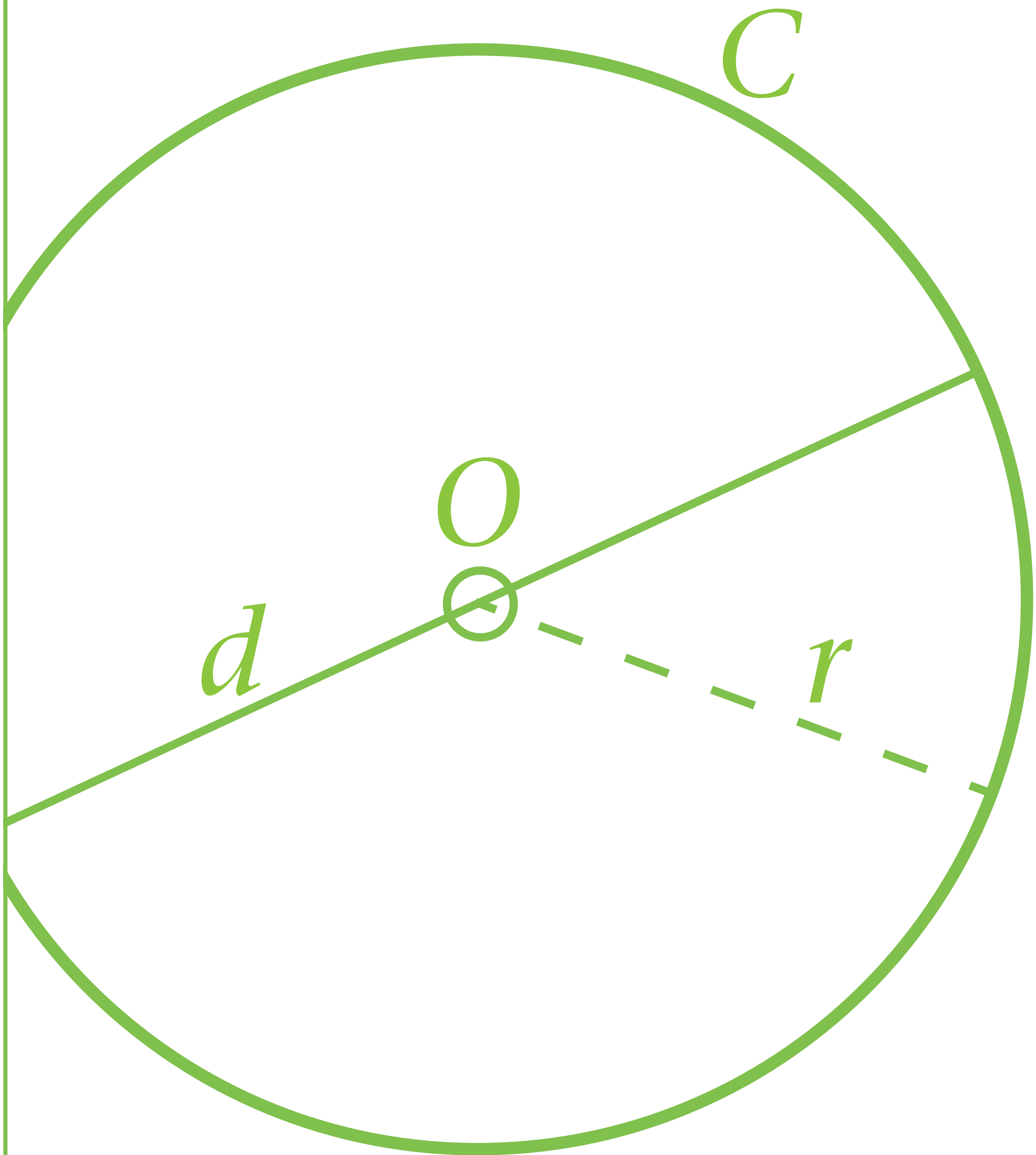
LITERACY FOUNDATIONS MATH



LEVEL 5
GEOMETRY

To the Student

This resource covers topics from the British Columbia Ministry of Education's Literacy Foundations Math Level 5. You may find this resource useful if you're a Literacy Foundations Math student, or a K-12 student in grades 7 – 9.

We have provided learning material, exercises, and answers for the exercises, which are located at the back of each set of related lessons. We hope you find it helpful.

Literacy Foundations Math Prescribed Learning Outcomes

The Literacy Foundations Math Prescribed Learning Outcomes (PLOs) are grouped into four areas: Number (A), Patterns and Relations (B), Shape and Space (C), and Statistics and Probability (D). For a complete list of the PLOs in Level 5, search for Literacy Foundations Math curriculum on the BC Ministry of Education's website.

PLOs Represented in This Resource

The PLOs represented in this Level 5 resource are as follows:

Number

All topics, A1 – A12

Patterns and Relations

All topics, B1 – B6

Shape and Space

All topics, C1 – C3

Statistics and Probability

D2

PLOs Not Represented in This Resource

The PLOs for which no material is included in this resource are as follows:

Statistics and Probability

There is no material for D1, line graphs from data sets.

Acknowledgements and Copyright

Project Manager: Christina Teskey
Writer: Angela Voll
Production Technician: Beverly Carstensen
Cover Design: Christine Ramkeesoon

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Lesson 1

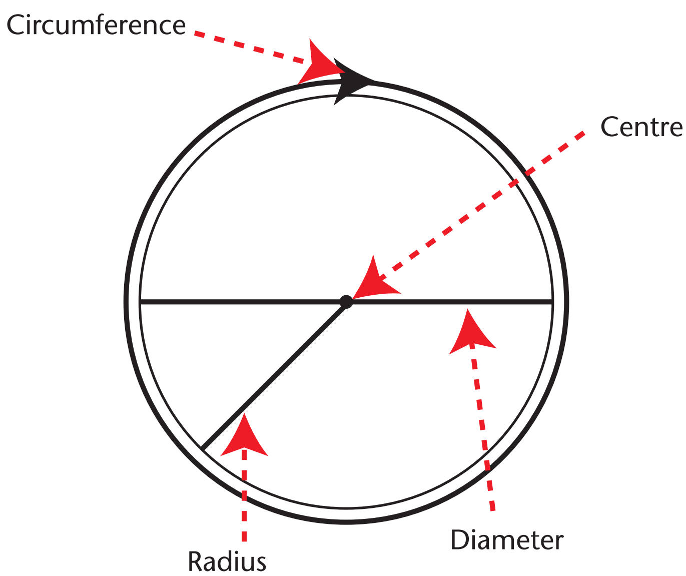
Circles

Learning Outcomes

By the end of this section you will be better able to:

- calculate the area of a circle
- identify parts of a circle using appropriate vocabulary
- calculate radius given diameter, and calculate diameter given radius
- calculate the length of the circumference given either radius or diameter

Before beginning this lesson, review some of the vocabulary you will need to study circles.



Circumference refers to the outside edge of the circle. The length of the circumference, the distance around the outside of the circle, is also called the circumference.

The **centre** of the circle is the same distance from every place on the circumference.

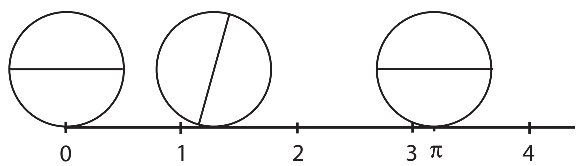
Radius is the distance from the centre of the circle to the circumference. **Radii** is the plural of radius.

Diameter is the distance from one side of the circle, through the centre, to the other side of the circle. The diameter is the same length as two radii.

Circumference

The number line in this diagram has been marked to measure distance by counting the diameters of the circle. The distance between 0 and 1 is the same as the diameter of the circle. The distance between 0 and 4 is four times as long as the diameter of the circle.

We want to measure the circumference of the circle. The circle rolls along the number line, and stops after rolling around exactly once. Every circle, big or small, stops at the same place. The circumference of every circle is a little more then three times as long as its diameter. Ancient mathematicians gave this place on the number line a name, pi π.



It's clear that π is a little bigger than 3, and definitely less than 4. But how big is it exactly? This is an interesting question, and maybe you'll explore it in other math courses. In this course, we will follow the example of physicists, architects, and engineers for the last 3000 years:

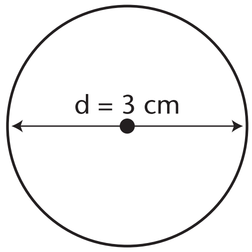
$$\pi \approx 3.14$$

To find the circumference of a circle, multiply its diameter by π.

$$\boxed{\mathbf{C = \pi \times d}}$$

Example:

Find the circumference of a circle with a diameter of 3.0 cm.



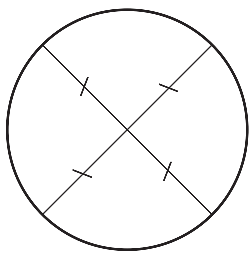
Step 1: Write down the formula, and include what you know.
$C = \pi \times d$
$C = \pi \times 3.0$

Step 2: You can press the π button on your calculator and multiply by 3, or
You can multiply $3.14 \times 3 = 9.42$

Step 3: Write down the circumference to one decimal place (because the diameter is given to one decimal place) and include units.
$C = 9.4$ cm

Calculate the Circumference of a Circle given the Radius

The radius is the length of a line segment from the centre of the circle to the circumference of a circle. All radii within a circle are equal.



If the diameter of a circle is 4 cm, what is the radius? Look at the diagram and definitions at the beginning of this lesson. The diameter is the same as two radii. That means that the radius must be half of the diameter.

Radius = diameter ÷ 2

Sometimes you know the radius of a circle and you want to find the circumference.

Start with the circumference formula that you already know	$C = \pi d$
Remember that d, for diameter, is the same as two radii. Take d out of the formula and write $2 \times r$ instead	$C = \pi (2 \times r)$

That's it. We could leave it just as it is, but it looks a little awkward. Multiplication doesn't care about order, so we can re-arrange the symbols any way we like. Usually, we write numbers first, then special constants (like pi), and finally the variable.

The formula is written as:

$$\boxed{C = 2\pi r}$$

Example:

Find the circumference of a circle with a radius of 4.0 cm.

Step 1: Write down the formula and include what you know.

$C = 2 \times \pi \times r$

$C = 2 \times \pi \times 4$

Step 2: You can press the π button on your calculator and multiply by 2 and then multiply by 4, or
You can multiply $2 \times 3.14 \times 4 = 25.1$

Step 3: Write down the circumference to one decimal place (because the diameter is given to one decimal place) and include units.

$C = 25.1$ cm

The work on your page should look like this:

$C = 2\pi r$

$C = 2 \times \pi \times 4$

$C = 25.1$ cm

Calculate the Diameter given the Circumference of a Circle

If you know what the circumference of a circle is, can you figure out what the diameter will be?

Before using the formula given below, look back at the lesson on ratios. Use ratios (and your calculator) to find the diameter of a circle with a circumference of 9 cm. The ratio of diameter to circumference is 1:π.

To find the diameter, you divide the circumference by π.

$$\boxed{\mathbf{d} = \frac{\mathbf{C}}{\pi}}$$

Example:

Find the diameter of a circle with a circumference of 9.0 cm.

Step 1: Write down the formula and include what you know.

$$d = \frac{C}{\pi}$$

$$d = \frac{9.0}{\pi}$$

Step 2: You can punch 9.0 into your calculator, then push the divide button followed by the π button on your calculator, or

You can divide 9 by 3.14 = 2.866.

Step 3: Write down the diameter to one decimal place (because the circumference is given to one decimal place) and include units.

d = 2.9 cm

The work on your page should look like this:

$$d = \frac{C}{\pi}$$

$$d = \frac{9.0}{\pi}$$

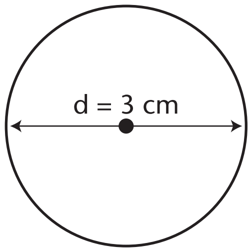
$$d = 2.9\text{ cm}$$

Exercises 1.1

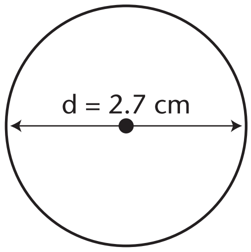
Round your answers to the nearest tenth.

1. Find the circumference of each circle.

   a.



   b.



2. Determine the diameter of each circle with a given radius:

   a. r = 3.0 cm

   b. r = 4.6 m

   c. r = 1.9 mm

3. Determine the radius of each circle with a given diameter:

   a. d = 4.0 m

   b. d = 12.4 cm

   c. d = 9.2 mm

4. Determine the circumference of a circle with each diameter:

   a. d = 10.0 cm

   b. d = 6.7 m

5. Determine the circumference of a circle with each radius:

   a. r = 5.0 cm

   b. r = 2.3 m

   c. r = 7.0 mm

6. What is the radius of a circle with a circumference of 31.4 cm?

7. What is the diameter of a circle with a circumference of 24.0 cm?

8. Fill in the following table:

RADIUS	DIAMETER	CIRCUMFERENCE
5.0 cm		
		28.3 cm
	8.0 m	
	4.8 mm	
1.6 cm		10.0 cm
	7.2 m	

Turn to the Answer Key at the end of the module to check your work.

Area

Area is the amount of surface within a shape. You know that the area inside a triangle is base × height ÷ 2. If a triangle has a base of 3 cm and a height of 10 cm, you know that the area is 3 × 10 ÷ 2, which equals 15 cm^2.

You also know that the area inside a parallelogram is base × height. If the base of a parallelogram is 6.2 cm and the height is 4.9 cm, then the area is 6.2 cm × 4.9 cm. The area is 30.4 cm^2.

A circle is round and doesn't have a base or height like the triangle or parallelogram. How can you determine the area of a circle without these measurements?

Try these ways to estimate the area of a circle:

Method 1. You can draw a circle on graph paper and count the squares.

Step 1: Draw a line segment 3 units long on graph paper. This will be the radius of your circle.

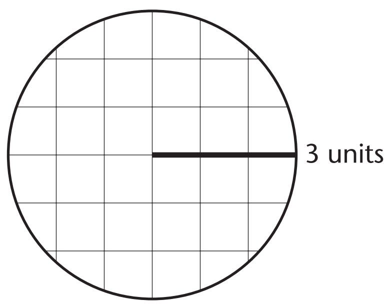
Step 2: Place your compass point on the beginning point of the line segment and your pencil on the end point of the line segment.

Step 3: Draw a circle with your compass.

Step 4: Count the whole squares inside the circle.

Step 5: Combine parts of squares to equal whole squares and add that number to your whole square count.

Step 6: The total number of squares inside the circle is the area.



Did you count approximately 28 square units inside the circle?

Method 2. Estimate the area by dividing a circle into triangles.

Follow the steps below on a separate piece of paper.

Step 1: Draw a line segment 3 units long on graph paper. This will be the radius of your circle.

Step 2: Place your compass point on the beginning point of the line segment and your pencil on the end point of the line segment.

Step 3: Draw a circle with your compass.

Step 4: Use your scissors to cut out the circle.

Step 5: Fold your circle in half, then again in half and again in half.

Step 6: Unfold the circle to find 8 triangular segments.

Step 7: Cut along the fold lines to get 8 triangles.

Step 8: Arrange the triangles on graph paper so that it makes a parallelogram shape.

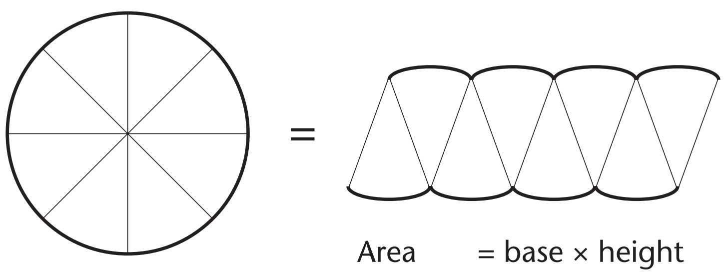
Step 9: Use your ruler and pencil to outline the base and height of the parallelogram.

Step 10: Count the squares to measure the base and height.

Step 11: Estimate the area by multiplying the base x height.

Step 12: Write down your answer in units.

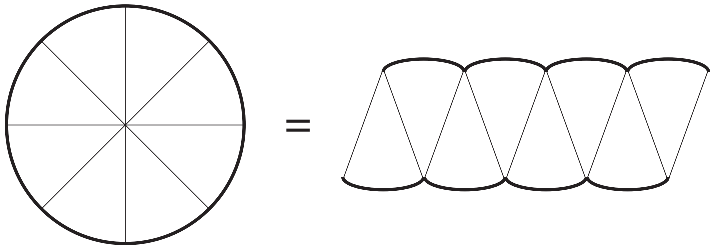
Example:



Area = base × height
= 9.25 squares × 3 squares
= 28 square units
= 28 units^2

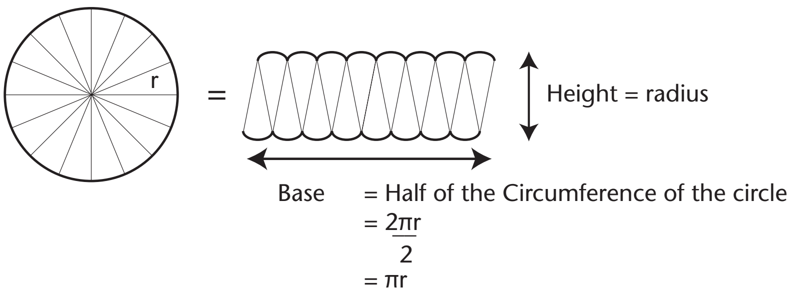
Using a Formula to Find the Area of a Circle

You know how to estimate the area of a circle by dividing a circle into triangles to make a parallelogram. By measuring the height and width of the parallelogram, you can determine the area. Look at the parallelogram below. How is the radius of the circle similar to the height of the parallelogram?



The height of the parallelogram is the same as the radius of the circle. The base of the parallelogram is half of the circumference of the circle.

If the circle is cut into more pieces, the base of the parallelogram gets straighter. In the previous diagram the circle was cut into eight pieces. In the following diagram, the circle is cut into sixteen pieces. If we kept going, and cut the circle into thirty-two or sixty-four pieces, the base of the parallelogram would be almost perfectly straight.



Area of a circle = area of our pizza-slice parallelogram

$= \text{base} \times \text{height}$

$= \dfrac{1}{2}$ of circumference × radius

$= \dfrac{1}{2}(2\pi r) \times r$

$= \pi r \times r$

$= \pi r^2$

Example 1:

A circle has a radius of 5.0 cm. Find the area of the circle.

Step 1: Use the formula to find area.
Area = $\pi \times r^2$
Area = 3.14×5^2
Area = $3.14 \times 5 \times 5$
Area = $78.5\ \text{cm}^2$

Step 2: Write down the area with units.
The area of the circle is $78.5\ \text{cm}^2$.

Example 2:

A circle has a diameter of 8.0 cm. Find the area of a circle.

Step 1: Find the radius of the circle
Radius = diameter ÷ 2
Radius = 8 ÷ 2 = 4

Step 2: Use the formula to find area.
Area = $\pi \times r^2$
Area = 3.14×4^2
Area = $3.14 \times 4 \times 4$
Area = $50.2\ \text{cm}^2$

Step 3: Write down the area including units.
The area of the circle is $50.2\ \text{cm}^2$.

Exercises 1.2

1. What is the area of each circle?

    a. r = 2.0 cm

    b. r = 2.3 m

2. What is the area of each circle?

    a. d = 8.0 m

    b. d = 3.8 cm

3. A pumpkin pie has a diameter of 28 cm.

    a. What is the radius of the pie?

b. What is the area of the pumpkin pie?

c. The pumpkin pie is cut into 4 pieces. What is the area of one piece of pie?

4. Hayden is painting a circular mural on the side of a building. He will need to paint two coats of paint. The radius of the mural is 1.5 m. How much area in total will he need to paint?

Turn to the Answer Key at the end of the module to check your work.

Lesson 2

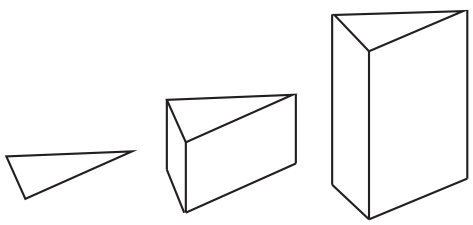
Volume of Prisms and Cylinders

Learning Outcomes

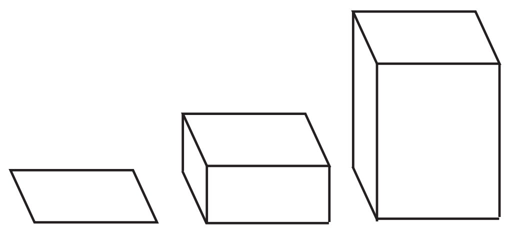
By the end of this section you will be better able to:

- calculate the volume of right prisms and cylinders

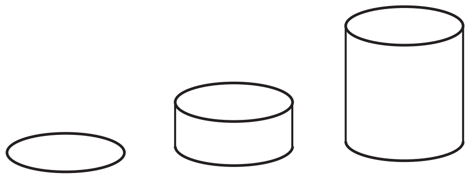
Imagine starting with a shape drawn on a piece of paper and lifting that shape up out of the page. The object that results is called a prism. A prism with a circular base is a cylinder.



The triangle on the left is the base for both of these **triangular prisms**.



The rectangle on the left is the base for these **rectangular prisms**.



The circle is the base for both of these **cylinders**.

Prisms can be formed with a base of any shape.

The top and bottom of a prism are the same shape and parallel to each other. In fact, every parallel slice through a prism or cylinder is the same shape.

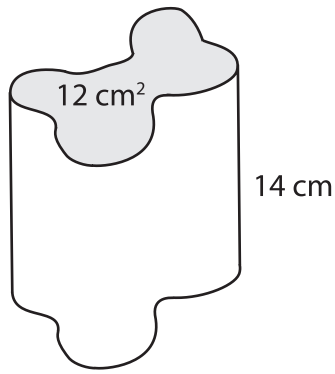
To find the volume of any prism or cylinder:

Step 1: Find the area of a face that has an identical face opposite it.

Step 2: Identify the distance between these two identical and parallel faces.

Step 3: Multiply the area of the face from the first step by the distance found in the second step to find the volume.

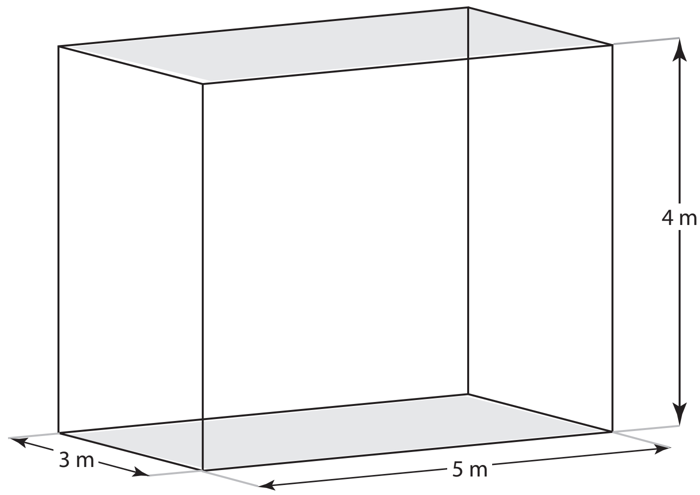
You can use these same steps to find the volume of an irregularly shaped prism, if you know the area of the base.



$$\begin{aligned} \text{Volume} &= 12 \text{ cm}^2 \times 14 \text{ cm} \\ &= 168 \text{ cm}^3. \end{aligned}$$

Volume of a Rectangular Prism

Use these steps to calculate the volume of a rectangular prism.



Step 1: Find the area of the rectangle at the base of the prism.

$$\begin{aligned}\text{Area of base} &= \text{length} \times \text{width}\\ &= 5\text{ m} \times 3\text{ m}\\ &= 15\text{ m}^2\end{aligned}$$

Step 2: Find the height of the prism.

$$\text{height} = 4\text{ m}$$

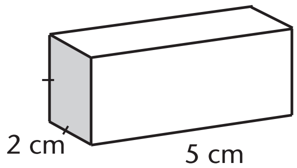
Step 3: Calculate the volume.

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height}\\ &= 15\text{ m}^2 \times 4\text{ m}\\ &= 60\text{ m}^3\end{aligned}$$

Exercises 2.1

Find the volume of each rectangular prism. Include units in your answer.

a.



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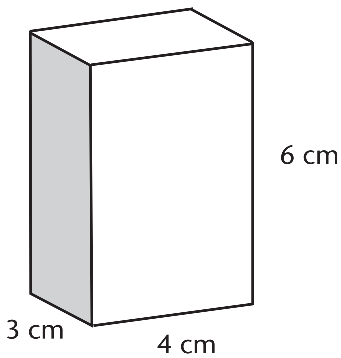
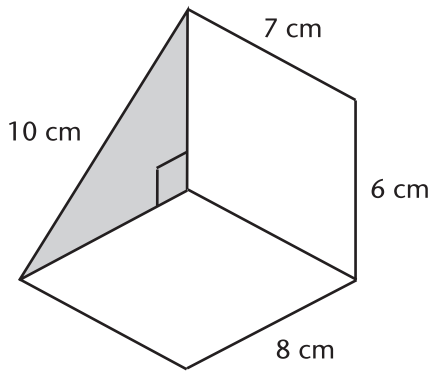


Turn to the Answer Key at the end of the module to check your work.

Volume of a Triangular Prism

Use the following steps to calculate the volume of a triangular prism.



Step 1: Figure out the area of the base of the object. In this example, the base is a triangle.

$$\begin{aligned}\text{Area of base} &= \frac{1}{2}bh \\ &= \frac{1}{2}(8)(6) \\ &= 24 \text{ cm}^2\end{aligned}$$

Step 2: Find the distance between the triangles. This distance is the height of the prism, even if the prism is on its side.

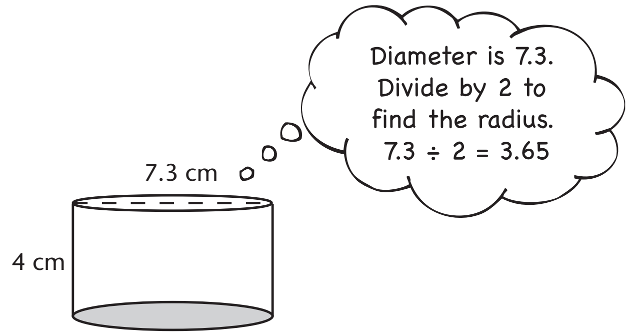
$$= 7 \text{ cm}$$

Step 3: Calculate the volume.

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height} \\ &= 24 \text{ cm}^2 \times 7 \text{ cm} \\ &= 168 \text{ cm}^3\end{aligned}$$

Volume of a Cylinder

You can find the volume of cylinders using these same steps.



Step 1: Figure out the area of the circle. This is the base of the cylinder.

$$\begin{aligned} \text{Area} &= \pi r^2 = \pi(r \times r) \\ &= \pi(3.65)(3.65) \\ &= 41.85\ \text{cm}^2 \end{aligned}$$

Step 2: Figure out the distance between the circles. This is the height of the cylinder.

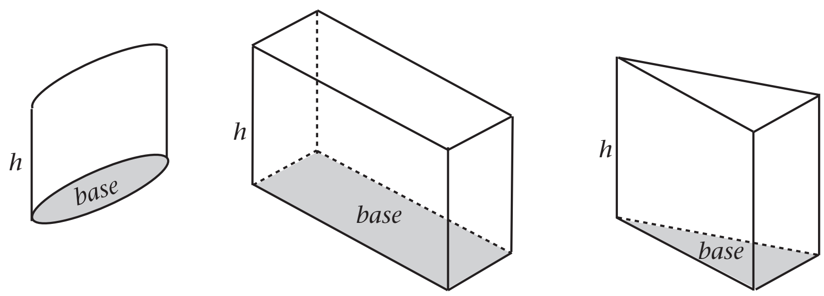
$$= 4\ \text{cm}$$

Step 3: Calculate the volume.

$$\begin{aligned} \text{Volume} &= \text{Area of base} \times \text{height} \\ &= 41.85\ \text{cm}^2 \times 4\ \text{cm} \\ &= 167.4\ \text{cm}^3 \end{aligned}$$

Also, notice that it doesn't matter if the shape is standing up or lying down—the volume is found the same way:

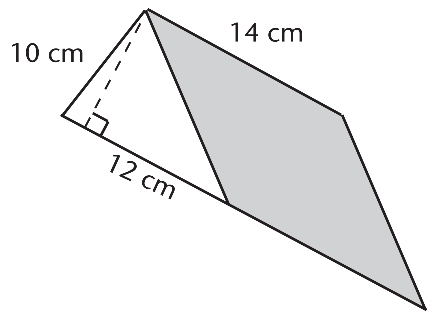
$$\text{Volume} = (\text{Area of Base}) \times h$$



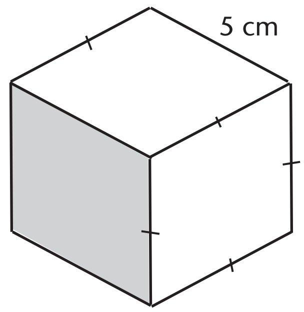
Exercises 2.2

1. Find the volume of each prism. Round your answers to the nearest whole cm^3.

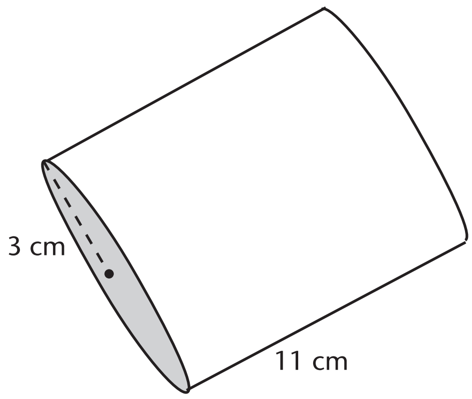
a.



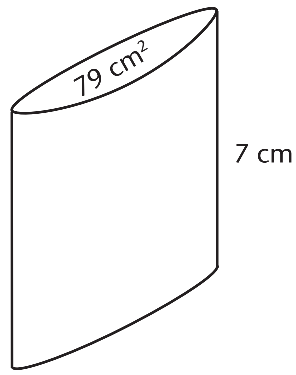
b.



c.



d.



2. Explain why the formula for finding the volume of a cylinder is:
   $V = \pi r^2 h$

____

____

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3. A box display needs to be set up. The employee decides to stack the boxes sideways rather than upright. Will the display have more volume upright or sideways? Explain your answer.

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Turn to the Answer Key at the end of the module to check your work.

Lesson 3

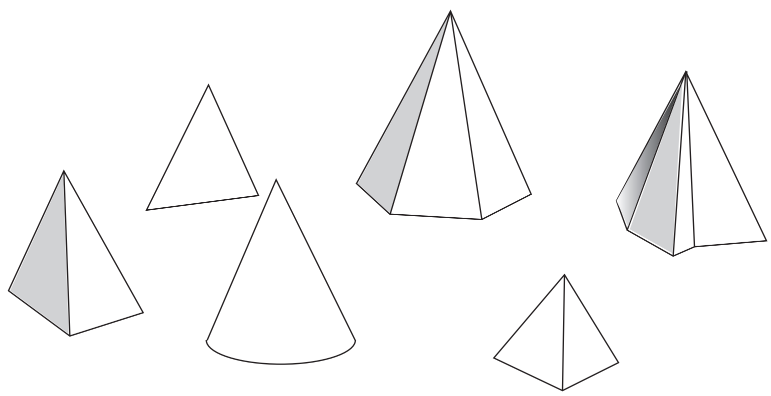
Volume of Pyramids and Cones

Learning Outcomes

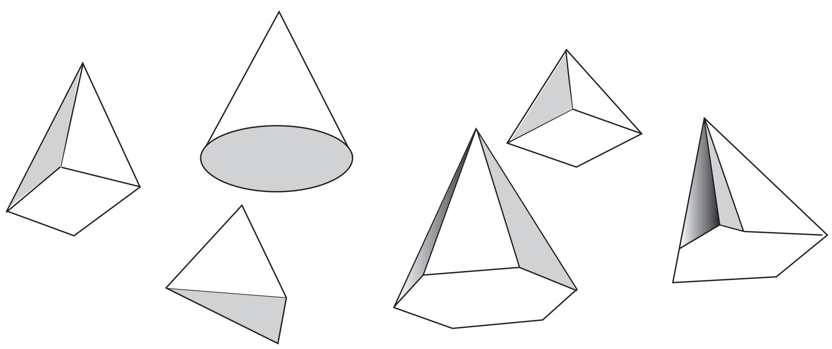
By the end of this section you will be better able to:

- calculate the volume of pyramids and cones

Pyramids and cones are three-dimensional solids. They are similar to the prisms and cylinders you studied in the previous lesson, but they rise up to a point.

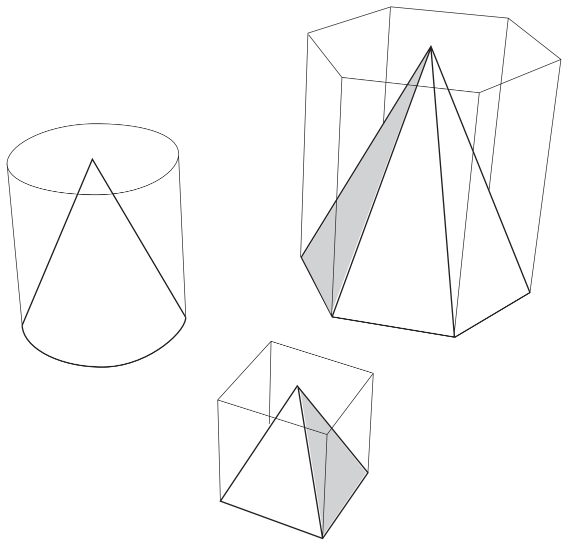


The base of a **pyramid** can be any shape with straight sides. If the base is a circle, the object is called a **cone**.



The volume of a pyramid (or cone) is exactly one-third the volume of the prism (or cylinder) with the same base and height. If you are interested in exploring *why* this is true, do an internet search for "Nrich Maths", and then search on the site for "Volume of a Pyramid".

In this diagram, you can see that the pyramids are much smaller than the prisms.



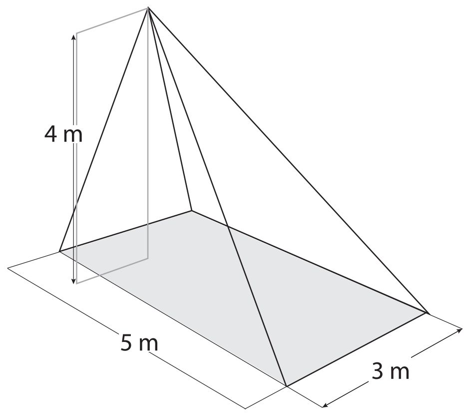
To calculate the volume of any pyramid or cone:

Step 1: Find the area of the base.

Step 2: Find the height of the pyramid (or cone). This is the perpendicular distance from the base to the point.

Step 3: Multiply the area of the base by the height. Then multiply by $\frac{1}{3}$ to find the volume.

Volume of a Pyramid



Step 1: Find the area of the base. The base of this pyramid is a rectangle.

$$\begin{aligned} \text{Area of base} &= \text{length} \times \text{width} \\ &= 5\text{ m} \times 3\text{ m} \\ &= 15\text{ m}^2 \end{aligned}$$

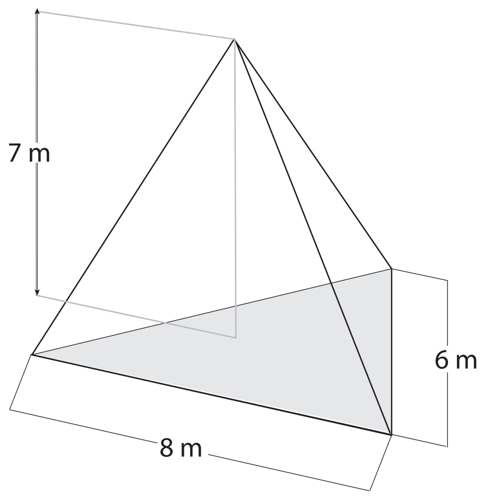
Step 2: Find the height of the pyramid.

$$\text{height} = 4\text{m}$$

Step 3: Calculate the volume.

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times \text{Area of base} \times \text{height} \\ &= \frac{1}{3} \times 15\text{ m}^2 \times 4\text{ m} \\ &= 20\text{ m}^3 \end{aligned}$$

Calculate the volume of all pyramids in exactly the same way, regardless of the shape of the base.



Step 1: Find the area of the base. The base of this pyramid is a triangle.

$$\text{Area of base} = \frac{1}{2}bh$$

$$= \frac{1}{2}\text{ (8 cm)(6 cm)}$$

$$= 24\text{ cm}^2$$

Step 2: Find the height of the pyramid.

height = 7 m

Step 3: Calculate the volume.

$$\text{Volume} = \frac{1}{3} \times \text{Area of base} \times \text{height}$$

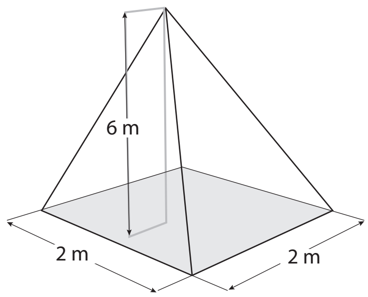
$$= \frac{1}{3} \times 24\text{ m}^2 \times 7\text{ m}$$

$$= 56\text{ m}^3$$

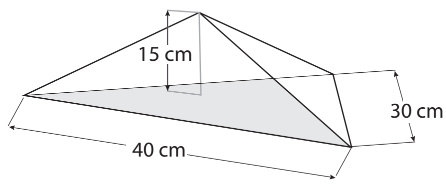
Exercises 3.1

Find the volume of each pyramid. Include units in your answer.

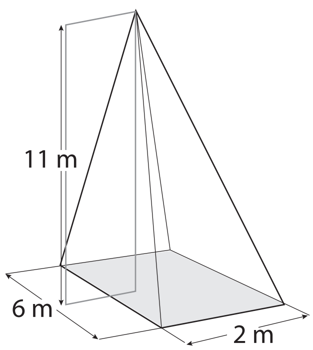
1.



2.

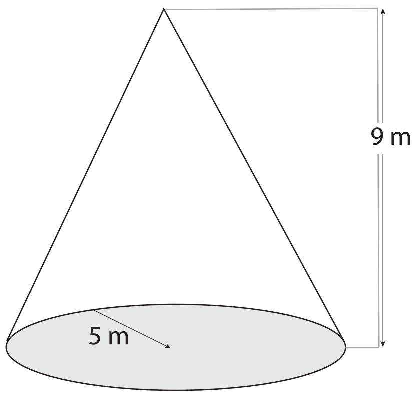


3.



Turn to the Answer Key at the end of the module to check your work.

Volume of a Cone



You can find the volume of a cone using the same steps you used to find the volume of a pyramid.

Step 1: Find the area of the base. The base of a cone is a circle.

$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= \pi(5)(5) \\ &= 78.6 \text{ m}^2 \end{aligned}$$

Step 2: Find the height of the pyramid.

$$\text{height} = 9 \text{ m}$$

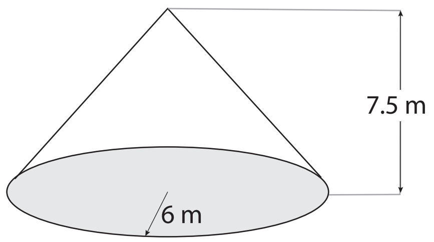
Step 3: Calculate the volume.

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times \text{Area of base} \times \text{height} \\ &= \frac{1}{3} \times 78.6\text{m}^2 \times 9 \text{ m} \\ &= 236 \text{ m}^3 \text{ (rounded to the nearest whole number)} \end{aligned}$$

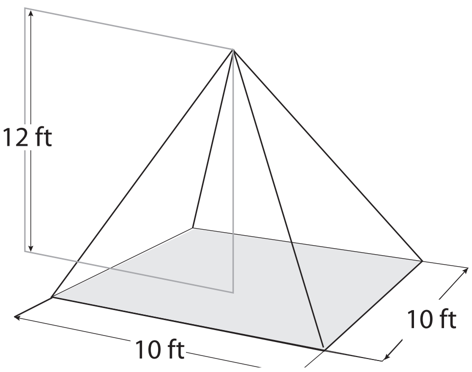
Exercises 3.2

Find the volume of each of these pyramids and cones. Round your answers to the nearest whole number.

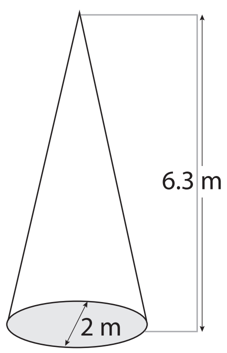
1.



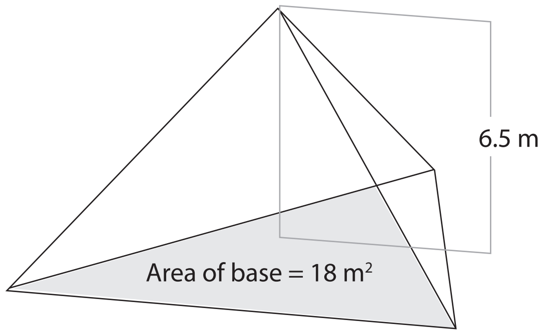
2.



3.



4.



Turn to the Answer Key at the end of the module to check your work.

Answer Key

Lesson 1: Circles

Exercises 1.1

1. a. 9.4 cm
   b. 8.5 cm

2. a. 6.0 cm
   b. 9.2 m
   c. 3.8 mm

3. a. 2.0 m
   b. 6.2 cm
   c. 4.6 mm

4. a. 31.4 cm
   b. 21.0 m

5. a. 31.4 cm
   b. 14.4 m or 14.5 m (using π)
   c. 44.0 mm

6. 5 cm

7. 7.6 cm

8.

Radius	Diameter	Circumference
5.0 cm	**10.0 cm**	**31.4 cm**
4.5 cm	**9.0 cm**	28.3 cm
4.0 m	8.0 m	**25.1 m**
2.4 mm	4.8 mm	**15.0 mm**
1.6 cm	**3.2 cm**	10.0 cm
3.6 m	7.2 m	**22.6 m**

Exercises 1.2

1. a. 12.6 cm^2
   b. 16.6 m^2

2. a. 50.2 m^2
   b. 11.3 cm^2

3. a. 14 cm
   b. 615.4 cm^2 or 615.8 cm^2 (using π)
   c. 153.9 cm^2

4. $7.065 \times 2 = 14.1$ m^2

Lesson 2: Volume of Prisms and Cylinders

Exercises 2.1

a. $V = lwh$
$V = (5)(2)(2)$
$V = 20$ cm^3

b. $V = lwh$
$V = (4)(3)(6)$
$V = 72$ cm^3

Exercises 2.2

1. a. Volume
   = (Area of base) $\times h$
   = 60 cm^2 × 14 cm
   = 840 cm^3

   b. Volume
   = (Area of base) $\times h$
   = 25 cm^2 × 5 cm
   = 125 cm^3

   c. Volume
   = (Area of base) $\times h$
   = 28.27 cm^2 × 11 cm
   = 311 cm^3

d. Volume
$= (\text{Area of base}) \times h$
$= 79 \text{ cm}^2 \times 7 \text{ cm}$
$= 553 \text{ cm}^3$

2. Answers will vary. The formula for the area of a circle is πr^2. The formula for volume is (Area of base) $\times h$ and if Area of base is replaced with πr^2 then the formula becomes $\pi r^2 \times h$, or $\pi r^2 h$.

3. Answers will vary. The orientation of the boxes does not affect the volume. The volume will be the same either way.

Lesson 3: Volume of Pyramids and Cones

Exercises 3.1

1. 8 m^3
2. 6000 cm^3
3. 44 m^3

Exercises 3.2

1. 283 cm^3
2. 400 ft^3
3. 7 m^3
4. 39 m^3