilaterals:

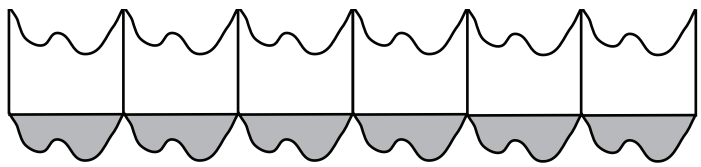


Thinking Space

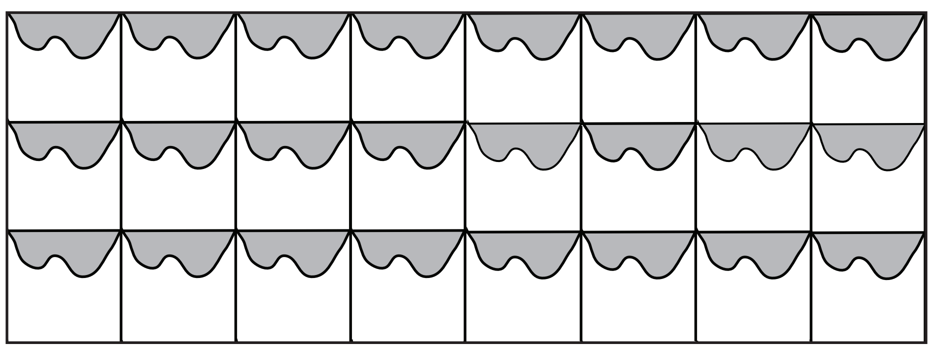
Translations can also be used to create a tessellation. By sliding a shape over and tracing it, the entire plane can be covered.



The first row is made by repeatedly sliding the original tile to the right and tracing it. The shape was translated across.



The second row is made by sliding the top row down and copying it. The shapes were translated down.

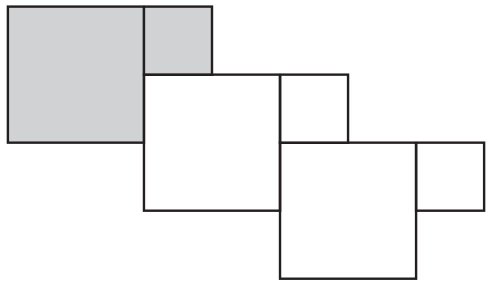


In this example, we covered the entire plane using an irregular shape.

Thinking Space

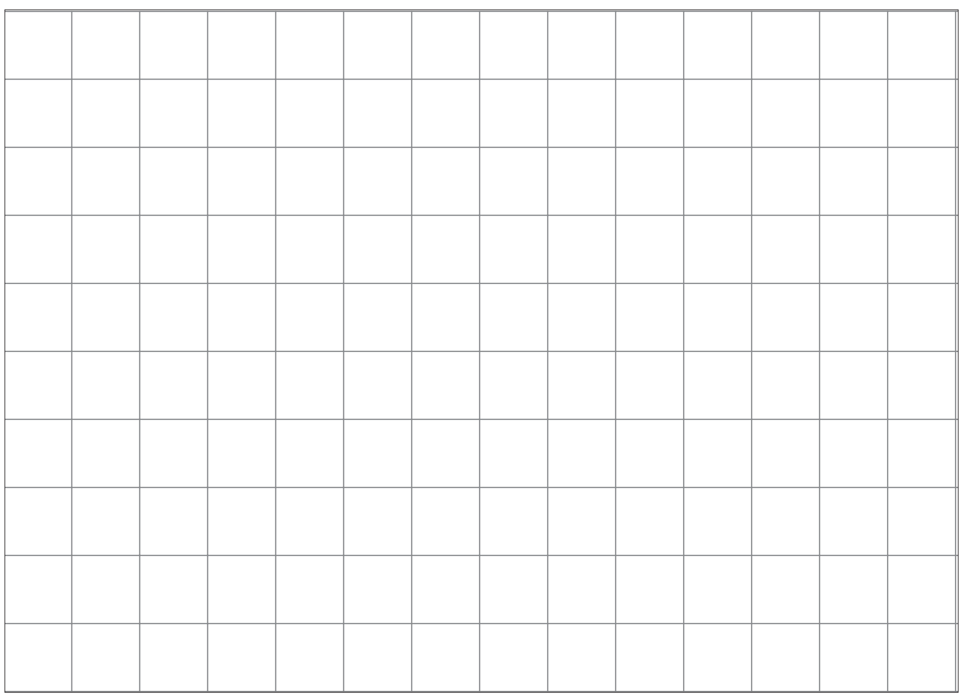
Try It!
Activity 1

1. The tessellation tile for this pattern is shaded.



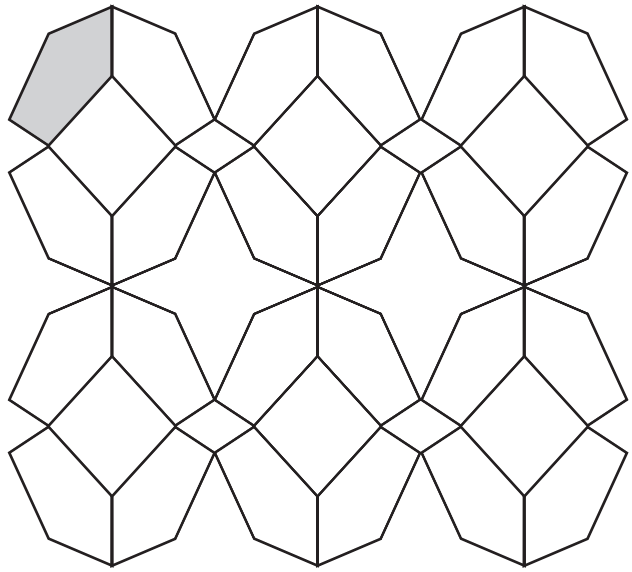
   a. Describe how this tiling was created using the shaded design.

   b. Use the shaded shape to cover the entire grid to create a tessellation.



Thinking Space

2. This design was created by repeatedly flipping the shaded shape.



   a. Describe how this entire design was creating using the shaded shape.

   b. Other than pentagons, what shapes are in this tessellation?

Turn to Solutions at the end of the module and mark your work.

Thinking Space

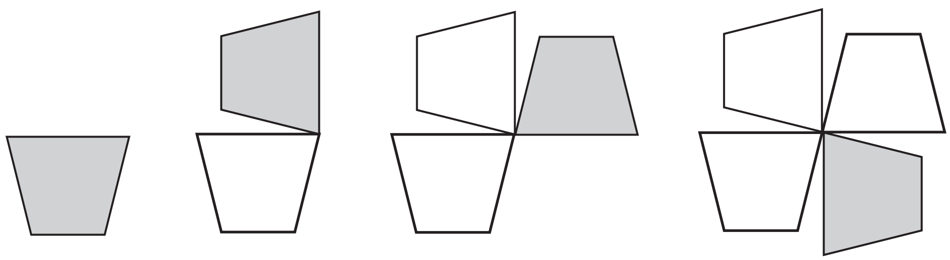
Explore

Using Rotations to Tessellate

Slides and flips are used in many creations.

Translations, reflections, and rotations are used in tiling designs. Can you see turns in some of these designs?

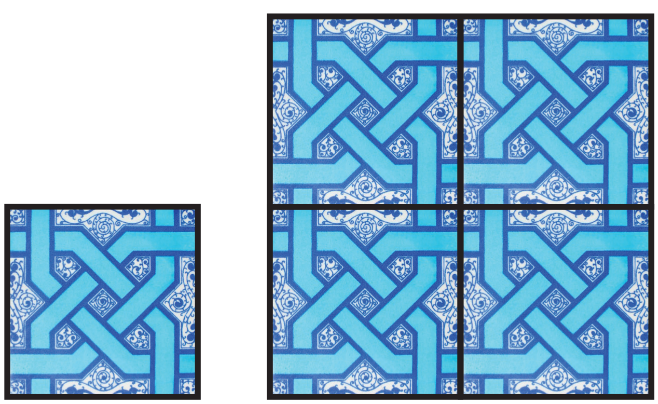
Rotations are easy to see when the shape is being turned around a point.



Original | **Original Rotated 90° Clockwise** | **Original Rotated 180° Clockwise** | **Original Rotated 270° Clockwise**

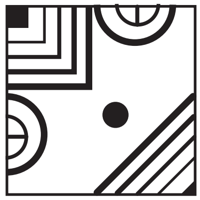
Following is a square tile that has been painted.

When you tile a floor or wall, different patterns can be formed by using a combination of translations, reflections, and rotations.

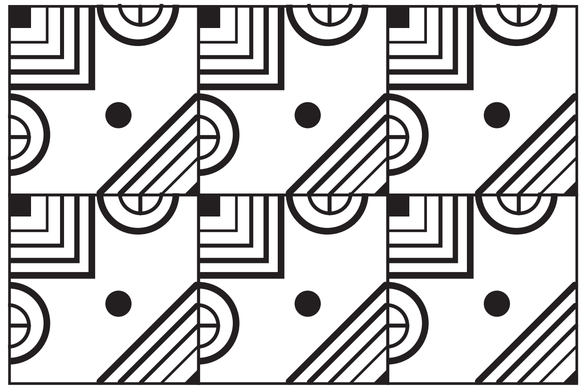


Thinking Space

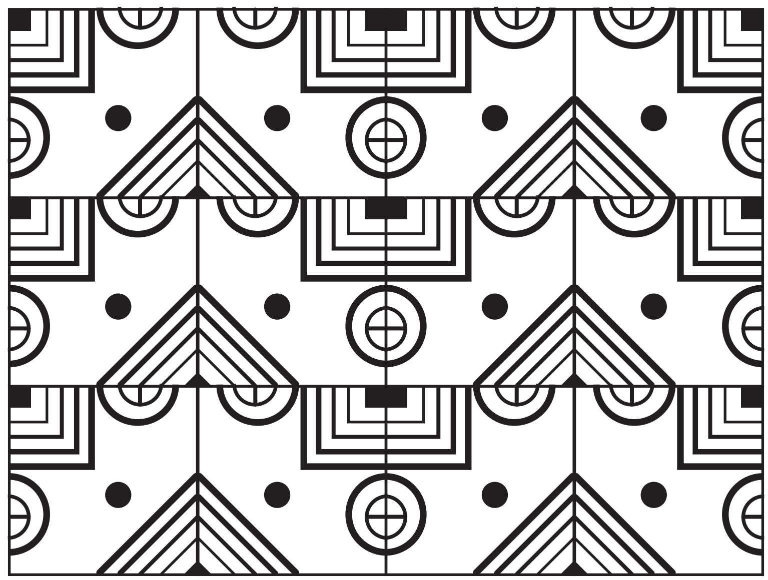
Here is a tile.



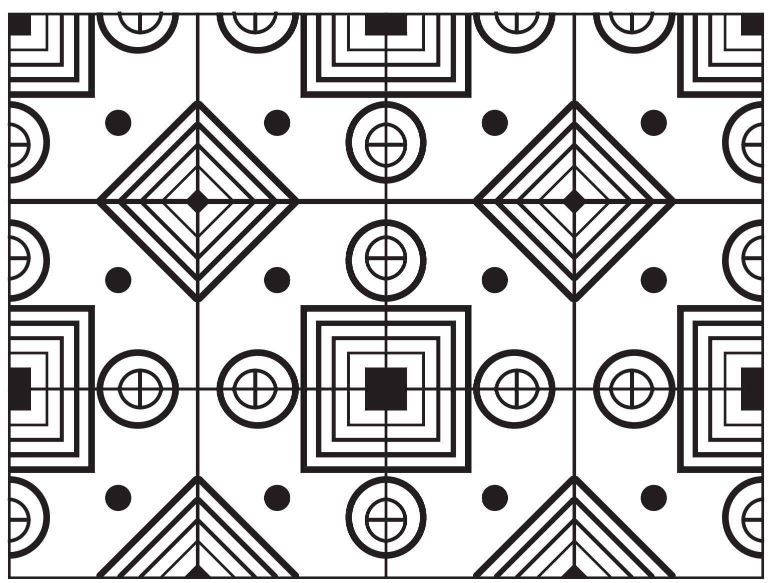
Tessellation formed by translation.



Tessellation formed by translation and reflection.

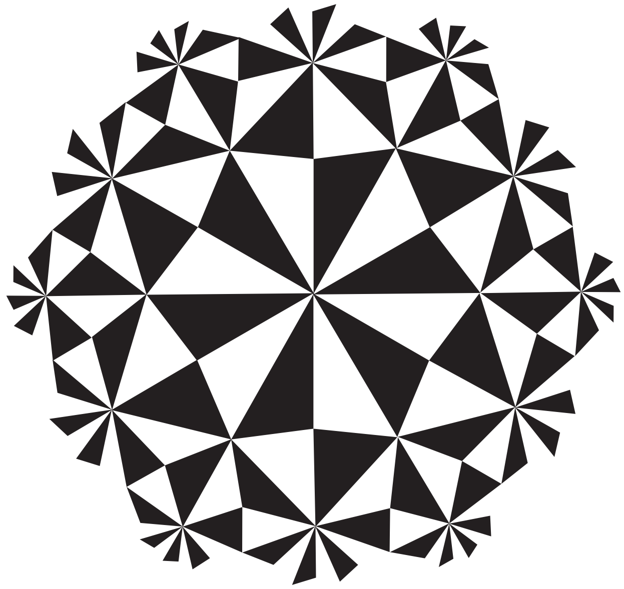


Tessellation formed by rotation and translation.

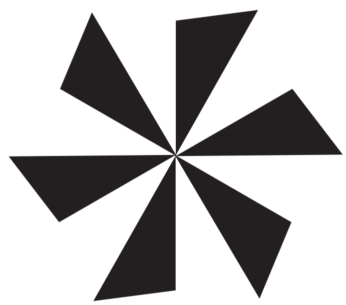


Thinking Space

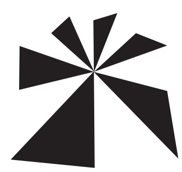
This tessellation is constructed within a circle.



The centre uses a triangular shape which is rotated 60 degrees five times, to form a flower pattern. The pivot point for the rotation is the bottom vertex of the triangle. The pivot point becomes the centre of the design.



Around this centre is a different flower shape. This shape is also rotated 60 degress five times around the centre point of the design. The outer ring uses the same shape, but shrunk in size. It too is rotated around the centre point.

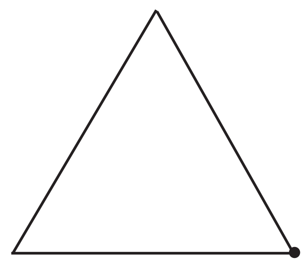


The tessellation looks like a dome. Shrinking the shapes and presenting the tessellation in a circle gives an illusion of depth as the smaller shapes appear further away.

Thinking Space

Try It!
Activity 2

1. Trace this triangle on another piece of paper. Draw a dot on the bottom right vertex of this triangle as shown. Rotate this triangle around the dot, and trace the triangle as you rotate it. Be sure not to overlap or leave any gaps.



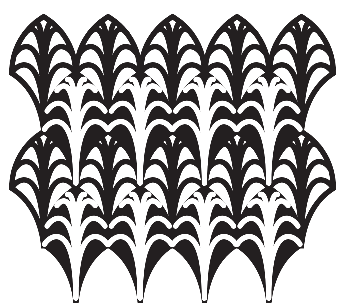
   a. How many triangles rotate around the point? ________

   b. All together these triangles have created another regular polygon. Which one?

   ____

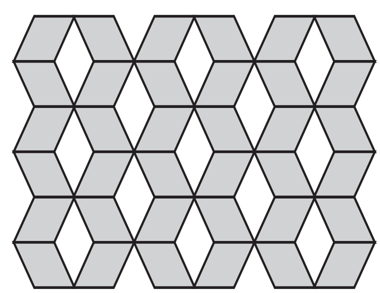
2. Which transformation(s) created each tessellation?

   a.



   ____

   b.



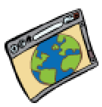
   ____

Thinking Space

c.

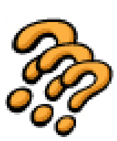
____

3. The artwork of M.C. Escher is very relevant to this module.



Find out more about his work on the Internet or by visiting the library. Then write four sentences about Escher's art.

Turn to Solutions at the end of the module and mark your work.



You've finished Lesson B. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Lesson C
Creating a Tessellation

For this lesson you will need:

- scissors
- tape
- protractor
- metric ruler
- construction paper, card stock or index card (optional)

When you turn the page over, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

When you're finished the lesson, go back to the Essential Questions page and fill in the second column. Be sure to keep your page in a safe place while you're working through the lesson.

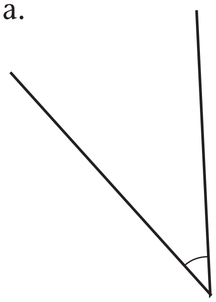
Essential Questions	Before the lesson: What I know	After the lesson: What I learned
How do I create a tessellation?		
How does the creation of a tessellation tile from a shape affect the area of that shape?		

Thinking Space

Warm-up

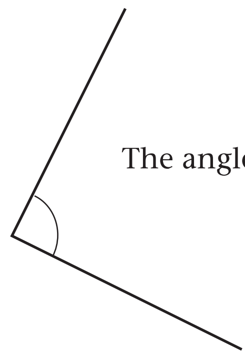
1. Measure each angle using a protractor. Then fill in the blank. The first one has been done for you.

a.



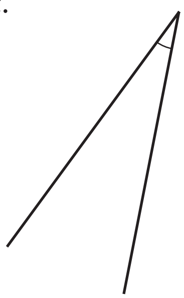
The angle is **39**°

b.



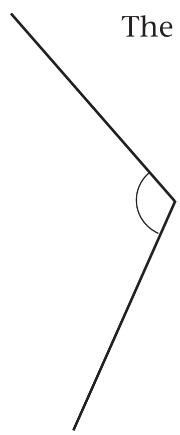
The angle is ____°

c.



The angle is ____°

d.



The angle is ____°

2. Use a protractor to draw a regular hexagon on another piece of paper. Each interior angle will measure 120º. Make each side measure 2 cm.

   Hint: Start with a line 2 cm long and draw an angle of 120° to make a second side.

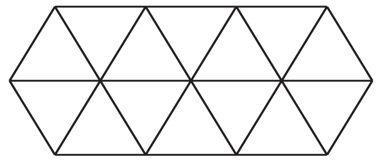
Turn to Solutions at the end of the module and mark your work.

Thinking Space

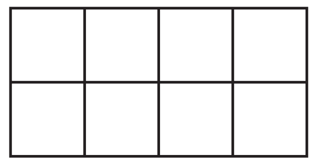
Explore
Using Shapes with Straight Edges

Three regular polygons can tessellate an entire plane: equilateral triangles, squares, and hexagons.

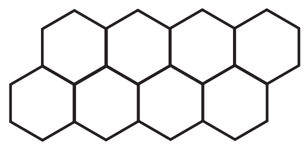
A tessellation of triangles:



A tessellation of squares:

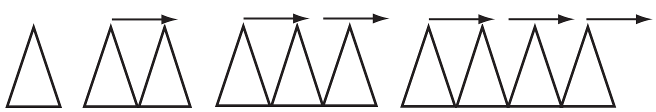


A tessellation of hexagons:

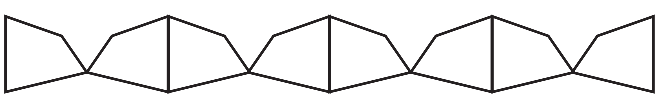


Transformations such as slides, flips, and turns are used to create tessellations of polygons.

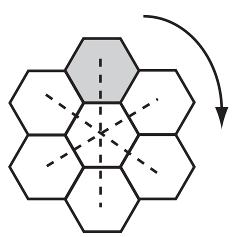
Translation:



Reflection:



Rotation:



Thinking Space

Notice that when any of these shapes are transformed the polygon stays the same in size and shape. This means the shapes are all **congruent**. Since the shapes are all congruent, this also means the area of the shapes does not change.

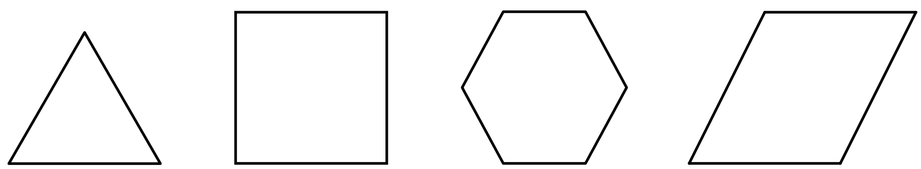
Remember, area is the number of square units covered by a shape. So if the size of the shape stays the same, then so does the area.

The area of a polygon does not change each time it is transformed.

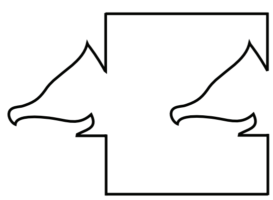
Thinking Space

Try It!
Activity 1

1. Create a tessellation that covers the entire grid using graph paper from the Appendix. Your tessellation should:
   - use all of the different shapes
   - use all the transformations: slides, flips and turns



2. Someone cut out a seahorse-head shaped piece from a rectangle and taped that piece on the other end. The seahorse-head shaped piece had an area of 3.2 cm^2. If the area of the original rectangle was 10 cm^2, what is the area of the new shape? Explain your answer.



____

____

Turn to Solutions at the end of the Module and mark your work.

Thinking Space

Explore

Using Shapes with Irregular Edges

For this Explore you will need:

- construction paper, card stock or index card (optional)
- ruler
- scissors
- tape

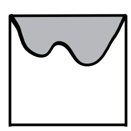
M. C. Escher created many interesting works of art in his tessellations. Most of the shapes that he translated, reflected, or flipped were shapes without straight edges.

Creating Escher-type Tessellations

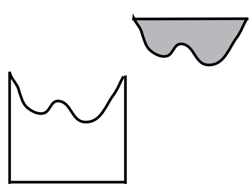
We'll explain how to create an Escher-type tessellation here. You'll have a chance to create some tessellation tiles in the activities.

Step 1: Draw and cut out a square.

Step 2: Begin cutting at one vertex to the next vertex beside it. Don't use a straight cut. Be creative. Here is an example. The cut was started at the vertex on the top left corner and ended at the vertex on the top right corner.

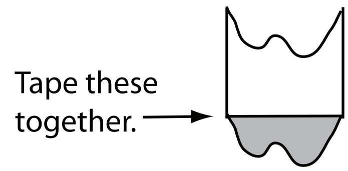


Step 3: Slide the piece out that you have cut. Do not flip it over or rotate it.



Thinking Space

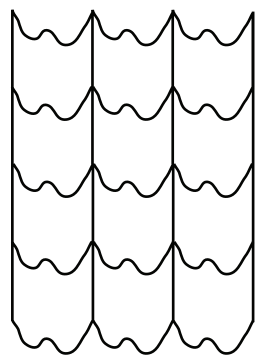
Move the piece to the OPPOSITE side of the original shape.



Tape the pieces together.

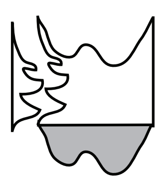
This is a tessellating tile.

When you trace your tile repeatedly, it will cover an entire paper with no overlaps or gaps! The result will look something like this.

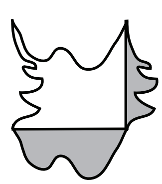


However, we were trying to create something as creative as Escher's design. Our tessellating tile still has straight edges. Here is one more step to make a more creative tessellating tile.

Step 4: Cut a piece from the left side (remember to not use a straight cut).



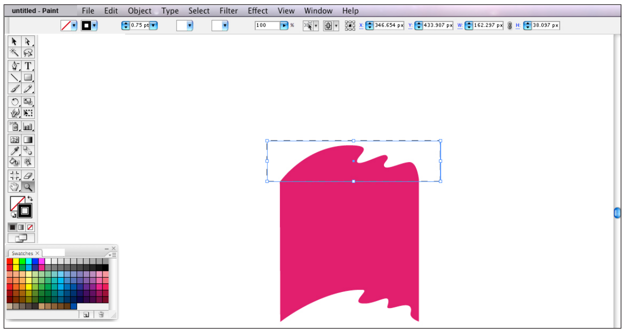
Step 5: Move the piece from the left side to the right side and tape it.



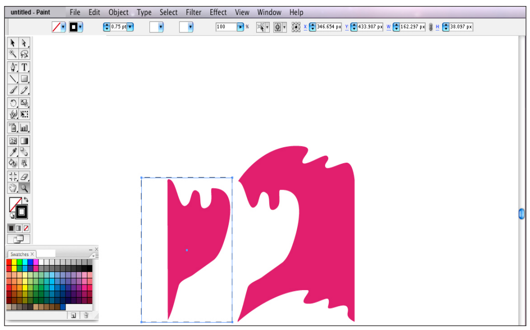
All of Steps 1–4 can be performed using a simple graphics program on the computer.

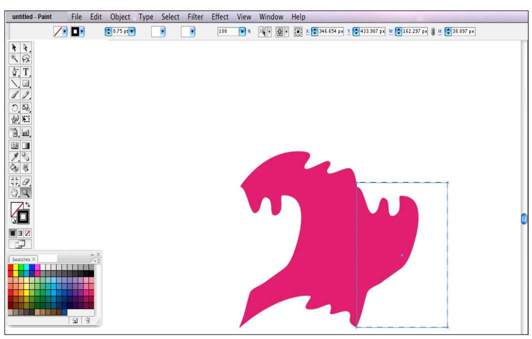
Thinking Space

Here are the results of Steps 1–2 for a slightly different tile.



Here is that same tile after performing Steps 3–4.



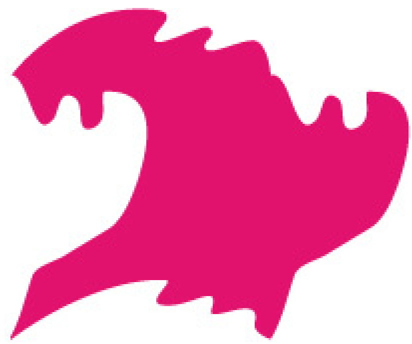


If you have Internet access, check out the links to drawing or paint programs on the Math 8 website at: http://www.openschool.bc.ca/courses/math/math8/mod1/.html.

Thinking Space

Try It!
Activity 2

1. a. Trace and cut out this tessellation tile.



   b. Use the tile to create a tessellation on another piece of paper.

2. Create a different tessellation tile using the following instructions.

   Go to **http://media.openschool.bc.ca/osbcmedia/ma08/course/html/ma0813c1f_tessellation.html** and open *Create a Translation Tessellation!*

   Trace your tessellation tile here.

Turn to Solutions at the end of the Module and mark your work.

Thinking Space

Explore

Creating Tessellations Using Rotation

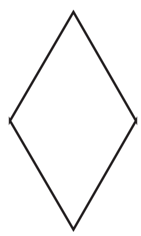
How can we create a rotation tessellation? We'll just explain it here, and you'll have a chance to create a tessellation tile in the activities. But you can follow along and do this activity if you like.

Choose one of the following shapes:

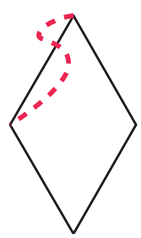
- regular hexagon
- rhombus (or square)

Draw your shape on a piece of paper. Now complete the following steps to create your tessellation.

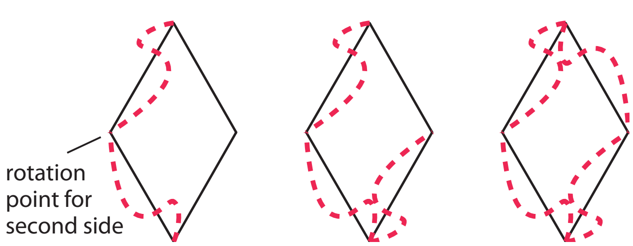
Step 1: Draw your shape. We'll start with a rhombus.



Step 2: Use a different colour and draw a curved line on one side, going from vertex to vertex.

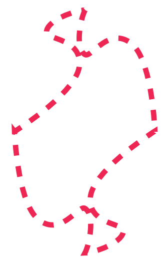


Step 3: Copy the curved line, and rotate it along to the next side. Use the bottom vertex as the rotation point. Repeat this for all sides.

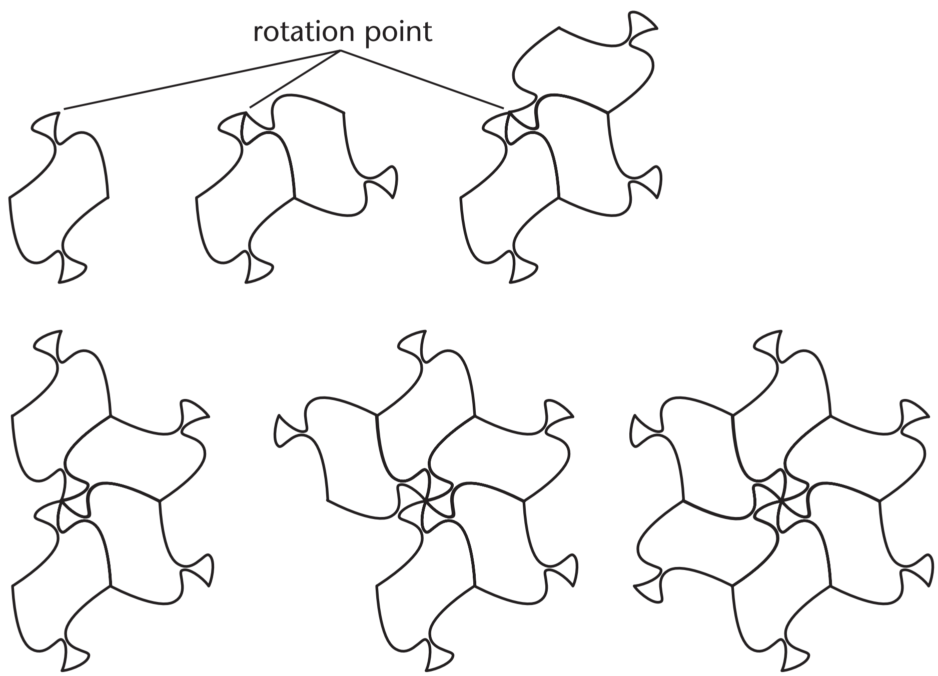


Thinking Space

Step 4: The curved lines together form the outline of your tessellation tile. If you're using paper, cut out your tile.

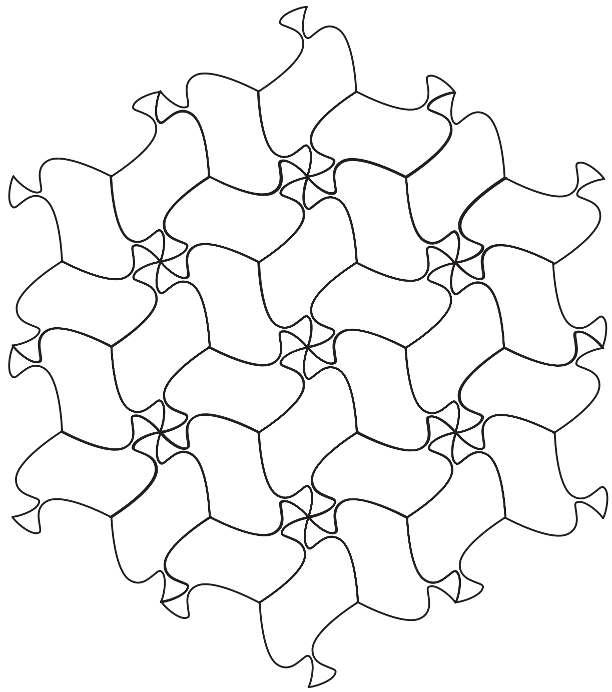


Step 5: Trace an outline of your shape on a new pice of paper or copy and paste your shape if using a computer. Pick a vertex on the shape and rotate your tile around the point until it matches up along one side. Draw or copy and paste another shape. Continue this until you have drawn in a complete rotation.

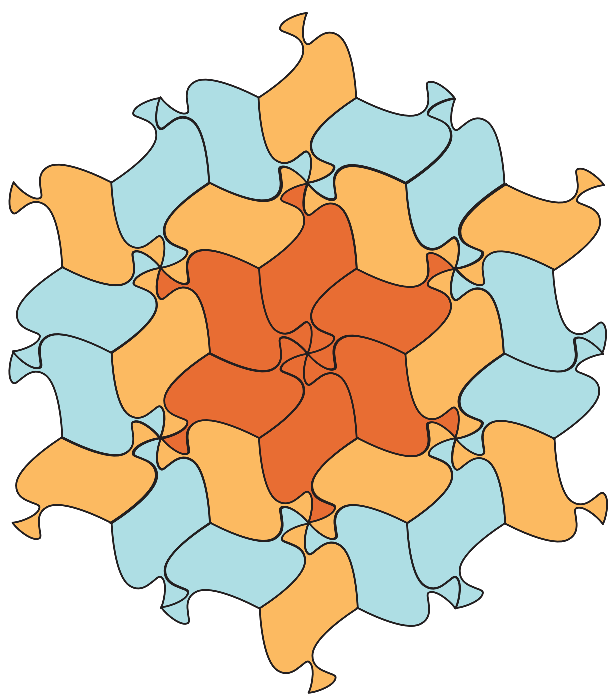


Thinking Space

Step 6: Once you have one complete rotation, you can add onto it by finding where your shape fits and adding more.



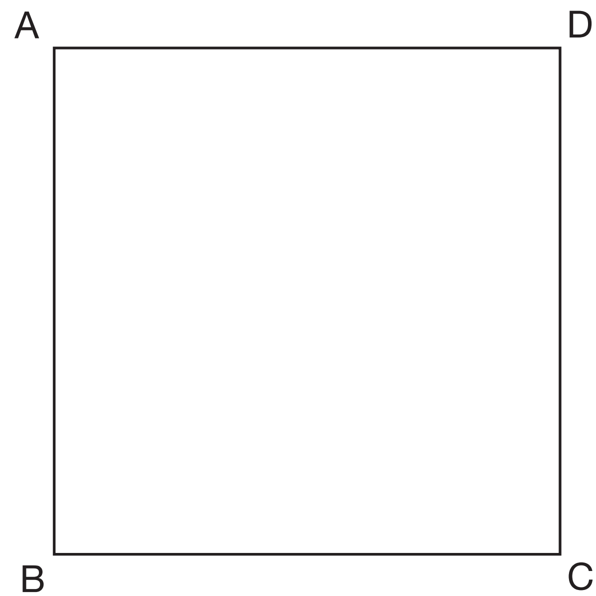
Step 7: As a final touch, add colour or line details to your tessellation.



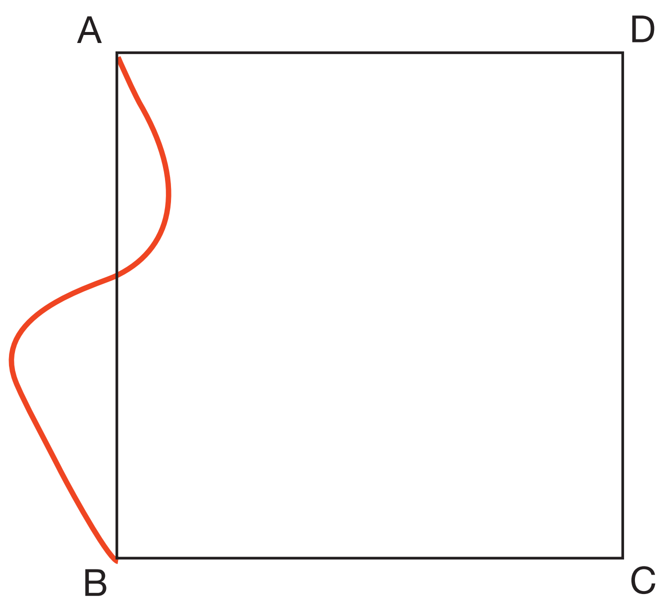
Thinking Space

Try It!
Activity 3

1. a. Draw a square. Label the corners A, B, C and D as shown on the diagram.



b. Draw a curved line all the way along the left side from A to B. The line can go outside of the square at some point.



Copy this drawing onto another piece of paper. You don't have to copy the straight line from A to B, just the curved one.

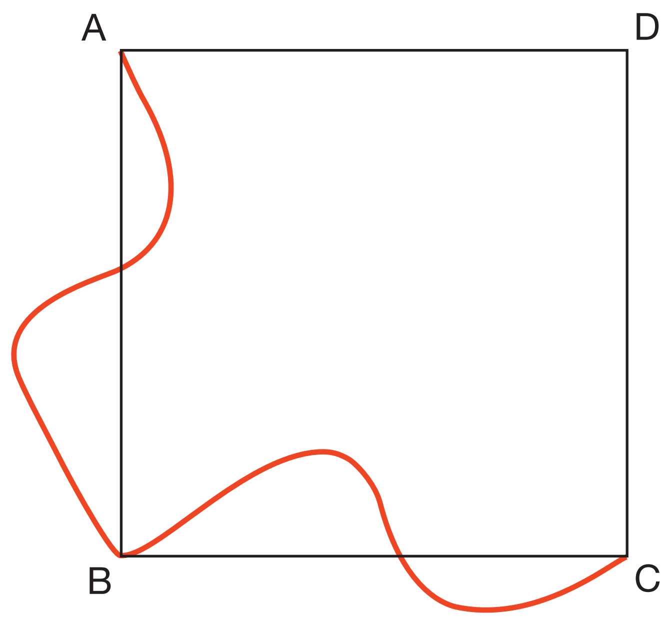
Thinking Space

c. Carefully cut out this figure. This will be your template to trace the curved line on to the other three sides of the square.

Place the template on top of your drawing so that it matches up perfectly.

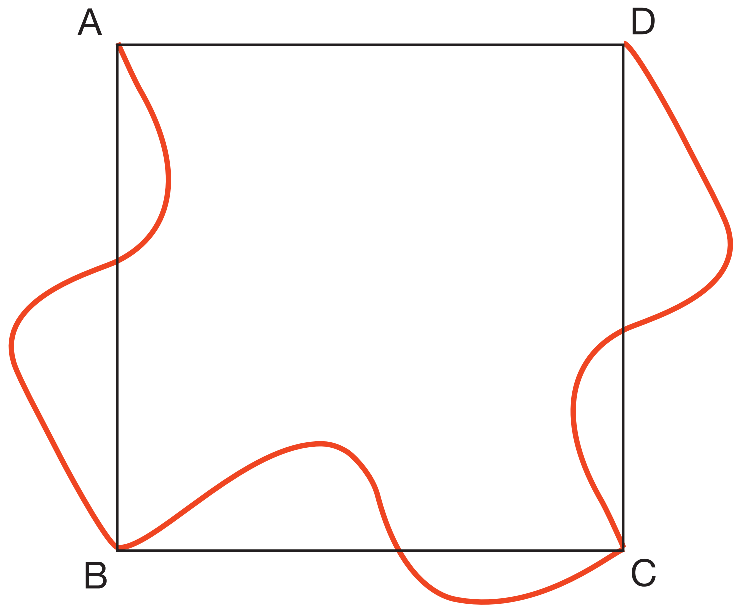
Using B as the pivot point, rotate the template clockwise 90°. The curved line should now be lying from B to C. Trace the curved line onto the drawing of the square.

You should now have two curved lines like this:

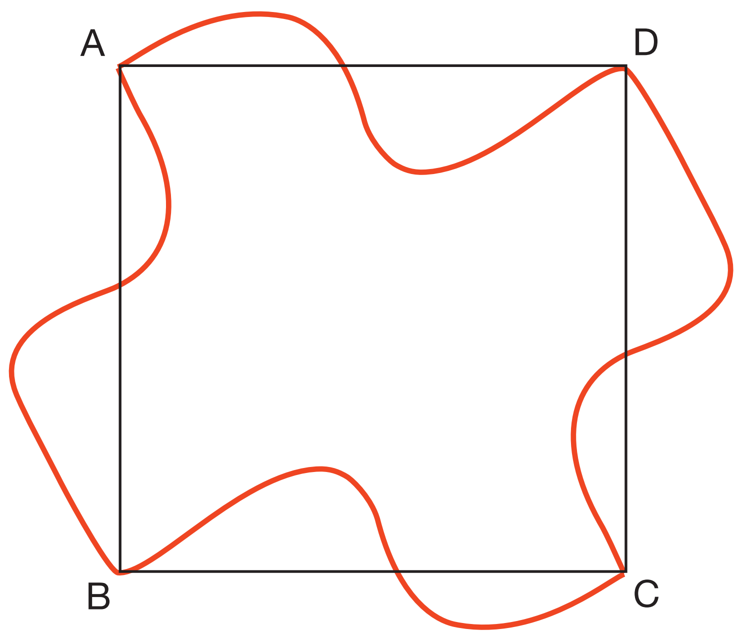


Thinking Space

d. Now repeat the last step, but use C as the pivot point and swing the template so the curved line is running from C to D. Trace.



e. Final side: Use D as the pivot point and swing the template so the curved line is running from D back to A. Trace.



Your tile is done!

If you like, cut out your tile on the curved lines (ignore the straight ones).

Thinking Space

On a fresh piece of paper, trace around your tile. Then choose a pivot point on your drawing and rotate your tile around this point. Eventually one side should match up perfectly to the drawing underneath it.

Repeat this step using the same rotation point. How many rotations do you need to match up with the original drawing?

Trace your tile as many times as you like, always matching up lines so your drawing has no gaps. Then add colour and admire your artwork!

2. You have seen various tessellations throughout this section and while researching.

   a. Describe the one tessellation you liked the best by referring to the colours or designs used in it.

   b. Describe what you liked about it.

   c. Describe what types of transformations were used.

Turn to Solutions at the end of the module and mark your work.

You've finished Lesson C. Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final column.

Thinking Space

Section Summary

Completing this section has helped you to:

- distinguish between polygons that will and won't tessellate
- create tessellations using flips, slides and turns
- create Escher-like tessellations tiles

Appendix

Solutions

Section 1

Pretest

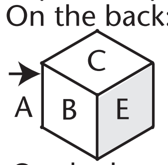
Lesson A: Visualizing in Three Dimensions

1.

2-D Shape	3-D Shape
rectangle	cube
oval	box
circle	sphere
	can of pop

2. a.

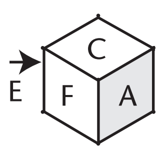
On the back: F



On the bottom is: D

b.

On the back: B

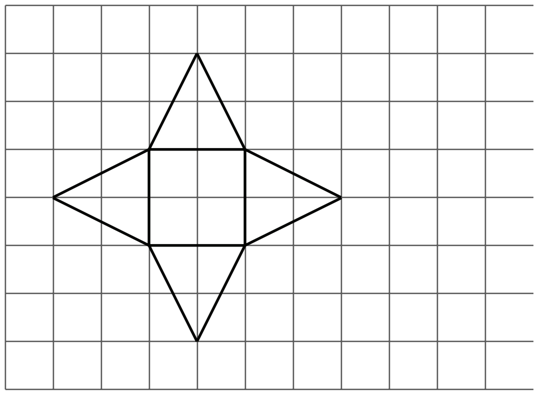


On the bottom is: D

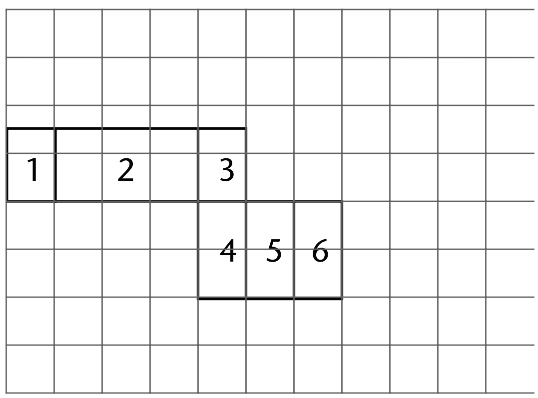
Lesson B: Prisms and Their Nets

3. a. triangular prism   rectangular prism
   b. cylinder
   c. cylinder
   d. triangular prism

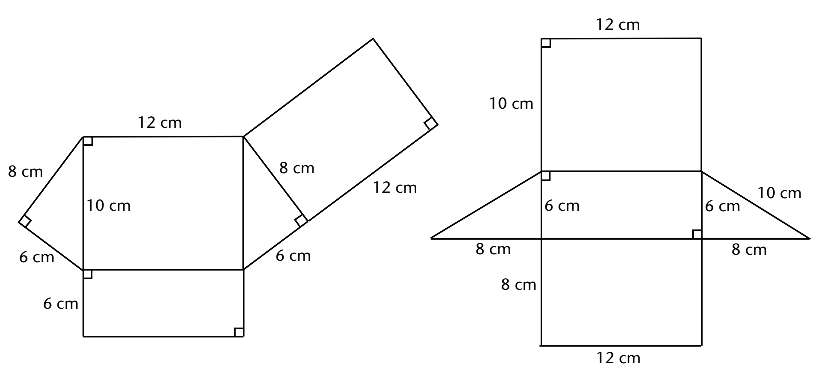
4. a. Make each triangle the same size.



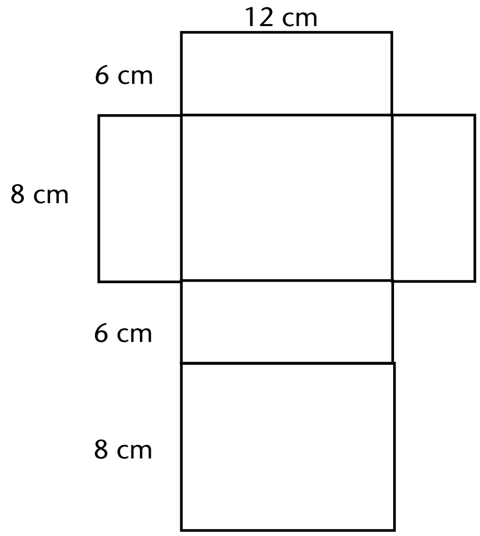
b. There are several solutions to thsi question. One solution is to make Side 5 the same as Side 2.



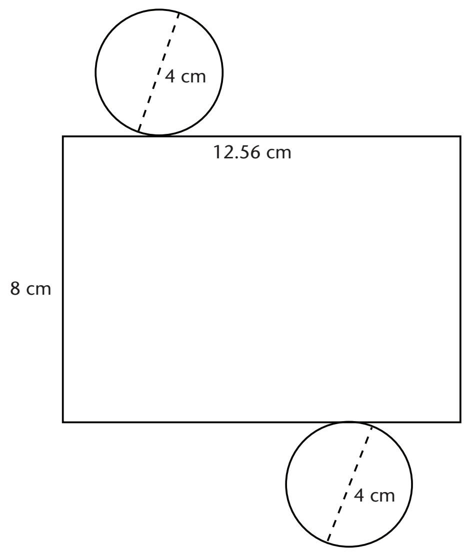
5. a. There is more than one solution to this question. Perhaps your answer looks like one of these:



b.

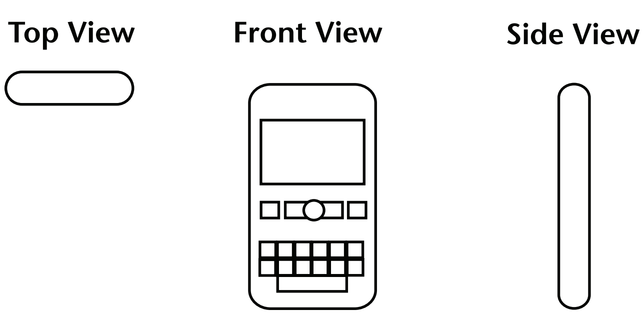


c.



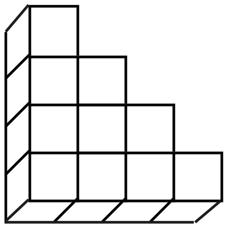
Lesson C: Top View, Side View, and Front View

6.

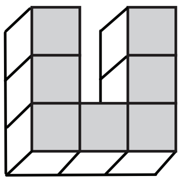


7.

a.



b.



Lesson A: Visualizing in Three Dimensions

Warm-up

1.

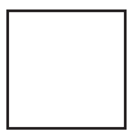
a. parallelogram

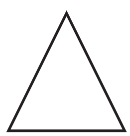
b. rectangle

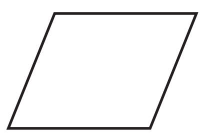
c. square

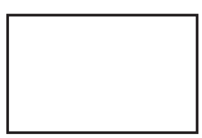
d. rhombus

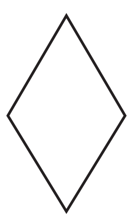
e. triangle

c. square 

e. triangle 

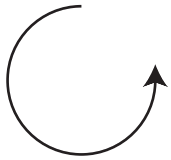
a. parallelogram 

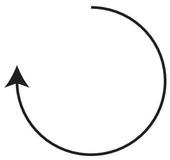
b. rectangle 

d. rhombus 

2. b

3.

d 

b 

a 

c 

a. moving right

b. turning clockwise

c. moving left

d. turning counter-clockwise

Try It! Activity 1

1. 2-D has length and width, 3-D has length, width, and depth

2. a. 3-D

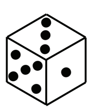
   b. 2-D

   c. 3-D

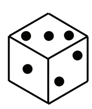
   d. 2-D

   e. 3-D

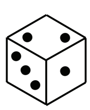
3. a.



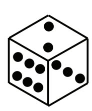
b.



4. a.

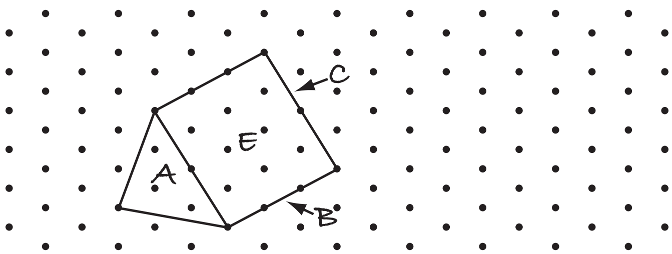


b.

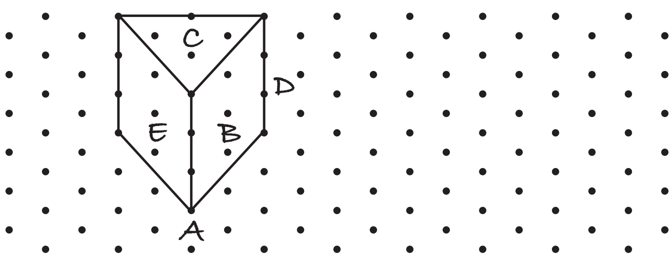


Try It! Activity 2

1. a.



b.



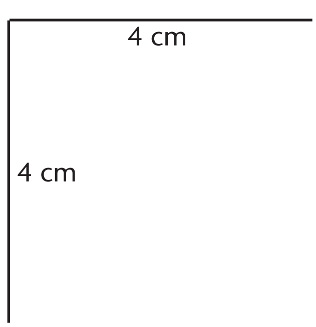
Lesson B: Prisms and Cylinders and Their Nets

Warm-up

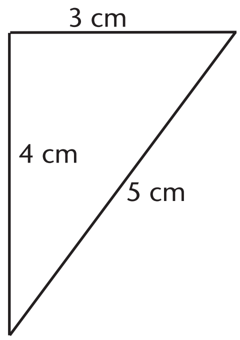
1. a.
   b.
   c.

2.

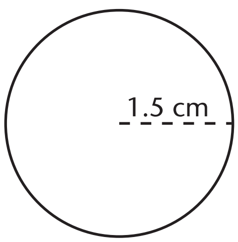
a.



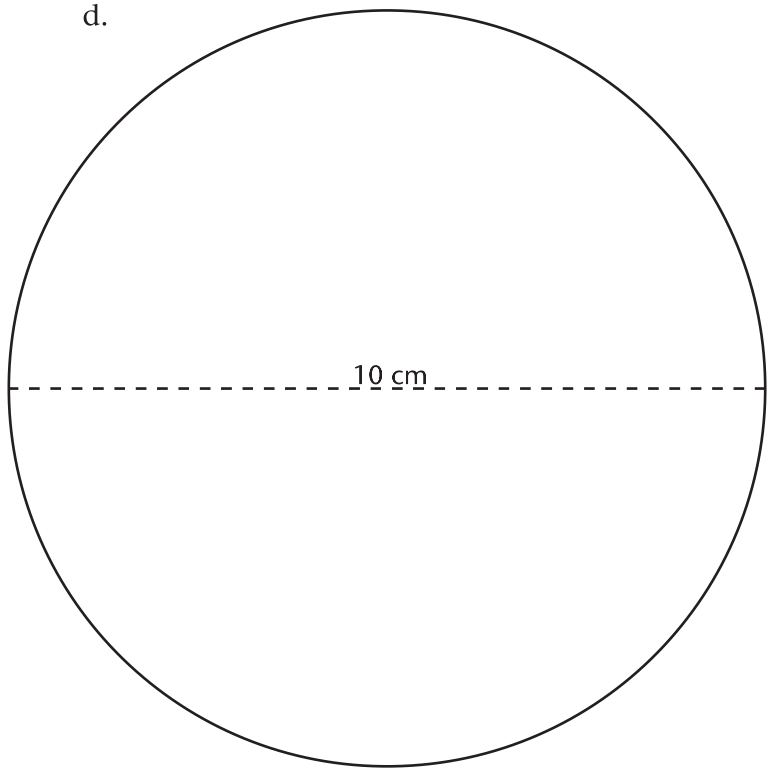
b.



c.



d.



3. a. equilateral triangle b. square c. isosceles triangle

Try It! Activity 1

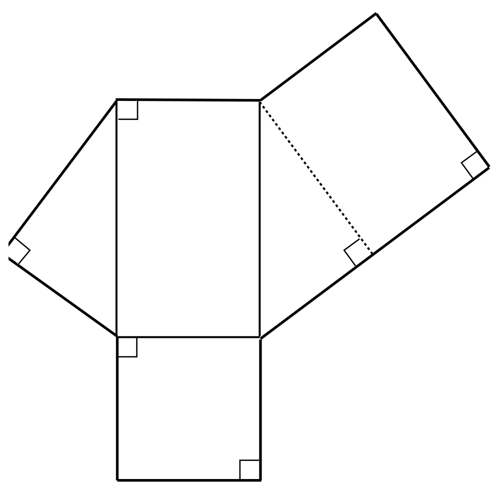
1. a. no b. no c. yes
2. Student is able to answer this when doing reconstructions, answers not required.
3. Student is able to answer this when doing reconstructions, answers not required.

Try It! Activity 2

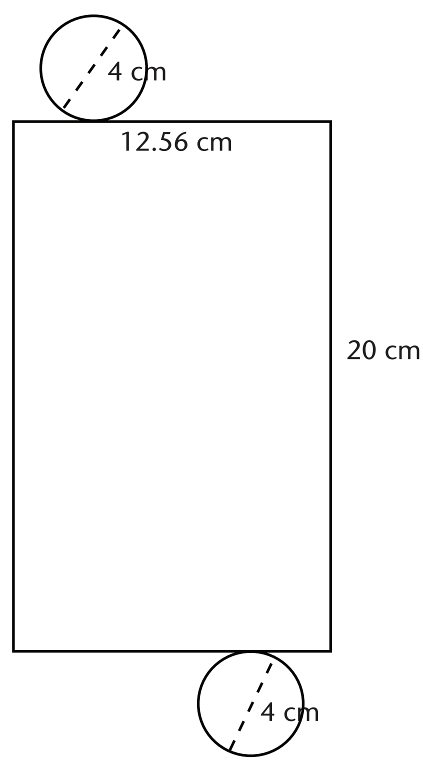
1.

b	art	a.	rectangular prism
c	art	b.	triangular prism
a	art	c.	cylinder prism
d	art	d.	rectangular and triangular prism

2. a. triangular prism b. rectangular prism c. cylinder
3. a.



b.

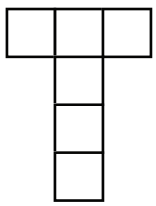


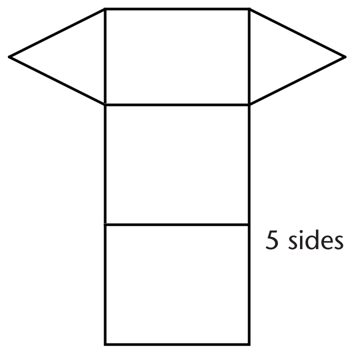
4. Student is able to answer this when doing reconstructions, answers not required.

Lesson C: Top View, Front View, and Side View

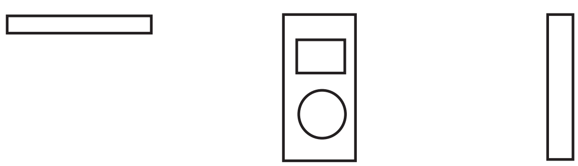
Warm-up

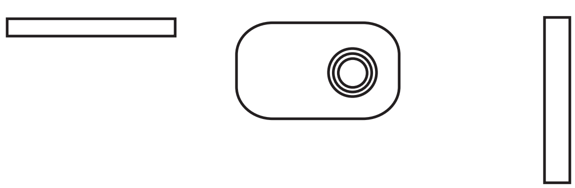
1. a. circle, square
   b. square, rectangle
   c. triangle, parallelogram, rectangle

2.
   a.
   
   6 sides

   b.
   
   5 sides

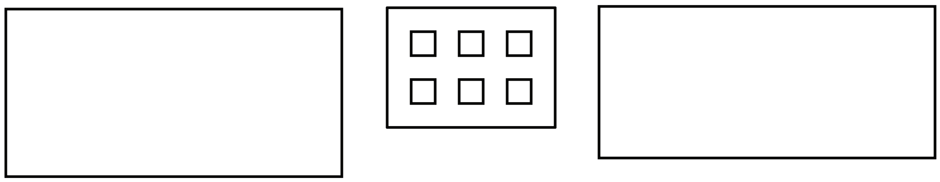
Try It! Activity 1

1.
   a. Top View   Front View   Side View
   

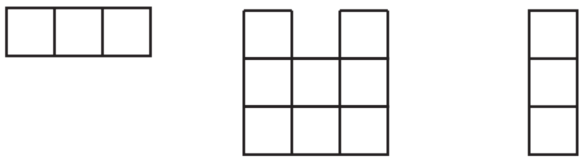
   b. Top View   Front View   Side View
   

2.

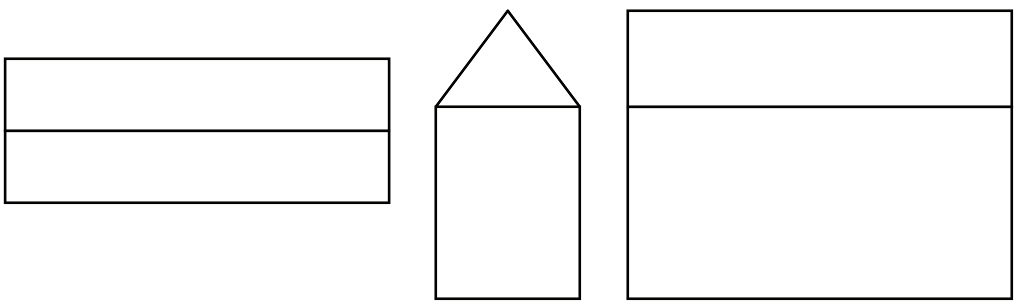
a. Top View — Front View — Side View



b. Top View — Front View — Side View

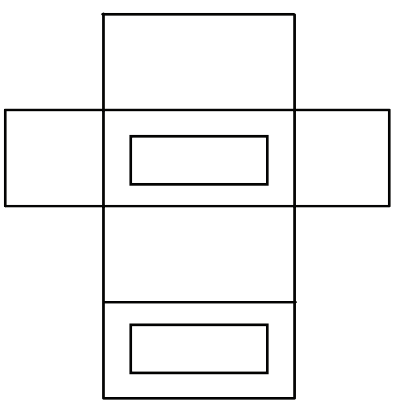


c. Top View — Front View — Side View



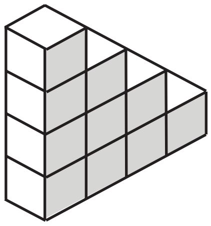
Try It! Activity 2

1. a.

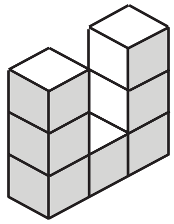


b. The net should create a 3-D brick-like object.

2.
   a.

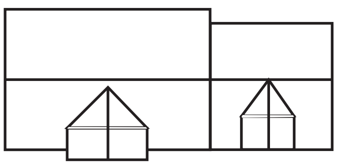


   b.



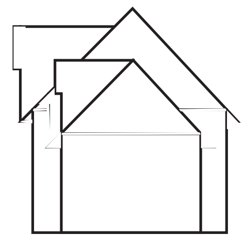
3. Answers will vary with size of building and placement of doors and windows, etc.
   Possible answers:

Top



Front View

Side View



Section 2

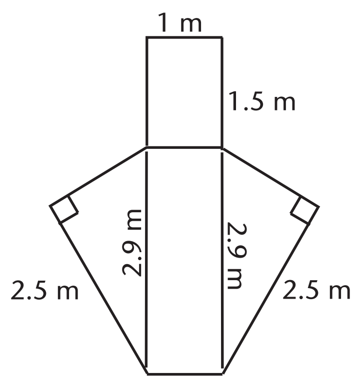
Pretest

Lesson A: Total Area in the Net

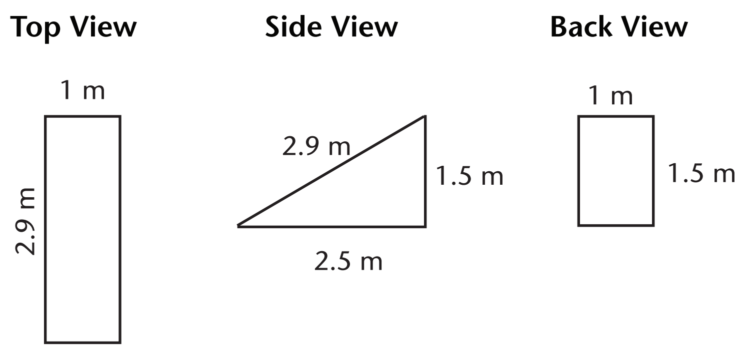
1. a. 112 cm^2 b. 822 cm^2 c. 164.9 cm^2
2. a. 17.6 cm b. 281.5 cm^2

Lesson B: More About Area in 2-D and 3-D

3. a. 27 cm^2 b. 45 cm^2 c. 71.7 cm^2
4. a.



b.



c. 8.15 m^2

d. Answers will vary. It will be enough if only one coat of paint is used. If two coats are used, then a second can is needed.

Lesson C: The Amount of Space

5. a. 64 cm^3 b. 1242 cm^3 c. 157.1 cm^3

6. a. yes, each has a volume of 60 cm^3

   b. Option 1: 94 cm^2 Option 2: 104 cm^2

c. Explanations will vary. Option 1 would be the better choice, since the surface area is less than Option 2, which means less material would be used.

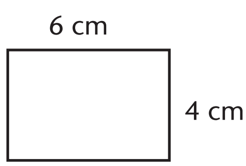
7. Explanations will vary. The surface area increases too. When the volume increases then the overall size of the 3-D shape gets bigger. This means the surface gets larger too.

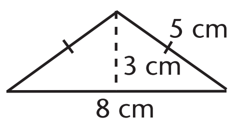
Lesson A: Total Area of the Net

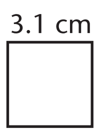
Warm-up

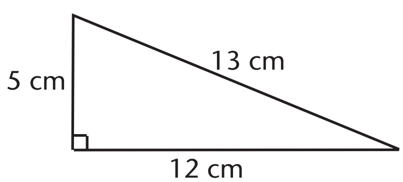
1. a. rectangular prism, triangular prism, cylinder, cube
   b. triangular prism
   c. cylinder
   d. cube

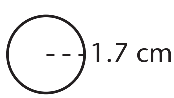
2.

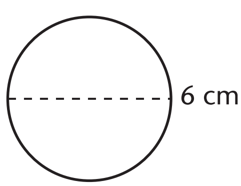
h 

g 

c 

d 

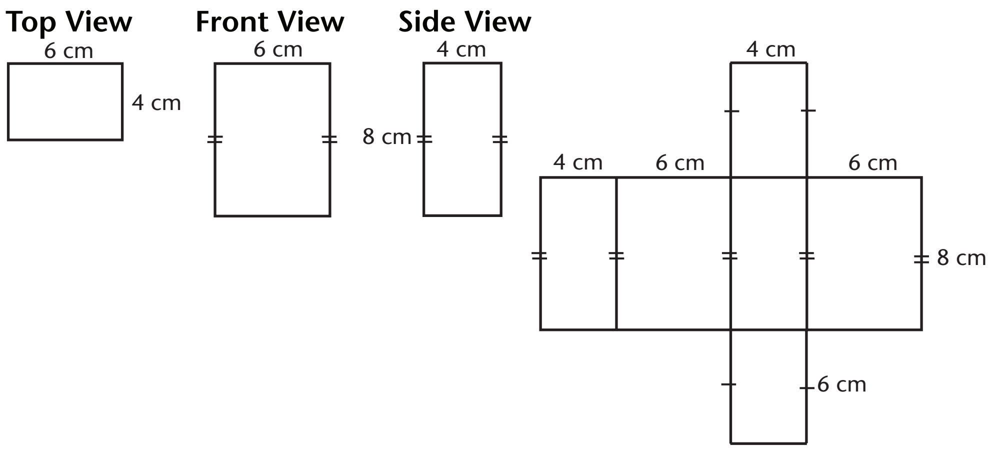
a 

f 

a. 9.1 cm²
b. 113 cm²
c. 9.6 cm²
d. 30 cm²
e. 60 cm²
f. 28.3 cm²
g. 12 cm²
h. 24 cm²

Try It! Activity 1

1. a. You can draw the shapes all connected or separately, but your drawing should contain:



b. Surface area = 208 cm²

area of 2 larger rectangles = 2 × (6 × 8) = 96 cm²

area of 2 medium sized rectangles = 2 × (4 × 8) = 64 cm²

area of 2 smallest rectangles = 2 × (4 × 6) = 48 cm²

2. For the prism in question 1:

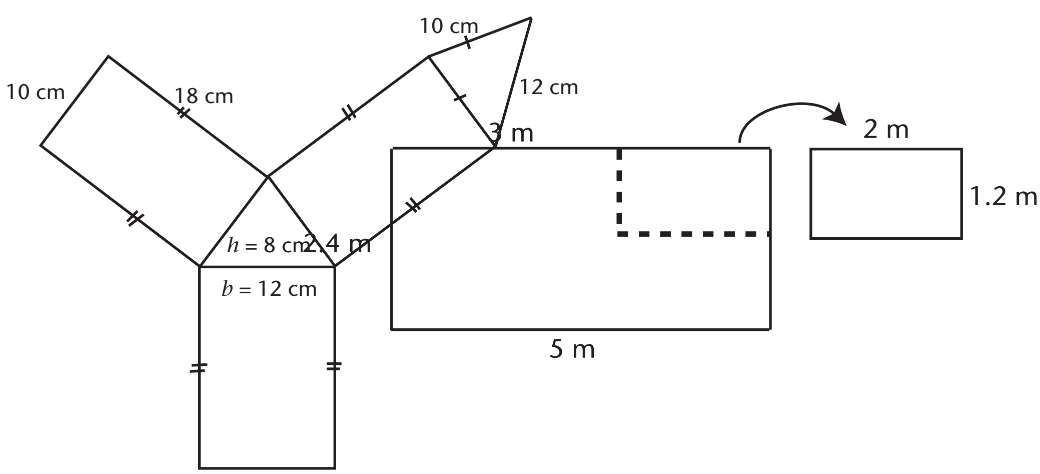
a. l = 6 cm w = 4 cm h = 8 cm (length is typically longer than width)

b. SA = $2lw + 2wh + 2lh$

SA = 2(6)(4) + 2(4)(8) + 2(6)(8)

SA = 208 cm²

3. a. Answers will vary. This one is a possibility:



b. Surface area = 672 cm2

area of 2 triangles = $2 \times (\frac{1}{2} \times 12 \times 8) = 96$ cm^2

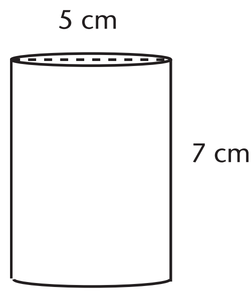
area of largest rectangle = $12 \times 18 = 216$ cm^2

area of 2 other rectangles = $2 \times (10 \times 18) = 360$ cm^2

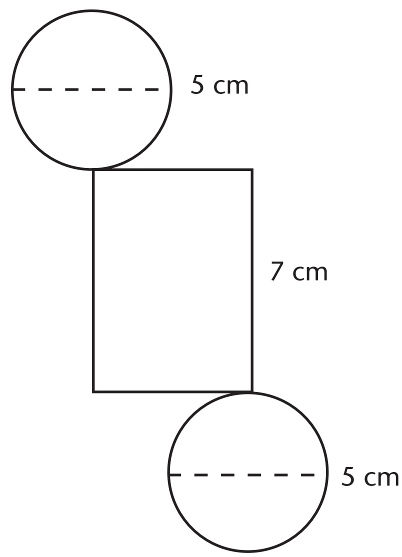
c. $SA = 2(\frac{1}{2}bh) + 2sl + bl$

Try It! Activity 2

1. a.



b.



2. a. The area of the 2 circles

$= 2 \times (\pi \times 2.5 \times 2.5)$

$= 39.3$ cm^2

b. Area of the rectangle in the net

= circumference of the circle × height

$= (\pi \times 5) \times 7$

$= 110$ cm^2

c. $SA = 2\pi r^2 + \pi dh$

$= 2\pi(2.5)^2 + \pi(5)(7)$

$= 149.2$ cm^2

3. a. They are the same except for the last part. One formula has πdh, the other formula has $2\pi rh$

b. Since the diameter of a circle is 2 times the radius ($d = 2r$), then the formulas πdh and $2\pi rh$ are the same.

Lesson B: More About Area in 2-D and 3-D

Warm-up

1. a. a triangle with a circle cut out of it
   b. half a circle
   c. a square with a rectangle added onto it, or a rectangle with a rectangle cut out of it

2. a. BC = 3 cm, CD = 7 cm, DE = 2 cm
   b. AC = 5.5 cm, AB = 2 cm, CD = 9 cm

Try It! Activity 1

1. a. area of the irregular shape

   = (area of the triangle) minus (area of the circle)
   $= 1/2bh \quad - \quad \pi r^2$
   $= 1/2(8)(6) \quad - \quad \pi(2.5)^2$
   $= 24 \quad - \quad 12.6$
   $= 11.4 \text{ cm}^2$

   b. area of the irregular shape
   = area of the rectangle minus area of the half circle
   $= lw \quad - \quad (\pi r^2) \div 2$
   $= (10)(5) \quad - \quad \pi(5)^2 \div 2$
   $= 50 \quad - \quad 39.3$
   $= 10.7 \text{ cm}^2$

2. a. One way to do this is by subtracting the window area from the wall area.

   Area to be painted =
   (area of the wall) – (area of the window)
   $= lw - lw$
   $= (5)(2.4) - (2)(1.2)$
   $= 12 - 2.4$
   $= 9.6 \text{ m}^2$

   b. no

c. two coats of paint = 9.6 + 9.6 = 19.2 m^2.

Two cans of paint covers 8 + 8 = 16 m^2, not enough.

Three cans of paint covers 8 + 8 + 8 = 24 m^2, more than enough.

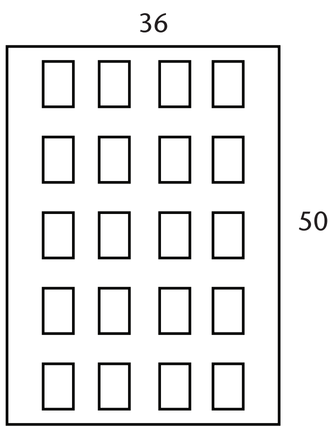
You need three small cans of paint.

d. Answers will vary. If you buy three small cans, it costs $30 and you have 3.2 m^2 of leftover paint. If you buy one large can, it costs $22, and you would have 30.8 m^2 of leftover paint. If the amount of savings is most important, you would buy one large can. If you are concerned about the paint waste, you may choose to buy the 3 small cans.

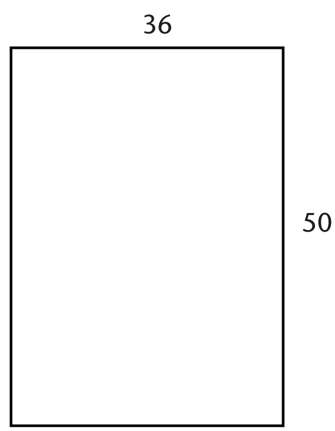
Try It! Activity 2

1. a.

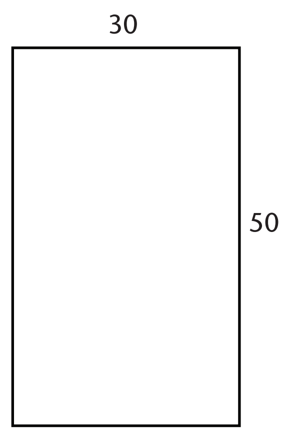
Front View



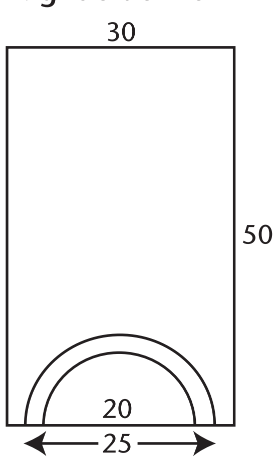
Back View



Left Side View



Right Side View



b. **Front View**

= area of front wall – area of all 20 windows

= 36 m × 50 m – 20 × 24 m^2

= 1800 m^2 – 480 m^2

= 1320 m^2

Back View

= 36 × 50 = 1800 m^2

Left Side View

= 30 × 50 = 1500 m^2

Right Side View

= same as left side view – area of parking garage entrance

= 30 × 50 – (π × 12.5^2) ÷ 2

= 1254.56 m^2

c. 1320 + 1800 + 1500 + 1254.56 = 5874.56 m^2

2. a. Answers will vary. The top view was labelled 25 by 15. Since the view is of a curved surface, the actual dimensions are 39.27 by 15. These dimensions give a larger area, which is why Painter One was wrong.

   b. Area that needs to be painted = 677.41 m^2

   Rectangle = 39.3 × 15 = 589.05 m^2

   Area of larger half circle – area of smaller half circle

   = (π × 12.5^2) ÷ 2 – (π × 10^2) ÷ 2

   = 88.36 m^2

Lesson C: The Amount of Space

Warm-up

1.

b	surface area	a. cm
c	volume	b. m^2
a	length	c. mm^3

2. a. V = *lwh*

   V = (5)(2)(2)

   V = 20 cm^3

b. $V = lwh$
$V = (4)(3)(6)$
$V = 72 \text{ cm}^3$

Try It! Activity 1

1. a. Volume
= (Area of base) × h
= 60 cm^2 × 14 cm
= 840 cm^3

b. Volume
= (Area of base) × h
= 25 cm^2 × 5 cm
= 125 cm^3

c. Volume
= (Area of base) × h
= 28.27 cm^2 × 11 cm
= 311 cm^3

d. Volume
= (Area of base) × h
= 79 cm^2 × 7 cm
= 553 cm^3

2. Answers will vary. The formula for the area of a circle is πr^2. The formula for volume is (Area of base) × h and if Area of base is replaced with πr^2 then the formula becomes πr^2, or $\pi r^2 h$.

3. Answers will vary. The orientation of the boxes does not affect the volume. The volume will be the same either way.

Try It! Activity 2

1. a. Option A: surface area = 112 cm^3
Option B: surface area = 94 cm^3
Option B would have the smallest surface area.

b. Answers will vary. The dimensions should multiply to 60.

2. a. Students choices and reasons will vary.

   b. Option 1 has more volume (251.32 cm^3) Option 2 has a volume of 480 cm^3.

   c. Option 1 has a surface area of 226.2 cm^2. Option 2 has a surface area of 408 cm^2.

   d. Answers will vary. Some possibilities are:

      Option 1 since it can hold the most and will cost the least in materials since the surface area is the smallest.

      Option 2 might pack up easier and cost less to ship. Also, it might be more difficult to cut out a circle than a triangle, so that might add to the cost.

Section 3

Pretest

Lesson A: What is a Tessellation Anyways?

1. a. irregular, all sides not equal

   b. regular, all sides are equal

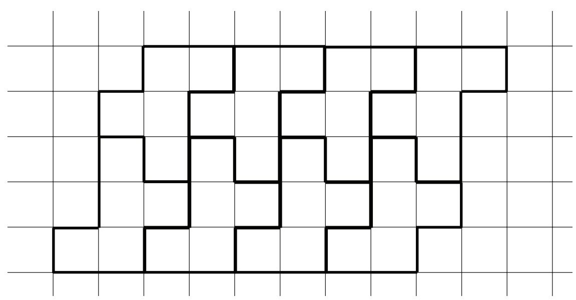
   c. irregular, all sides are not equal

   d. regular, all sides are equal

2. a. yes b. no c. yes

3. Yes. Explanations will vary. Joining all three vertices at a single point adds up to 180°. Creating three more duplicates of this triangle and joining it to the same point will have the vertices add up to 360°.

4.



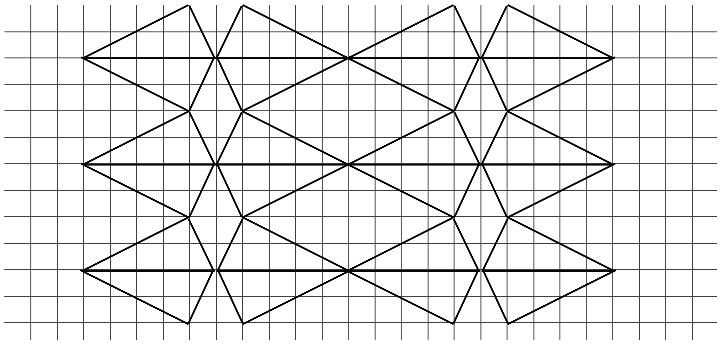
Lesson B: Transformations in Tessellations

5. a. translations

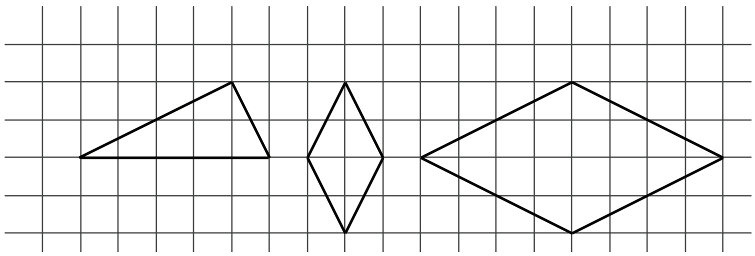
   b. translations, rotations (since shapes are symmetric, could also say reflections)

   c. reflection (or rotation, translation)

6. a.

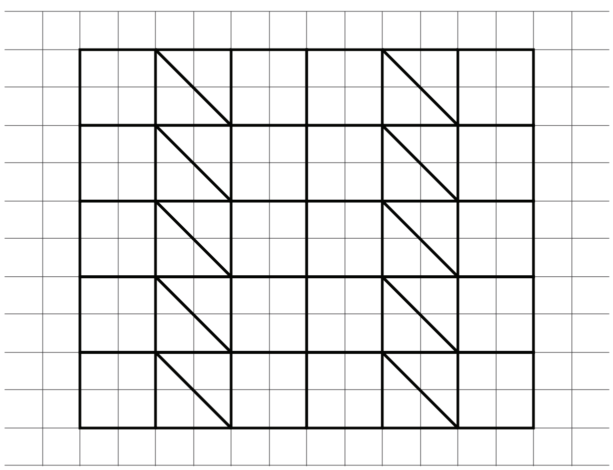


b.



Lesson C: Creating a Tessellation

7.



8. a. yes

b. yes

Lesson A: What is a Tessellation Anyway?

Warm-up

1. a. regular hexagon: has six equal sides and six equal angles
   b. regular octagon: has eight equal sides and eight equal angles

2. a. irregular pentagon: An irregular pentagon has five sides that are not all equal and five angles that are not all equal
   b. An irregular hexagon has eight sides that are not all equal and eight angles that are not all equal

3. A square is a regular polygon since all four sides are equal and all four angles are equal.

Try It! Activity 1

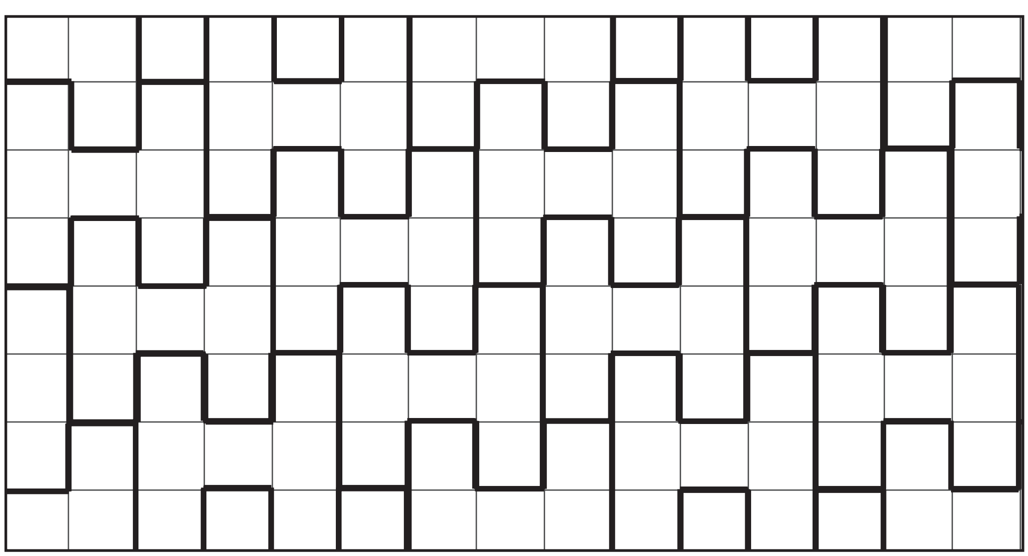
1. equilateral triangles, hexagons

2. Each interior angle in a pentagon is 108º. 108 + 108 + 108 = 324, which is less than 360, so the regular pentagons would leave a gap. 108 times 4 = 432, which is more than 360—so this arrangement would overlap.

3. Yes, by creating six of these polygons and joining the interior angles 50, 50, 60, 60, 70, 70 they will add up to 360º, so there will be no overlap or gaps.

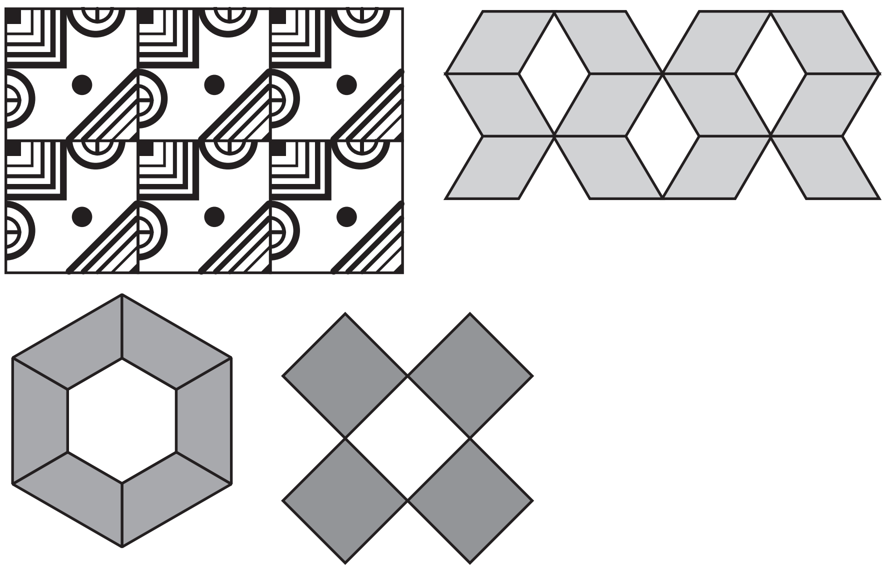
Try It! Activity 2

1.



2. Examples will vary. Regular polygons may include squares, equilateral triangles, or hexagons. Irregular polygons may include rectangles.

   a.



   b. Answers will vary, but each shape identified should have sides that are all equal in length.

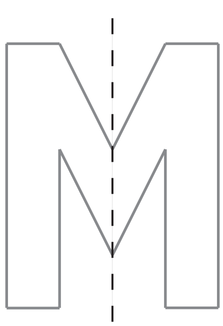
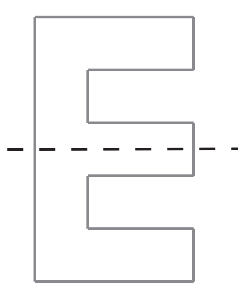
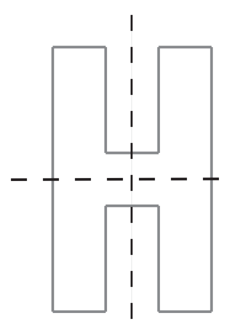
   c. Answers will vary, but each shape identified should have sides that are not all equal in length.

Lesson B: Transformations in Tessellations

Warm-up

1. a. reflection b. translation c. reflection

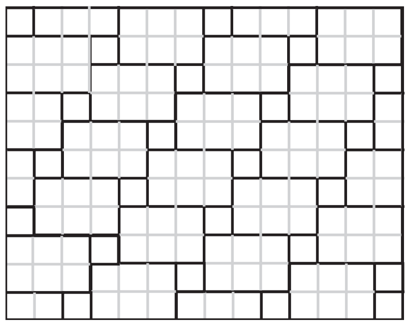
2. a. rotation b. translation c. rotation, reflection

3.
   a.  b.  c. 

Try It! Activity 1

1. a. The shaded design was translated down and to the right.

   b. Answers may vary. The grid should be entirely covered with the rectangle/square pattern. Cut the edges that hang off the grid space.



2. a. Answers will vary. Repeatedly flip the pentagon vertically, then flip the row horizontally.

   b. star, diamond (or quadrilateral)

Try It! Activity 2

1. a. six

   b. regular hexagon

2. a. translation

   b. reflection and translation

   c. rotation and translation

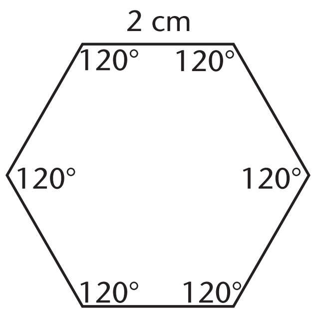
3. Answers will vary but should include references to the fact that this art uses tessellations as well as optical illusions. Escher was also not strong in mathematics in school and yet he was considered a mathematician by many.

Lesson C: Creating a Tessellation

Warm-up

1. b. 90º c. 25º d. 115º

2.



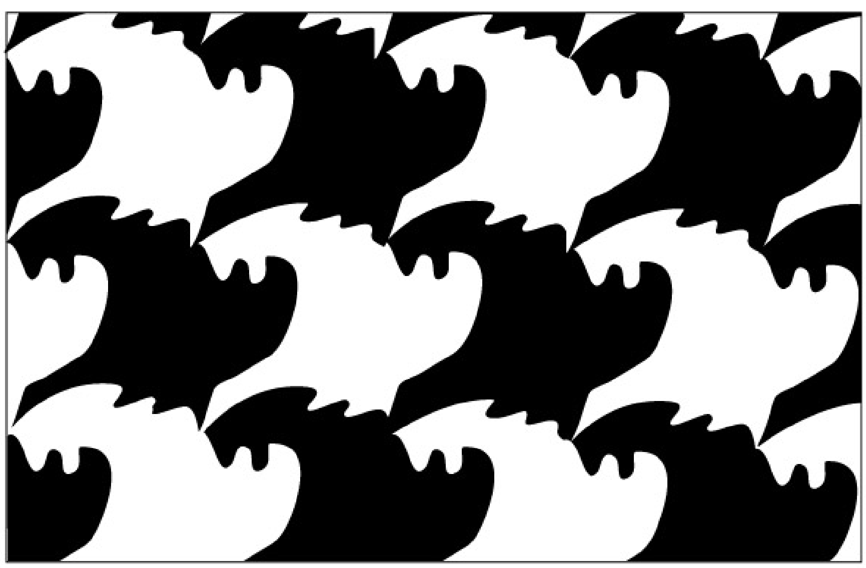
Try It! Activity 1

1. The tessellation will contain regular polygons and should include slides, flips, and turns.

2. The area is still 10 cm² because the area that was cut out was just moved.

Try It! Activity 2

1. Answers should look similar to this:



2. Answers will vary. Shape should have begun as a rectangle, and have a piece on one edge cut out from vertex to another vertex, then slid to the opposite side.

Try It! Activity 3

1. Students art work. Answers will vary.

2. Students art work. Answers will vary.

Glossary

area
Area is the number of square units that fit inside a 2-D shape.

axis (axes)
The axes are the lines that show the number scale on a graph. The x-axis is horizontal, and the y-axis is vertical. Axis is singular and axes is plural.

bar graph
A bar graph is a graph that uses vertical or horizontal rectangular bars to show the quantity being measured. The longer (or higher) the bar, the higher value it represents.

basic operations
Basic operations include addition, subtraction, multiplication, and division.

bias
Bias occurs when a particular outcome is favoured over another.

circle graph/pie chart
A circle graph or pie chart are visual representations of data amounts that together form a total amount or a single quantity.

circumference
Circumference is the distance around a circle.

coefficient
A coefficient is a number that multiplies a variable in a mathematical expression.

For example, in the expression $3x - 7$, the number 3 is a coefficient. In the expression $\frac{x}{5} + 8$, the coefficient is $\frac{1}{5}$.

constant/constant term
A constant or constant term is a number in a mathematical expression that has no variable attached to it. The number can't be changed.

For example, in the expression $3x - 7$, the constant is 7. In the expression $\frac{x}{5} + 8$, the constant is 8.

continuous data

Continuous data is data that is part of a set of numbers that can be infinitely divided into smaller and smaller fractions.

For example, time or distance information can be thought of as continuous because they exist in units smaller than we can measure.

coordinates

Coordinates are a set of numbers that can be used to describe a location of a point on a coordinate plane.

coordinate plane or Cartesian plane

A coordinate plane or Cartesian plane is a rectangular area with one or more axes. The plane is designed to show data in a visual way. It is named after its inventor, Rene Descartes.

congruent

Congruent means "equal to."

cross section

A cross section is a section cut from a prism or a cylinder. The cut is made parallel to the base.

cylinder

A cylinder is a three-dimensional or 3-D shape which has two circular bases that are parallel to each other and the same distance apart.

data

Data are numbers that represent measurements in the real world. Data may represent money, time, distances, or any other amounts.

degrees

Degrees are the measurement of the size of an angle or part of a circle. A full circle is 360 degrees.

denominator

The denominator is the bottom number in a fraction. It represents the total number of equal parts.

For example, in the fraction $\frac{3}{4}$ where 4 is the denominator, an object or group has been divided into 4 equal parts. (See also **numerator**.)

diameter
In a circle, the diameter is a straight line from one edge of the circle to the other, which passes through the centre of the circle.

discrete data
Discrete data is data that is grouped into separate categories, with no information existing between the categories.

equation
An equation is a pair of mathematical expressions that are joined by an equals sign (=), and so they represent the same amounts. An equation is a mathematical "complete sentence".

equilateral triangle
An equilateral triangle is a triangle with three equal sides. In an equilateral triangle, all of the interior angles are 60°.

equivalent
When two things are equivalent they have the same value.

For example $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent expressions

event
An event is a specific outcome from the sample set of all possible outcomes.

For example, drawing a five of hearts from a normal deck of 52 cards is an event.

expression
An expression is a mathematical phrase. An expression is made of terms. Terms are joined by the mathematical operators plus or minus (+ or –) into expressions.

For example, $5x - 7$ is a two-term expression.

extrapolate
To extrapolate means to estimate quantities or data beyond the last amounts measured; to extend a graph line beyond the last data point. (See also **interpolate**.)

favourable outome
A favourable outcome means achieving a desired result in a probability experiment.

fraction
A fraction is a number that represents part of a whole.

For example, $\frac{1}{2}$ represents one part out of a total of two parts.

graph
A graph is a visual representation of data using lines, bars, symbols, or areas.

heptagon
A heptagon is a seven-sided closed figure.

hexagon
A hexagon is a six-sided closed figure.

histogram
A histogram is a vertical bar graph.

hypotenuse
1. the side of a right triangle that is not a leg.
2. the longest side of a right triangle.
3. the side of a right triangle that is opposite the right angle.

icon
An icon is a small symbol that represents a quantity of items for a pictograph or in a graph legend. Usually a picture or line drawing of the item is used as an icon.

improper fraction
An improper fraction is a fraction where the numerator is larger than the denominator.

For example, $\frac{7}{5}$ is an improper fraction.

independent event
In a probability experiment, an independent event is when the outcome of one event does not influence or change the possible outcome of another event.

intercept
The intercept is the location where a line graph intersects an axis.

interior angles
Interior angles are angles that are inside a figure. For polygons, interior angles are at each vertex.

interpolate
To interpolate means to estimate the data amounts between data points that were measured. (See also **extrapolate**.)

interval
An interval is the amount between two values; their difference.

irregular polygon
An irregular polygon is a closed figure where all the sides are not equal and all the angles are not equal.

isosceles triangle
An isosceles triangle is one with two equal sides.

legs
Legs refer to:
1. the sides of a right triangle that form the right angle.
2. the part of the body that the feet are attached to.

line graph
A line graph is a graph using a straight, bent, or curved line to show continuous data.

linear equation/linear relation
A linear equation or linear relation is an equation, table, description or graph that shows the relationship between two variables and forms a straight-line graph.

misinterpret
To misinterpret means to misunderstand or to gain a false impression from a conversation, picture, data or text.

misleading information
Misleading information is information (such as a graph) that is technically correct but would give most viewers an inaccurate impression.

misrepresent
To misrepresent is to present information falsely, visually or in words.

mixed number
A mixed number is a number composed of a whole number and a fraction.

For example, $2\frac{1}{3}$ is a mixed number.

model
1. To model is to create a representation of real-life data.
2. A model is the graph, map, computer program or another item that represents data.

net
A net is a two-dimensional or 2-D construction of a three-dimensional or 3-D object.

numerator
The numerator is the top number in a fraction. It represents the number of equal parts you are working with.

For example, in the fraction $\frac{3}{4}$ where 3 is the numerator, you are working with only 3 of the parts out of 4 total. (See also **denominator**.)

octagon
An octagon is an eight-sided closed figure.

operations
When we do something with a number or numbers, it is called an operation. Addition, subtraction, multiplication, and division are basic operations.

ordered pair
An ordered pair is a pair of numbers (x, y) that represent the values that satisfy a relation and also represent a location on the graph of the relation.

origin
The origin is the point (0,0) on a two-dimensional graph at which the axes intersect.

outcome
The outcome is the result of a single trial or experiment.

pentagon
A pentagon is a five-sided closed figure .

percent
A percent is a fraction of a whole, expressed as a fraction out of 100.

perfect square
A perfect square is a number that represents the area of a square whose sides are whole numbers.

For example, if a square has sides of length 3, its area is 9, and 9 is a perfect square.

It is also the result when a whole number is multiplied by itself.
For example, $5 \times 5 = 25$, and 25 is a perfect square.

perspective
Perspective is the viewer's perception, visually or psychologically.

pictograph
A pictograph is a graph that uses icons or symbols to represent the amount measured in each category, instead of using an axis to show the measurements.

pie chart
See **circle graph**.

plane
A plane is a two-dimensional or 2-D surface.

point
A point is a location on a coordinate plane which can be represented by an ordered pair (x, y).

polygon
A polygon is a closed geometric shape made of 3 or more line segments

prism
A prism has three-dimensional or 3-D shapes that have the same cross section along a length.

proper fraction
A proper fraction is a fraction whose denominator is greater than its numerator.

For example, $\frac{2}{3}$ is a proper fraction.

probability
Probability is the chance or likelihood that a particular event will occur. Probabilities are often listed as ratios (e.g. 1:2 or 2 to 5), fractions (e.g. $\frac{3}{5}$) or percents (e.g. 15%)

proportion
A proportion is a pair of equal ratios.

pythagorean theorem
The Pythagorean Theorem states that $a^2 + b^2 = c^2$

pythagorean triple
A Pythagorean Triple are three whole numbers that satisfy the Pythagorean Theorem.

For example, the numbers 3, 4, and 5 form one Pythagorean Triple. The first two numbers in a Pythagorean Triple are the measurements of the legs, and the third (the largest number) is the measurement of the hypotenuse.

quadrilateral
A quadrilateral is a four-sided closed figure.

radius
In a circle, the radius is the distance from the center to the edge of the circle.

random experiment
A random experiment is a a process leading to at least two outcomes with some uncertainty about which will occur.

rate
A rate is a comparison of two quantities in which each quantity is measured in different units. For example \$8 per dozen roses (or \$8.00/12 roses) is a rate. (See also **unit rate**.)

ratio
A ratio is a comparison of two or more numbers. Ratios are written with a ":" (e.g. 2:3), using words (e.g. 2 to 3), or as a fraction (e.g. $\frac{2}{3}$).

reciprocal
A reciprocal is a number that you multiply a fraction by so that the result equals one. If you start with a whole number, put it over 1 first. The easiest way to find it is to just flip the fraction over. (e.g., The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.)

rectangular prism
A rectangular prism is a six-sided three-dimensional or 3-D shape made up of rectangles.

regular polygon
A regular polygon is a closed figure with all sides equal and all angles equal.

right angle
A right angle is an angle that measures 90°.

right triangle
A right triangle has one right angle.

round/round off
To round or round off is to remove unwanted place values at the right end of a number, adjusting the first remaining place value if necessary. (See also **truncate**.)

sample space
A sample space includes all the possible outcomes resulting from a probability experiment.

satisfy
To satisfy means to replace variables with values that make an equation into a true statement.

For example, $y = 3x$ can be satisfied with the ordered pair (2, 6), but cannot be satisfied with (4, 9).

square root
The square root symbol tells us to take the square root of the number that's inside.

For example, $5^2 = 25$. The square root of 25 is 5.

square root symbol
√ This symbol tells us to take the square root of the number that's inside.

For example: $\sqrt{4} = 2$

surface area
Surface area refers to the total area of the net of a three-dimensional or 3-D object. The units are squared, for example, cm^2, m^2.

term
A term is an item in an expression that is a constant, or variable, or coefficient-and-variable combination. (See also **expression**.)

tessellation
Tessellation is a tiling pattern that covers an entire plane without overlapping or leaving gaps.

three-dimensional (3-D)
Three-dimensional refers to an object that has length, width and depth, or a representation of an object that has the appearance of depth.

triangular prism
A triangular prism is a five-sided three-dimensional or 3-D shape with two triangles that are parallel and equal to each other and joined by rectangles.

truncate
To truncate means to remove unwanted place values at the right end of a number without adjusting the remaining place value. (See also **round/round off.**)

two-dimensional (2-D)
Two-dimensional refers to an object that has length and width, but no depth.

unit rate
A unit rate is a rate where the second term is 1.

For example, wages are often given as a unit rate.

$10.00/hr represents $10.00 earned for every 1 hour worked.

unknown
An unknown is the value(s) that provide the solution to an equation. (See also **variable.**)

variable
A variable is a value that is unknown or that could change. It is often represented in an expression by a letter such as x, but could be represented by a word or other symbol. (See also **unknown**.)

vertex (vertices)
In a closed figure, the vertex refers to the point where two sides meet. Vertex is singular and vertices is plural.

view
The view refers to a two-dimensional or 2-D drawing of a three-dimensional or 3-D object from one particular position—front view, side view, top view, bottom view, etc.)

volume
The volume is the amount of space an object takes up. The units are cubed, for example, cm^3, m^3.

x-axis
The x-axis is the horizontal axis of a coordinate plane. (See also **coordinate plane** and **axis**.)

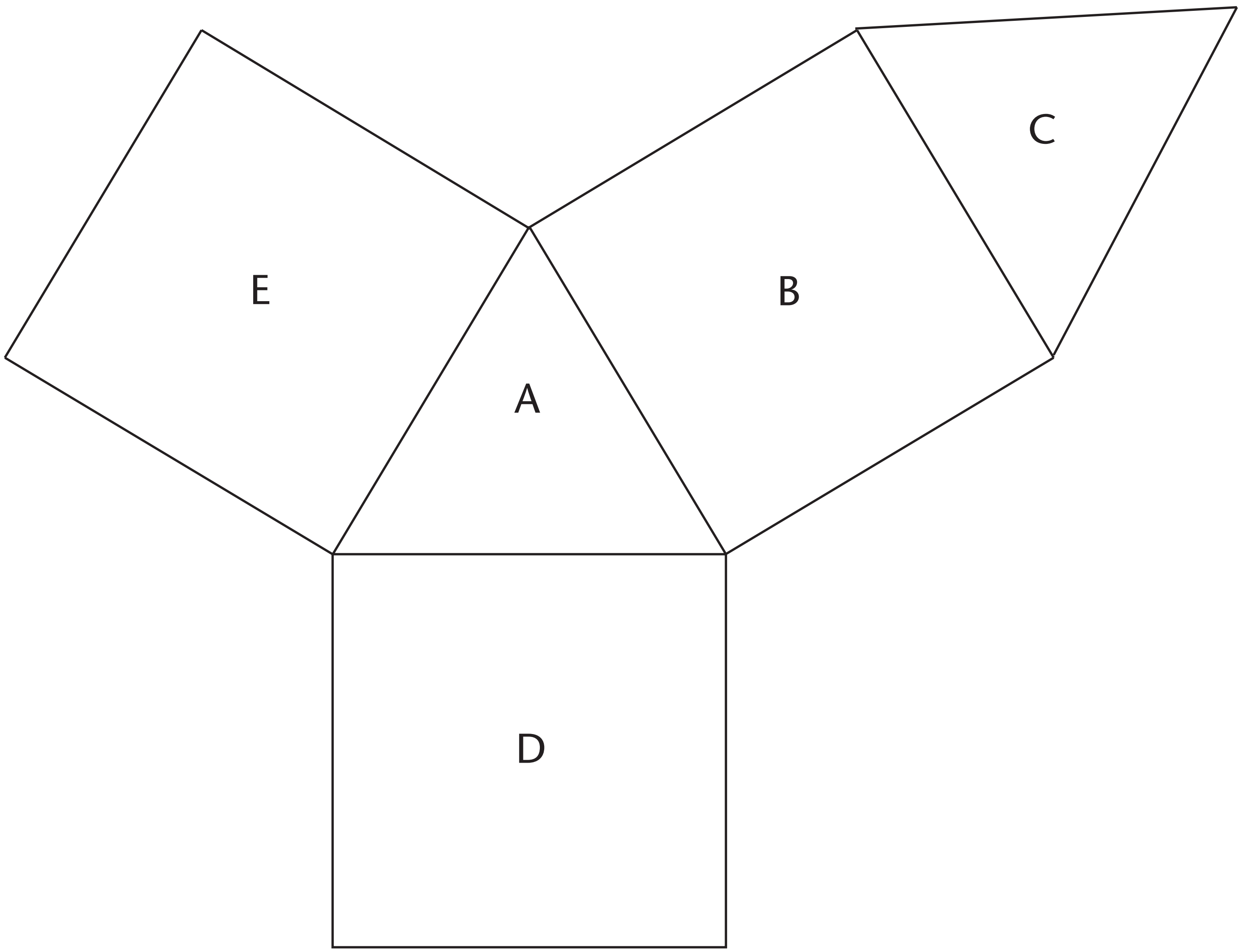
y-axis
The y-axis is the vertical axis of a coordinate. See also **coordinate plane** and **axis**.)

Templates

Section 1

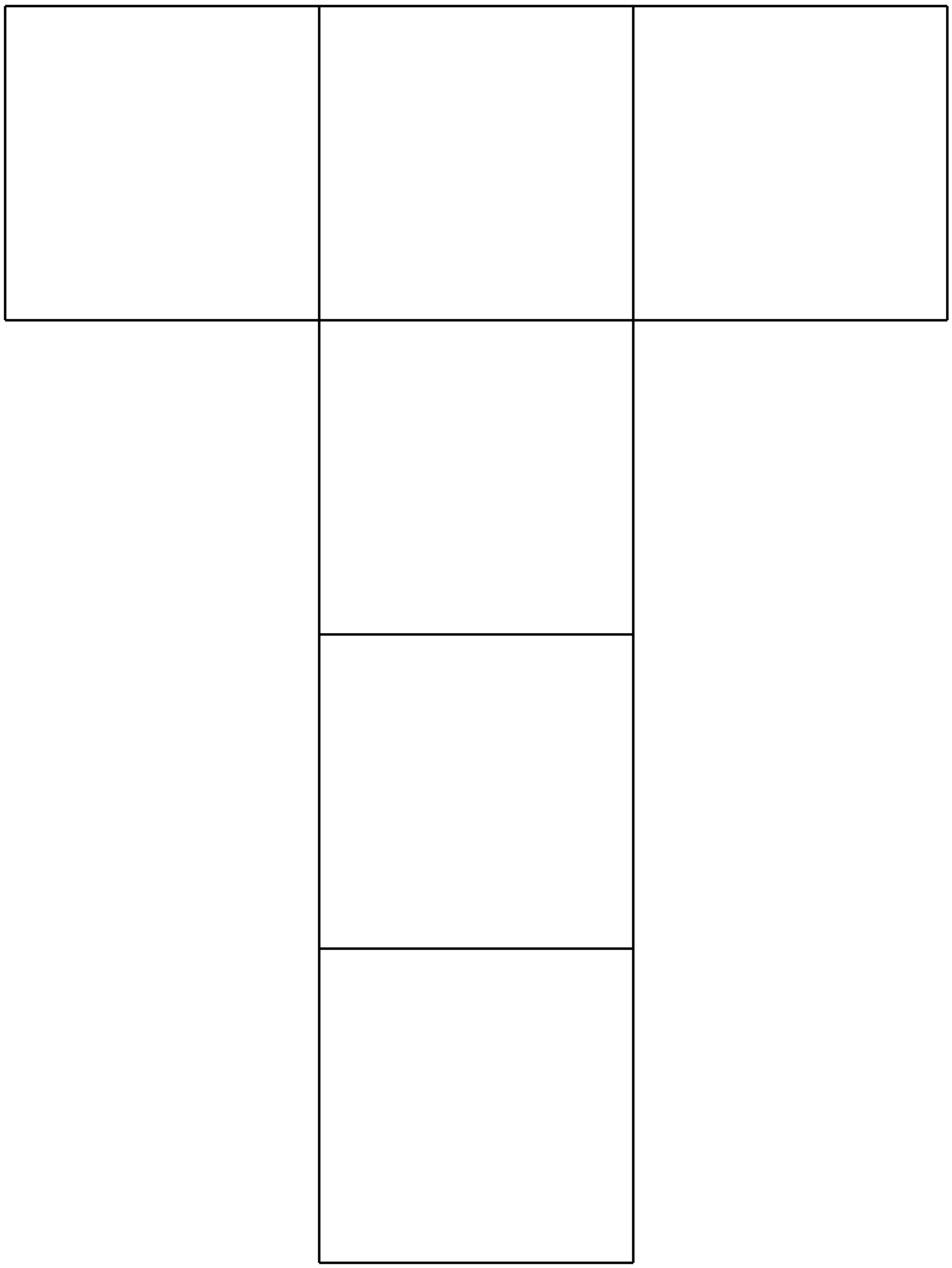
Lesson A: Visualizing in Three Dimensions

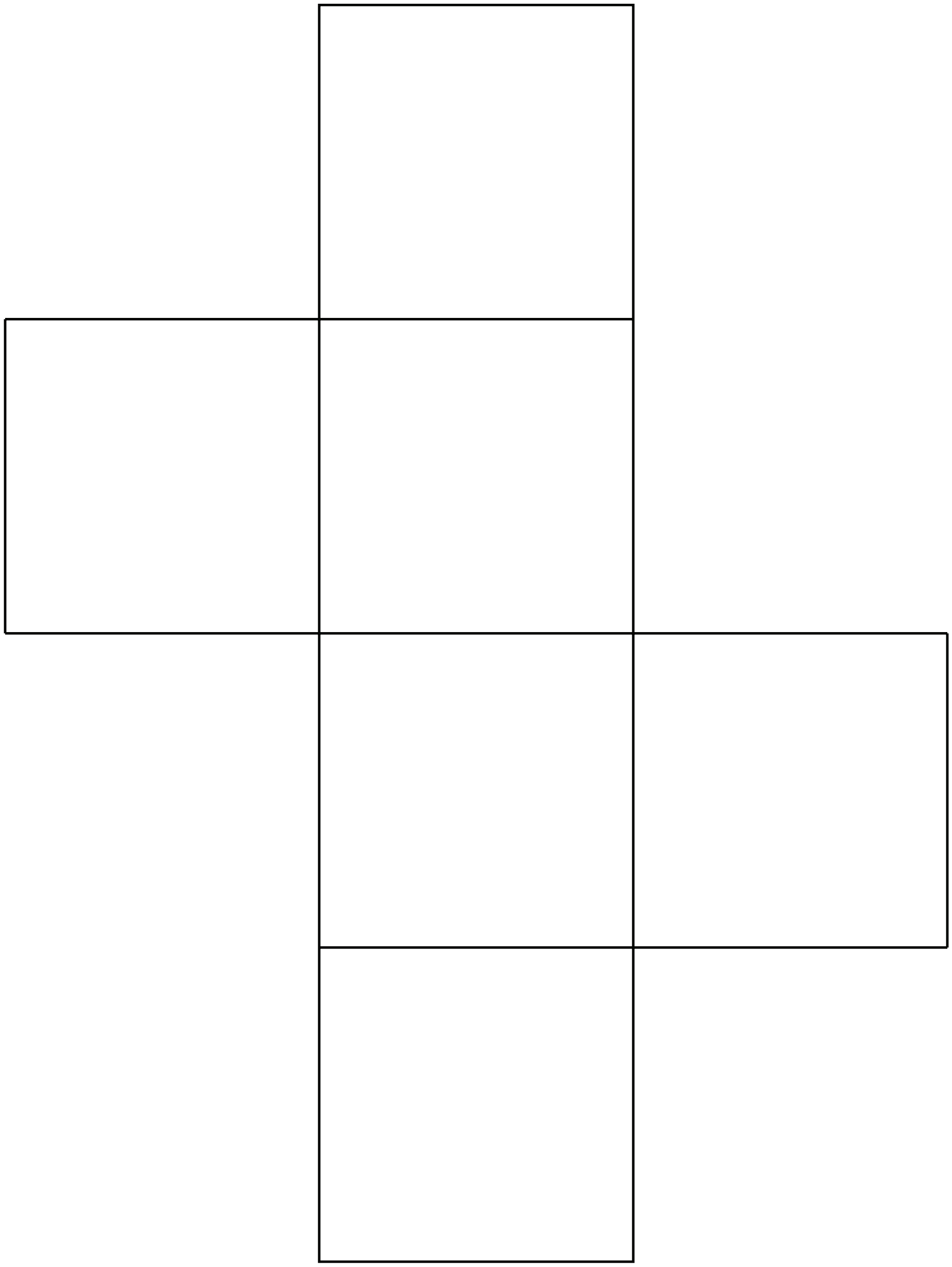
Try It! Activity 2

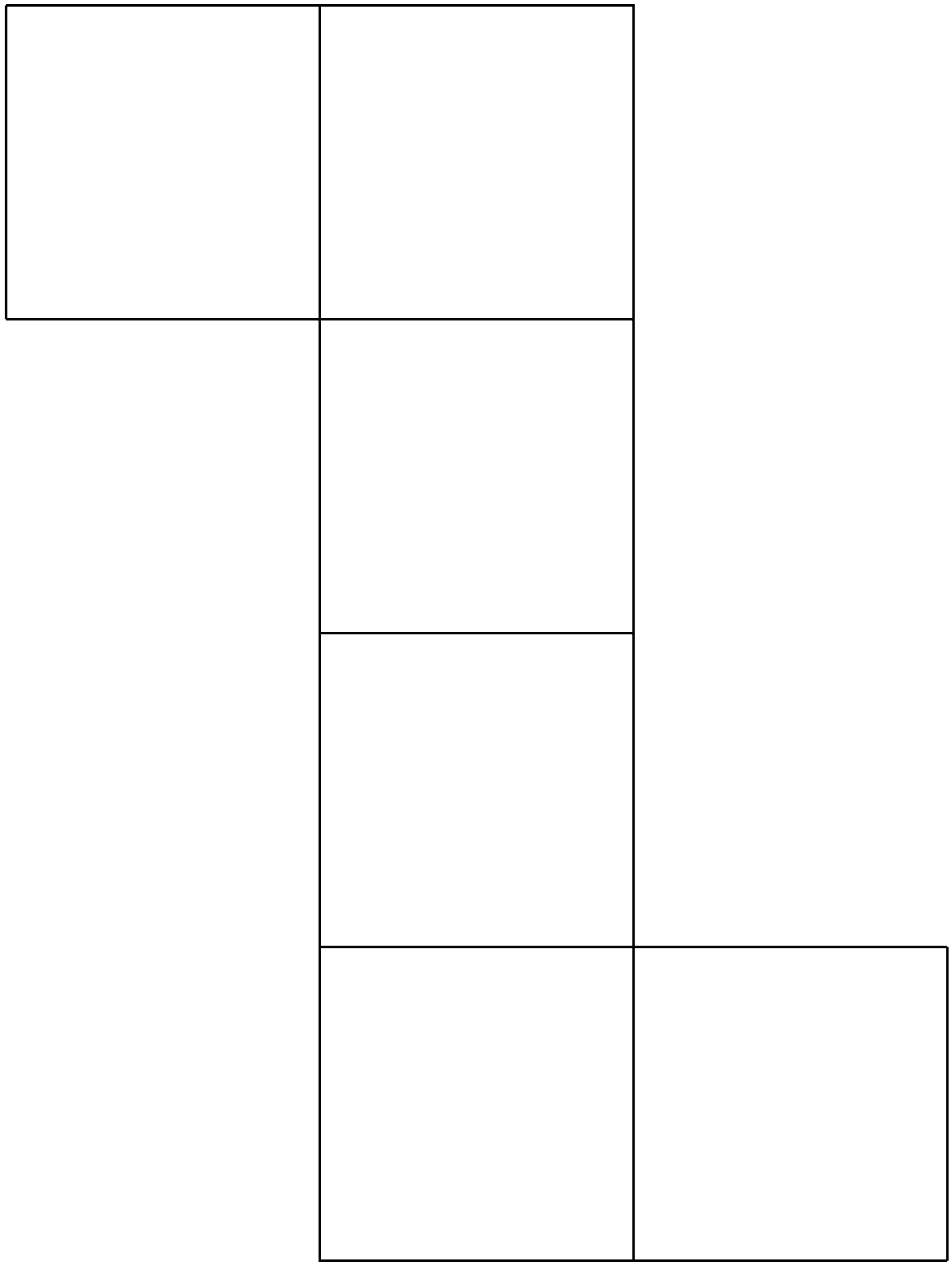


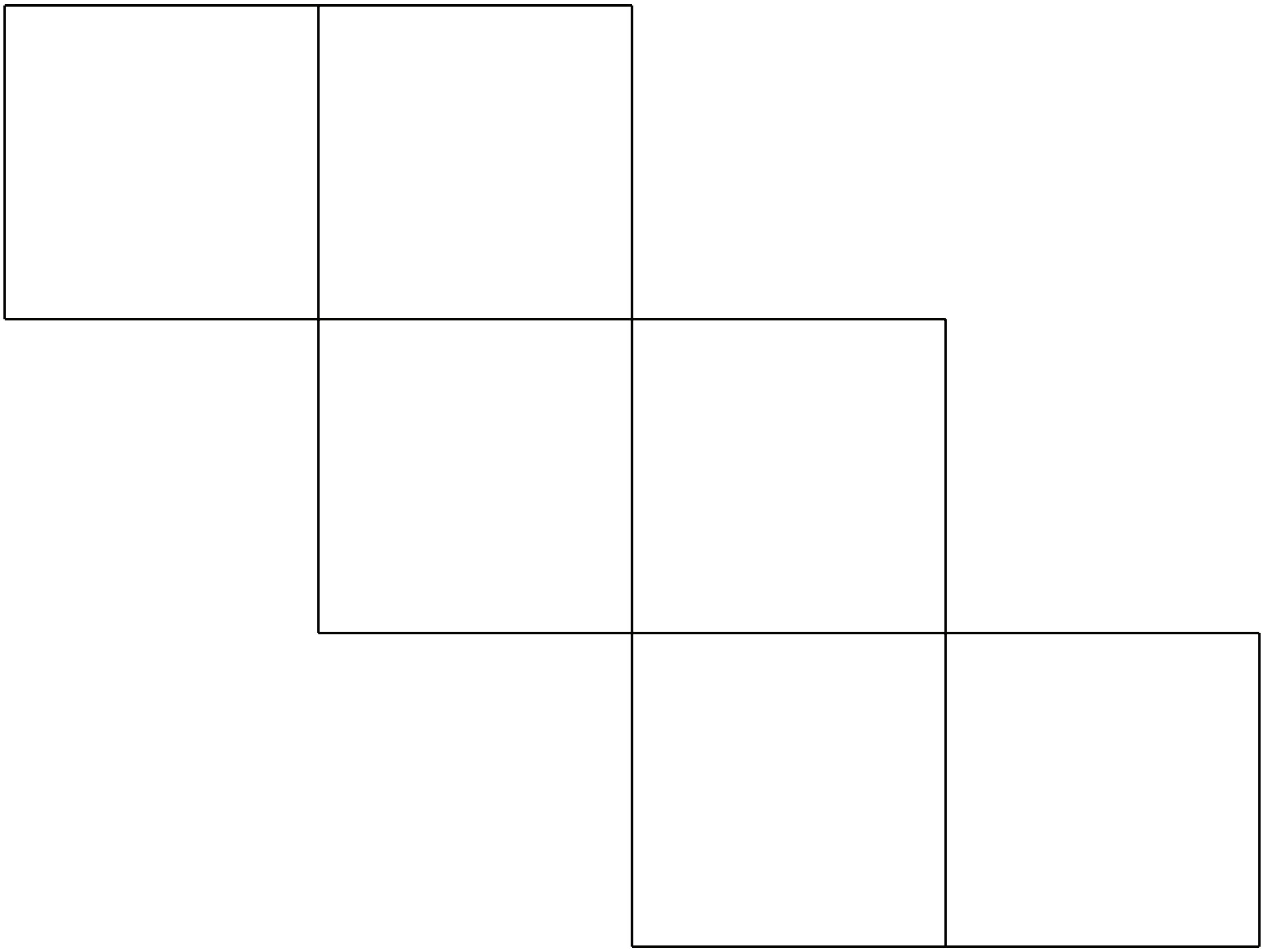
Lesson B: Prisms and Cylinders and Their Nets

Explore Nets and Rectangular Prisms



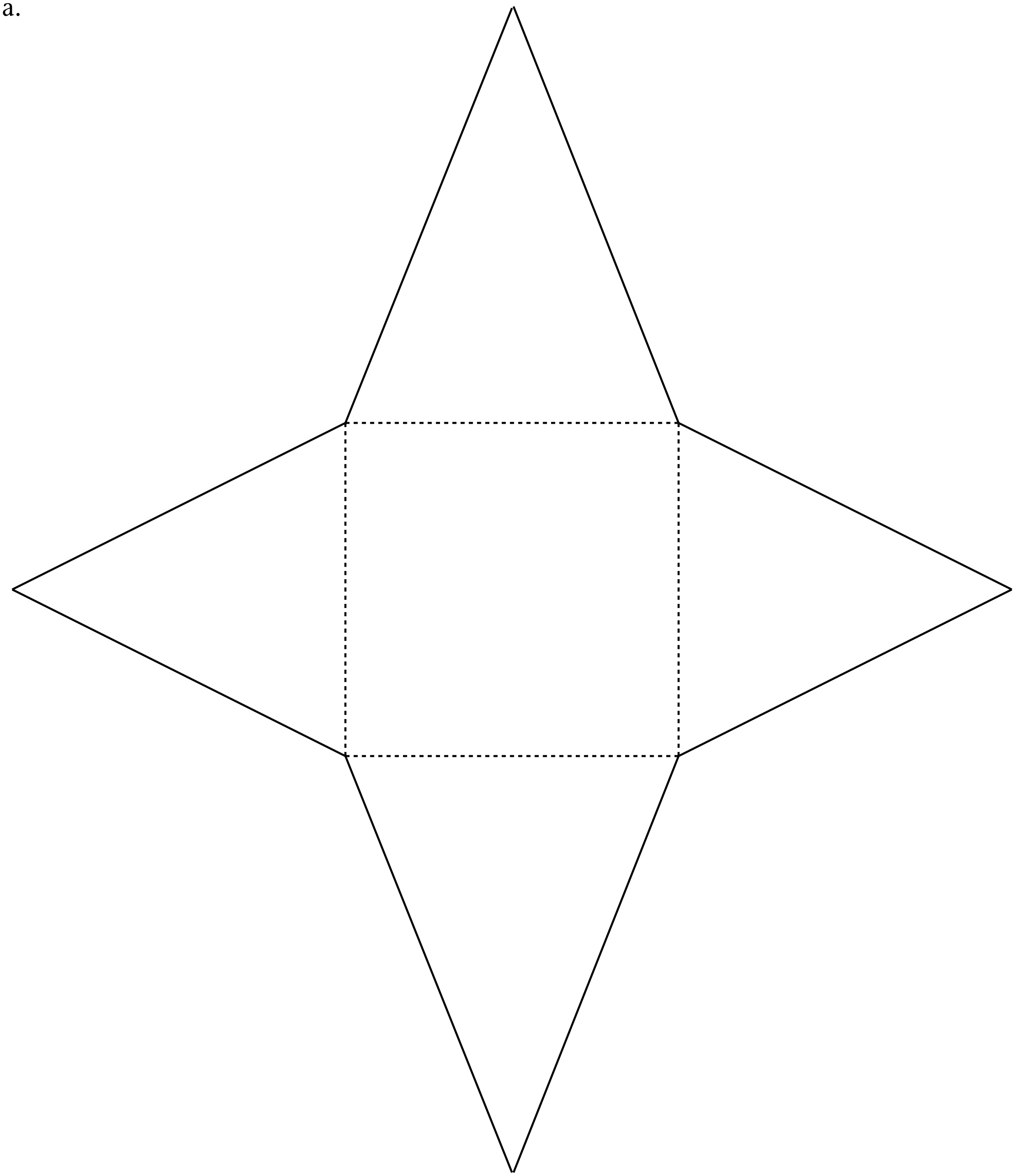






Try It! Activity 1

a.



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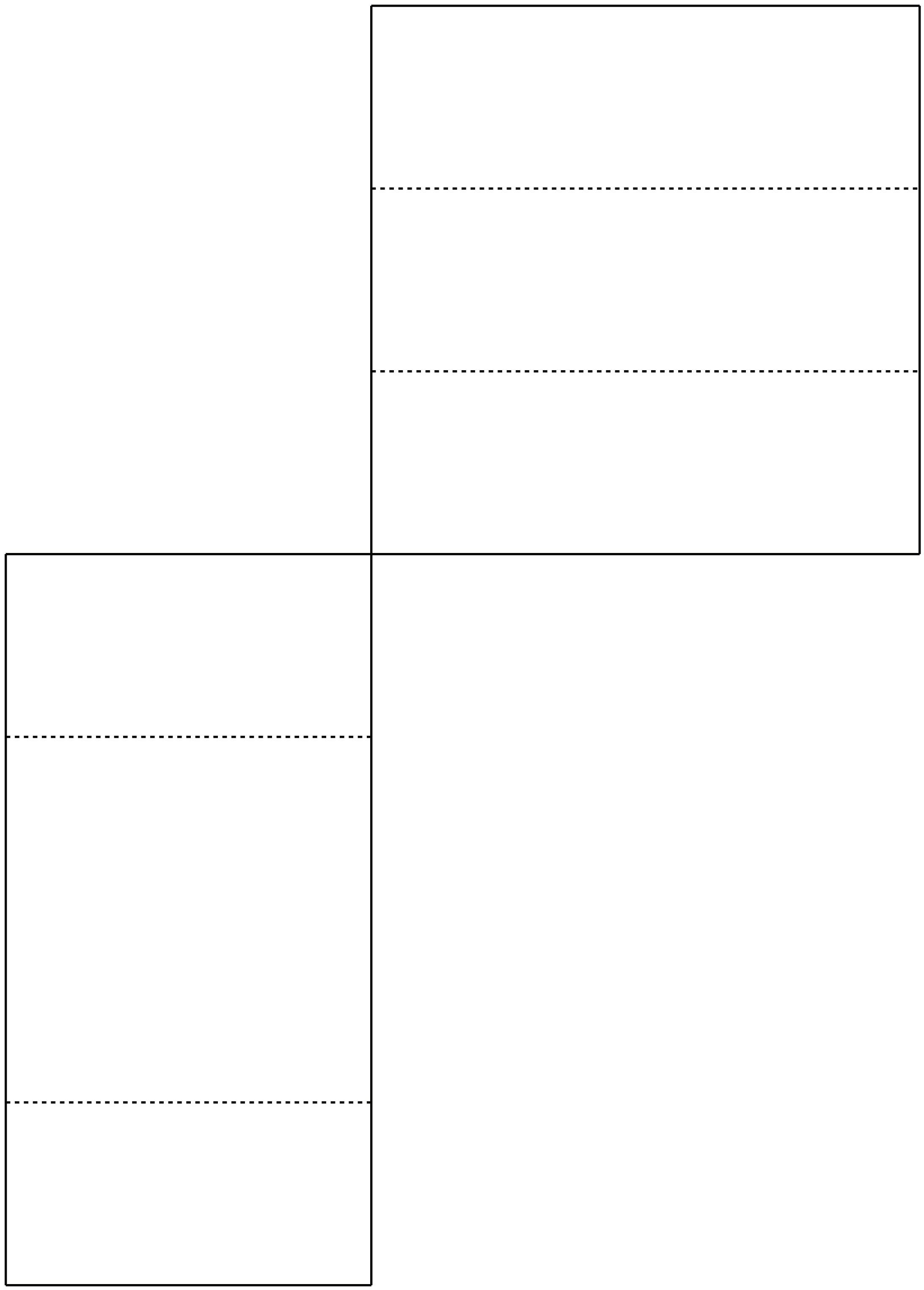

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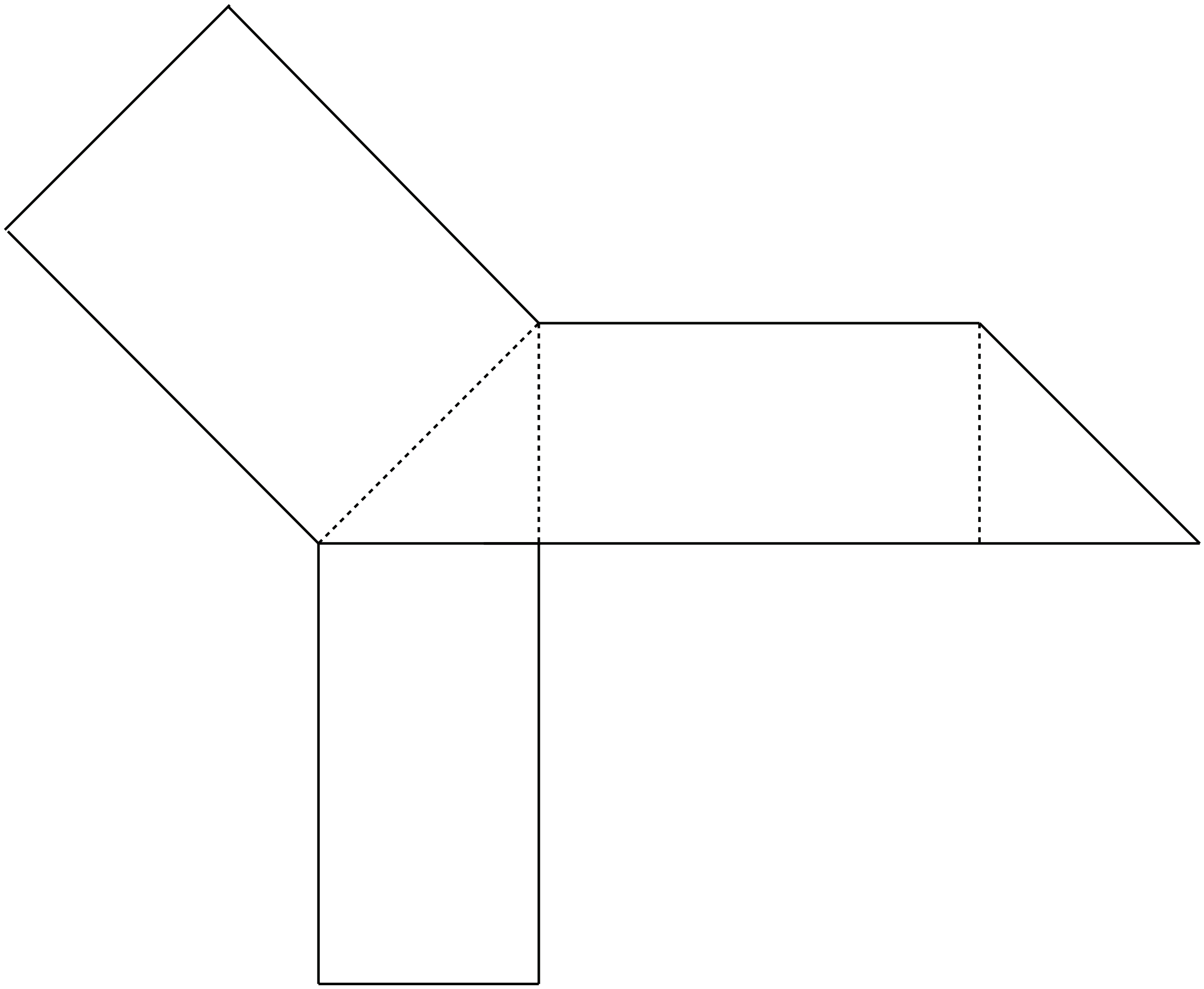
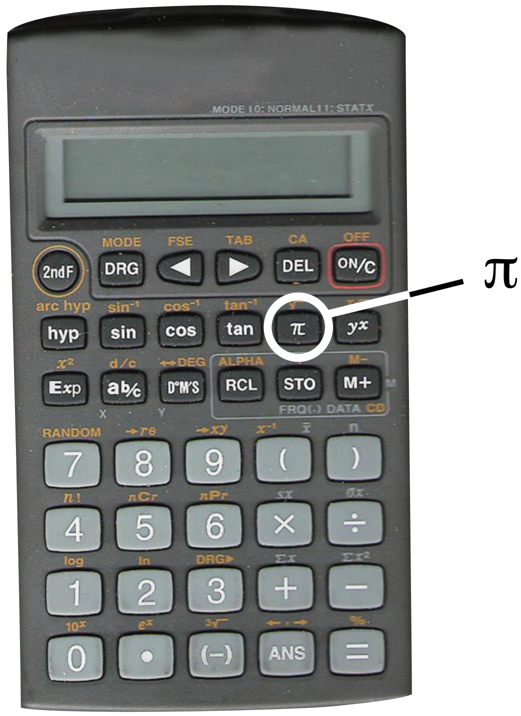


Section 2

Pretest



Section 3

Lesson B: Transformations in Tessellations